Source Separation Tutorial Mini-Series III: Extensions and Interpretations to Non-Negative Matrix Factorization

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Roadmap of Talk

1. Review
2. Further Insight
3. Supervised and Semi-Supervised Separation
4. Probabilistic Interpretation
5. Extensions
6. Evaluation
7. Future Research Directions
8. Matlab
Roadmap of Talk

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Non-Negative Matrix Factorization

\[
\begin{bmatrix}
V
\end{bmatrix} \approx \begin{bmatrix} W \\ H \end{bmatrix}
\]
Non-Negative Matrix Factorization

\[
\begin{bmatrix}
V \\
\end{bmatrix} \approx
\begin{bmatrix}
W \\
\end{bmatrix}\begin{bmatrix}
H \\
\end{bmatrix}
\]

- A matrix factorization where everything is non-negative
- \( V \in \mathbb{R}^{F \times T}_+ \) - original non-negative data
- \( W \in \mathbb{R}^{F \times K}_+ \) - matrix of basis vectors, dictionary elements
- \( H \in \mathbb{R}^{K \times T}_+ \) - matrix of activations, weights, or gains
- \( K < F < T \) (typically)
  - A compressed representation of the data
  - A low-rank approximation to \( V \)
NMF With Spectrogram Data

NMF of *Mary Had a Little Lamb* with $K = 3$

- The basis vectors capture prototypical spectra [SB03]
- The weights capture the gain of the basis vectors
Factorization Interpretation I

Columns of $\mathbf{V} \approx$ as a weighted sum (mixture) of basis vectors

$$\begin{bmatrix} v_1 & v_2 & \ldots & v_T \end{bmatrix} \approx \begin{bmatrix} \sum_{j=1}^{K} H_{j1} w_j & \sum_{j=1}^{K} H_{j2} w_j & \ldots & \sum_{j=1}^{K} H_{jT} w_j \end{bmatrix}$$
Factorization Interpretation II

\[ \mathbf{V} \text{ is approximated as sum of matrix "layers"} \]

\[
\begin{bmatrix}
\mathbf{v}_1 & \mathbf{v}_2 & \ldots & \mathbf{v}_T
\end{bmatrix}
\approx
\begin{bmatrix}
\mathbf{w}_1 & \mathbf{w}_2 & \ldots & \mathbf{w}_K
\end{bmatrix}
\begin{bmatrix}
\mathbf{h}_1^T \\
\mathbf{h}_2^T \\
\vdots \\
\mathbf{h}_K^T
\end{bmatrix}
\]

\[ \mathbf{V} \approx \mathbf{w}_1 \mathbf{h}_1^T + \mathbf{w}_2 \mathbf{h}_2^T + \ldots + \mathbf{w}_K \mathbf{h}_K^T \]
General Separation Pipeline

1. STFT
2. NMF
3. FILTER
4. ISTFT

\[ \hat{x}_1, \hat{x}_2, \ldots, \hat{x}_S \]

\[ V = |X| \]

\[ W, H \]

\[ \hat{X}_1, \hat{X}_2, \ldots, \hat{X}_S \]
Algorithm KL-NMF

initialize $W, H$

repeat

$H \leftarrow H \cdot \frac{W^T V WH}{W^T WH}$

$W \leftarrow W \cdot \frac{V WH H^T}{WH H^T}$

until convergence return $W, H$
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Question: Why do we get a ’parts-based’ representation of sound?
Non-Negativity

• Question: Why do we get a ‘parts-based’ representation of sound?

