

# Virtual Electric Guitars and Effects Using Faust and Octave

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LAC-2008

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# Outline

- Overview and Demo
- Extended Karplus Strong (EKS) Elements
- Overdrive
- Amplifier Feedback
- Coupled Strings
- Wah Pedal
- Faust Libraries:
  - `filter.lib`
  - `effect.lib`
  - `osc.lib`



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(Extended Karplus Strong, Sullivan extensions, early waveguide)

- Some patents have expired
- Useful free methods are not in wide use
- Reference implementations in a modern framework



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# Extended Karplus-Strong (EKS) Algorithm



# Karplus-Strong (KS) Algorithm (1983)

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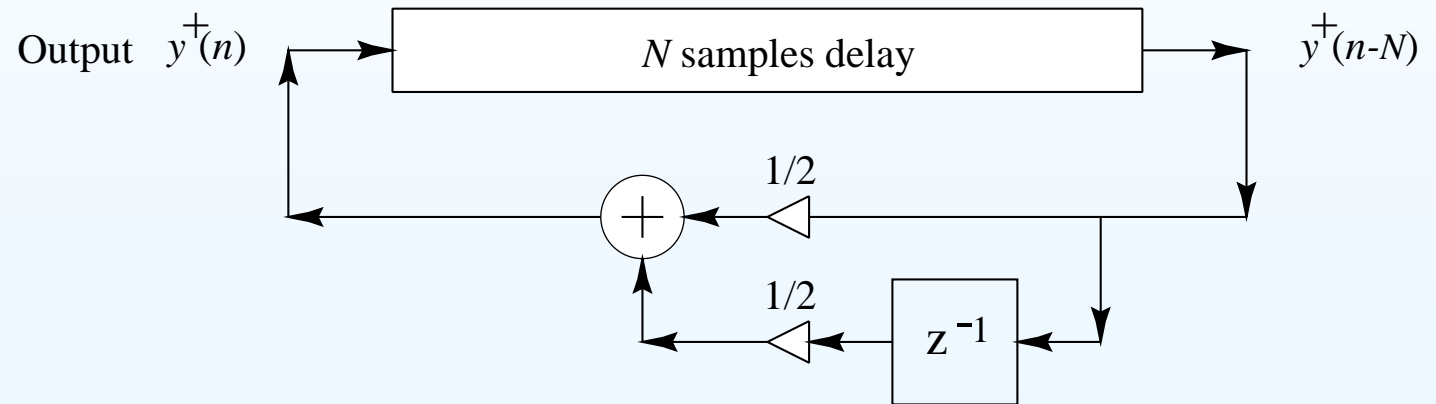
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## Digital Filter Interpretation:



- Discovered as “self-modifying wavetable synthesis”
- Wavetable is preferably initialized with random numbers
- Patents now expired



# Karplus-Strong (KS) Algorithm (1983)

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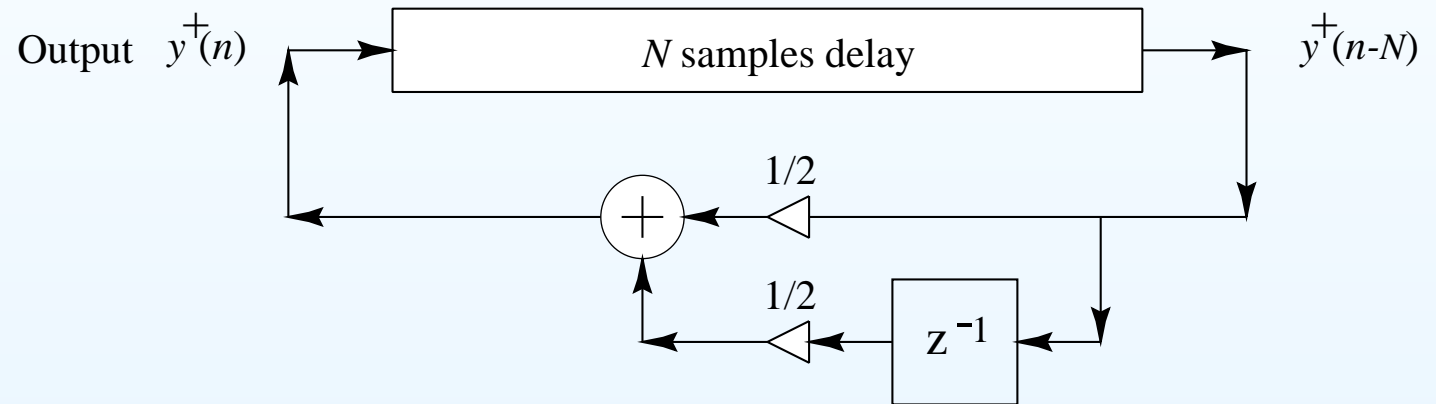
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## Karplus-Strong (KS) Algorithm (1983)

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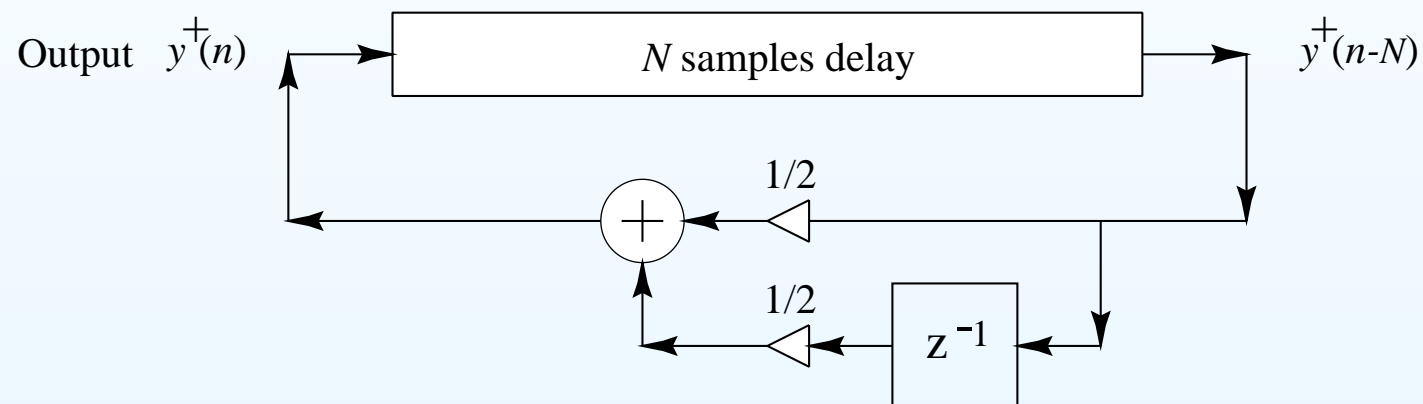
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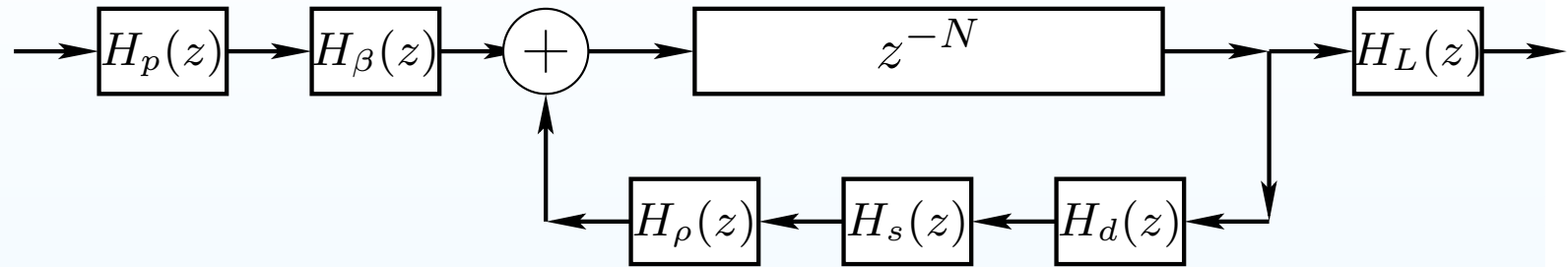
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$N$  = pitch period ( $2 \times$  string length) in samples

$H_p(z) = \frac{1 - p}{1 - p z^{-1}}$  = pick-direction lowpass filter

$H_\beta(z) = 1 - z^{-\beta N}$  = **pick-position comb filter**,  $\beta \in (0, 1)$

$H_d(z)$  = **string-damping filter** (one/two poles/zeros typical)

$H_s(z)$  = string-stiffness allpass filter (several poles and zeros)

$H_\rho(z) = \frac{\rho(N) - z^{-1}}{1 - \rho(N) z^{-1}}$  = first-order **string-tuning** allpass filter

$H_L(z) = \frac{1 - R_L}{1 - R_L z^{-1}}$  = **dynamic-level lowpass filter**





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# Pick Position Comb Filter



# String Model Excited Externally at One Point

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● Physical Excitation

● Pick Position FFCF

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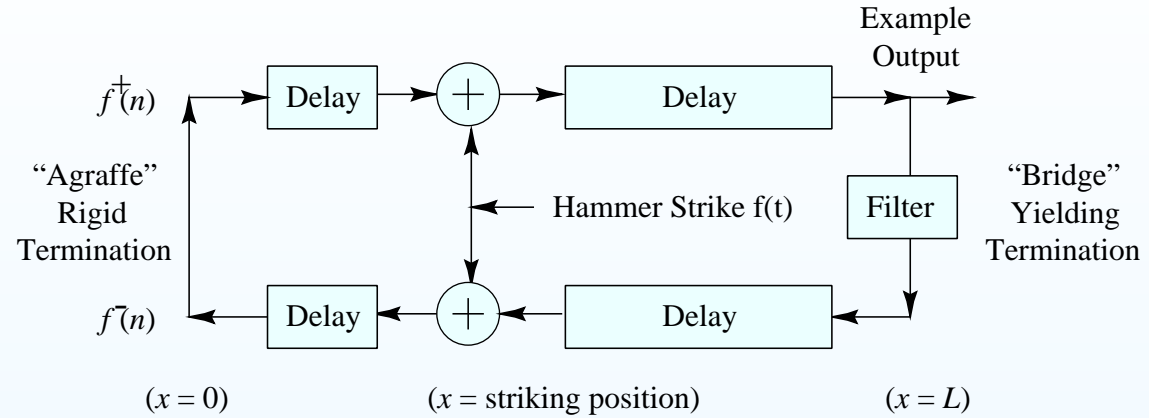
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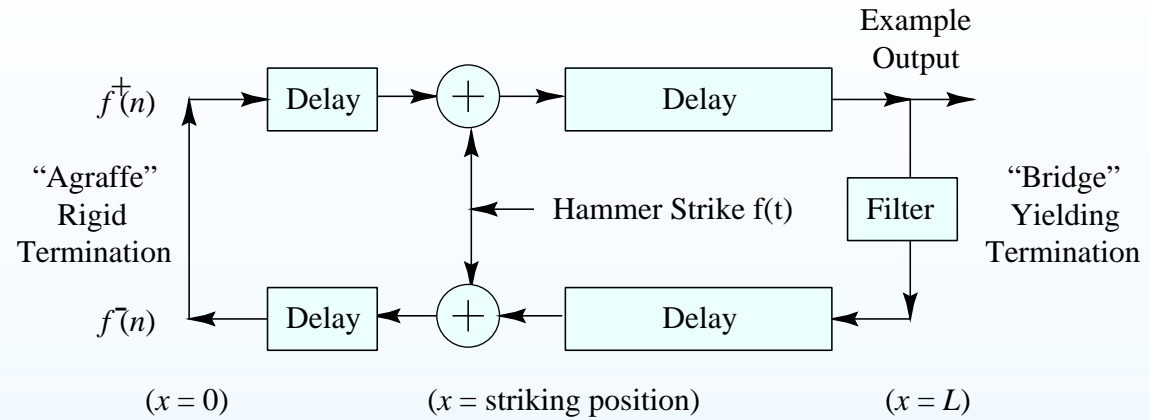


“Waveguide Formulation”



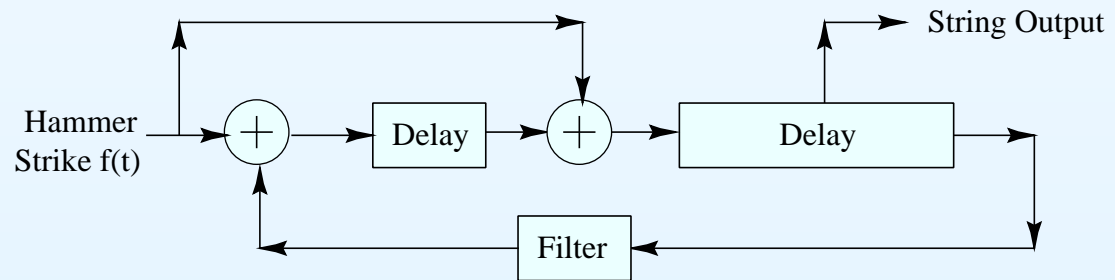
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## "Waveguide Formulation"

