# An Exploration of Guitar Neck Admittance Measurements Taken at Different String Stopping Locations

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## ABSTRACT

String instrument synthesis models have typically ignored the influence of the string stopping position when modeling the string boundary conditions at the neck. This simplification neglects the differences between frequency-dependent wave reflection phenomena taking place at the string boundary when it is stopped at different positions along the neck. Driving-point admittance measurements were taken at the bridge of an electric guitar, and also at various stopping positions along the neck: open string, 2nd, 4th, 6th, 8th, and 10th fret. We explore the acquired data and compare boundary reflectance functions as computed for the bridge and the various stopping positions. As a test case, we compare measured and theoretical string decay times for one particular note.

## 1. INTRODUCTION

Stringed instruments, such as guitars, involve the transfer of vibrational energy from a set of strings into a body, which serves as a more efficient radiator of acoustic energy. The energy of vibrating guitar strings is transferred to the instrument at the string termination points, the bridge and the stopping location along the neck. The mechanical "input admittance" or "driving point mobility" is defined in the frequency domain as the velocity of the structure divided by the input force driving the structure at a given location and in a given direction. The admittance shows the ability of the guitar body to be displaced as a function of frequency, providing insight about the vibrational modes and string decay time [1, 2]. Input admittance is often measured at the bridge on a hollow instrument as this will give a good estimate of the instrument's vibrational characteristics which will result in sound [2]. Admittance is the inverse of impedance which can be used to calculate the string reflection coefficient

$$r = \frac{Z_b - Z_c}{Z_b + Z_c} \tag{1}$$

where $Z_c$ and $Z_b$ respectively represent the characteristic string and bridge impedance functions at the boundary. This amplitude reflection coefficient is the ratio of an incoming wave amplitude to the reflected wave amplitude. The string termination along the rigid neck will not result in a significant amount of sound compared to the termination at the mobile bridge, so this termination is often ignored when considering the instrument's radiated sound [3]. However, this termination will affect the stopping location string reflectance as there are vibrational modes of the neck, effecting the string decay rates [3].

The purpose of this work is to use input admittance measurements to study the boundary conditions at the bridge and stopping location along the neck of an electric guitar. We perform a first exploration of how much energy is lost at the string ends by computing the corresponding reflectances, with the aim of foreseeing the importance of including a model for the neck losses to attain accurate string decay times in a future synthesis model.

The rest of the paper is organized as follows. Section 2 gives details on the experimental setup used to measure neck and bridge admittances as well as string decay times. In Section 3 we present the admittance measurements as well as an example of computed reflectance functions. A simplified method to predict string decay times is presented and compared to an example measured string decay. Section 4 discusses the measurements, missing factors in the decay time prediction, and proposes future steps.

## 2. MEASUREMENTS

Admittance measurements were made on a guitar to observe the body and neck vibrations. String decay measurements were then made to explore the relationship between the neck vibrations and the string decay. A hollow body electric guitar with a floating bridge and two electromagnetic pickups was measured. The measurements were taken in a semi-anechoic room with the guitar hung from the ceiling by the tuning pegs, and lightly resting against foam for stability. The setup for the admittance measurements is shown in Figure 1. The guitar was strung with flatwound strings which were tuned to pitch.

### 2.1. Admittance

To measure the admittance of the guitar, the instrument was struck with a force sensing hammer (B&K 8203) while a laser Doppler vibrometer (Polytec PDV 100) was used to measure the velocity. The hammer strike and laser measurement locations are made as close as possible to obtain a driving point admittance. The signals were pre-amplified to the appropriate level and recorded through a National Instruments data acquisition card with a sample rate of 44.4k Hz and 16-bit


**Figure 1:** Experimental setup for admittance measurements.

sample resolution. All strings were damped using a combination of thick card stock and elastic bands. Each measurement was triggered as soon as the impact hammer came in contact with the guitar, and lasted for 2 seconds. For each location, 5 consecutive measurements were made and averaged, observing the coherence between the measurements and discarding any erroneous measurements. The bridge admittance was measured in the vertical (normal to the guitar top plate) and horizontal (along the direction of the bridge) directions. The impacts were made near the low E string; this position was chosen because the low E is the thickest string, thus providing strong coupling to the body. Admittance measurements were also made on the neck at the open position, 2nd, 4th, 6th, 8th, and 10th frets. The open position admittance was measured with a driving point on the nut in the vertical and horizontal directions. The admittance was measured at each fret position on the fret for the vertical direction, and near the fret on the side of the neck for the horizontal direction. All admittance measurements were also recorded with calibrated microphones to later calculate the instruments radiativity for synthesis purposes.