• Answer: Non-negativity avoids destructive interference
Constructive and Destructive Interference

Constructive Interference

\[ x + x = 2x \]
Constructive and Destructive Interference

**Constructive Interference**

\[ x + x = 2x \]

**Destructive Interference**

\[ x + (-x) = 0 \]
Non-Negative Constructive and Destructive Interference

Constructive Interference

\[ |x| + |x| = 2|x| \]
Non-Negative Constructive and Destructive Interference

Constructive Interference

\[ |x| + |x| = 2|x| \]

Destructive Interference

\[ |x| + |-x| = 2|x| \]
Non-negativity Avoids Destructive Interference

- With non-negativity, destructive interference cannot happen
Non-negativity Avoids Destructive Interference

- With non-negativity, destructive interference cannot happen

- Everything must cumulatively add to explain the original data
Non-negativity Avoids Destructive Interference

- With non-negativity, destructive interference cannot happen
- Everything must cumulatively add to explain the original data
- But ...
Approximation I

In doing so, we violate the superposition property of sound

\[ x = x_1 + x_2 + \ldots + x_N \]

and actually solve

\[ |X| \approx |X_1| + |X_2| + \ldots + |X_N| \]
Approximation II

Alternatively, we can see this approximation via:

\[
\mathbf{x} = \mathbf{x}_1 + \mathbf{x}_2 + \ldots + \mathbf{x}_N
\]

\[
|\mathbf{X}| e^{j\phi} = |\mathbf{X}_1| e^{j\phi_1} + |\mathbf{X}_2| e^{j\phi_2} + \ldots + |\mathbf{X}_N| e^{j\phi_N}
\]

\[
|\mathbf{X}| e^{j\phi} \approx (|\mathbf{X}_1| + |\mathbf{X}_2| + \ldots + |\mathbf{X}_N|) e^{j\phi}
\]

\[
|\mathbf{X}| \approx |\mathbf{X}_1| + |\mathbf{X}_2| + \ldots + |\mathbf{X}_N|
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Unsupervised Separation I

Single, simultaneously estimation of \( W \) and \( H \) from a mixture \( V \)

\[
V \approx WH
\]

What we’ve seen so far
Unsupervised Separation II

- Complex sounds need more than one basis vector.
Unsupervised Separation II

- Complex sounds need more than one basis vector
- Difficult to control which basis vector explain which source
Unsupervised Separation II

• Complex sounds need more than one basis vector

• Difficult to control which basis vector explain which source

• No way to control the factorization other than $F$, $T$, and $K$
Supervised Separation

General idea:

1. Use isolated training data of each source within a mixture to pre-learn individual models of each source [SRS07]
Supervised Separation

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1. Use isolated training data of each source within a mixture to pre-learn individual models of each source [SRS07]

2. Given a mixture, use the pre-learned models for separation
Supervised Separation I

Example:

*Drum and Bass Loop*
Supervised Separation II

Use isolated training data to learn factorization for each source

\[ V_1 \approx W_1 H_1 \]
Supervised Separation II

Use isolated training data to learn factorization for each source

**Bass Loop**

**Drum Loop**

\[ V_1 \approx W_1 H_1 \quad \text{and} \quad V_2 \approx W_2 H_2 \]
Supervised Separation III

Throw away the activations $\mathbf{H}_1$ and $\mathbf{H}_2$

**Bass Loop**

**Drum Loop**

$$\mathbf{V}_1 \approx \mathbf{W}_1 \mathbf{H}_1$$

$$\mathbf{V}_2 \approx \mathbf{W}_2 \mathbf{H}_2$$
Supervised Separation IV

Concatenate basis vectors of each source for complete dictionary

\[ W \approx [ W_1 \quad W_2 ] = \]
Supervised Separation V

Now, factorize the mixture with $\mathbf{W}$ fixed (only estimate $\mathbf{H}$)

\[
\mathbf{V} \approx \mathbf{W} \approx \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 \end{bmatrix} \mathbf{H} = \begin{bmatrix} \mathbf{H}_1^T \\ \mathbf{H}_2^T \end{bmatrix}
\]
Supervised Separation V

Now, factorize the mixture with $\mathbf{W}$ fixed (only estimate $\mathbf{H}$)

$$\mathbf{V} \approx \mathbf{W} = \begin{bmatrix} \mathbf{W}_1 & \mathbf{W}_2 \end{bmatrix} \begin{bmatrix} \mathbf{H}_1^T \\ \mathbf{H}_2^T \end{bmatrix}$$
Complete Supervised Process

1. Use isolated training data to learn a factorization \((W_s H_s)\) for each source \(s\)
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4. Hold \(W\) fixed, and factorize unknown mixture of sources \(V\) (only estimate \(H\))
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5. Once complete, use \(W\) and \(H\) as before to filter and separate each source
Sound Examples

Mixture sound (left) and separated drums and bass.

