A Survey of Piano Modeling

Mengfan Zhang

Abstract
The physical modeling approach provides semantic sound representations for the parameters having physical interpretations. In this paper, piano modeling approach is surveyed and discussed. A typical piano modeling structure is presented. First the excitation part, then the string and radiation modeling. A multi-rate soundboard model is specifically discussed towards the high computational complexity of physical modeling.

Keywords: physical modeling, audio signal processing, piano

1. Introduction

Physical modeling has gained more and more interest in the past decades. In contrast to traditional synthesis methods which only model the resulting sound signal itself, physical modeling provides a straightforward physical interpretation in terms of masses, springs, dashpots, etc. Hence, modifying the control parameters allows for intuitions towards the virtual instrument.

To develop a physical model for piano, the first task is to investigate the acoustical properties of piano, so the physical structure should be studied. Then we should select some parts of the features to be modeled, which depends on the part of structure's contribution to the generation of piano sound. Finally, finding efficient modeling methods to implement these selected features. The physical modeling approach is implemented based on the instrument's structure and its sound generation mechanism. Consequently, this white-box modeling of system identification can directly interpret the real world. Not like the black-box modeling using abstract algorithms or modifying prerecorded samples. So the big advantage of physical modeling is the reveal of the underlying sound generation mechanism. But the drawbacks are the lack of generality and high computational costs. The black-box method can be used to model many kinds of instruments, but a physical model is valid only for one specific instrument. Physical modeling has much higher computational cost than traditional techniques. One study of computational cost towards sound board modeling is discussed in Section 6.

This paper is organized as follows. First the piano structure is described. Then a typical model for piano modeling is discussed. After presenting the basic idea of piano modeling, each component of the modeling structure is modeled separately. Finally, the conclusion is presented.

2. Piano structure

The main parts of a piano are the keyboard, the action, the strings, the sound-board, and the frame. A simplified diagram of a piano is shown in Fig. 1. The strings extend from the pin block across the bridge to the hitch-pin rail at the far end. When a key is depressed, the damper is raised, and the hammer is thrown against the string, setting it into vibration. Vibrations of the string are transmitted to the soundboard by the bridge. [1]

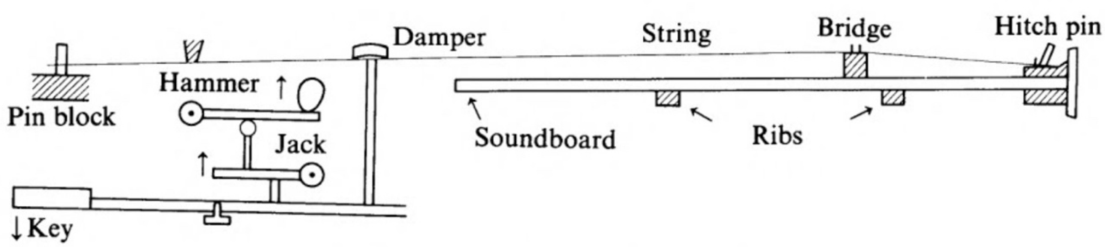

Fig. 1 A simplified diagram of the piano

3. Model structure

Piano sounds are the final product of a complex synthesis process. Using physical models to mimic the piano acoustics can be developed. Physical modeling approach simulates the structure of the instrument. The main elements of the piano to be modeled are the hammer, string, bridge, soundboard, enclosure, acoustic space, mic placement, etc [2].

A typical model structure was proposed by Bank [3-5]. The sound production mechanism of the piano can be divided into three model blocks, as shown in Fig. 2. The first model block is the excitation, the hammer strike exciting the string. The kinetic energy given by the piano player is transformed to kinetic energy of the hammer. Then the hammer hits the string and transform its energy to vibrational energy of the string. Some of the energy is dissipated due to internal losses, the remaining energy gets to the sound board through the bridge. The soundboard converts the vibrational energy to acoustical energy, the audible sound. The string determines the fundamental frequency of the tone and its quasi-periodic output signal is filtered through a post-processing block, covering the radiation effects of the soundboard.

