Introduction to digital filters

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1 The simplest low-pass filter

The difference equation for the simplest low-pass filter is given by:

$$y(n) = \frac{1}{2}(x(n) + x(n-1)) \tag{1}$$

In the frequency domain this filter becomes:

$$Y(z) = \frac{1}{2}(X(z) + z^{-1}X(z)) = \frac{1}{2}X(z)(1 + z^{-1}) \tag{2}$$

the frequency response of this filter is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{2}(1 + z^{-1}) \tag{3}$$

Considering that $z = e^{j\omega}$, where $j = \sqrt{-1}$, we have:

$$H(e^{j\omega}) = \frac{1}{2}(1 + e^{-j\omega}) \tag{4}$$

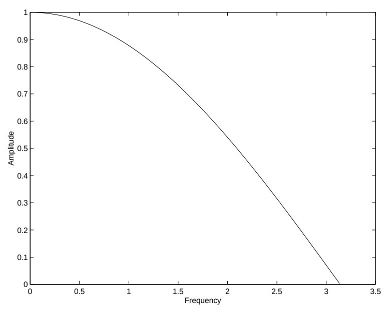

Figure 1: *Frequency response of the simplest low-pass filter.*

2 The simplest high-pass filter

The difference equation for the simplest low-pass filter is given by:

$$y(n) = \frac{1}{2}(x(n) - x(n-1)) \tag{5}$$

Derive the frequency response.

3 A feedforward comb filter

The difference equation for a feedforward comb filter is given by:

$$y(n) = b_0 x(n) + b_M x(n - M) \tag{6}$$

where M is a delay and b_0, b_M are called *filter coefficients*. In the frequency domain this filter becomes:

$$Y(z) = (b_0 X(z) + b_M z^{-M} X(z)) \tag{7}$$

Let's put $b_0 = 1$ and $b_M = 1$.
the frequency response of this filter is:

$$H(z) = \frac{Y(z)}{X(z)} = (1 + z^{-M}) \tag{8}$$

Considering that $z = e^{j\omega}$, where $j = \sqrt{-1}$, we have:

$$H(e^{j\omega}) = (1 + e^{-j\omega M}) = |e^{j\omega M/2}||e^{j\omega M/2} + e^{-j\omega M/2}| = 2|cos(\omega \frac{M}{2})| \tag{9}$$

(since $|e^{j\omega M/2}| = e^{j\omega M/2} e^{-j\omega M/2} = 1$).

4 One pole filters

$$y(n) = gx(n) + b_1 y(n - 1) \tag{10}$$

The Z transform is:

$$Y(z) = gX(z) + b_1 z^{-1} Y(z) \tag{11}$$

Stable iff $|b_1| < 1$ Transfer function:

$$H(z) = g/(1 - b_1 z^{-1}) \tag{12}$$