Masking filters used to process mixture into the separated sources.
Question

- What if you don’t have isolated training data for each source?
Question

- What if you don’t have isolated training data for each source?

- And unsupervised separation still doesn’t work?
Semi-Supervised Separation

General Idea:

1. Learn supervised dictionaries for as many sources as you can [SRS07]
Semi-Supervised Separation

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1. Learn supervised dictionaries for as many sources as you can [SRS07]

2. Infer remaining unknown dictionaries from the mixture (only fix certain columns of $W$)
Semi-Supervised Separation I

Example:

Drum + Bass

Drum and Bass Loop  play  stop
Semi-Supervised Separation II

Use isolated training data to learn factorization for as many sources as possible (e.g. one source)

*Bass Loop*  play  stop

\[ V_1 \approx W_1 H_1 \]
Semi-Supervised Separation III

Throw away the activations $\mathbf{H}_1$

$\mathbf{Bass\ Loop}$

$$V_1 \approx W_1 H_1$$
Semi-Supervised Separation IV

Concatenate *known* basis vectors with *unknown* basis vectors (initialized randomly) for complete dictionary.

\[
W \approx [W_1 \ W_2] = \]

Known bass basis vectors

Unknown drum basis vectors (initialized randomly)
Semi-Supervised Separation V

Now, factorize the mixture with $W_1$ fixed (estimate $W_2$ and $H$)

$$V \approx WH \approx [W_1 \quad W_2] \begin{bmatrix} H_1^T \\ H_2^T \end{bmatrix}$$
Semi-Supervised Separation V

Now, factorize the mixture with $W_1$ fixed (estimate $W_2$ and $H$)

$$V \approx W H \approx \begin{bmatrix} W_1 & W_2 \end{bmatrix} \begin{bmatrix} H_1^T \\ H_2^T \end{bmatrix}$$
Complete Semi-Supervised Process

1. Use isolated training data to learn a factorization \((W_s H_s)\) for as many sources \(s\) as possible.
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4. Hold the columns of \(W\) fixed which correspond to known sources, and factorize a mixture \(V\) (estimate \(H\) and any known column of \(W\)).
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Supervised the bass.

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Probabilistic Interpretation

Some notation:
\( z \) indexes basis vectors, \( f \) frequency bins, and \( t \) time frames.

The model:
For each time frame \( t \), repeat the following:
- Choose a component from \( p(z|t) \).
- Choose a frequency from \( p(f|z) \).

The spectrogram \( V_{ft} \) are the counts that we obtain at the end of the day. We want to estimate \( p(z|t) \) and \( p(f|z) \).
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\[ z \rightarrow t \rightarrow f = H \]
\[ z \rightarrow f = W \]
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For each time frame $t$, repeat the following:

- Choose a component from $p(z|t)$. $z \rightarrow t \rightarrow H$

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The spectrogram $V_{ft}$ are the counts that we obtain at the end of the day. We want to estimate $p(z|t)$ and $p(f|z)$. 
Probabilistic Interpretation

Is this realistic?

- We’re assuming the spectrogram contains counts. We sample “quanta” of spectral energy at a time.
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- This model is popular in topic modeling, where we assume documents are generated from first sampling a topic from $p(z|d)$ and then a word from $p(w|z)$. 

