Discretizing the Casio SK-1 Bass and Chord Filters (and "circuit-bent" extensions)

Kurt James Werner

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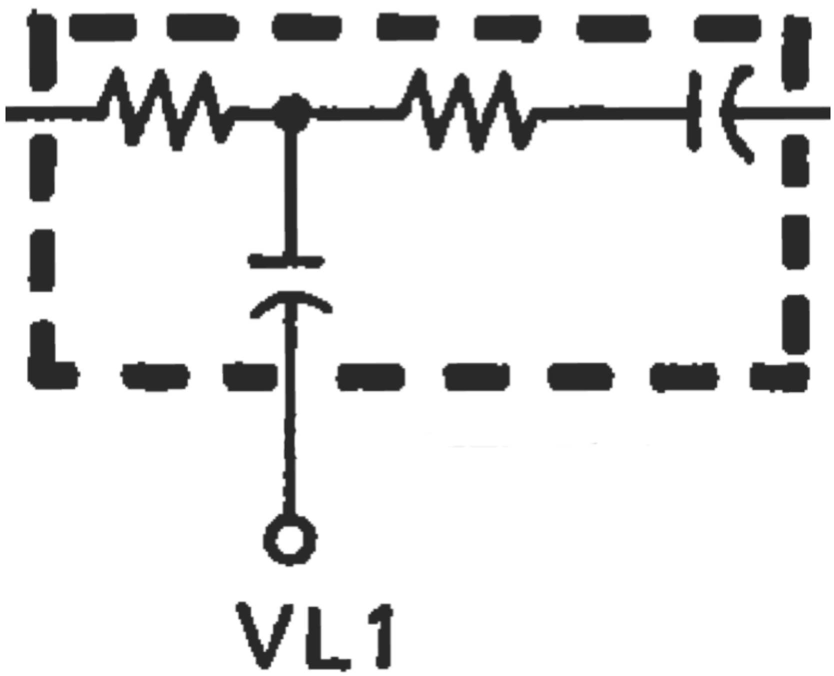
Part I
Bass Filter

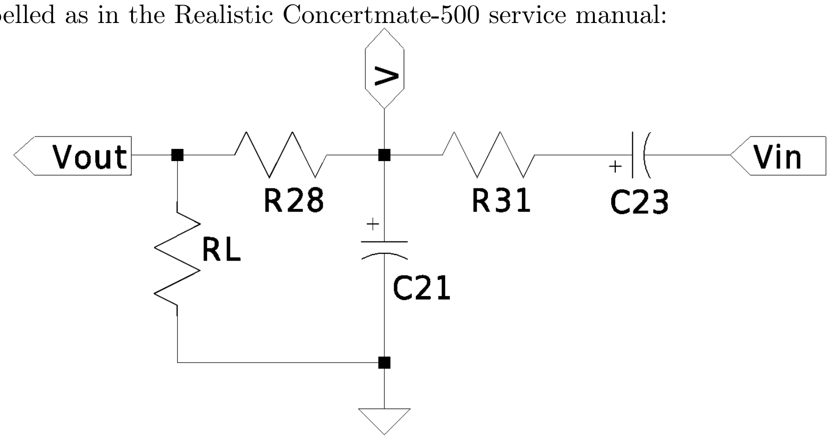
1 Overview

It is desirable to find the digital transfer function $H(z)$ approximating the analog circuitry of the Casio SK-1. Two of these is the Bass Filter and the Chord Filter. Schematically, these filters are identical. But, they have different component values. Analysis will be performed on the Bass Filter, then component values substituted in to obtain an analysis of the Chord Filter. $H(z)$ can be obtained by performing the bilinear transform on the continuous-time transfer function $H(s)$. $H(s)$ can be obtained by manipulating the continuous-time expressions that describe the circuit's behavior. This analysis assumes linear / ideal circuit behavior.

A schematic diagram of the Bass Filter circuit, as found in the Realistic Concertmate-500 service manual (the alternate name of the Casio SK-1, when it was sold through RadioShack). $VL1$ is ground, V_{in} enters on the right, and V_{out} leaves on the left.



