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Analysis of jet instability in flute-like instruments by means of image processing: effect of the excitation amplitude.

Patricio de la Cuadra[1], *Benoît Fabre*[2]

[1]CCRMA, Center for Computer Research in Music and Acoustics, Stanford University, USA
[2]LAM, Laboratoire d'acoustique musicale, Universite Paris 6, 75252 Paris Cedex O5, France
[1]pdelac@ccrma.stanford.edu,[2]fabreb@ccr.jussieu.fr

Abstract

We are studying experimentally the behavior of jets in flute-like instruments. Presently most models for jet oscillations are based on linear jet instability analysis described by Rayleigh. These models show two weak points in the application to flute modeling: the initial perturbations induced by transverse acoustic velocity is not clearly described, and the jet transverse oscillation may reach amplitudes which can not be described by a linear model in most flutes. Aiming to understand and describe the origin of the perturbation as well as the limit of linear models, we are currently investigating the jet motion under transverse acoustic perturbation generated by loudspeakers. The jet behavior is analyzed through flow visualization and image processing using a cross-correlation technique. The paper will present results for jet behavior under different excitation amplitudes.

1. Introduction

Instruments from the flute family share a principle of operation that consists of an unstable jet issuing from either a channel or the lips of a player. The jet flows in the direction of a sharp edge that we call the labium. The acoustic field due to the presence of the resonator triggers the jet instability and therefore, the jet oscillates at the frequency of the acoustic field. The interaction of the perturbed flow with the labium provides the necessary acoustic energy to sustain the acoustic oscillation in the pipe.

Most models for jet oscillations in flute-like instruments are developed on a linear theory based on Rayleigh's approach [1]. This theoretical model can only be valid for small oscillations of the jet, small compared to the order of the thickness of the jet. Flow visualizations in flutes show an amplitude of the traverse jet displacement larger that the jet thickness [2]. Furthermore, there is no clear model for the initial amplitude of the perturbation on the jet.

This paper presents experimental results on the limit of the linear description of the oscillating jet, as well as some insights on the influence of the acoustic excitation amplitude in the jet behavior. Analyses are carried out using a non-intrusive technique based on image analysis of flow visualization obtained by the Schlieren method [3].

2. Jet oscillation

A jet submitted to transverse acoustic field oscillates at the same frequency of the excitation. The transverse displacement of the jet is very small at the flue exit, slowly growing as it travels away from it, up to the point where the flow rolls up and finally breaks into vortices and later turbulence as shown in fig.1.

It appears from the literature that the jet oscillation can be described by two models : a linear model or a vortex street.

2.1. Linear model

Due to the intrinsic instability of the jet, the oscillation is amplified while being convected downstream. The perturbation travels at a velocity about one half of the jet speed [4]. In a linear model, we consider a perturbation described by the stream function ψ:

$$\psi = \varphi(y)e^{i(\omega t - \alpha x)} \tag{1}$$

where φ is the complex amplitude, ω the angular pulsation, t time, α the wave number and x, y the horizontal and vertical coordinates. Spatial analysis has shown to provide better fit to measurements. It assumes real ω, while looking for a complex wave number $\alpha = \alpha_r + i\alpha_i$. The velocity of perturbation corresponds to $c_p = \omega/\alpha_r$ while the spatial amplification is given by the coefficient α_i.

In order to calculate the transverse displacement $\eta(x)$ of the jet (streakline) at a distance x from the flue exit, one has to integrate over the particle path, from the flue exit to the actual position, the transverse velocity. This transverse velocity is the sum of the acoustic velocity and the velocity of the instability wave, calculated by differentiation of the stream function $\partial\psi/\partial x$ [5]. The problem of determining the amplitude of the excitation at the flue exit ($x = 0$), or the response of the jet to the acoustic perturbation, hasn't yet found an adequate description [6].