• probabilistic latent semantic indexing, or pLSI [Hof99]
• latent Dirichlet allocation, or LDA [BNJ03]
• In audio, this model is called probabilistic latent component analysis, or PLCA [SRS06]
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Latent Variable Model

We only observe the outcomes $V_{ft}$. But the full model involves unobserved variables $Z$. 
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$$p(f|z)$$

$$p(z|t)$$
Latent Variable Model

We only observe the outcomes $V_{ft}$. But the full model involves unobserved variables $Z$.

The **Expectation-Maximization (EM) algorithm** is used to fit latent variable models. It is also used in estimating Hidden Markov Models, Gaussian mixture models, etc.
Maximum Likelihood Estimation

To fit the parameters, we choose the parameters that maximize the likelihood of the data. Let’s zoom in on a single time frame:

\[
p(v_1, \ldots, v_F) = \frac{(\sum f v_f)!}{v_1! \ldots v_F!} \prod_{f=1}^{F} p(f|t)^{v_f}
\]
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\[
p(v_1, ..., v_F) = \frac{(\sum_f v_f)!}{v_1!...v_F!} \prod_{f=1}^{F} p(f|t)^{v_f}
\]

According to the model on the previous slide, the frequency could have come from any of the latent components. We don’t observe this so we average over all of them.

\[
p(f|t) = \sum_z p(z|t)p(f|z)
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\]

Putting it all together, we obtain:

\[
p(v_1, \ldots, v_F) = \frac{(\sum_f v_f)!}{v_1! \ldots v_F!} \prod_{f=1}^{F} \left( \sum_z p(z|t)p(f|z) \right)^{v_f}
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Maximum Likelihood Estimation

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- We want to maximize this over \( p(z|t) \) and \( p(f|z) \).
Maximum Likelihood Estimation

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- We want to maximize this over \( p(z|t) \) and \( p(f|z) \).
- In general, with probabilities it is easier to maximize the log than the thing itself:

\[
\log p(v_1, \ldots, v_F) = \sum_{f=1}^F v_f \log \left( \sum_z p(z|t)p(f|z) \right) + \text{const.}
\]
Maximum Likelihood Estimation

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- **Remember from last week:** First thing you should always try is differentiate and set equal to zero. Does this work here?
The Connection to NMF

- Last week, we talked about minimizing the KL divergence between $V$ and $WH$.

$$D(V||WH) = - \sum_{f,t} V_{ft} \log \left( \sum_z W_{fz} H_{zt} \right) + \sum_{f,t} \sum_z W_{fz} H_{zt} + \text{const.}$$
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- Compare with maximizing the log-likelihood:

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subject to $\sum_z p(z|t) = 1$ and $\sum_f p(f|z) = 1$. 
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subject to $\sum_z p(z|t) = 1$ and $\sum_f p(f|z) = 1$.

- Last week, we used majorization-minimization on $D(V||WH)$:

\[
- \log \left( \sum_z \phi_{ftz} \frac{W_{fz} H_{zt}}{\phi_{ftz}} \right) \leq - \sum_z \phi_{ftz} \log \frac{W_{fz} H_{zt}}{\phi_{ftz}}
\]
The Connection to NMF

- Last week, we talked about minimizing the KL divergence between $V$ and $WH$.

\[ D(V||WH) = -\sum_{f,t} V_{ft} \log \left( \sum_z W_{fz} H_{zt} \right) + \sum_{f,t} \sum_z W_{fz} H_{zt} + \text{const.} \]

- Compare with maximizing the log-likelihood:

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\[ -\log \left( \sum_z \phi_{ftz} \frac{W_{fz} H_{zt}}{\phi_{ftz}} \right) \leq -\sum_z \phi_{ftz} \log \frac{W_{fz} H_{zt}}{\phi_{ftz}} \]