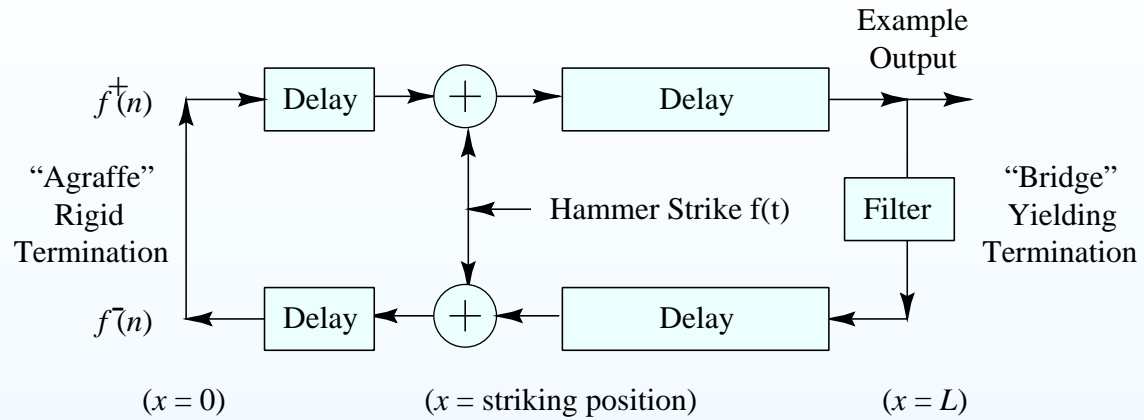
### Equivalent System by Delay Consolidation:





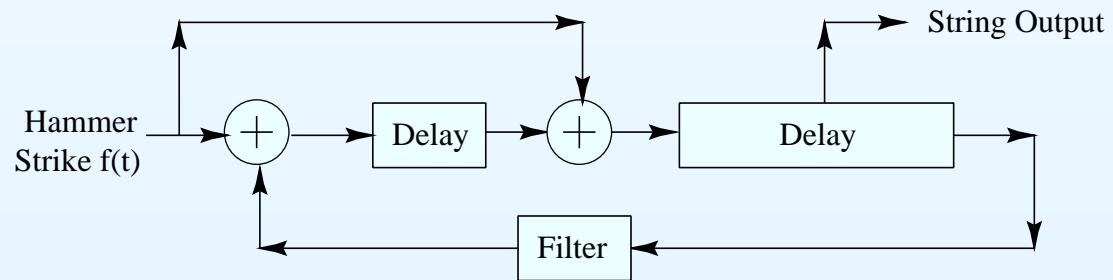
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## "Waveguide Formulation"

### Equivalent System by Delay Consolidation:



Finally, we "pull out" the comb-filter component:





# Pick-Position Comb Filter

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- **Pick Position FFCF**
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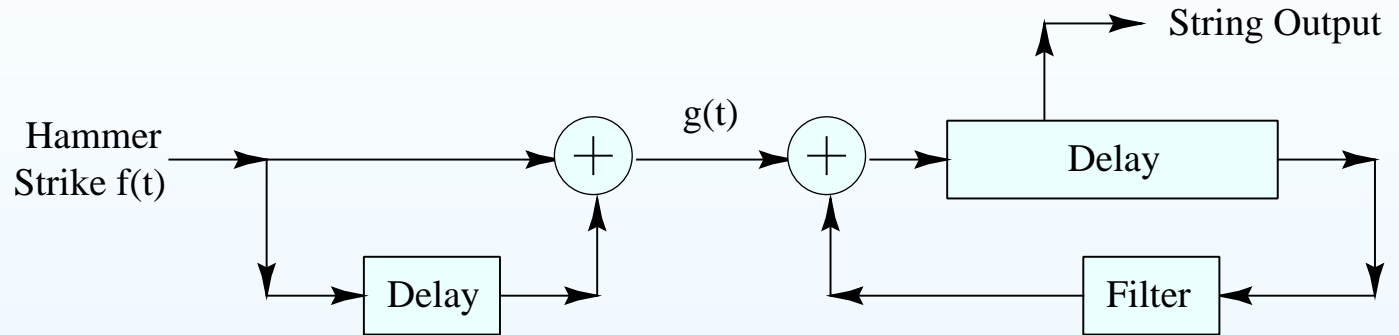
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## Equivalent System: Comb Filter Factored Out



$$H(z) = z^{-N} \frac{1 + z^{-2M}}{1 - z^{-(2M+2N)}} = (1 + z^{-2M}) \frac{z^{-N}}{1 - z^{-(2M+2N)}}$$

- *Excitation Position* controlled by left delay-line length
- *Fundamental Frequency* controlled by right delay-line length
- Derived originally (1982) by transfer-function factorization



## Pick-Position Comb Filter in Faust

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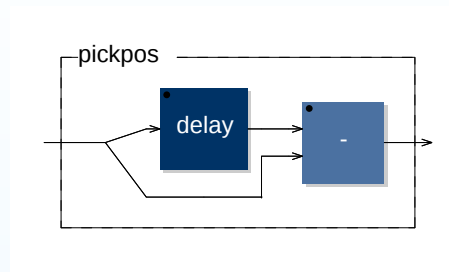
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```
beta = hslider("pick_position",  
              0.13, 0.02, 0.5, 0.01);
```

```
P = SR/freq; // fundamental period in samples  
Pmax = 4096; // maximum P (delay-line allocation)  
ppdel = beta*P; // pick-position delay
```

```
ffcombfilter(maxdel,del,g) =  
  _ <: delay(maxdel,del) : *(g) : + ;
```

```
pickposfilter = ffcombfilter(Pmax,ppdel,-1);
```





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# EKS Damping Filter



## EKS Damping Filter

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**Original:**

$$H_d(z) = (1 - S) + Sz^{-1}, \quad S \in \left[ \frac{1}{2}, 1 \right]$$

**Later:** Symmetric FIR (delay always one sample):

$$H_d(z) = h_1 + h_0z^{-1} + h_1z^{-2} = z^{-1} [h_0 + h_1(z + z^{-1})].$$

**Faust Implementation:**

```
t60 = hslider("decaytime T60", 4, 0, 10, 0.01);
B = hslider("brightness", 0.5, 0, 1, 0.01); // 0-1

rho = pow(0.001, 1.0/(freq*t60));
h0 = (1.0 + B)/2;
h1 = (1.0 - B)/4;
dampingfilter(x) = rho * (h0 * x' + h1*(x+x''));
```



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# EKS Tuning Filter



## Original EKS Tuning Filter

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● Lagrange Test

● Lagrange1-4 AR

● Lagrange4 AR

● Lagrange4 PD

● Thiran PD 2

● Thiran AR 2

● Lagrange5 AR

● Lagrange5 PD

● Lagrange4E AR

● Lagrange4E PD

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$$H_{\eta}(z) = -\frac{\eta - z^{-1}}{1 - \eta z^{-1}}$$

where the phase delay  $\Delta_{\eta}(\omega)$  has dc limit

$$\Delta_{\eta}(0) = \frac{1 + \eta}{1 - \eta}$$

$$\Rightarrow \eta \approx \frac{\Delta - 1}{\Delta + 1}$$

where  $\Delta =$  desired delay

Faust Implementation:

```
fdelay1a(n,d,x) = delay(n,id,x) : tf1(eta,1,eta);
```





# Tuning by Lagrange Interpolation (HUT)

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● Lagrange4 AR

● Lagrange4 PD

● Thiran PD 2

● Thiran AR 2

● Lagrange5 AR

● Lagrange5 PD

● Lagrange4E AR

● Lagrange4E PD

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Transfer function:

$$H_{\Delta}(z) = \sum_{n=0}^N h_{\Delta}(n) z^{-n}$$

Coefficient formula:

$$h_{\Delta}(n) = \prod_{\substack{k=0 \\ k \neq n}}^N \frac{\Delta - k}{n - k}$$

Useful maxima function:

```
hd(n,d,N) :=  
  product(if k=n then 1 else (d-k)/(n-k),k,0,N);
```





# Lagrange Interpolation in Faust

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- Lagrange4 AR
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- Lagrange5 AR
- Lagrange5 PD
- Lagrange4E AR
- Lagrange4E PD

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Second-order case:

$$\begin{aligned} \text{fdelay2}(n, d, x) &= \text{delay}(n, \text{id}, x) * (1 - \text{fd}) * (2 - \text{fd}) / 2 \\ &+ \text{delay}(n, \text{id} + 1, x) * (2 - \text{fd}) * \text{fd} \\ &+ \text{delay}(n, \text{id} + 2, x) * (\text{fd} - 1) * \text{fd} / 2 \end{aligned}$$

with {

```
o = 0.49999; // center delay-range in polynomial
dmo = d - o; // assumed nonnegative
id = int(dmo);
fd = o + frac(dmo);
```

};

More robust than allpass under rapidly time-varying conditions





# Testing Lagrange Interpolation in Faust

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- Allpass Interpolation
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```
> cat tlagrange1-4.dsp
```

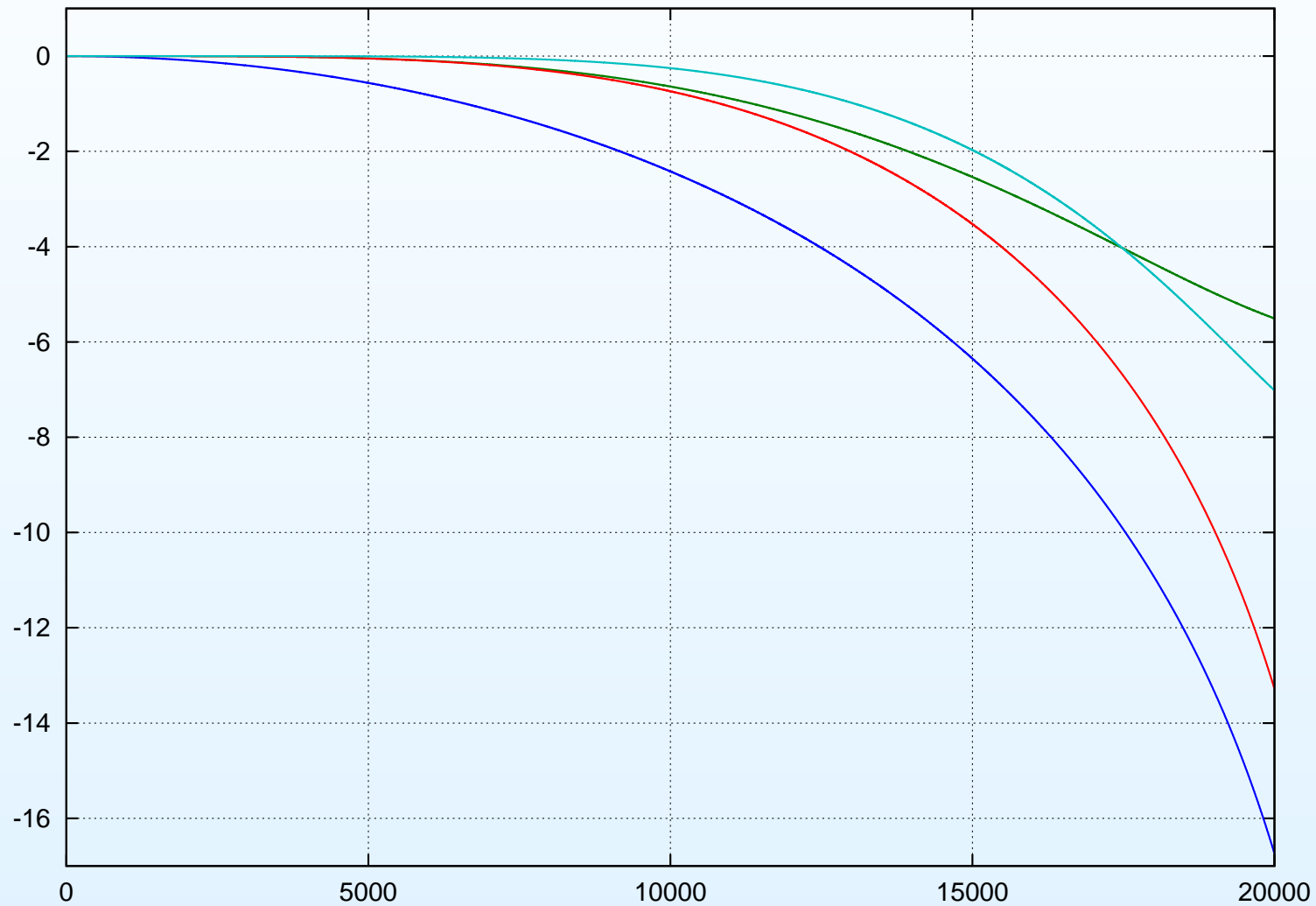
```
import("filter.lib");  
N = 16;  d = 1.5;  impulse = 1-1';  
process = impulse <: (fdelay1(N,d),  
                      fdelay2(N,d),  
                      fdelay3(N,d),  
                      fdelay4(N,d));
```

```
> faust2octave tlagrange1-4.dsp  
octave:1> \  
plot(20*log10(abs((fft(faustout,1024)(1:512,:))))))
```