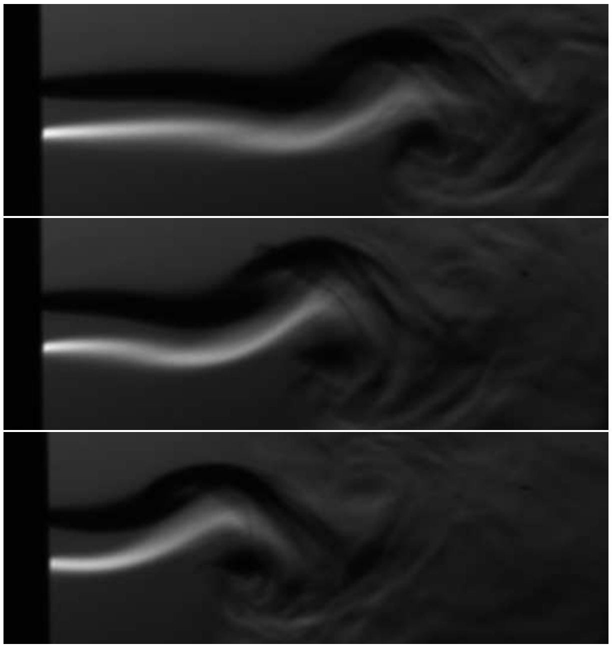

Figure 1: Images obtained via Schlieren technique. Reinolds: 500, Jet speed: 7.5 m/s, Excitation freq.: 448 Hz, Strouhal: 0.37. V_{ac}/U_b : $Top = 0.0737\%; Center = 0.1484\%; Bottom = 1.1889\%.$ *The two shear layers appear with opposite light refractive gradient, so that the upper shear layer gets darker while the lower shear layer gets lighter than the average gray level.*

The central speed of the jet U_b at the channel exit can be estimated using Bernoulli's equation:

$$U_b = \sqrt{\frac{2p_f}{\rho_0}} \tag{2}$$

where p_f is the pressure in the cavity before the channel, and ρ_0 the air density.

2.2. Discrete vortices model

In the context of edgetone analysis [7], Holger developed a non-linear model to describe the oscillations beyond the boundaries of the linear model. In his description vortices in the shear layers of the jet are growing exponentially downstream from the flue exit until they reach an ultimate strength K, obtained for a stable vortex street.

Although the model is conceived for a much smaller transverse perturbation of the jet (about a factor of 100) the overall description is in good agreement with our visualization. This paper concentrates on the transition between formation area (linear) and the vortex street (non-linear) jet oscillations. The position of this transition may depend on the Strouhal number, but also on the amplitude of the perturbation which is very significant in most flutes.

3. Experimental setup

A jet issuing from a squared flue exit is submitted to an acoustic excitation generated by loudspeakers, whose amplitude is changed. Fig.1 shows three images obtained using the Schlieren method [3] with three different excitation amplitudes.

Sequential images of the jet are taken with a digital camera [PCO,Sensicam, fast shutter] with exposure time set to one microsecond. Since the frequencies of jet oscillation are much higher than those provided by any modern camera, and moreover, we want to be able to have an arbitrary number of images per cycle, a stroboscopic light is used. Its frequency is set such that multiples of its frequency are slightly out of phase with the excitation frequency providing an aliased representation of the oscillation. By changing this phase we can produce as many images per cycle as we want. Certainly they come from different cycles of the jet, but they can be collapsed to the same cycle since the oscillation is stationary.

The frequency of the camera was set to 14 fps, the acoustic excitation frequency was 448 Hz giving a Strouhal of 0.37 (Strouhal = fh/U_b, with the channel height $h = 1mm$ and U_b the jet speed at the exit deduced using eq.2). Approximately 100 images are taken covering two cycles of jet oscillation. Images are captured in raw bmp files, with size 1280 x 448, and 8 intensity bits.

The speed of the jet is controlled with a manual valve measuring the pressure in the cavity p_f just before the jet formation. A Reynolds number based on the channel height h of 500 was chosen (Reynolds = $U_b h/\nu$, ν: kinematic viscosity) giving a jet velocity = 7.5 m/s which corresponds to normal blowing conditions for a recorder and assures a laminar behavior.

The amplitude of the acoustic excitation is measured through the acoustic velocity amplitude, which is created with two speakers in opposed phase, providing a more uniform field. The amplitude of the acoustic velocity was varied from 0.03% of the jet velocity up to 1.2%. These values are considerably below the 10% observed in real flutes, but it is a restriction imposed by the possibilities of the speakers.

The phase and amplitude of the acoustic excitation are measured using a velocity sensor (Microflown Technologies) and double checked with gradient pressure measurements done with two microphones aligned along the propagation direction of the acoustic waves.

4. Image processing

4.1. Schlieren method

Changes in the fluid density cause variations in the refractive index that can be visualized using the Schlieren technique [3]. With a particular setup, the density gradient in the direction perpendicular to the jet plane is converted into intensity differences, making the jet visible (Fig.1).

A CO_2 jet is used with mass density higher than the air ($CO_2 = 1.98kg/m^3$, air = $1.2kg/m^3$). This choice is a compromise between having enough difference to allow good optical results and being close enough to emulate the behavior found in flute-like instruments.