- Now watch what we do with the log-likelihood....
• Suppose we observed the latent component for a frequency quanta. Then we wouldn’t need to average over the components; its log-likelihood would be:

\[ \log p(z|t)p(f|z) \]
Suppose we observed the latent component for a frequency quanta. Then we wouldn’t need to average over the components; its log-likelihood would be:

$$\log p(z|t)p(f|z)$$

But we don’t know the latent component, so let’s average this over our best guess of the probability of each component:

$$\sum_z p(z|f, t) \log p(z|t)p(f|z)$$
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But we don’t know the latent component, so let’s average this over our best guess of the probability of each component:

$$\sum_z p(z|f,t) \log p(z|t)p(f|z)$$

In summary, we’ve replaced

$$\log \left( \sum_z p(z|t)p(f|z) \right)$$

by

$$\sum_z p(z|f,t) \log p(z|t)p(f|z)$$

Look familiar?
EM Algorithm

**E-step**: Calculate

\[ p(z|f, t) = \frac{p(z|t)p(f|z)}{\sum_z p(z|t)p(f|z)} \]

**M-step**: Maximize

\[ \sum_{f, t} V_{ft} \sum_z p(z|f, t) \log p(z|t)p(f|z) \]
EM Algorithm

**E-step**: Calculate

\[ p(z|f, t) = \frac{p(z|t)p(f|z)}{\sum_z p(z|t)p(f|z)} \]

**M-step**: Maximize

\[ \sum_{f,t} V_{ft} \sum_z p(z|f, t) \log p(z|t)p(f|z) \]

**Majorization**: Calculate

\[ \phi_{ftz} = \frac{W_{fz}H_{zt}}{\sum_z W_{fz}H_{zt}} \]

**Minimization**: Minimize

\[ -\sum_{f,t} V_{ft} \sum_z \phi_{zft} \log W_{fz}H_{zt} + \sum_{f,t,z} W_{fz}H_{zt} \]
EM Algorithm

**E-step**: Calculate

\[ p(z|f, t) = \frac{p(z|t)p(f|z)}{\sum_z p(z|t)p(f|z)} \]

**M-step**: Maximize

\[ \sum_{f, t} V_{ft} \sum_z p(z|f, t) \log p(z|t)p(f|z) \]

**Majorization**: Calculate

\[ \phi_{ftz} = \frac{W_{fz}H_{zt}}{\sum_z W_{fz}H_{zt}} \]

**Minimization**: Minimize

\[ -\sum_{f, t} V_{ft} \sum_z \phi_{zt} \log W_{fz}H_{zt} + \sum_{f, t, z} W_{fz}H_{zt} \]

The EM updates are exactly the multiplicative updates for NMF, up to normalization!
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The EM algorithm is a special case of MM, where the minorizing function is the expected conditional log likelihood.
• We can think of the basis vectors $p(f|z)$ as lying on a probability simplex.
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The possible sounds for a given source is the convex hull of the basis vectors for that source.
Geometric Interpretation

In supervised separation, we try to explain time frames of the mixture signal as combinations of the basis vectors of the different sources.
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1. Review
2. Further Insight
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Extensions

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In high-dimensional settings, it is useful to impose additional structure.

We will look at two ways to do this: priors and regularization.
Priors

- Assume the parameters are also random, e.g., $H = p(z|t)$ is generated from $p(H|\alpha)$. This is called a **prior** distribution.
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\[
p(f|z) = p(H, V|\alpha) p(V|\alpha) = p(H|\alpha) p(V|H) p(V|\alpha)
\]
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- Bayes’ rule: $p(H|\alpha, V) = \frac{p(H, V|\alpha)}{p(V|\alpha)} = \frac{p(H|\alpha)p(V|H)}{p(V|\alpha)}$
Bayesian Inference

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- An easier option is the **posterior mode** (MAP):

$$\max_{H} \log p(H|\alpha, V) = \log p(H|\alpha) + \log p(V|H) - p(V|\alpha)$$
Bayesian Inference

