Music 420
Winter 2005-2006
Homework #6
Digital Waveguide Theory, Acoustic Guitar Synthesis
155 points
Due in one week (March/07/2006)

Theory Problems

1. Consider a semi-infinite ideal string with wave impedance R terminated on one end by an ideal mass m which slides vertically along a "frictionless vertical guide rod". The other end of the string stretches to infinity with uniform tension K.
   (a) (5 pts) Find a formula for the *mass density* of the string.
   (b) (20 pts) Find the *reflectance* (reflection transfer function) $S_v(s) = V^-(s)/V^+(s)$ of the terminating mass, as seen from the string, for *velocity waves*.
   (c) (5 pts) Find the reflectance $S_f(s) = F^-(s)/F^+(s)$ of the terminating mass for *force waves*.
   (d) Find the reflectance of the terminating mass for
      i. (5 pts) *displacement waves*
      ii. (5 pts) *acceleration waves*
      iii. (5 pts) *slope waves*
   (e) (5 pts) What is the magnitude of the force reflectance as a function of frequency, $|S_f(j\omega)|$?
   (f) (5 pts) What is the phase of the force reflectance as a function of frequency, $\angle S_f(j\omega)$?

2. Suppose an infinitely long ideal string has wave impedance R and is sitting at rest with displacement $y(0, x) = 0$ for all $x \in (-\infty, \infty)$. Along comes a mass m which hits the string at time 0 with velocity v_0.
   (a) (10 pts) Find the mass velocity $v_m(t)$ for $t \geq 0$.
   (b) (5 pts) Find the mass position $y_m(t)$ for $t \geq 0$.
   (c) (10 pts) Assuming $R = m = c = 1$, sketch the shape of the string at times $t = 0, 1/4, 1/2, 1, \infty$, labeling all interesting points and asymptotes.

Lab Assignments

All the Matlab part and theory questions related to lab problems are to be submited on PAPER (including Matlab functions/scripts/figures) with the rest of the theory problems.

STK/C++ Lab assignments parts are to be submitted electronically at http://coursework.stanford.edu. For each assignment, submit a single archive (`zip` or `tar.gz`) file containing all code/output/soundfiles for the assignment. For each problem in the assignment, create a sub-directory in the archive named `hwX_pY`, where `X` is the homework number, and `Y` is the problem number, and place all code/output/soundfiles for that problem in that sub-directory.

1. An acoustic guitar simulation needs to take into account the resonances of the body, since they impart a characteristic "color" to the plucked tone. In a commuted waveguide synthesis model, the impulse response of the body can simply be fed in as the excitation to the guitar string. Sometimes it becomes impractical to use the full impulse response due to memory constraints. One technique to shorten the impulse response is to factor out the least damped resonances in the response and implement them separately as second-order filters.

   (a) (10 pts) Find the frequencies of the three least-damped modes in the guitar body impulse response (gtrbody.wav[1]). A quick way to find these is to take an FFT of a portion of the response where most of the modes have decayed considerably (e.g., after 125 ms) and only a few strong modes are left. To get a more accurate estimate of the peak frequency, you can zero-pad and/or use parabolic fitting on a dB scale. This is documented at `http://ccrma.stanford.edu/~jos/parshl/Peak_Detection_Steps_3.html`.

   (b) (20 pts) Using the code at `http://ccrma.stanford.edu/~jos/waveguide/Matlab_Code_Inverse_Filtering.html` as a *starting point*, successively remove the lowest mode (the "Helmholtz mode") and the other two modes. Make sure that the mode removals do not affect the spectral content of the rest of the frequency response. You can estimate the peak bandwidths with any method, or just use 2, 9, and 9 Hz and manually tweak if needed.

   Plot the time and frequency responses of the original body response and the three mode-extracted responses. Include a close-up plot of the frequency region where the modes were removed for the 4 cases.

   (c) (5 pts) Which of the mode-extracted responses has the shortest effective length? Approximately how long is the response (in ms)?

   (d) (10 pts) Restore the mode-extracted guitar body impulse responses by using biquads as needed to model the missing modes. Compare these against the original

[1] http://ccrma.stanford.edu/~jos/hw420/hw6/gtrbody.wav

response. Is there an audible difference between the four versions? (You can play sound directly from Matlab using `sound` or `soundsc`.) Plot the amplitude error between the original and mode-restored responses.

(e) (5 pts) How might this decomposition of the guitar body impulse response allow for additional controls or effects for the commuted acoustic guitar model? Give a few examples.

2. (20 pts) The manner of termination for the guitar string at the bridge results in slightly different effective lengths for horizontal and vertical transverse string vibration. In the horizontal plane, the string terminates at the bridge, but it terminates slightly beyond that in the vertical plane of vibration. The non-rigid, yielding bridge also allows for coupling between the two polarizations.

   Turn your electric guitar model into an acoustic guitar by using a guitar body impulse response soundfile as the excitation source and implementing coupled strings as diagrammed at `http://ccrma.stanford.edu/~jos/CoupledStrings/Linearly_Coupled_Planes_Vibration.html`. Include a plucking angle control parameter. Some design restrictions are:

   (a) $N_v > N_h$ *and/or* $P_{vv} > P_{hh}$, where P_x denotes phase delay of filter x.

   (b) $H_{vh}(z) = H_{hv}(z)$, with lowpass characteristics (can be modeled after $H_{vv}(z)$)

   (c) $H_{hh}(z)$ should be nearly lossless (e.g., -1 for velocity waves if the nut reflection has not been commuted through)

   (d) $H_{vv}(z)$ should be a symmetric second-order FIR loop filter with parametric control (as used in the previous assignment)

   (e) $H_l(z)$ should be nearly lossless (weakly lowpass)

   Produce an illustrative example of your acoustic guitar consisting of at least a few notes, and submit it with the rest of your code. Turn in any relevant program listings and document the parameter values you used to generate the example.