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AN INVESTIGATION OF THE IMPACT OF TORSION WAVES AND FRICTION CHARACTERISTICS ON THE PLAYABILITY OF VIRTUAL BOWED STRINGS

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ABSTRACT

"Playability" is measured for variations in a bowed-string simulation model. The variations studied are (1) the effect of torsion waves, and (2) the effect of the choice of friction model. It is found that (1) elimination of torsion-wave simulation does not degrade playability, and (2) the more recently developed "plastic" bowed-string friction model, in which the frictional force is a function of temperature, is more "playable" than prior friction models which depend only on relative sliding velocity.

1. INTRODUCTION

Time-domain models of bowed-string dynamics have achieved a level of completeness that makes them potentially suitable for answering fundamental questions about the effects of various physical parameters on the "playability" of a bowed-string instrument [1, 2], [3], [4, 5]. Such information could be of great interest to the instrument maker and scientifically inclined performer. Additionally, digital sound synthesis algorithms have been developed based on these models [6, 7]. Since the goal in sound synthesis is to provide the most cost-effective computation for a given quality level, it is of great interest to determine the relative value of the various model components for synthesis (both on the sound and on the "playability" of the "virtual instrument"). Questions of this nature can also be addressed using computer-based time-domain simulation models.

In this paper, we present results of an initial study aimed at evaluating (1) the importance of including explicit simulation of torsion (twisting) waves on the string, and (2) the importance of using more realistic friction models in the simulated bow-string interaction.

In Section 2, we discuss the quality metric used, and Section 3 summarizes the simulation model. Section 4 presents the simulation results, and Section 5 summarizes our conclusions.

2. QUALITY MEASURES

The quality of a bowed-string instrument is more reliably determined by the player than the listener [5]. While the "tone" is clearly an important component of quality, a "poor tone" can be compensated in many ways. A more intrinsic quality which is less easily compenated is the "playability" of the instrument.

2.1. Evaluating Playability

"Playability" can be loosely defined as the "volume" of the mutidimensional parameter space in which "good tone" is produced.

The "playability" evaluation technique, described in [3, 4, 5], includes two high-level components: (1) a bowed-string software model [2] which is calibrated by measured and/or inferred physical data, and (2) an algorithm for evaluating the quality of the model's output [3, p. 149],[5].

In this particular study, we define playability in terms of the minimum and maximum bowing force over a range of bowing positions for steady bowing (constant bow force and velocity). The type of bowed-string motion is automatically classified [3] for a reasonable range of bow forces and positions along the string, and these are used to produce a kind of empirical "Schelleng diagram" [8]. As discussed in the following subsection, a Schelleng diagram displays at a glance the region of "good behavior" for the bowed string model, *i.e.*, the region of the parameter space in which simple "Helmholtz motion" is obtained.

2.2. Schelleng Diagram

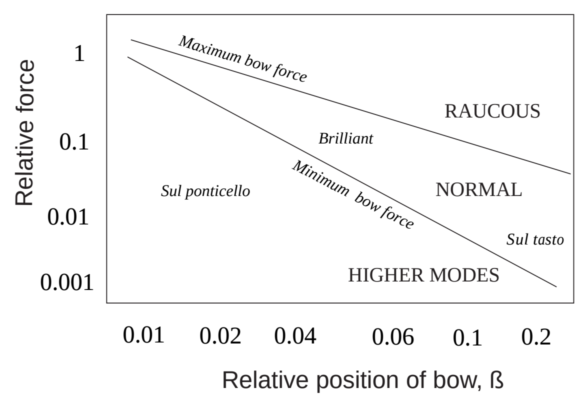

Figure 1: *Theoretical Schelleng diagram.*

Figure 1 shows the classical "Schelleng diagram" [9] indicating the theoretical minimum and maximum bow force as a function of bow position along the string. Between the bow-force limits, "Helmholtz motion" is possible. Helmholtz motion is characterized by a single "corner" traveling back and forth on the string under an approximately parabolic envelope. While the corner is between the bow and the nut or finger, the string is sticking to the bow. When the corner is on the shorter part of its journey, between the bow and the bridge, the string is slipping under the bow. This fundamental picture of normal bowed-string behavior was first discovered and described by Helmholtz in the mid-nineteenth century [10]. Further details of possible bowed-string motion are summarized in [5], and a review of theoretical models can be found in [11].

Below the minimum bow force, a second Helmholtz corner is likely to appear (or more), due physically to *multiple slips* per period. This regime is is often referred to as "surface sound" and is common in "sul ponticello" playing.