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$$\text{maximize } \frac{1}{H} \log p(H|\alpha, V) = \underbrace{\log p(H|\alpha)}_{\text{log prior}} + \underbrace{\log p(V|H)}_{\text{likelihood}} - \underbrace{p(V|\alpha)}_{\text{penalty}}$$

- We can choose priors that encode structural assumptions, like sparsity.
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Another way is to add another term to the objective function:

$$\minimize_{W, H \geq 0} D(V \parallel WH) + \lambda \Omega(H)$$

$\Omega$ encodes the desired structure, $\lambda$ controls the strength.
Regularization Viewpoint

- Another way is to add another term to the objective function:

\[
\underset{W,H \geq 0}{\text{minimize}} \quad D(V \| WH) + \lambda \Omega(H)
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\(\Omega\) encodes the desired structure, \(\lambda\) controls the strength.

- We showed earlier that \(D(V \| WH)\) is the negative log likelihood. So:

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\]

- Some common choices for \(\Omega(H)\):
  - sparsity: \(\|H\|_1 = \sum_{z,t} |H_{zt}|\)
  - smoothness: \(\sum_{z,t} (H_{z,t} - H_{z,t-1})^2\)
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Evaluation Measures

- Signal-to-Interference Ratio (SIR)
- Signal-to-Artifact Ratio (SAR)
- Signal-to-Distortion Ratio (SDR)

We want all of these metrics to be as high as possible [VGF06]
To compute these three measures, we must obtain:

- \( s \in \mathbb{R}^{T \times N} \) original unmixed signals (ground truth)
- \( \hat{s} \in \mathbb{R}^{T \times N} \) estimated separated sources

Then, we decompose these signals into:

- \( s_{target} \) — actual source estimate
- \( e_{interf} \) — interference signal (i.e. the unwanted source)
- \( e_{artif} \) — artifacts of the separation algorithm
To compute $s_{target}, e_{interf},$ and $e_{artif}$

- $s_{target} = P_{s_j} \hat{s}_j$
- $e_{interf} = P_s \hat{s}_j - P_{s_j} \hat{s}_j$
- $e_{artif} = \hat{s}_j - P_s \hat{s}_j$

where $P_{s_j}$ and $P_s$ are $T \times T$ projection matrices.
Signal-to-Interference Ratio (SIR)

A measure of the suppression of the unwanted source

\[
\text{SIR} = 10 \log_{10} \frac{||s_{\text{target}}||^2}{||e_{\text{interf}}||^2}
\]
Signal-to-Artifact Ratio (SAR)

A measure of the artifacts that have been introduced by the separation process

\[
\text{SAR} = 10 \log_{10} \frac{||s_{target} + e_{interf}||^2}{||e_{artif}||^2}
\]
Signal-to-Distortion Ratio (SDR)

An overall measure that takes into account both the SIR and SAR

\[
SDR = 10 \log_{10} \frac{\|s_{target}\|^2}{\|e_{artif} + e_{interf}\|^2}
\]
Selecting Hyperparameters using BSS Eval Metrics

- One problem with NMF is the need to specify the number of basis vectors $K$. 
Selecting Hyperparameters using BSS Eval Metrics

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Even more parameters if you include regularization.

BSS eval metrics give us a way to learn the optimal settings for source separation.
Selecting Hyperparameters using BSS Eval Metrics

• One problem with NMF is the need to specify the number of basis vectors $K$.
• Even more parameters if you include regularization.
• BSS eval metrics give us a way to learn the optimal settings for source separation.
• Generate synthetic mixtures, try different parameter settings, and choose the parameters that give the best BSS eval metrics.
BSS Eval Toolbox

A Matlab tool box for source separation evaluation [VGF06]:

http://bass-db.gforge.inria.fr/bss_eval/
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Research Directions

- Score-informed separation - sheet music
- Interactive separation - user-interaction
- Temporal dynamics - how sounds change over time
- Unsupervised separation - grouping basis vectors, clustering
- Phase estimation - complex NMF, STFT constraints, etc.
- Universal models - big data for general models of sources
Demos