## Lagrange Interpolation Orders 1-4

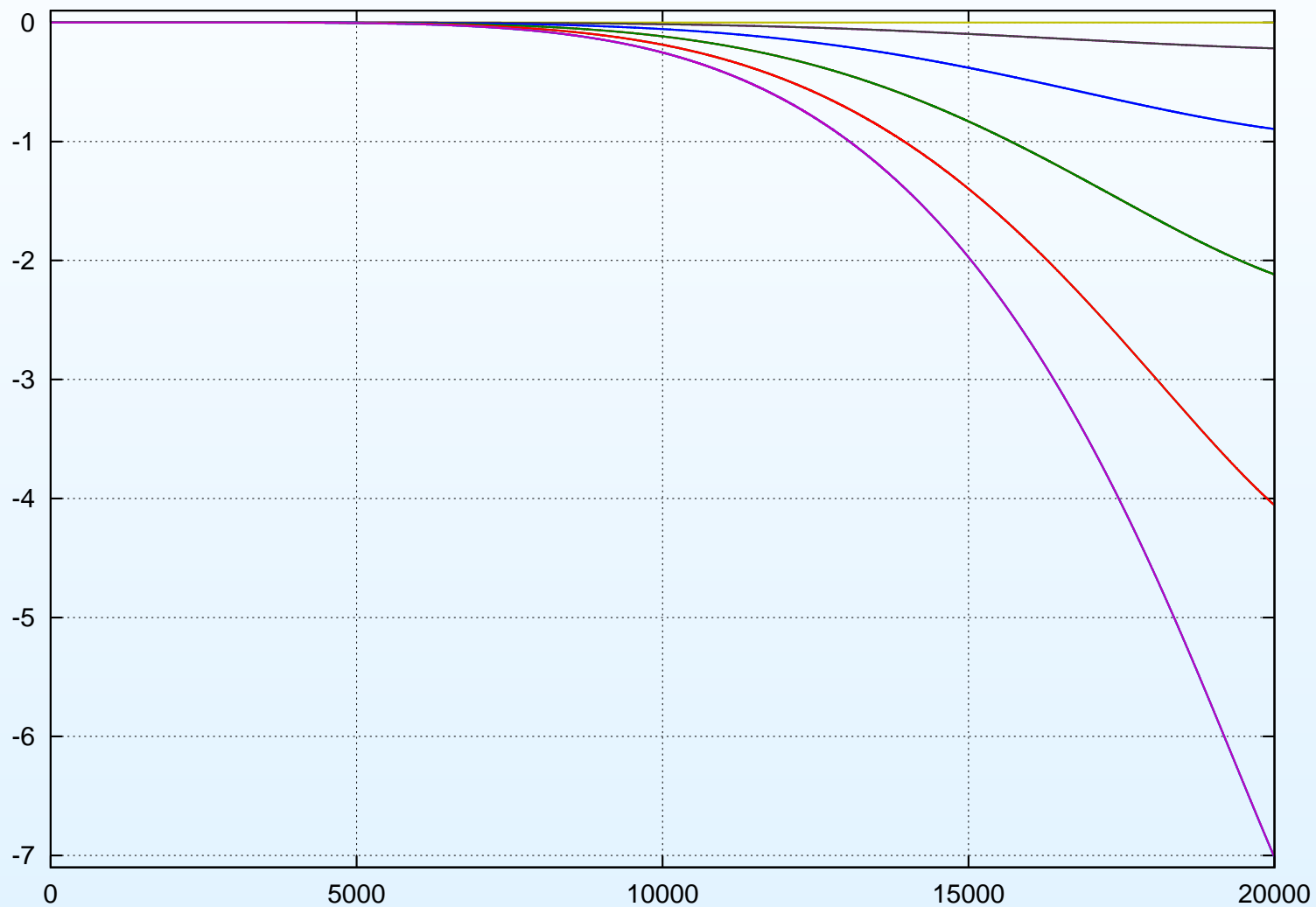
Amplitude Responses for requested delay = 1.5 samples:





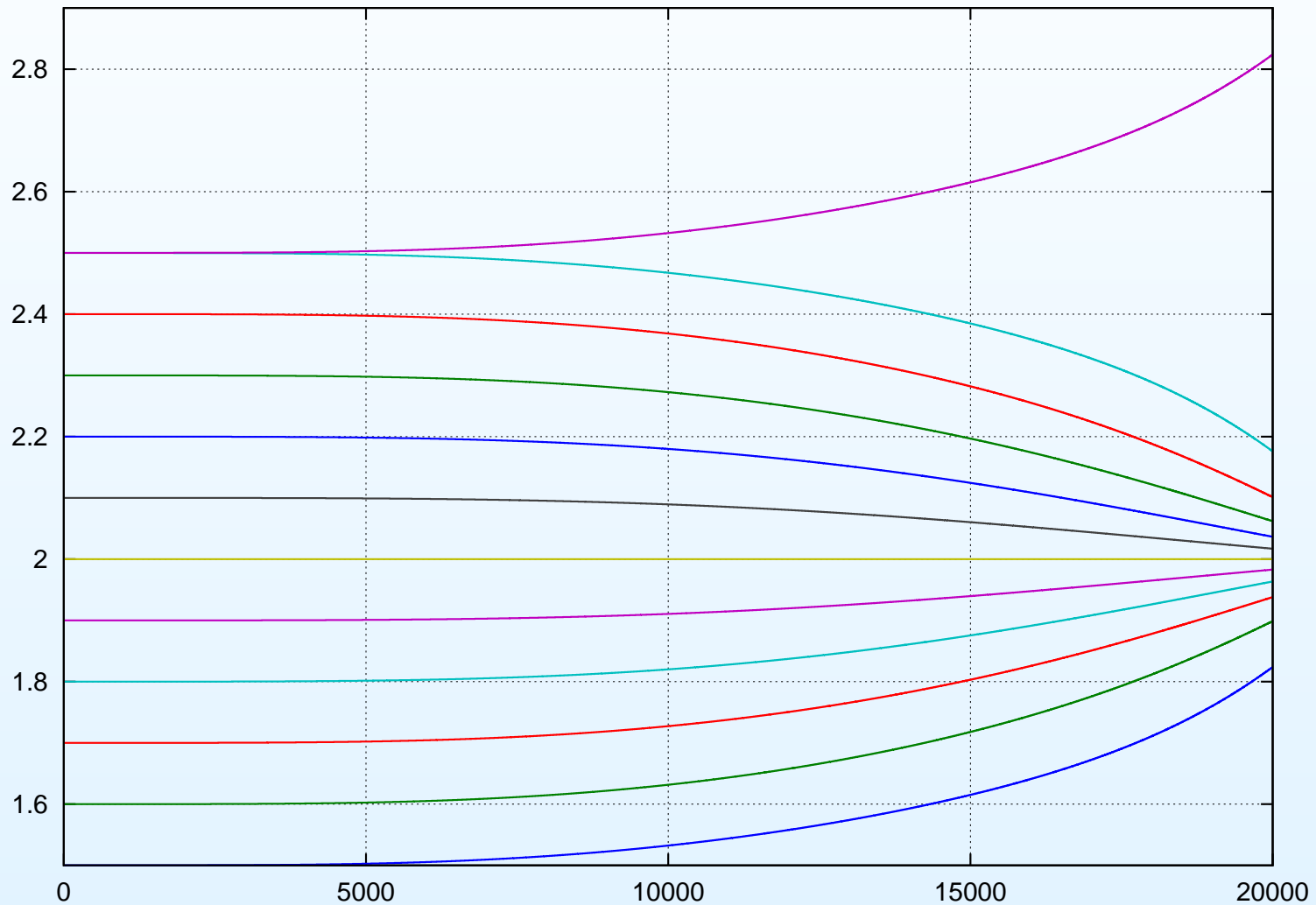
## Lagrange Interpolation Order 4

Amplitude Responses for a range of requested delays [1.5 : 0.1 : 2.5]:



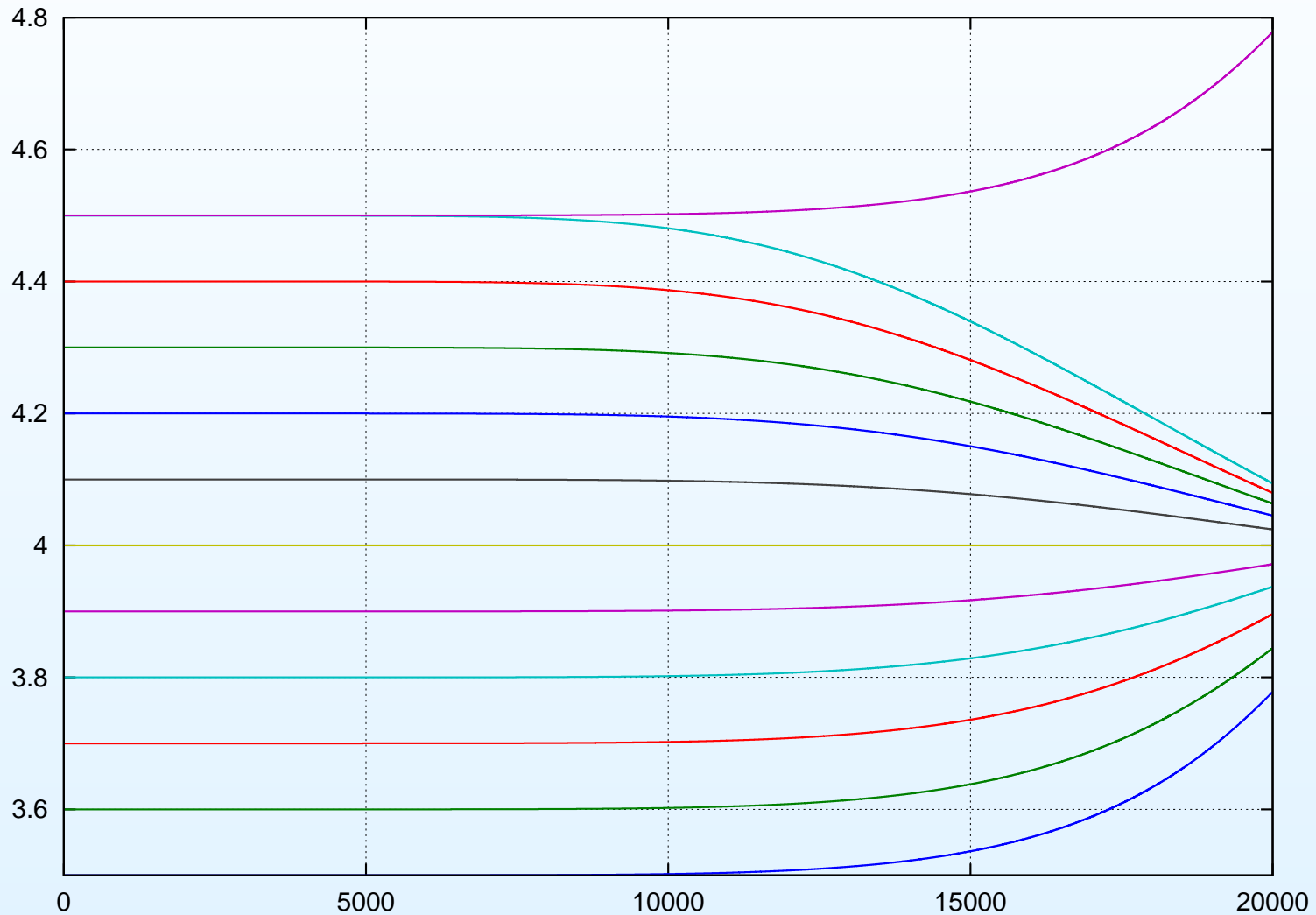
# Lagrange Interpolation Order 4

Phase Delays for a range of requested delays [1.5 : 0.1 : 2.5, 2.5001]:



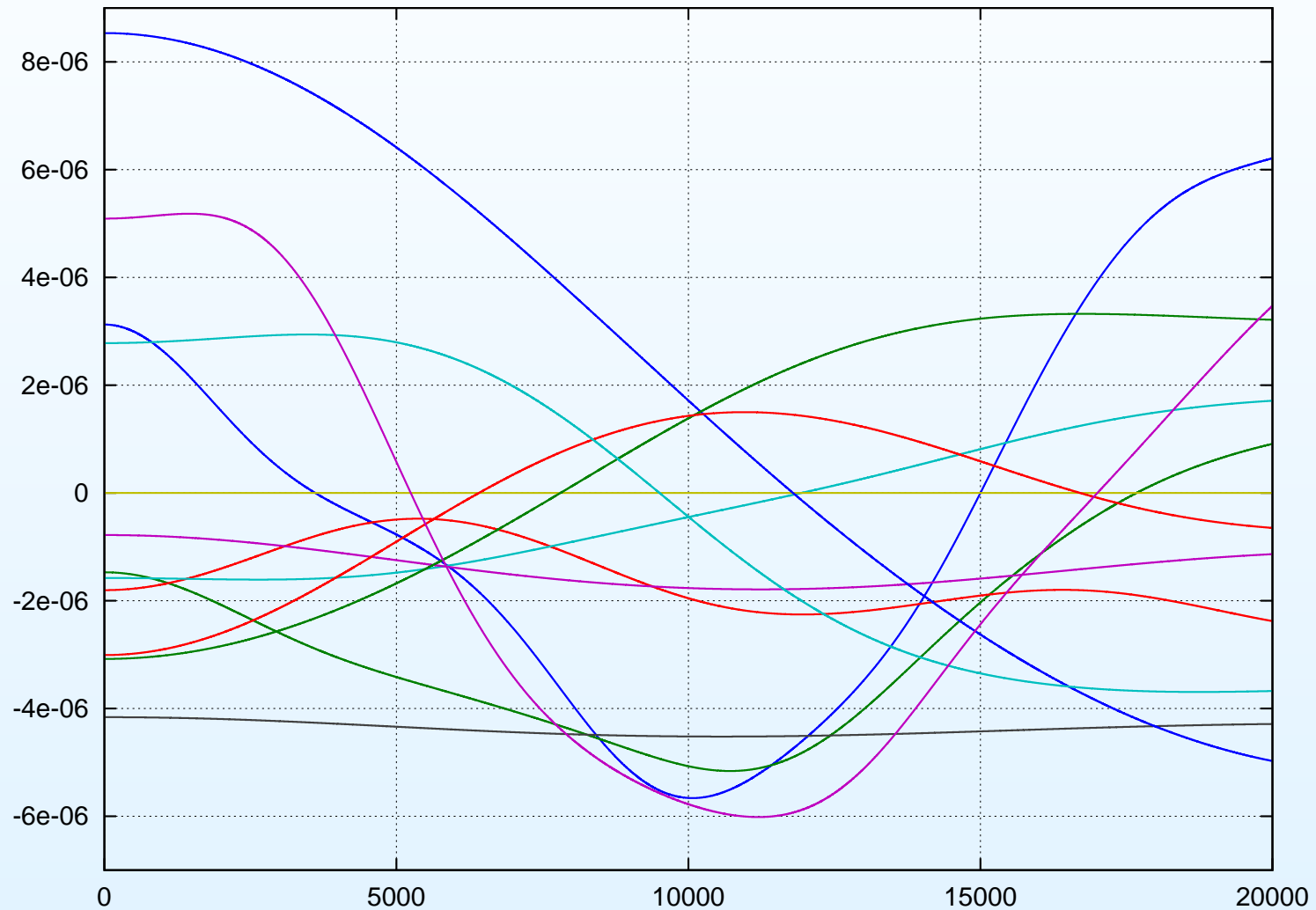
# Thiran Allpass Interpolation Order 4

Phase Delays for a range of requested delays [1.5 : 0.1 : 2.5, 2.5001]:



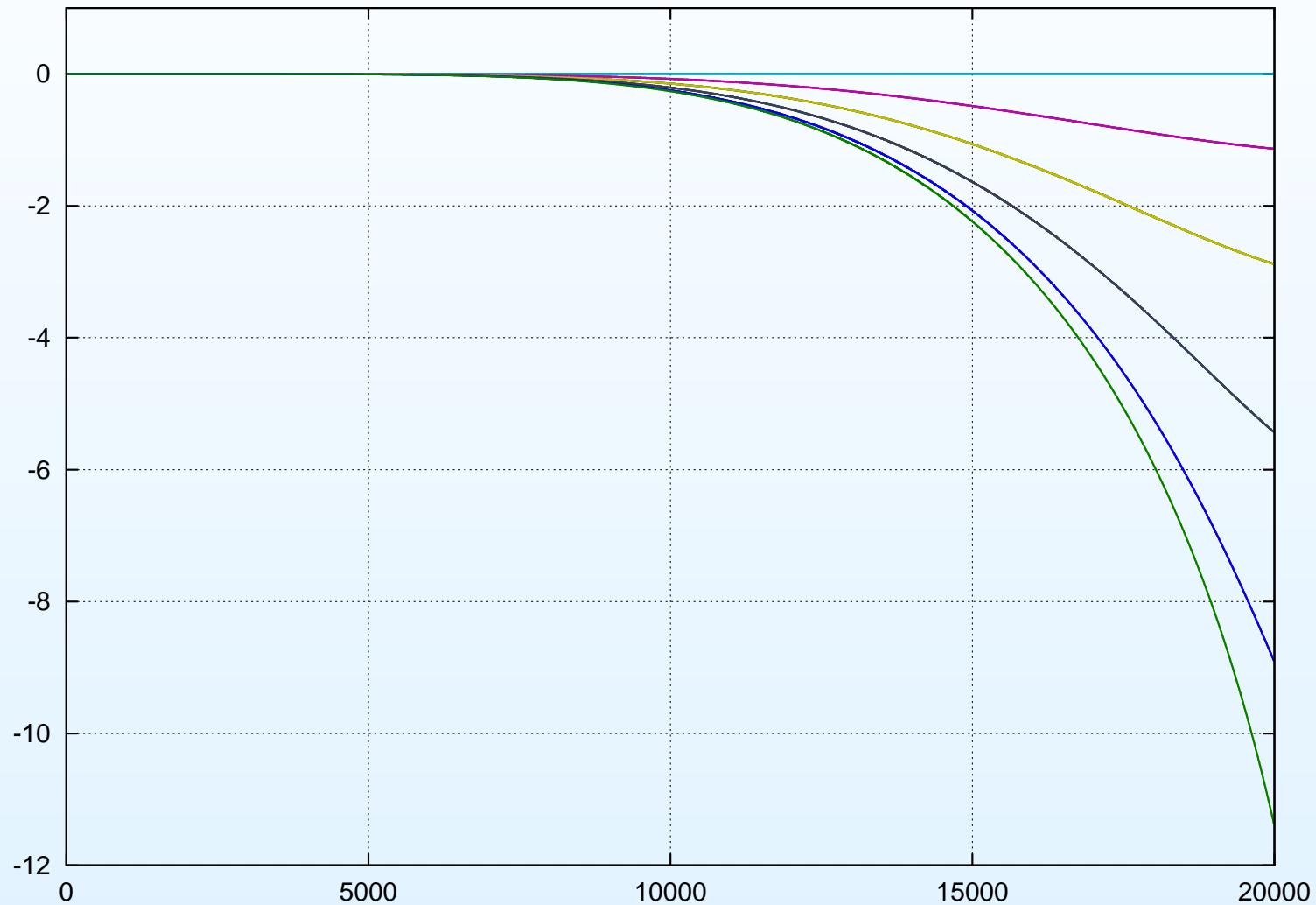
# Thiran Allpass Interpolation Order 4

Amplitude Responses for a range of requested delays [1.5 : 0.1 : 2.5]:



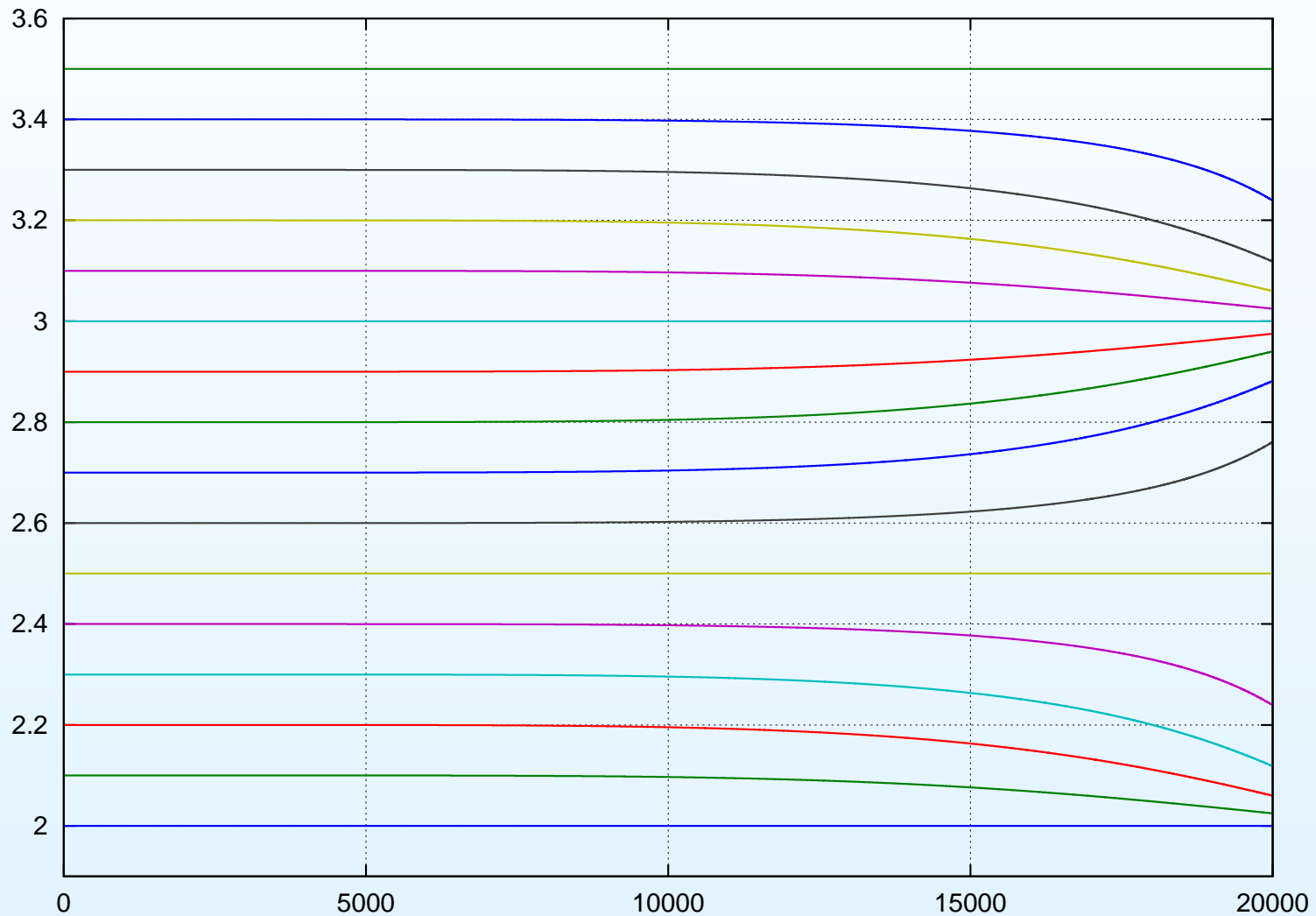
## Lagrange Interpolation Order 5

Amplitude Responses for a range of requested delays [2 : 0.1 : 3.5]:



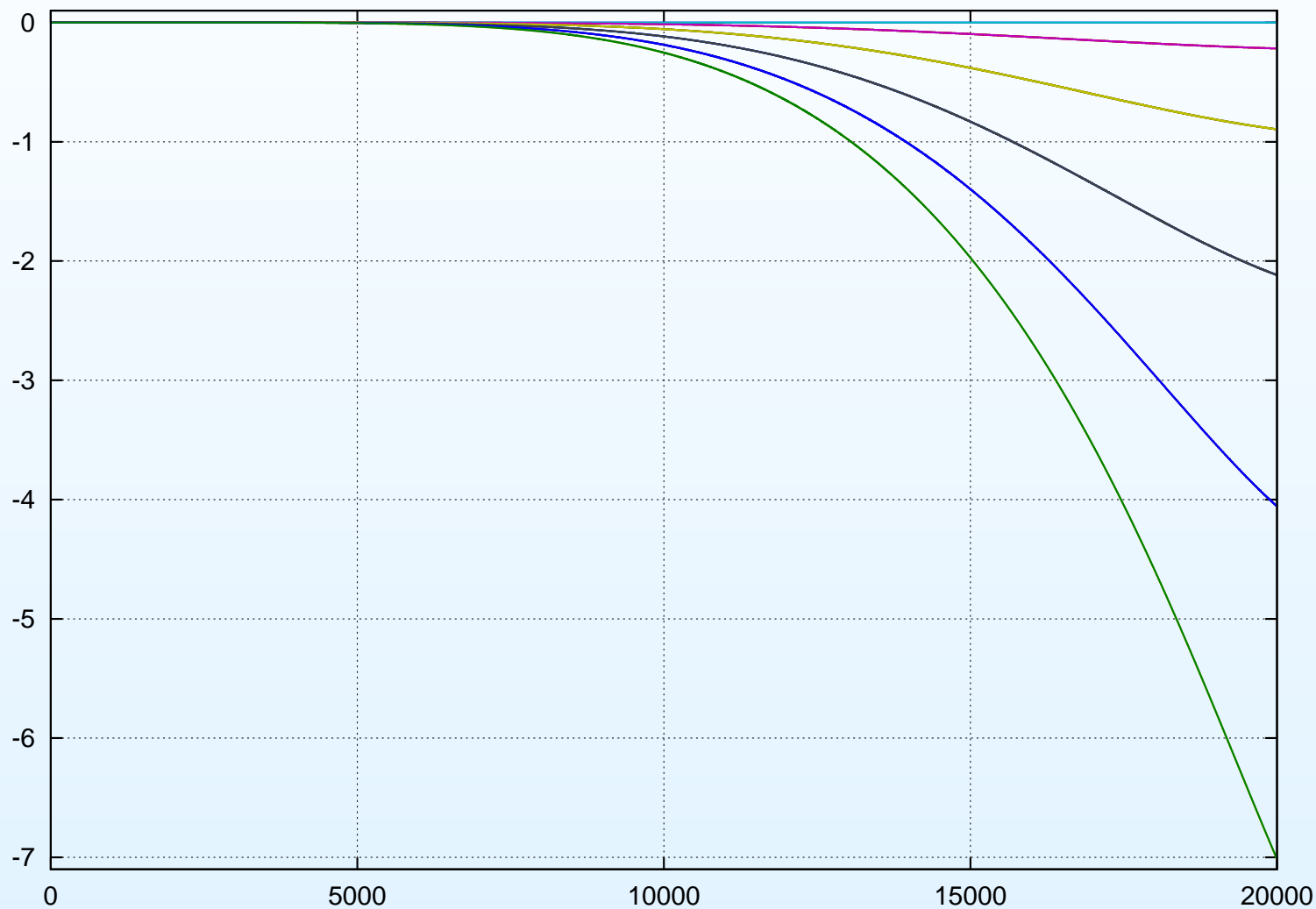
# Lagrange Interpolation Order 5

Phase Delays for a range of requested delays [2 : 0.1 : 3.5]:



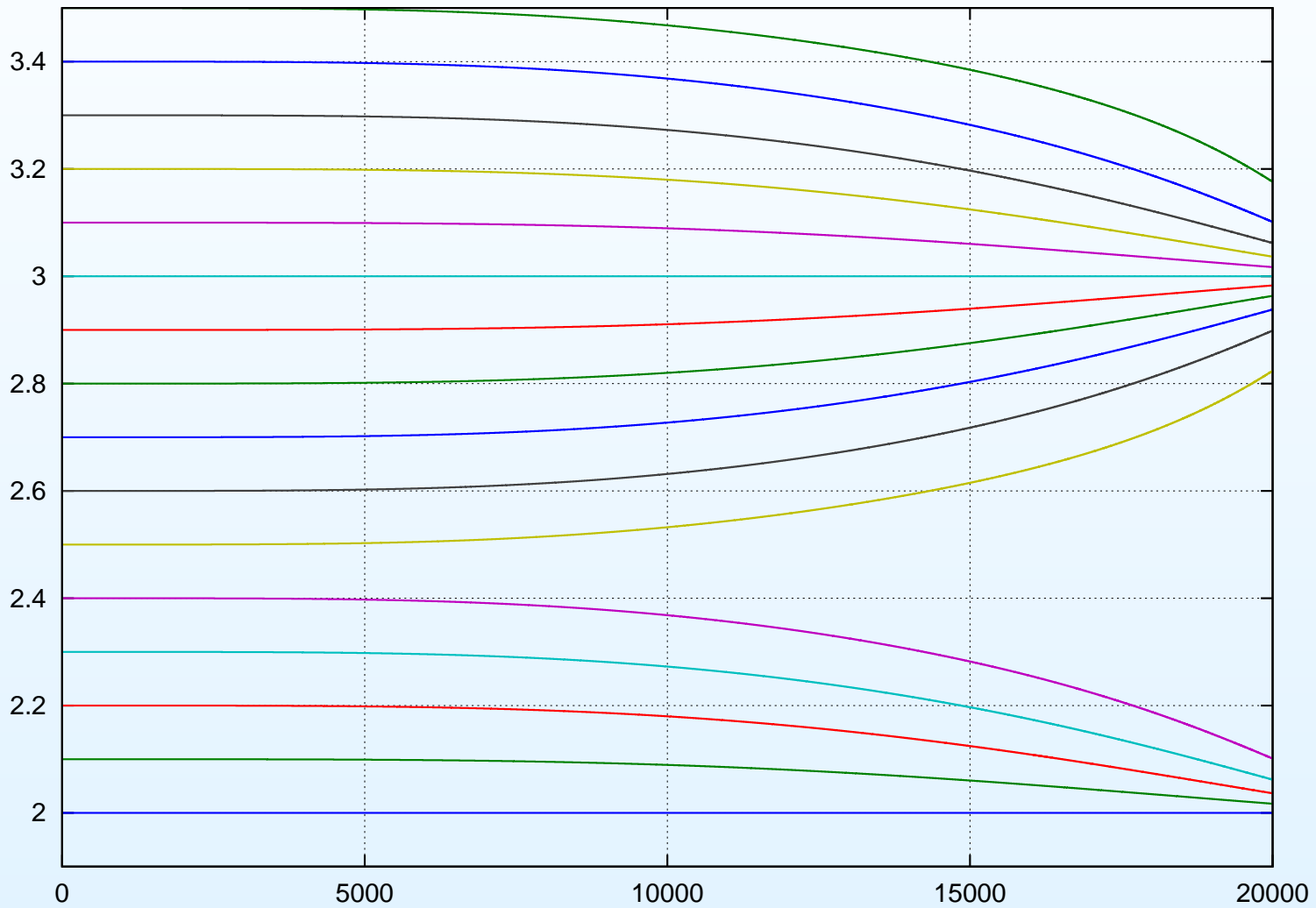
## Lagrange Interpolation Order 4

Amplitude Responses for a range of requested delays [2 : 0.1 : 3.5]:



# Lagrange Interpolation Order 4

Phase Delays for a range of requested delays [2 : 0.1 : 3.5]:







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# EKS Dynamic Level Filter



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In real strings, the *spectral centroid* typically rises as plucking/striking becomes more energetic.

The EKS dynamic-level lowpass filter

$$H_{L,\omega_1}(z) = \frac{1 - R_L(\omega_1)}{1 - R_L(\omega_1)z^{-1}}$$

qualitatively models this phenomenon

- “Spectral modeling filter”
- Not needed in a true physical model
- Useful with noise-burst string excitation (KS, EKS)



## Dynamic Level Filter

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## Dynamic Level Filter

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## Dynamic Level Filter

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## DLF Derivation

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$$H_{L,\omega_1}(z) = L \cdot L_0(L) + (1 - L) \cdot H_1(z)$$

where  $H_1(z) \approx$  analog 1st-order Butterworth lowpass:

$$H_1(s) \triangleq \frac{\omega_1}{s + \omega_1}$$

- Unity dc gain
- $-3$  dB gain at  $s = j\omega_1$
- $\omega_1 = 2\pi f_1 =$  fundamental frequency (rad/sec)
- Rolls off  $-6$  dB/octave for  $\omega \gg \omega_1$ .
- $L$  (“dynamic level”) = gain at Nyquist limit
- $L_0 =$  low-frequency attenuation:

$$L_0(L) = L^{1/3}$$



# Bilinear Transform

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$$s = \alpha \frac{1 - z^{-1}}{1 + z^{-1}}$$

$\Rightarrow$

$$H_1(z) = \frac{\omega_1}{\alpha \frac{1-z^{-1}}{1+z^{-1}} + \omega_1} = \frac{\omega_1}{\alpha + \omega_1} \cdot \frac{1 + z^{-1}}{1 - pz^{-1}}$$

where

$$p = \frac{\alpha - \omega_1}{\alpha + \omega_1}$$

To map analog break frequency exactly:

$$\alpha = \frac{\omega_1}{\tan\left(\frac{\omega_1 T}{2}\right)}$$



# Dynamic Level Filter for Various Levels

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● Bilinear Transform

● DLF Response

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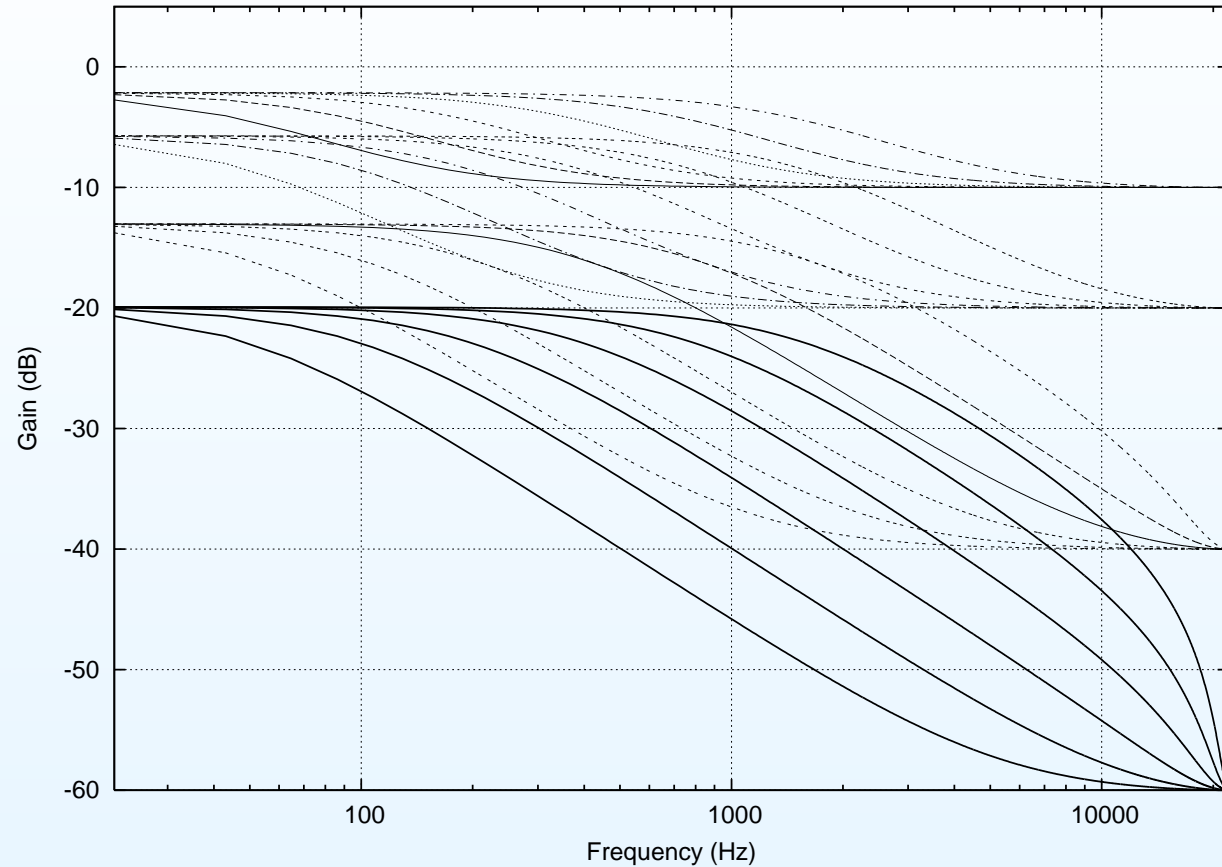
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- $L = -60 \text{ dB}, -40 \text{ dB}, -20 \text{ dB}, \text{ and } -10 \text{ dB}$
- DC level = 1/3 Nyquist-limit level in dB





## Dynamic Level Lowpass in Faust

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```
levelfilter(L,freq,x) = (L * L0 * x) + ((1.0-L)
                        * lp2out(x))
```

```
with {
```

```
    L0 = pow(L,1/3);
```

```
    Lw = PI*freq/SR; // = w1 T / 2
```

```
    Lgain = Lw / (1.0 + Lw);
```

```
    Lpole2 = (1.0 - Lw) / (1.0 + Lw);
```

```
    lp2out = *(Lgain) : + ~ *(Lpole2);
```

```
};
```

To intensify the effect,  $N_d$  units can be used in series, with the desired Nyquist-limit level divided by  $N_d$  for each section:

```
levelfilterN(Nd,freq,L) =
    seq(i,Nd,levelfilter((L/Nd),freq));
```



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# Overdrive and Feedback



# Cubic Nonlinearity

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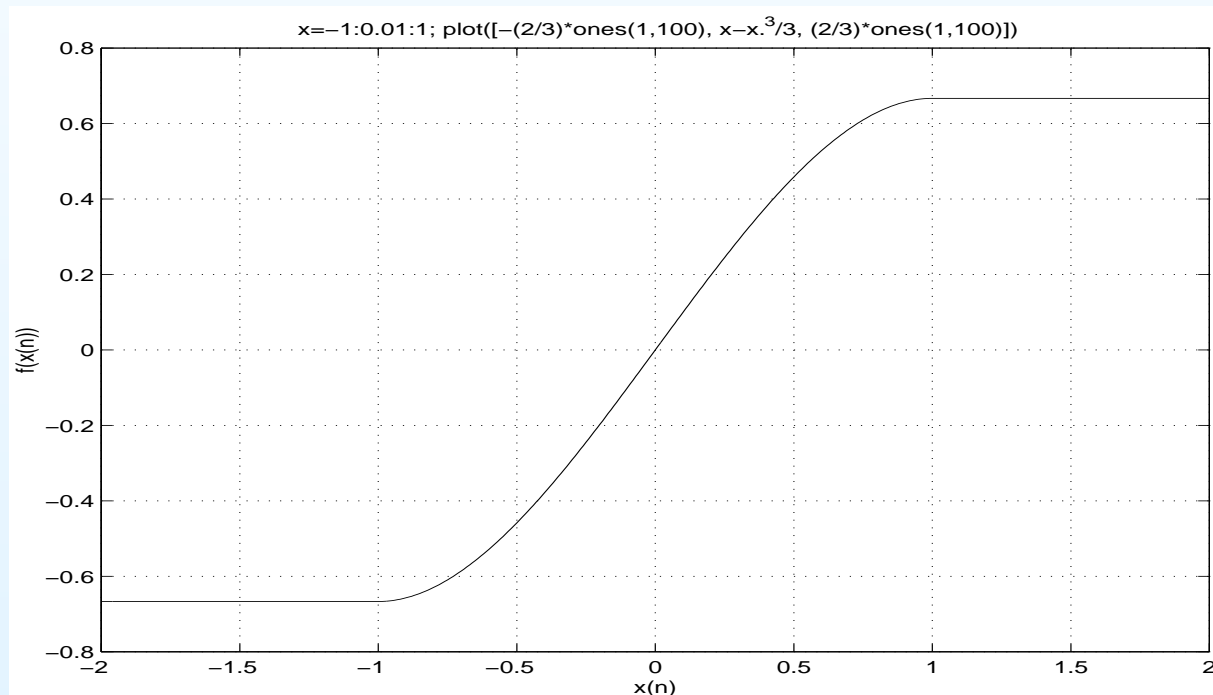
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$$f(x) = \begin{cases} -\frac{2}{3}, & x \leq -1 \\ x - \frac{x^3}{3}, & -1 \leq x \leq 1 \\ \frac{2}{3}, & x \geq 1 \end{cases}$$





# Clipping Cubic Nonlinearity in Faust (effect.lib)

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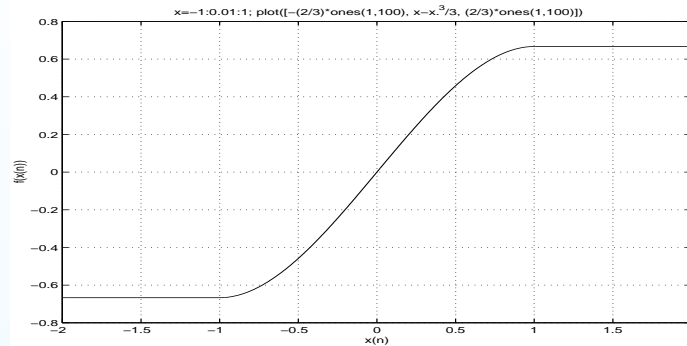
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```
cubicnl(drive,offset) =  
    +(offset) : *(pregain) : clip(-1,1)  
    : cubic : dcblocker  
with {  
    pregain = pow(10.0,2*drive);  
    clip(lo,hi) = min(hi) : max(lo);  
    cubic(x) = x - x*x*x/3;  
};
```