Images obtained with Schlieren method are extremely sensitive to variations in the setup. Small differences in the parameters produce enormous image variations. Therefore, even under attentive care, images obtained from different trials are different, requiring a robust analysis method.

4.2. Data analysis

A cross-correlation method for finding the center position of the jet has been developed and is explained in detail in the companion paper [8]. Fig.2 illustrates the results that can be obtained with it. The position of the detected jet is shown for a sequence of images covering two cycles of oscillation as described in section 3.

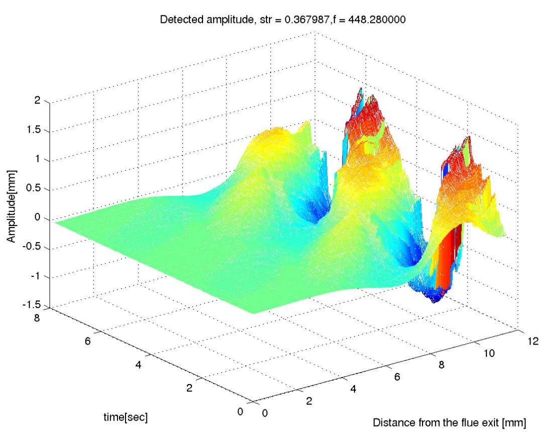

Figure 2: Position of jet found with cross-correlation method, $Reynolds = 500$ and $Strouhal = 0.37$, excitation frequency = 448 Hz, $V_{ac}/U_b = 0.0737\%$. Time axis corresponds to frame times in seconds.

All columns of the image oscillate in time at the frequency of the excitation. Therefore, for each column we fit a sinusoid at that frequency:

$$Y(f) = \frac{1}{N} \sum_{i=0}^{N-1} X_i e^{\frac{-j2\pi f}{f_s}} \tag{3}$$

Where, N is the number of pixels per column. Fig 3 shows an example of the position detection for one image together with the amplitudes $|Y(f)|$ and phases $\angle Y(f)$ for a complete sequence.

The linear model predicts for the perturbation an exponential growth and a constant convection velocity, obtained respectively from an exponential fit to the amplitude data (Fig.3 center, thin curve), and the slope of a fit to the phase data (Fig.3 center, thin curve)

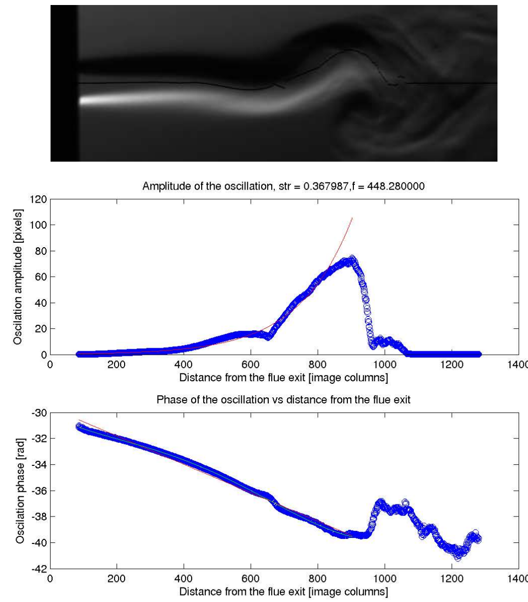

Figure 3: Top: Schlieren image with the detection of position done with the cross-correlation method. Center: (thick) Amplitude of the detected sinusoids, (thin) exponential fit. Bottom: (thick) Phase of detected sinusoids, (thin) linear fit.
$Reynolds = 500$ and $Strouhal = 0.37$. Excitation frequency = 448 Hz, $V_{ac}/U_b = 0.0737\%$

The sinusoidal fit described by eq.3 works properly only when the oscillation of the column analyzed is periodic. This is the case for the laminar portion of the jet, from the flue exit to the point where the jet breaks into discrete vortices. Beyond this point intensity of the image columns becomes incoherent, columns don't oscillate periodically, and sinusoids don't fit data properly. In fig.3 we observe the amplitude growing exponentially until approximately the point where the jet breaks into vortices. The maximum of this curve is considered the breaking point between the two models outlined by Holger.

5. Results and discussion

5.1. Amplitude of the perturbation at the flue exit

Our analysis show that the jet oscillation can be described, at the beginning of the oscillation, by :

$$\eta(x,t) = \eta_0 e^{\gamma x} e^{i\omega(t - x/c_p)} \tag{4}$$

with an exponential growth of the oscillating amplitude $\hat{\eta}(x) = \eta_0 e^{\gamma x}$. This exponential behaviour ends at a distance x_t from the flue exit that we call the transition point.