- Universal Speech Models
- Interactive Source Separation
  - Drums + Bass
  - Guitar + Vocals + AutoTune
  - Jackson 5 Remixed
STFT

\[ x_1 = \text{wavread('bass')}; \]
\[ x_2 = \text{wavread('drums')}; \]
\[ [x_m \text{ fs}] = \text{wavread('drums+bass')}; \]
\[ \text{FFTSIZE} = 1024; \]
\[ \text{HOPSIZE} = 256; \]
\[ \text{WINDOWSIZE} = 512; \]

\[ X_1 = \text{myspectrogram}(x_1,\text{FFTSIZE},\text{fs},\text{hann}(\text{WINDOWSIZE}),-\text{HOPSIZE}); \]
\[ V_1 = \text{abs}(X_1(1:(\text{FFTSIZE}/2+1),:)); \]
\[ X_2 = \text{myspectrogram}(x_2,\text{FFTSIZE},\text{fs},\text{hann}(\text{WINDOWSIZE}),-\text{HOPSIZE}); \]
\[ V_2 = \text{abs}(X_2(1:(\text{FFTSIZE}/2+1),:)); \]
\[ X_m = \text{myspectrogram}(x_m,\text{FFTSIZE},\text{fs},\text{hann}(\text{WINDOWSIZE}),-\text{HOPSIZE}); \]
\[ V_m = \text{abs}(X_m(1:(\text{FFTSIZE}/2+1),:)); \]
\[ \text{maxV} = \text{max}([\text{max}([\text{db}(V_m)])]); \]

\[ \text{F} = \text{size}(V_m,1); \]
\[ \text{T} = \text{size}(V_m,2); \]

- [https://ccrma.stanford.edu/~jos/sasp/Matlab_listing_myspectrogram_m.html](https://ccrma.stanford.edu/~jos/sasp/Matlab_listing_myspectrogram_m.html)
- [https://ccrma.stanford.edu/~jos/sasp/Matlab_listing_invmyspectrogram_m.html](https://ccrma.stanford.edu/~jos/sasp/Matlab_listing_invmyspectrogram_m.html)
K = [25 25]; % number of basis vectors
MAXITER = 500; % total number of iterations to run
[W1, H1] = nmf(V1, K(1), [], MAXITER, []);
[W2, H2] = nmf(V2, K(2), [], MAXITER, []);
[W, H] = nmf(Vm, K, [W1 W2], MAXITER, 1:sum(K));

function [W, H] = nmf(V, K, W, MAXITER, fixedInds)
F = size(V,1); T = size(V,2);
rand('seed',0)
if isempty(W)
    W = 1+rand(F, sum(K));
end
H = 1+rand(sum(K), T);
inds = setdiff(1:sum(K),fixedInds);
ONES = ones(F,T);
for i=1:MAXITER
    % update activations
    H = H .* (W'*( V./(W*H+eps))) ./ (W'*ONES);
    % update dictionaries
    W(:,inds) = W(:,inds) .* ((V./(W*H+eps))*H(inds,:)') ./ (ONES*H(inds,:))';
end
% normalize W to sum to 1
sumW = sum(W);
W = W*diag(1./sumW);
H = diag(sumW)*H;
%% get the mixture phase
phi = angle(Xm);
c = [1 cumsum(K)];
for i=1:length(K)
  % create masking filter
  Mask = W(:,c(i):c(i+1))*H(c(i):c(i+1),:)./(W*H);
  % filter
  XmagHat = Vm.*Mask;
  % create upper half of frequency before istft
  XmagHat = [XmagHat; conj( XmagHat(end-1:-1:2,:))];
  % Multiply with phase
  XHat = XmagHat.*exp(1i*phi);
  % create upper half of frequency before istft
  xhat(:,i) = real(invmyspectrogram(XmagHat.*exp(1i*phi),HOPSIZE))';
end
References I