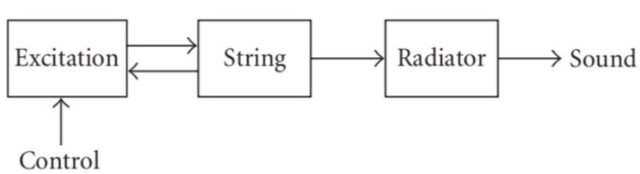

Fig. 2 A model structure for piano modeling

4. Hammer Modeling

Many different approaches have been presented in the literature for piano hammer modeling. A point-mass adding to a damped string is much closer to a realistic piano hammer [2], as shown in Fig. 3. The impedance of this plucking system is the parallel combination of the mass impedance ms and the damped spring impedance $\mu + \mathrm{k/s}$.

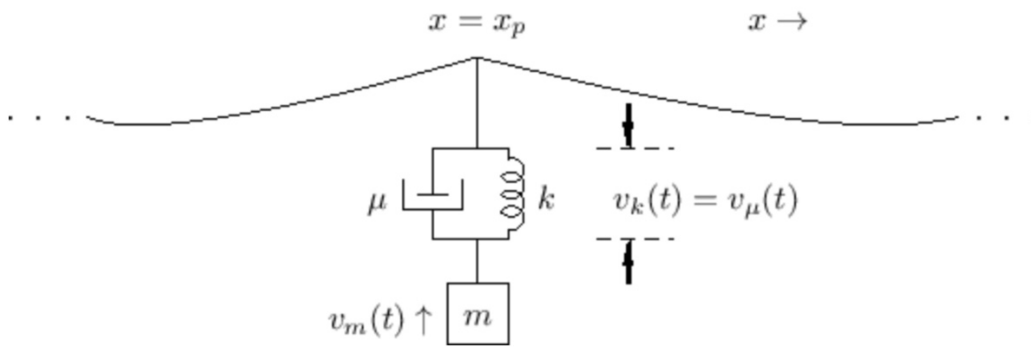

Fig. 3 Ideal string excited by a mass and damped spring

The driving-point impedance of the hammer at the string contact-point is

$$R_h(s) = ms\,||\left(\mu + \frac{k}{s}\right) = \frac{\mu s^2 + ks}{s^2 + \frac{\mu}{m}s + \frac{k}{m}}.$$

5. String modeling

Digital waveguide model is an ideal solution to model the piano string. The velocity distribution of the string can be calculated as the sum of two traveling waves:

$$\mathrm{v(x,t)} = v^+(x - ct) + v^-(x + ct),$$

where x denotes the spatial coordinate, t is time, c is the propagation speed, and v^+ and v^- are the traveling wave components.

An example of initial conditions for the ideal struck string is shown in Fig. 4. The hammer strike itself could be considered to take zero time. The ideal struck string involves a zero initial string displacement but a nonzero initial velocity distribution. In concept, a “hammer strike” transfers an “impulse” of momentum to the string at time 0 along the striking face of the hammer.

In a physical sense, the hammer strikes the string between the agraffe and the bridge. This can be simulated by delay lines, as shown in Fig. 5. The delay lines contain samples of traveling force waves, and the bridge is allowed to vibrate, which results in a filtered reflection at the bridge. The modeling could be fine turned using delay line interpolation, such as first-order allpass interpolation, linear interpolation and Lagrange interpolation.