### 2.2. String Decay

In order to justify the stopping location admittance method and evaluate later synthesis, string decay measurements were made. A "wire break" method was used as it is more reproducible than a human pluck. A copper wire is looped around and pulled across the string such that it abruptly snaps at a repeatable level of stress, imparting an approximate step function (being more flat at low frequencies and rolling off at high frequencies above 10k Hz) in a known direction. It is difficult to know the exact force the wire exerts when it snaps but it is consistent between tests and the measurements are only concerned with the relative amplitude of vibrations [4]. Copper wire of gauge 40 on the American wire gauge scale (0.0799 mm) was chosen as it produced a string displacement similar to that of an average guitar plectrum. The signals were pre-amplified to the appropriate level and recorded through a National Instruments data acquisition card with a sample rate of 44.4k Hz and 16-bit sample resolution. Each measurement was triggered as soon as the wire broke, and lasted for 10 seconds. The output from the electromagnetic pickup nearest to the neck was recorded with the guitar's tone and volume controls at a maximum value, imparting no additional filtering. The vibrational axis of the strings will change with time [5], but guitar pickups are most sensitive in the direction perpendicular to the top plate of the guitar [6], so the measurements will approximate the vertical transverse string motion. The string plucks were also recorded with two calibrated microphones to later compare the radiated sound to that of the strings alone, as measured by the electromagnetic pickups. The strings were plucked at a constant distance away from the bridge at an angle of 45° between the vertical and horizontal axes. In order to measure the fretted notes, a "capo" (a device that clamps all of the strings to a particular fret) was placed on the corresponding fret. All strings that were not being measured were damped using heavy card stock and elastic bands.

Once recorded, the string decays measured through the electromagnetic pickup were analyzed. A short-time Fourier transform was used to determine the amplitudes of the first five harmonics at each time frame. The peaks of the first five harmonics were determined in a general range based on their theoretical frequency. Since the string frequencies will vary slightly over the duration of the decay, the frequency and amplitudes were calculated more carefully by using parabolic interpolation. The harmonics were then analyzed to determine the time each took to decay by 15dB assuming an exponential decay.

## 3. PRELIMINARY OBSERVATIONS

Bridge admittance measurements in the vertical and horizontal directions are shown in the right columns of Figures 2 and 3. The left columns of Figures 2 and 3 shows the vertical and horizontal admittance measurements made at the stopping locations corresponding to the open position, 4th fret, and 8th fret. When observing Figures 2 and 3, it can be seen that there are three modes of high amplitude at 85, 175, and 332 Hz which are likely caused by the neck bending modes. The amplitudes of the modes at 175 and 332 Hz decrease at positions along the neck approaching the bridge, suggesting that they are near the nodes of the bending modes.

Figure 4 shows the magnitude responses of the bridge and open position admittance in the vertical and horizontal directions as well as string decay plots which show the time for the first 5 harmonics to decay by 15dB in the vertical direction

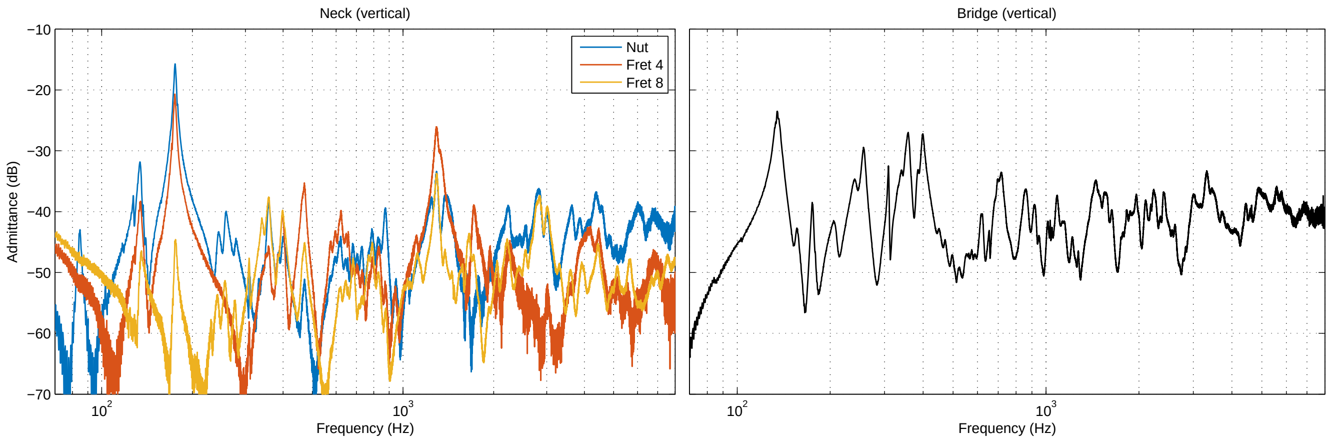


**Figure 2:** Magnitude response of the vertical admittance measurements made at the bridge, open position, 4th fret, and 8th fret.