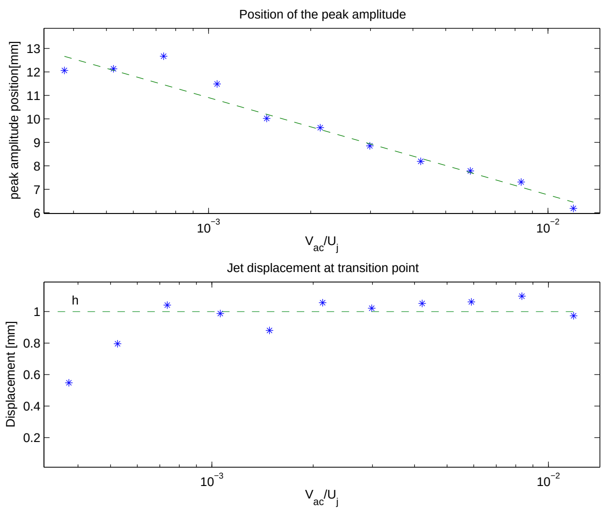

Figure 4: Top: Distance from the flue exit to the transition point on the jet; linear fit (dashed). Bottom: Displacement of the jet at the transition point; channel height h (dashed)

The position of this transition point as well as the oscillating amplitude $\hat{\eta}_t$ reached at this point are prensented figure 4 as function of the dimensionless amplitude of the acoustic velocity V_{ac}/U_b. As a first approximation, it appears from data analysis that the oscillating amplitude $\hat{\eta}_t$ at the transition point x_t is independant of the amplitude of the acoustic excitation V_{ac}/U_b, and equal to the channel height $\hat{\eta}_t = h$; while the position x_t of the transition point follows a logarithmic decay, that can be expressed in a dimensionless form as :

$$\frac{x_t}{h} = \frac{x_0}{h} - k \log \frac{V_{ac}}{U_b} \tag{5}$$

Combining relations 4 and 5, together with the parameter values deduced from the data presented in figure 4 and 5 (see after), allows to estimate the initial amplitude of the perturbation :

$$\eta_0 \approx \frac{V_{ac}}{U_b} h \tag{6}$$

5.2. Jet perturbations

The convection velocity of the perturbation c_p as well as the amplification γ are shown dimensionless in fig.5. Only the linear parts of the jet ($x \leq x_t$) have been included in the calculation of these values.

Although there is some dispersion in the data, we observe that both variables remain constant in the range of excitations studied. The slight tendency observed in the convection velocity c_p to increase with excitation requires further study to be validated.

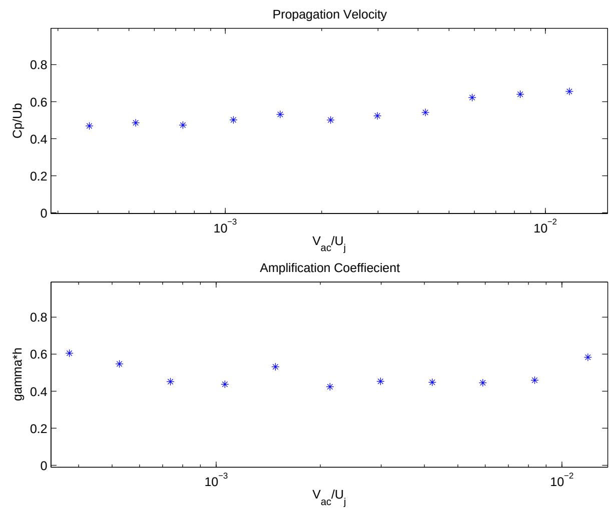

Figure 5: Dimensionless convection velocity (Top) and amplification coefficient (Bottom) as functions of the dimensionless acoustic velocity excitation V_{ac}/U_b

5.3. Conclusions

Considering the transition point between the two models described in section 2 it has been observed that:

1. Amplitude of the jet oscillation at the transition point is independent of the amplitude of excitation and is approximately equal to the jet height (fig.4, Bottom).

2. The transition point relates linearly to the amplitude of the excitation (fig.4, Top). Using that result, a first-order model for the amplitude of the excitation at the origin η_0 has been proposed. Further work is needed to relate this result to the initial amplitude of the perturbation in the linear model (eq.1).

It has also been observed that neither the amplification of the perturbation nor its velocity are affected when the amplitude of the excitation varies (fig.5).

For the moment there is no evidence that our observations can be generalized and the behavior could be different for values of V_{ac}/U_b greater than those studied. Further experimentation is needed to check the possible dependecy of our observations on Reynolds and Strouhal numbers.

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