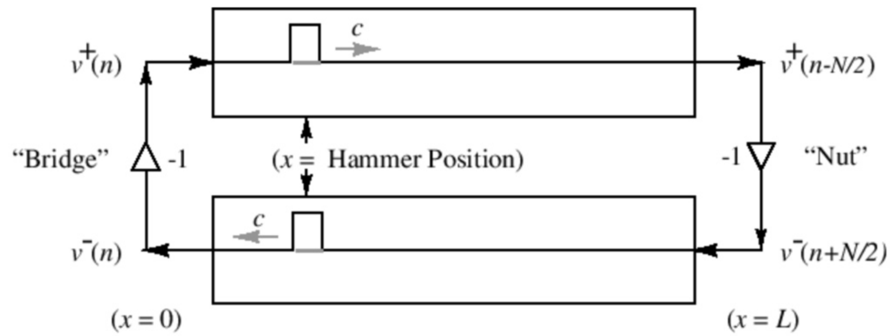

Fig. 4 Initial conditions for the ideal struck string

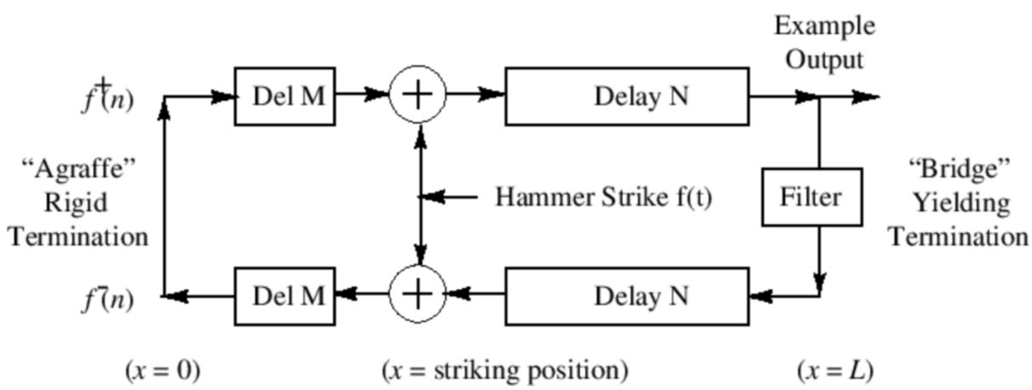

Fig. 5 Model of a piano string struck in its interior by a hammer

6. Radiation modeling

The soundboard radiation can be addressed as a linear filtering operation upon the signal coming from the string model. The problem can be solved as a filter design. However, the complex structure of the instrument body makes the transfer function exhibit high model density [6]. To obtain high quality sound, high order filters are needed. Fig. 6 shows the pressure-force transfer function of a piano sound board. The soundboard was excited by hitting the bridge using an impact hammer. The excitation force and the sound pressure at 2m distance from the piano were simultaneously recorded. The ratio of their spectra is depicted as the transfer function.

The FIR filter could be used to obtain high quality sound, but the filter order needs to be between 1000 and 2000 in 44.1kHz sampling rate for example. The computation cost is 10 to 100 times higher compared to the cost of the string and excitation models. IIR filters perform nearly the same as FIR filters with the same computational cost. Bank proposed a multi-rate approach [6] to address the computational issue. In filter design, FIR filter could preserve the sound characteristics because it maintains not only the overall magnitude response but also the phase information. The multi-rate approach avoids the high computational cost of the FIR filter but preserve its benefits of retaining sound quality.

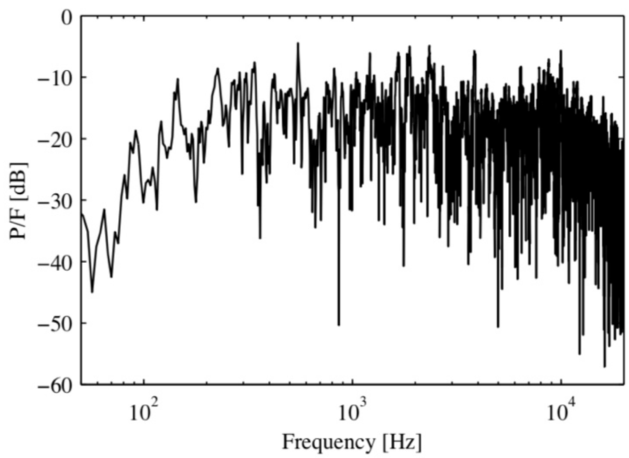

Fig. 6 The transfer function of a piano soundboard

Fig. 7 shows the diagram of multi-rate model. The string signal F_s is split into two frequency bands. $H_{di}(z)$ is a polyphase FIR filter used for decimation and interpolation filters. The filters $H_l(z)$ and $H_h(z)$ are computed using the measured force-pressure transfer function $H_t(z)$ of a real piano soundboard. The frequency below 2.2kHz is down-sampled by a factor of 8 to produce the FIR filter $H_l(z)$. The impulse response of the low-frequency chain is subtracted from the target response $H_t(z)$ providing a residual response containing frequencies above 2.2kHz. In the high frequency band, only the magnitude response is modeled using a low-order filter $H_h(z)$. The signal-flow is delayed by N samples to compensate the down-sampling and up-sampling operations. Since the human ear is less sensitive to the high frequency bands, the high frequency model could be simplified.