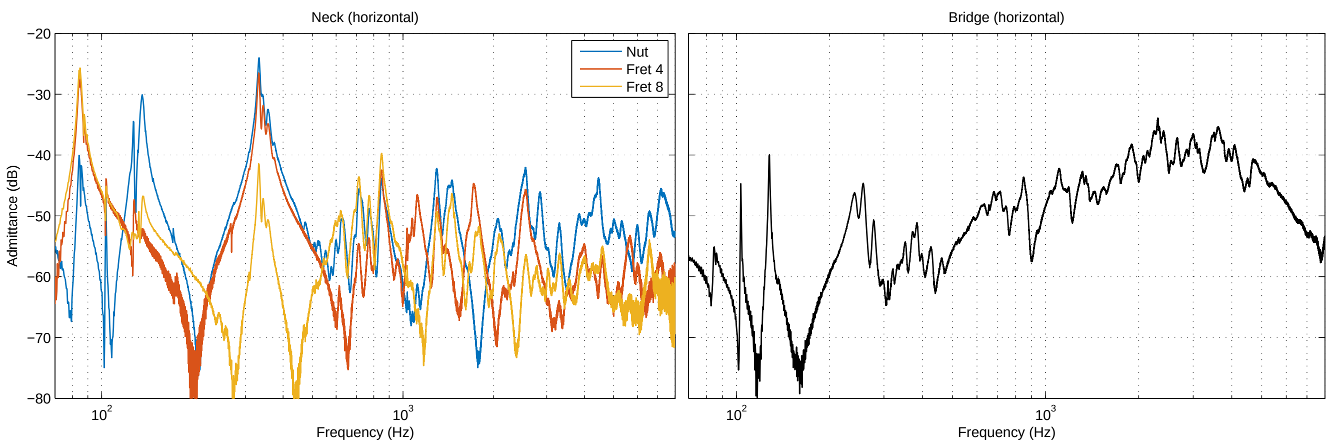


**Figure 3:** Magnitude response of the horizontal admittance measurements made at the bridge, open position, 4th fret, and 8th fret.

as measured through the electromagnetic pickup. Each circle colour corresponds to the first five harmonics of an individual string. This serves as a way to observe the overall decay rate of the strings with respect to the measured admittance.

The reflection coefficient functions for the bridge, open position, 4th fret, and 8th fret of the low E string were calculated as described in equation 1, and the amplitude is shown in Figure 5. The string characteristic impedance was calculated from manufacturer data to be to 0.8702 N m $s^{-1}$ [7]. As it can be observed, losses at the neck should not be neglected: for some frequencies they are comparable to those at the bridge.

Assuming that energy is only lost through the bridge admittance, neck admittance, and propagation losses, the change in amplitude ratio $G$ over one period can be calculated as

$$G = \frac{A}{A_0} = r_B \cdot r_N \cdot r_S \qquad (2)$$

where $A_0$ is the original amplitude, $A$ is the final amplitude after one period, $r_B$ is the bridge reflection coefficient, $r_N$ is the stopping location reflection coefficient, $r_S$ is the propagation loss coefficient for one period of vibration. The reflection coefficients $r_B$ and $r_N$ represent the losses through one reflection from each end respectively. This can be arranged to calculate the decay time $t_L$ for the transverse string motion to decay by a certain amplitude as follows

$$t_L = \frac{\gamma_L}{f_0 20 \log_{10}(r_B r_N r_S)} \qquad (3)$$

where $f_0$ is the fundamental frequency in Hertz, and $\gamma_L$ is the decay amplitude ratio in dB. This method can be used to predict the vertical and horizontal transverse string decay by using the respective vertical and horizontal reflection coefficients.