# Amplifier Overdrive and Feedback

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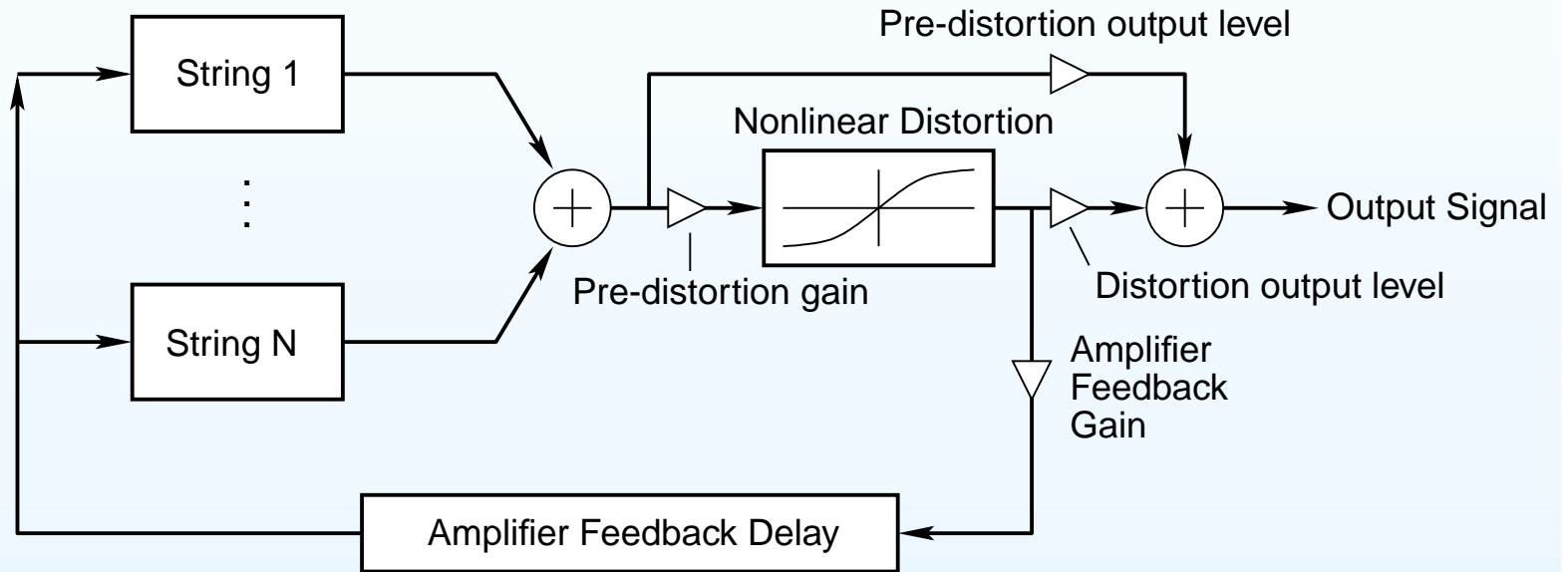
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Distortion output signal often further filtered by an *amplifier cabinet filter*, representing speaker cabinet, driver responses, etc.



## Distortion and Amplifier Feedback in Faust (freeax.dsp)

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```
D = hslider("distortion drive",0.629,0,10,0.01);

ampfbgain = hslider("Amp feedback gain",
                    0.001,0,0.01,0.001);
ampfbfres = hslider("Amp feedback 1st resonance Hz",
                    100,40,2000,1);

ampfbdelay = int(SR/ampfbfres); // no interpolation
ampfb = *(ampfbgain) : delay(4096,ampfbdelay);
...
process = filtered_excitation : (+ : stringloop
    : cubicnl(D,0) : speakerbp(sf1,sf2))
~ ampfb : *(level) <: _,_ : widthdelay : panner;
```



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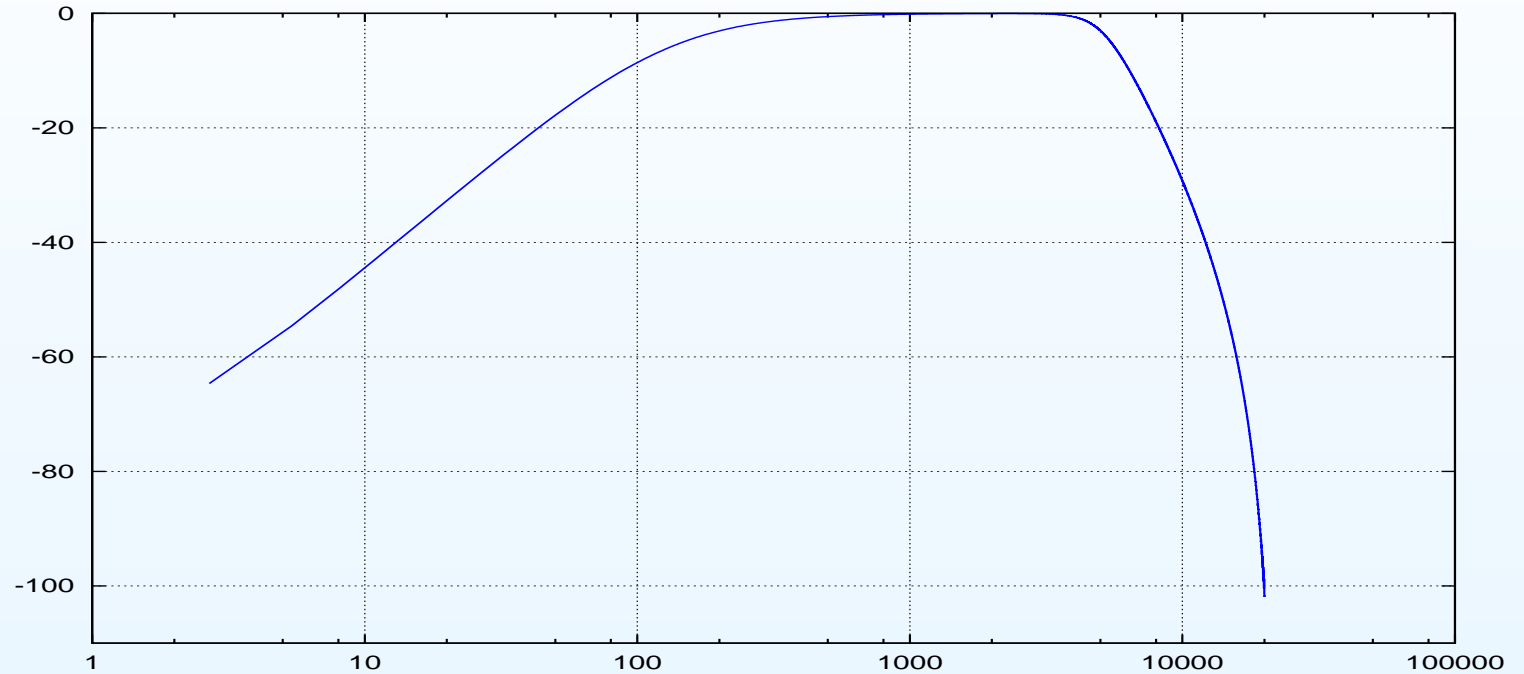
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# Speaker Bandpass



## Speaker Bandpass

General “spectral window” of a Celestion 12” speaker:



```
import("filter.lib");  
dcblockerat(130) : dcblockerat(130) : lowpass4(5000);
```

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# Coupled Strings



# Two Ideal Coupled Strings at a Common Bridge Impedance

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• Waveguide Model

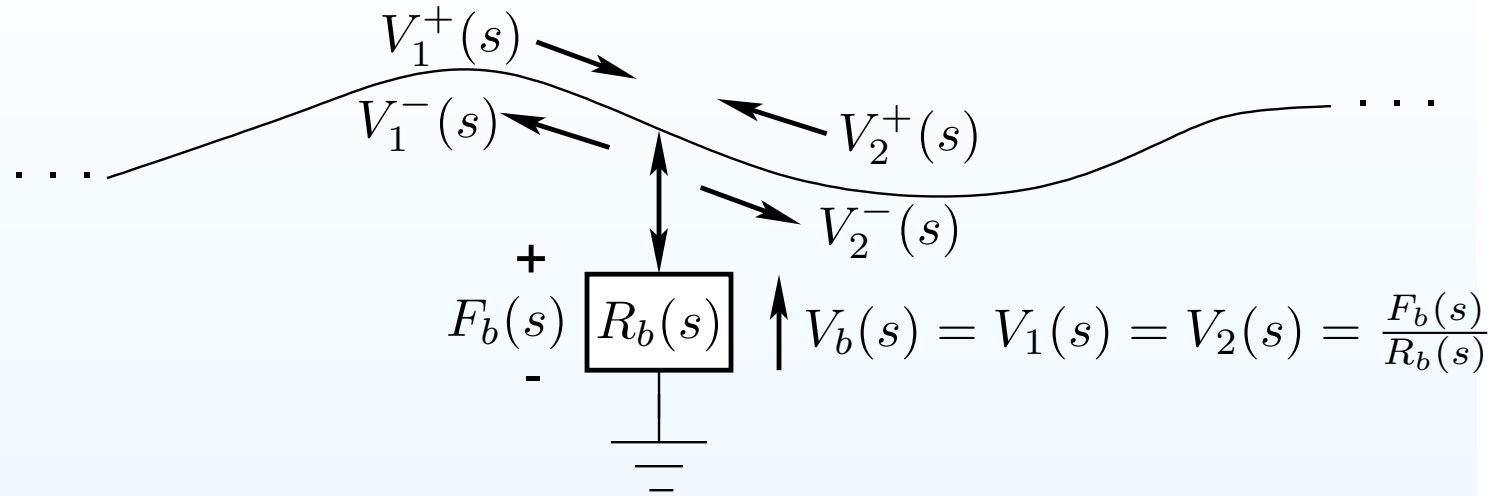
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$$v_1^-(t) = v_b(t) - v_1^+(t)$$

$$v_2^-(t) = v_b(t) - v_2^+(t)$$

$$V_b(s) = H_b(s)[R_1 V_1^+(s) + R_2 V_2^+(s)], \text{ where}$$

$R_i$  = wave impedance of string  $i$ , and

$$H_b(s) \triangleq \frac{2}{R_b(s) + R_1 + R_2}$$



# Digital Waveguide Model

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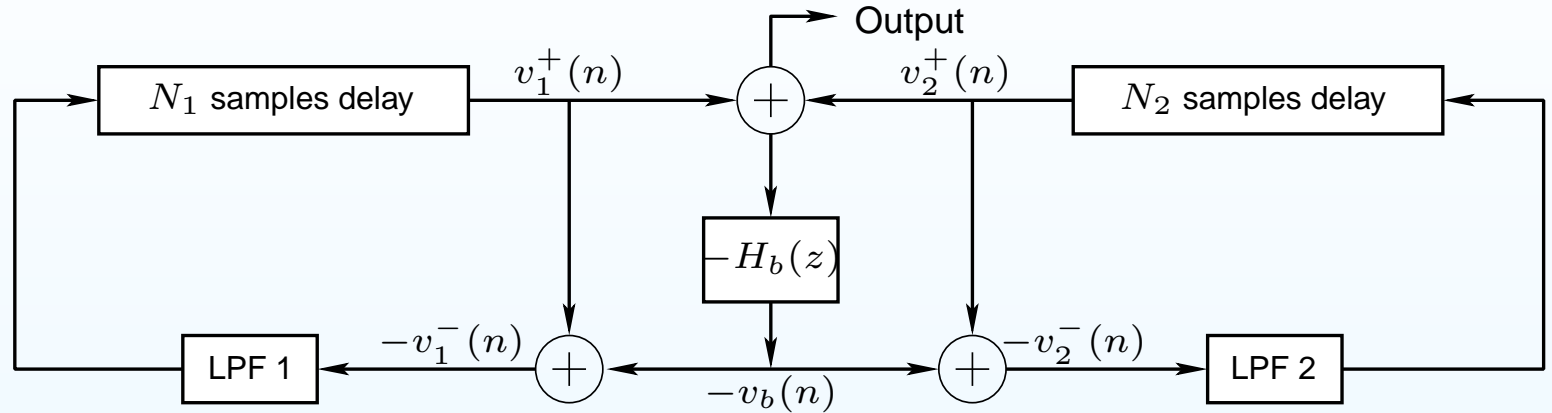
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## Faust Code

// Detuning of coupled planes of vibration:

```
detune = hslider("detuning percent", 1, 0, 10, 0.1);
```

```
d1 = fdelay1(Pmax, P-2); // P = period in samples
```

```
d2 = delay(Pmax, P*(1.0 - 0.01*detune)-2);
```

```
stringloop = coupledstrings(d1,d2);
```



# Coupled Strings in Faust

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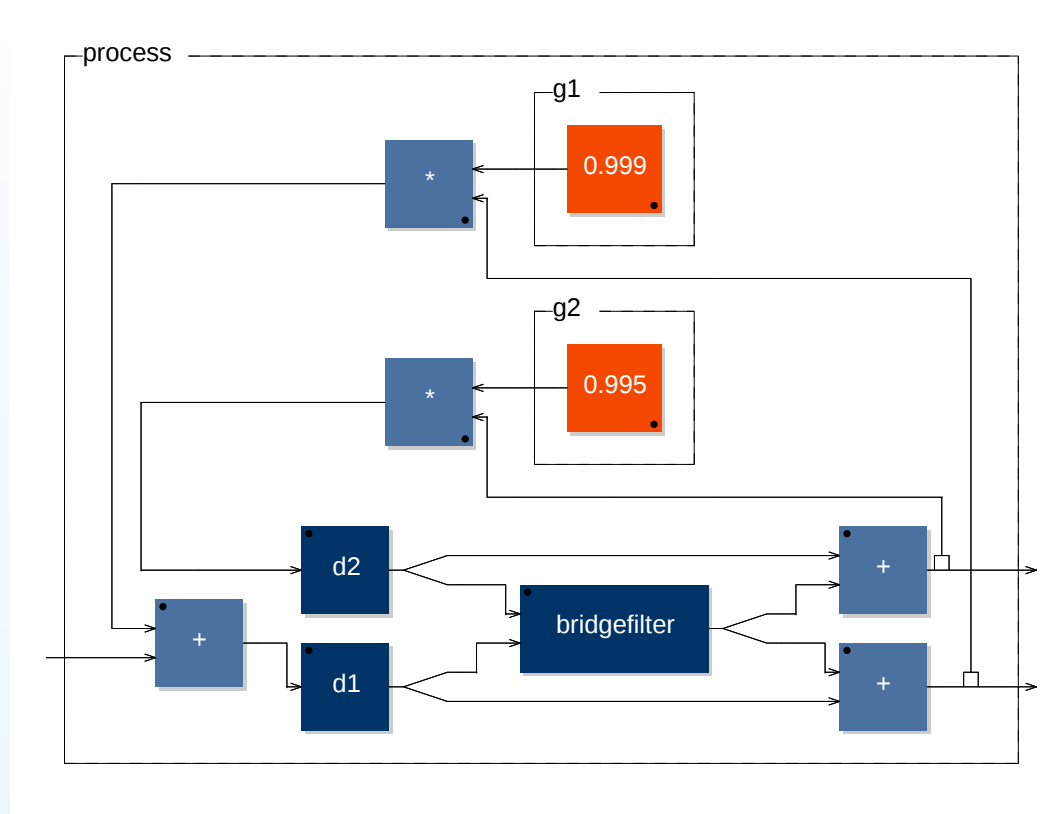
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```

d1 = fdelay1(Pmax, P-2); // P = period in samples
d2 = delay(Pmax, P*(1.0 - 0.01*detune)-2);
stringloop = ( _,+ : ((d2 <: _,_), (d1 <: _,_))
              : (_, (bridgefilter <: _,_) ,_)
              : +,+) ~ (*(g2),*(g1)) ;

```



# Simplified String Coupling

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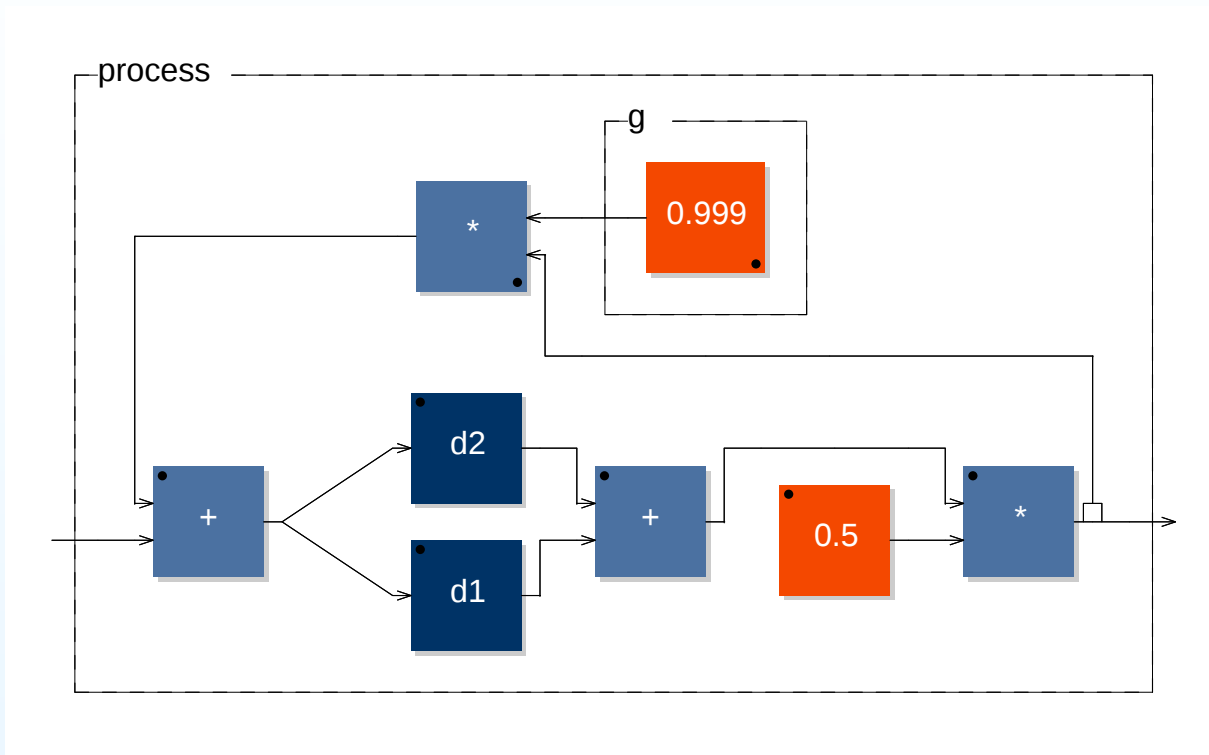
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```
stringloop = (+ <: d1,d2 : + : *(0.5))
             ~ dampingfilter;
```



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# Wah Pedal



# CryBaby Measurements at Three Pedal Angles

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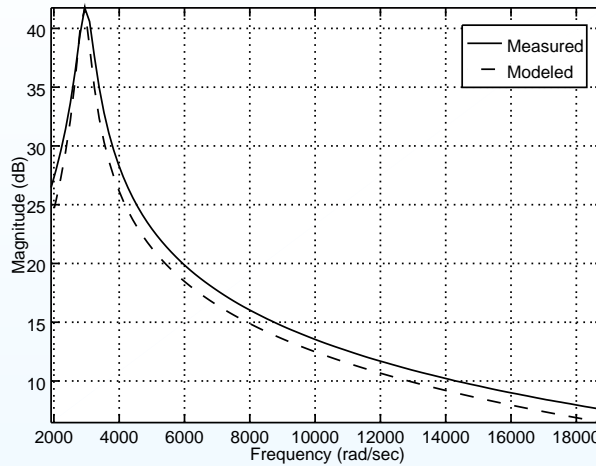
● minphaseir 1

● minphaseir 2

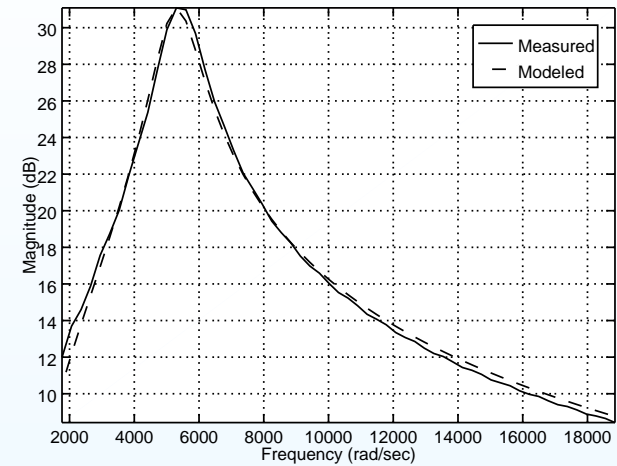
● Faust CryBaby

● Moog VCF

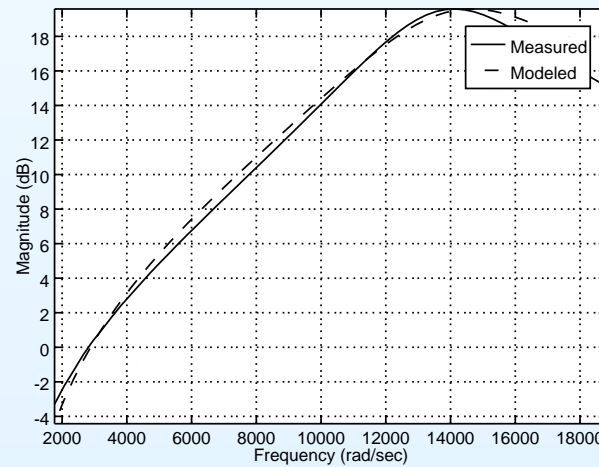
● Faust Implementation



(a) Rocked Back Full



(b) Middle Angle



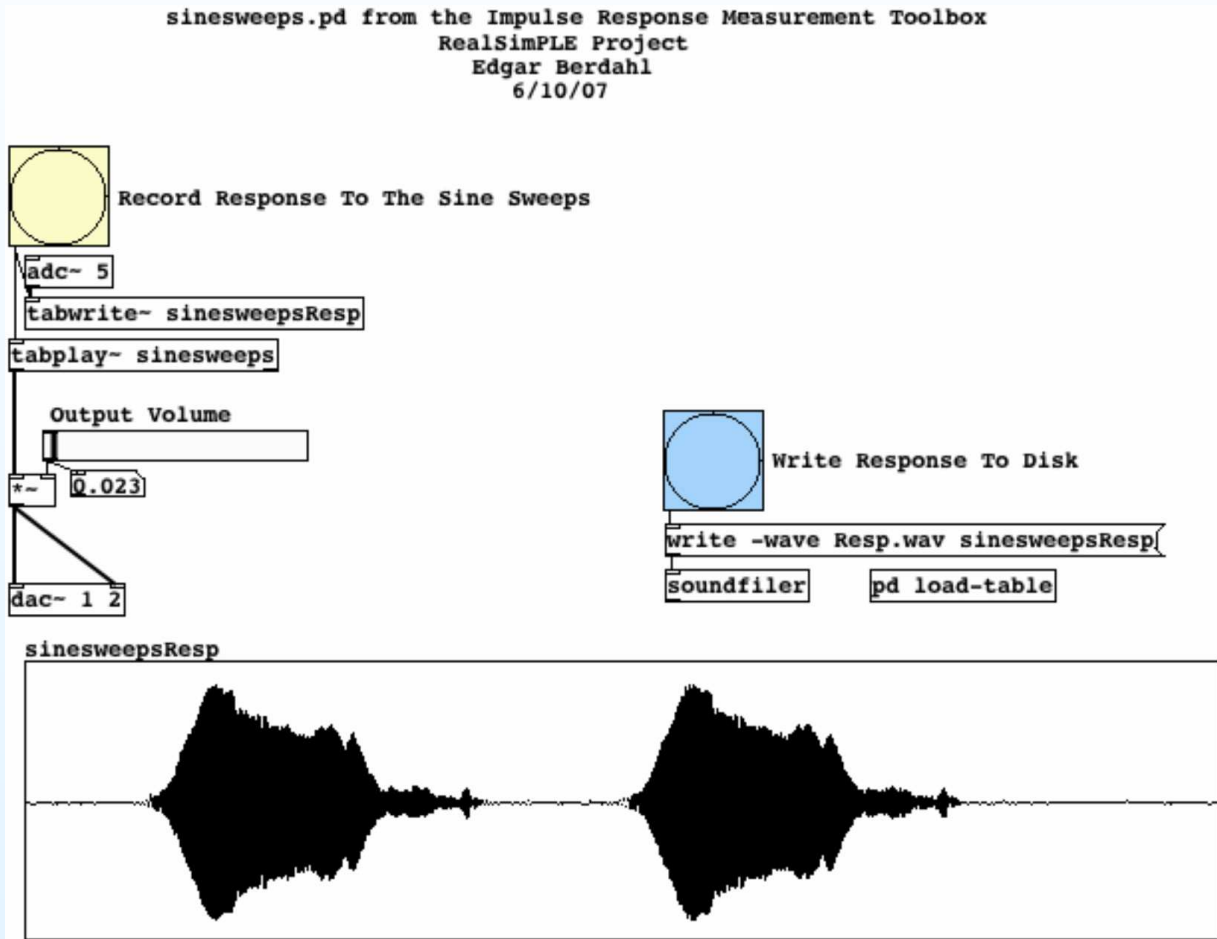
(c) Rocked Forward Full



# RealSimple Transfer Function Measurement Toolbox

Measures acoustic impulse responses using Pd and Octave:  
[http://ccrma.stanford.edu/realsimple/imp\\_meas/](http://ccrma.stanford.edu/realsimple/imp_meas/)

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sinesweeps.pd







## Octave Code for CryBaby Q and Resonance Measurement

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```
f1 = 40; f2 = 10000; % used in filename
f1z = 300; f2z = 3000; % zoom-in range
del = [3000 2000 2000]; % system delay (samples)
dur = [2048 1024 1024]; % impulse-response duration
dir = sprintf('wah-2sec-%dHz-%dkHz',f1,f2/1000);
Q = zeros(1,3);
wp = zeros(1,3);
for i=1:3
    ifn = sprintf('%s/wah%dImpResp.wav',dir,i-1);
    [wahir,fs] = wavread(ifn);
    wi = wahir(del(i)+1:del(i)+dur(i));
    [Qi,wpi,Hp,Hd,w] = invfreqsmethod(wi,f1z,f2z,fs);
    Q(i) = Qi; wp(i) = wpi;
    disp('PAUSING - RETURN to continue'); pause;
end
Q % print out estimated Q values
fp = wp/(2*pi) % and estimated pole frequencies
```