Above the maximum bow-force, the Helmholtz corner may not be strong enough to initiate slipping when passing the bow toward the bridge. In this case, the time-keeping function of the traveling corner may be disrupted, leading to aperiodic, even "raucous" sound.

Part of the "playability" of a bowed-string instrument is the ease with which Helmholtz motion can be achieved. Skilled players strive to achieve Helmholtz motion as quickly as possible in "smooth" playing [12]. It is even possible to hit Helmholtz motion immediately on the first period, which is especially desirable on a double bass for which a single period can be tens of milliseconds long.

3. BOWED STRING SIMULATION MODEL

The bowed-string simuation model, described in [2, 3, 5], includes simulation of transverse waves on a stiff string in the plane of the bow, torsion waves, constant-Q string resonances for the transverse and torsional waves, and bow-string interaction by means of one of a variety of friction models.

All simulations were run on a Pentium II 333Mz. We noticed that different computers gave slightly different results, due presumably to the extreme sensitivity of the results to precision in "chaotic" regimes.

All runs in this study simulate a cello D string with bending stiffness $B = 0.0004\ \mathrm{N\,m}^2$, where B is the coefficient of the fourth derivative of string displacement with respect to position in the wave equation for stiff vibrating strings [3, p. 133]. The sampling rate in all cases was 200 kHz.

In all simulations the string, starting from rest, is excited by a constant bow velocity v_b of 0.05 m/s.

In each computed Schelleng diagram, the bow force f_b is varied between 0.005 and 5 N, and the normalized distance β of the bow from the bridge is varied between 0.02 and 0.4 (where 0.5 would be at the string midpoint).

The torsional wave speed is 5.2 times the transverse wave speed; the transversal and torsional impedances are 0.55 and 1.8 kg/s, respectively. At the nut and bridge side, transversal and torsional wave losses are modeled by reflection functions with constant Q-factors of 500 and 45, respectively. These values were measured on a nylon-core cello D string tuned to 147 Hz [13].

Three friction models were compared [14]. For convenience, we will refer to them as *exponential*, *hyperbolic*, and *plastic*.

In the first two, the coefficient of friction depends only on sliding speed. The third introduces dependency on temperature, which in turn gives hysteresis in the friction curve.

The exponential model is given by [14]

$$\mu = 0.4\, e^{\frac{v-v_b}{0.01}} + 0.45\, e^{\frac{v-v_b}{0.1}} + 0.35$$

where v and v_b represent the velocity of the string and the bow, respectively. This model fits a sum of two exponentials to friction measurements made during steady sliding. As a result, any dynamic behavior is neglected in the establishment of frictional force after a velocity change.

The hyperbolic model for the coefficient of bow-string friction is given by

$$\mu = \mu_d + \frac{(\mu_s - \mu_d)v_0}{v_0 + v - v_b}$$

where v, v_b and v_0 are the string velocity, bow velocity and initial bow velocity, respectively, and $\mu_d = 0.3$ and $\mu_s = 0.8$ are the dynamic and static coefficients of friction, respectively. This model has been used for many years as a convenient mathematical approximation which yields closed-form results for the bow-string interaction [1, 2, 3].

The plastic friction model is given by [14]

$$\mu = \frac{Ak_y(T)}{N}\mathrm{sgn}(v)$$

where A is the contact area between the bow and the string, N is the normal load, and $k_y(T)$ is the shear yield stress as a function of temperature T. The temperature T of the shearing rosin layer

can be estimated from the current sliding velocity v by passing it through an appropriate linear filter [14]. Here, the rosin is modeled as exhibiting "plastic" deformations at the bow-string contact. Since there is a time delay associated with heat flow, the plastic model exhibits *hysteresis*, unlike the other two friction models.

4. RESULTS

The simulation results are summarized as follows. First, we consider the effect of torsion waves for the plastic friction model (which is believed to be the most physically accurate). Next, we look at the other choices of friction model.

4.1. Effect of Torsion-Wave Simulation on Playability

Figure 2 shows the empirical Schelleng diagram obtained by running the simulation with the plastic friction model installed. The darker shaded region including the squares is defined as the "playable" region of the parameter space, where Helmholtz motion is established. The region including the circles is the one in which multiple slips are established. We see that there is good qualitative agreement with the theoretical Schelleng diagram, as desired and expected.