Table 1 shows the measured decays as well as the computed vertical transverse decays of the first 5 harmonics of a pluck of the open low E string. The amount of decay, $\gamma_L$ was set to -15 dB and the string air propagation loss $r_S$ is assumed to be equal to 1 at all frequencies as an initial simplification. As expected from the imposed simplifications, we observe significant differences.

| Harmonic | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Measured Decay (s) | 5.45 | 4.49 | 0.86 | 0.87 | 1.12 |
| Predicted Decay (s) | 13.29 | 0.68 | 0.24 | 0.33 | 0.33 |

**Table 1:** Measured and predicted decay times for the first 5 harmonics of the open low E string to decay by 15dB.

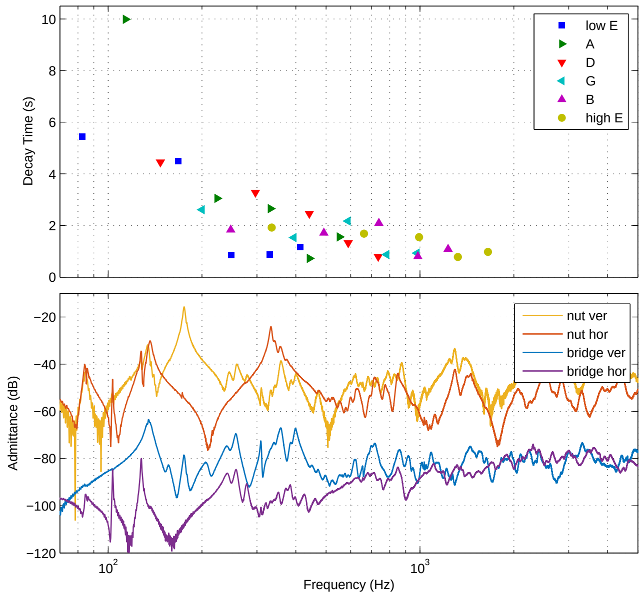


**Figure 4:** Magnitude responses (lower) of the bridge and open position admittance in the vertical and horizontal directions. The bridge admittance measurements are scaled by -40dB for clarity. String decay times (upper plot) for the first 5 harmonics to decay by 15dB.

## 4. DISCUSSION AND FUTURE WORK

Observing the admittance measurements, it is clear that the neck modes absorb a significant amount of energy and will affect the string decays. The stopping location reflection coefficient will change as the note is stopped at different positions along the string, suggesting that multiple stopping location transmission coefficients should be used for accurate string decay synthesis across the instrument.

A practical method for measuring the propagation losses in the string from simple measurements is not known, and it would be useful if these could be learned from string decay and boundary admittance measurements. We hoped that we would be able to approximate these frequency-dependent losses from the above analysis, but some of the predicted decay times are shorter than the measured decays. This suggests that the boundary conditions are more complicated than stateless reflectances, and that horizontal-vertical transverse motion coupling, and string-body and string-neck coupling should not be ignored as was done in this preliminary simplification. Future work will look into predicting the amount of energy transferred back into the string from the motion of the body and neck terminations, if modeled as resonant systems.

Along these lines, further steps will be to perform mode fitting on the admittance measurements made at the bridge as well as the neck stopping locations. This will result in efficient digital filters which are different at each end of the string termination, providing more accurate string decay rates for a digital waveguide model using the technique described in [8]. Multiple stopping location filters will be used with a different filter corresponding to each fretted note position. Synthesis examples will be computed, both including the stopping location filters, and without them. The synthesis will be compared to

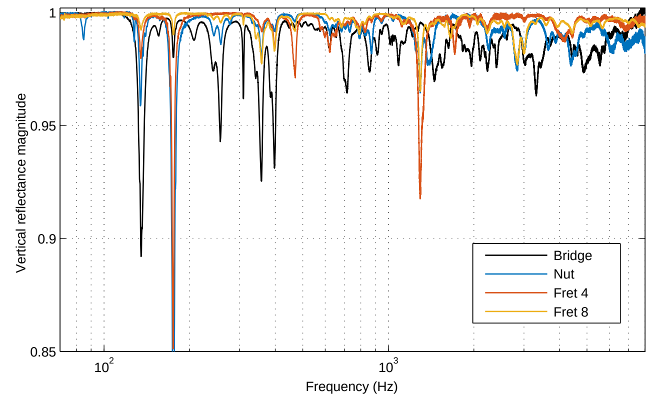


**Figure 5:** Low-E string reflectance magnitudes for the vertical direction, corresponding to the bridge and different stopping locations.

the measured decay rates to further validate the effectiveness of this method at correctly synthesizing the string decay.

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