# CryBaby Q and Resonance Measurement Results

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Estimated Q values:

$$Q = [9.4, 4.0, 1.9]$$

Estimated pole frequencies:

$$f_p = [464, 838, 2252] \text{ Hz}$$

Analog CryBaby model ( $\omega_p = 2\pi f_p$ ):

$$H(s) = \frac{s - \xi}{\left(\frac{s}{\omega_p}\right)^2 + \frac{2}{Q} \left(\frac{s}{\omega_p}\right) + 1}$$

Digital model derived using  $z = \exp(sT) \approx 1 + sT$   
(low-frequency resonance assumed)

## Octave function invfreqsmethod

```
function [Q,wp] = invfreqsmethod(h,f1,f2,fs);  
%  
% INVFREQSMETHOD - use invfreqs to estimate Q and resonance  
%           frequency from a resonator impulse response  
% USAGE:  
%           [Q,wp] = invfreqsmethod(h,f1,f2,fs);  
% where  
%  
% h = impulse response (power of 2 length preferred)  
% f1 = lowest frequency of interest (Hz)  
% f2 = highest frequency of interest (Hz)  
% fs = sampling rate (Hz)  
% Q = estimated resonator Quality factor  
% wp = estimated pole frequency (rad/sec)
```



## Octave function `invfreqsmethod`, continued

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```
h = minphaseir(h);
```

```
H = fft(h);
```

```
...
```

```
[Bh,Ah] = invfreqs(H(zoom),w,2,2,1 ./w.^2);
```

```
% Denominator to canonical form
```

```
%  $A(s) = s^2 + (wp/Q) s + wp^2$ :
```

```
Ahn = Ah/Ah(1);
```

```
wp = sqrt(Ahn(3));
```

```
Q = wp/Ahn(2);
```

```
...
```

## Octave function minphaseir

```
function [hmp] = minphaseir(h)
%
% MINPHASEIR - Convert a real impulse response to its
%             minimum phase counterpart
% USAGE:
%         [hmp] = minphaseir(h)
% where
%
% h      = impulse response (any length - will be zero-padded)
% hmp    = min-phase impulse response (at zero-padded length)
```



## Octave function minphaseir, continued

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```
nh = length(h);  
nfft = 2^nextpow2(5*nh);  
Hzp = fft(h,nfft);  
Hmpzp = exp(fft(fold(ifft(log(clipdb(Hzp,-100))))));  
hmpzp = ifft(Hmpzp);  
hmp = real(hmpzp(1:nh));
```



## Faust Digital CryBaby (effect.lib)

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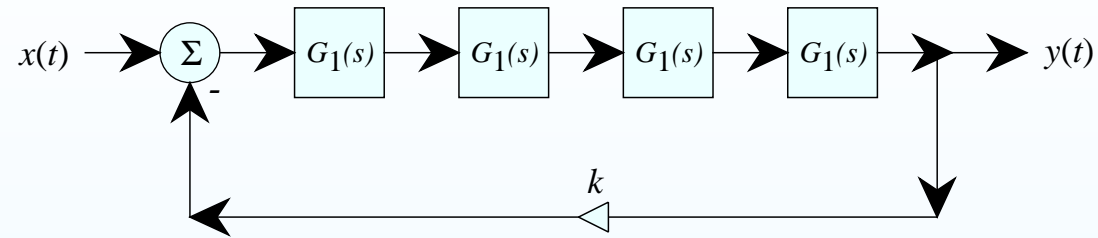
```
crybaby(wah) = *(gs(s)) : tf2(1,-1,0,a1s(s),a2s(s))
with { // wah = pedal angle in [0,1]
    s = 0.999; // smoothing parameter (pole location)
    Q = pow(2.0,(2.0*(1.0-wah)+1.0)); // Resonance Q,
    fr = 450.0*pow(2.0,2.3*wah); // tuning, and
    g = 0.1*pow(4.0,wah); // gain (optional)

    // Biquad fit using z = exp(s T) -> 1 + sT
    frn = fr/SR; // pole frequency (cycles per sample)
    R = 1 - PI*frn/Q; // pole radius
    theta = 2*PI*frn; // pole angle
    a1 = 0-2.0*R*cos(theta); // biquad coeff
    a2 = R*R; // biquad coeff

    a1s(s) = a1 : smooth(s); // "dezippering"
    a2s(s) = a2 : smooth(s);
    gs(s) = g : smooth(s);
};
```



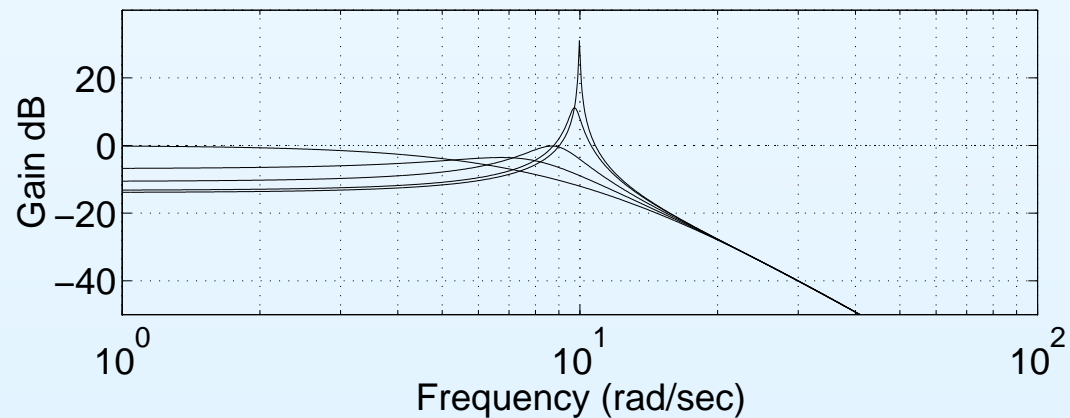
# Moog VCF



$$G_1(s) = \frac{1}{1 + s/\omega_c}$$

## Controls

- Pole location  $s = -\omega_c$ : controls *cut-off frequency*
- Feedback gain  $k \leq 4$ : controls *resonance*



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## Moog VCF in Faust (effect.lib)

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```
onepole(p) = *(1.0-p) : + ~ *(p);
```

```
moogvcf(mk,p) =
```

```
(+ : onepole(p) : onepole(p)
```

```
: onepole(p) : onepole(p)) ~ *(mk);
```

### 4th Order Wah Effect:

```
wah4(fr) = moogvcf(-3.8,pole(fr))
```

```
with { pole(fr) = 1.0-fr*2.0*PI/SR; };
```

$Q = 3.8$  can be changed as preferred



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# Faust Signal Processing Libraries



# Faust Signal Processing Libraries

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- `filter.lib`,  
continued

- `filter.lib`,  
continued

- `filter.lib`,  
continued

- `effect.lib` —

Current Contents

- `osc.lib` — Current  
Contents

- `osc.lib` —  
continued

Julius Smith

References

The following Faust libraries were developed in the course of this work:

- `filter.lib` - digital filters of various types
- `effect.lib` - digital audio effects
- `osc.lib` - filter-based oscillators (sine and sawtooth)



## filter.lib

### Simple first-order filters (mainly for readability):

```
zero(z) = _ <: _,mem : _,*(z) : -;  
pole(p) = + ~ *(p);  
integrator = + ~ _ ;  
tau2pole(tau) = exp(-1.0/(tau*SR));  
smooth(s) = *(1.0 - s) : + ~ *(s);  
dcblocker = zero(1) : pole(0.995);  
dcblockerat(fb); // pole set for desired "break frequency"
```

### Comb Filters:

```
ffcombfilter(maxdel,del,g) = _ <: delay(maxdel,del) : *(g): +;  
fbcombfilter(maxdel,idel,g) = (+ : delay(maxdel,idel)) ~ *(g);
```

## filter.lib, continued

### Elementary Transfer Functions in Direct Form:

```
tf1(b0,b1,a1) = _ <: *(b0), (mem : *(b1)) :> + ~ *(0-a1);  
tf2(b0,b1,b2,a1,a2) = sub ~ conv2(a1,a2) : conv3(b0,b1,b2) ...  
tf3(b0,b1,b2,b3,a1,a2,a3) = sub ~ conv3(a1,a2,a3) : conv4( ...  
tf4(b0,b1,b2,b3,b4,a1,a2,a3,a4) = sub ~ conv4(a1,a2,a3,a4) ...
```

### Numerically Robust Quadrature Resonators (for Oscillators):

```
nlf2(f,r,x); // 2nd-order normalized ladder resonator  
wgr(f,r,x); // Transformer-normalized waveguide resonator
```

## filter.lib, continued

### Digital filters specified by their ANALOG transfer functions:

```
tf1s(b0,a0,w1); // 1st-order section  $H(s) = b0 / (s + a0)$   
tf2s(b1,b0,a1,a0,w1); // 2nd-order section (biquad)
```

### Variable cut-off Butterworth lowpass filters:

```
lowpass1(fc) = tf1s(1,1,2*PI*fc); // 1st-order  
lowpass2(fc) = tf2s(0,1,sqrt(2),1,2*PI*fc); // 2nd-order  
lowpass3(fc) = tf2s(0,1,1,1,w1) : tf1s(1,1,w1)... // 3rd order  
lowpass4(fc) = tf2s(0,1,a11s,1,w1) : tf2s( ... // 4th order
```

### Parametric equalizer section:

```
pareq(Q,F,G) = tf2(...); // (adapted from bandfilter.dsp)
```

## filter.lib, continued

### Interpolating delay lines generalizing `fdelay(n,d,x)`

#### Lagrange polynomial interpolation:

```
fdelay1(n,d,x); // 1st-order Lagrange interpolation
fdelay2(n,d,x); // 2nd-order
fdelay3(n,d,x); // 3rd-order
fdelay4(n,d,x); // 4th-order
```

#### Thiran allpass interpolation:

```
fdelay1a(n,d,x); // 1st-order allpass interpolation
fdelay2a(n,d,x); // 2nd-order
fdelay3a(n,d,x); // 3rd-order
fdelay4a(n,d,x); // 4th-order
```

## effect.lib — Current Contents

### Dynamic Level Lowpass Filter

```
levelfilter(level,freq); // from EKS  
levelfilterN(N,freq,L) = seq(i,N,levelfilter((L/N),freq));
```

### Overdrive

```
cubicnl(drive,offset);
```

### Wah Pedals and VCFs

```
crybaby(wah);
```

```
moogvcf(Q,fr);
```

```
wah4(fr) = moogvcf(3.8,fr);
```

### Audio Speaker Bandpass

```
speakerbp(f1,f2) // "Loudspeaker Bandpass"  
= dcblockerat(f1) : dcblockerat(f1) : lowpass4(f2);
```





## osc.lib — Current Contents

### Misc.

```
impulse = 1-1';  
sawtooth(freq);
```

### Filter-Based oscillators osc\*(f)

```
oscb(f) = impulse : tf2(1,0,0,a1,1) ... ; // ringing biquad
```

```
oscs(f); // ringing state-variable filter  
          // = "magic circle", or "modified coupled form"
```

## osc.lib — continued

### Undamped Second-Order Normalized Ladder Resonator:

```
oscrcs(f) = impulse : nlf2(f,1) : _,!; // sine
oscrc(f) = impulse : nlf2(f,1) : !,_; // cosine
oscrcq(f) = impulse : nlf2(f,1); // quadrature
oscr = oscrcs; // default = sine
```

### Undamped Second-order Transformer-Normalized Waveguide Resonator:

```
oscwc(fr) = 1-1' : wgr(fr,1) : _,!; // cosine (1 mpy/sample)
oscws(fr) = 1-1' : wgr(fr,1) : !,_; // sine (2nd scaling mpy)
oscq(fr) = 1-1' : wgr(fr,1); // phase quadrature outs
OSCW = OSCWC;
```



## Online Resources

Outline

Why Now?

EKS Intro

Pick Position Comb

Damping Filter

Tuning Filter

Dynamic Level Filter

Overdrive, Feedback

Speaker Bandpass

Coupled Strings

Wah Pedal

Faust Libraries

References

- RealSimple module (including software download):  
[http://ccrma.stanford.edu/realsimple/faust\\_strings/](http://ccrma.stanford.edu/realsimple/faust_strings/)
- Online Book (physics-based synthesis of musical instruments):  
<http://ccrma.stanford.edu/~jos/pasp/>