Figure 3 shows the same case of Fig. 2 except without including simulation of torsional waves. We find that the playability region is not altered very much when torsional waves are removed. Looking only at the Helmholtz region, there are 65 pixels of Helmholtz-motion in Fig. 2, and 63 in Fig. 3. On the whole, the results are fairly comparable. However, the Helmholtz region is more contiguous without torsion waves. Evidently, torsion waves can reflect at a "bad time" so as to disturb the Helmholtz motion, as indicated by the '○' amidst the '□'s in Fig. 2.

The good news for synthesizer builders is that the added expense of torsional wave simulation (which basically adds a coupled

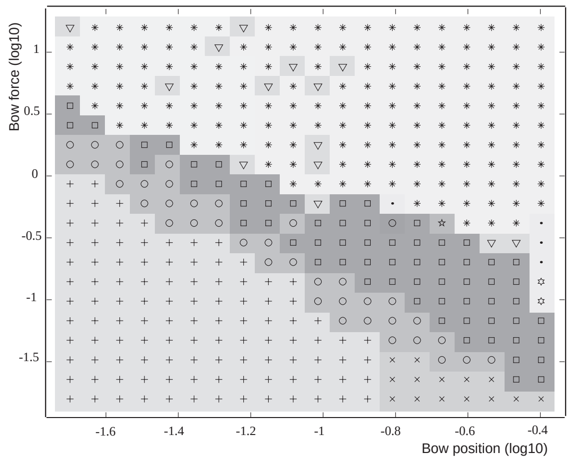

Figure 2: *Measured Schelleng diagram for the plastic friction model case. High quality Helmholtz motion is indicated by open squares (□), and multiple slipping is plotted using open circles (○). All other symbols denote generally less desirable modes of string motion.*

"second string") does not appear to improve playability. Since torsion waves are not prominent in the radiated sound either, it seems warranted to leave them out of synthesis models, even in the highest quality instances.

4.2. Effect of the Bow-String Friction Model

Up to now we have only looked at the plastic friction model which is believed to be the most accurate physically. We now look at the effect of using the older simpler models labeled "exponential" and "hyperbolic" above.

4.2.1. Exponential Friction Model

Figure 4 shows the Schelleng diagram obtained using the classic exponential friction model with torsion-wave simulation included. We see that the plastic friction model (Fig. 2) "plays better" close to the bridge, and it has a larger region of Helmholtz motion, especially when bowing somewhat away from the bridge. The combined areas of Helmholtz and multiple-slipping motion, however, are somewhat larger with the exponential friction model.

Figure 5 show the exponential friction case with torsion-wave simulation removed (completely). As in the plastic friction-model case, the playability is comparable, and arguably even improved. Note the greater "reliability" of playing near the upper bow-force limit. While there are 51 Helmholtz pixels in the full-simulation case, and only 47 in the case without torsion simulation, the Helmholtz region is more contiguous and solid, having fewer interior pitfalls.

4.2.2. Hyperbolic Friction Model

Figure 6 and Figure 7 show the Schelleng diagram obtained by running the simulation using the hyperbolic friction model. The results are quite similar to the preceding exponential model case.

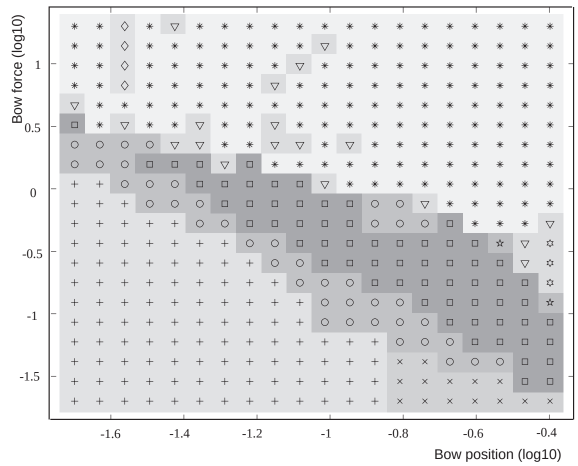

Figure 3: *Measured Schelleng diagram for the plastic friction model case, with torsion-wave simulation removed. As before, classic Helmholtz motion is indicated by '□', and multiple slipping by '○'.*