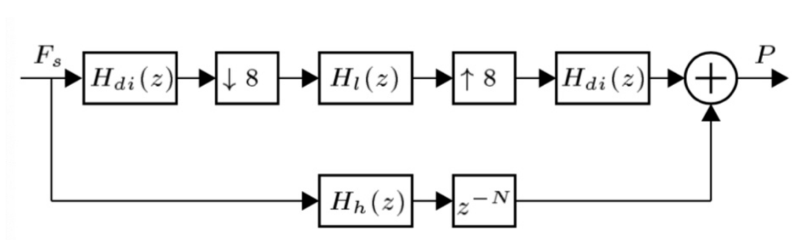

Fig. 7 The multi-rate soundboard model

The result of the multi-rate soundboard model is shown in Fig. 8. It can be seen that the magnitude is well reconstructed below the 2.2kHz. Above 2.2kHz, only the overall magnitude response is obtained.

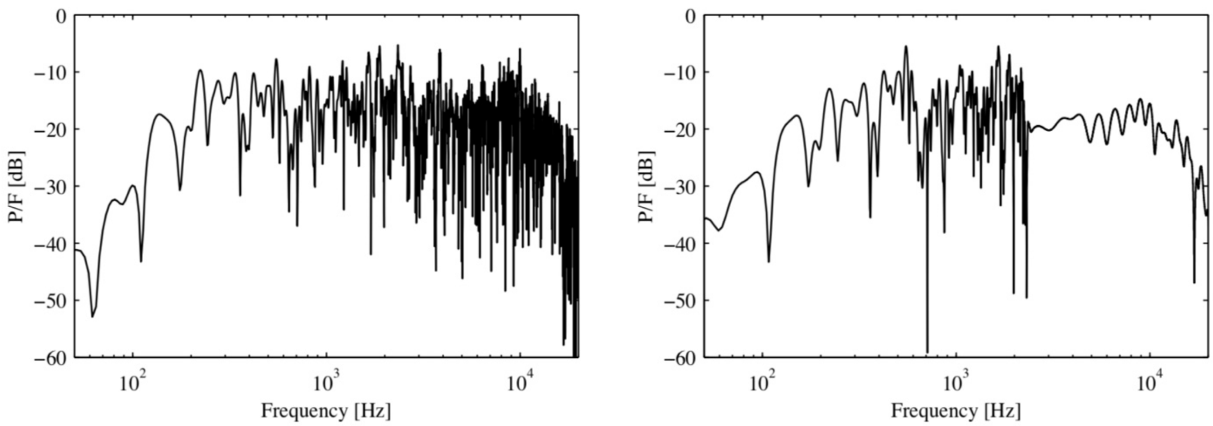

Fig. 8 The magnitude transfer function of the measured filter (left) and the modeled filter (right)

7. Conclusion

This paper made a survey of the main stages for developing a physical model of piano. It includes the hammer, string, and radiation modelling. Various approaches have been discussed for filter design in soundboard model. A multi-rate soundboard model is specifically discussed in terms of reducing the high computational complexity. For the future study, a black box model, such as neural network models, could be introduced to address the computational issue in physical modeling.

References

[1] N. H. Fletcher and T. D. Rossing, The Physics of Musical Instruments, Springer-Verlag, New York, NY, USA, 1991.
[2] J.O. Smith, Physical Audio Signal Processing, http://ccrma.stanford.edu/~jos/pasp/, online book, 2010 edition, accessed <date>.
[3] Federico Avanzini, Balázs Bank, Gianpaolo Borin, Giovanni De Poli, Federico Fontana, and Davide Rocchesso, Musical Instrument Modeling: The Case of the Piano, Proceedings of the MOSART Workshop on Current Research Directions in Computer Music, pp. 124-133, Barcelona, Spain, Nov. 2001.
[4] Balázs Bank, Stefano Zambon, and Federico Fontana, A Modal-Based Real-Time Piano Synthesizer, IEEE Trans. on Audio, Speech and Language Processing, Special Issue on Virtual Analog Audio Effects and Musical Instruments, vol. 18, iss. 4, pp. 809-821, May 2010.
[5] Balázs Bank, Federico Avanzini, Gianpaolo Borin, Giovanni De Poli, Federico Fontana, and Davide Rocchesso, Physically Informed Signal-Processing Methods for Piano Sound Synthesis: a Research Overview, EURASIP Journal on Applied Signal Processing, Special Issue on Digital Audio for Multimedia Communications, vol. 2003, No. 10, pp. 941-952, Sep. 2003.

[6] B. Bank, G. De Poli, and L. Sujbert, A multi-rate approach to instrument body modeling for real-time sound synthesis applications, in Proc. 112th AES Convention, Munich, Germany, May 2002, preprint no. 5526.