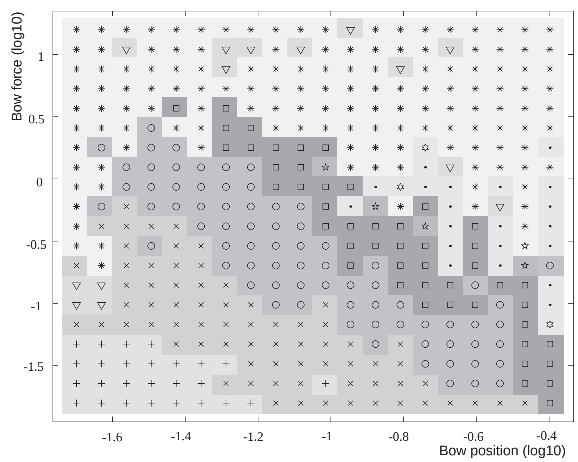

Figure 4: *Measured Schelleng diagram for the exponential friction model case.*

5. CONCLUSIONS

The simulations indicate that eliminating torsion waves from the model does not degrade playability, and the playable region tends to become somewhat more convex. These results suggest that bowed-string synthesizers which neglect torsion wave simulation should not be negatively impacted by this approximation.

The simulations also indicate that the more physically accurate temperature-dependent friction model improves playability relative to the previously used velocity-dependent friction models. Since one of the main distinguishing features of the temperature-dependent model is hysteresis, perhaps it will be possible in future work to devise simplified hysteretic friction models which can increase the playability and reduce computational expense, taking the present model as a starting point.

It is important to stress that playability has been examined only for steady bowing with the string starting from rest. Some addi-

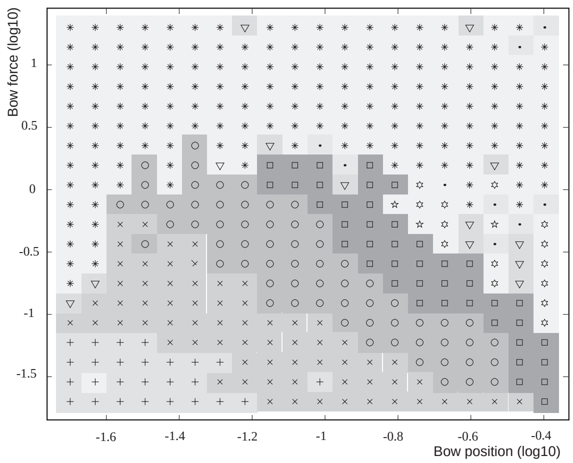

Figure 5: *Measured Schelleng diagram for the exponential friction model case, no torsion waves.*

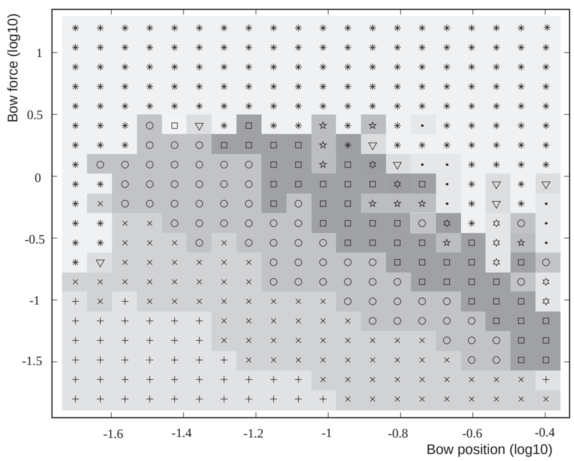

Figure 6: *Measured Schelleng diagram for the hyperbolic friction model case.*

tional simulations were tried in which the string was pre-initialized with Helmholtz motion, and the impact of torsion waves on playability was found to be similar to the steady-bowing case reported here. The results might differ, however, in cases not tried here, such as when simulating more realistic bowing transients.

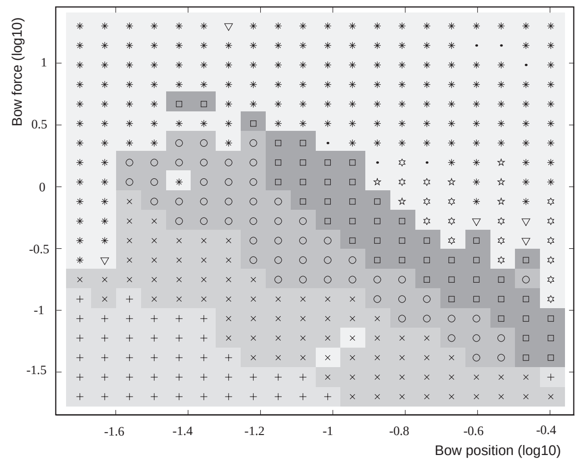

Figure 7: *Measured Schelleng diagram for the hyperbolic friction model case, no torsion waves.*

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