To analyze the circuit, a load resistor R_L will be added from V_{out} to ground. This represents the load that the rest of the circuit will contribute to the Bass Filter. For now, a "reasonable" value of $1M\,\Omega$ will be used. This can be replaced by a more accurate value once it is obtained. A node V is defined to make algebraic manipulations simpler (it will cancel out early on). Components are labelled as in the Realistic Concertmate-500 service manual:



Component values as given in the Realistic Concertmate-500 service manual:

$$
\begin{aligned}
R_{28} &= 15\,k\Omega \\
R_{31} &= 22\,k\Omega \\
C_{21} &= 0.047\,\mu F
\end{aligned}
$$

$$C_{23} = 0.1\,\mu F$$

2 The Continuous-Time Tranfer Function

2.1 Voltage Divider

Once expressions for V_{in} and V_{out} are obtained, their ratio will give the continuous-time transfer function $H(s)$. V_{in} and V_{out} can be found using voltage dividers. In a voltage divider, the input voltage is proportional to the series impedance of $Z_1 + Z_2$, and the output voltage is proportional to the impedance Z_2.

$$V_i = i * (Z_1 + Z_2) \to i = \frac{V_i}{Z_1 + Z_2}$$

$$V_o = i * Z_2 \to i = \frac{V_o}{Z_2}$$

$$\frac{V_i}{Z_1 + Z_2} = \frac{V_o}{Z_2} \to V_o = V_i \frac{Z_2}{Z_1 + Z_2}$$

2.2 Finding the Transfer Function

Expressions for V_{in} and V_{out} are obtained by application of the voltage divider equation. Keep in mind that the impedance of a resistor is the same as its resistance ($Z_R = R$) and that the impedance of a capacitor is inversely proportional to its capacitance multiplied by s ($Z_C = \frac{1}{sC}$), where s is the differentiation operator on the s-plane, R is the value of the resistor in Ohms (Ω), and C is the value of the capacitor in Farads(F).

$$V_{out} = V \frac{R_L}{R_{28} + R_L}$$

$$V = V_{in} \frac{Z}{Z + R_{31} + \frac{1}{sC_{23}}} \to V_{in} = V \frac{Z + R_{31} + \frac{1}{sC_{23}}}{Z}$$

Where Z is the impedance of C_{21} in parallel with the series impedance of R_{28} and R_L.

$$\begin{aligned} Z &= C_{21} \| (R_{28} + R_L) \\ &= \frac{\frac{1}{sC_{21}} (R_{28} + R_L)}{R_{28} + R_L + \frac{1}{sC_{21}}} \\ &= \frac{(R_{28} + R_L)\, sC_{21}}{(R_{28} + R_L)\, sC_{21} + 1} \\ &= \frac{Z_{num}}{Z_{denom}} \end{aligned}$$

With expressions for V_{in} and V_{out} in terms of impedences, $H(s)$ is easily obtained:

$$\begin{aligned} H(s) &= \frac{V_{out}}{V_{in}} \\ &= \frac{V \frac{R_L}{R_{28}+R_L}}{V \frac{Z+R_{31}+\frac{1}{sC_{23}}}{Z}} \end{aligned}$$

Node voltage V drops out of the expression:

$$\begin{aligned} H(s) &= \frac{\frac{R_L}{R_{28}+R_L}}{\frac{Z+R_{31}+\frac{1}{sC_{23}}}{Z}} \\ &= \left(\frac{R_L}{R_{28}+R_L}\right)\left(\frac{Z}{Z+R_{31}+\frac{1}{sC_{23}}}\right) \end{aligned}$$

Avoiding compound fractions, simplifying, and collecting terms:

$$\begin{aligned} H(s) &= \left(\frac{R_L}{R_{28}+R_L}\right)\left(\frac{ZsC_{23}}{ZsC_{23}+(R_{31}sC_{31}+1)}\right) \\ &= \left(\frac{R_L}{R_{28}+R_L}\right)\left(\frac{\frac{Z_{num}}{Z_{denom}}sC_{23}}{\frac{Z_{num}}{Z_{denom}}sC_{23}+(R_{31}sC_{31}+1)}\right) \\ &= \left(\frac{R_L}{R_{28}+R_L}\right)\left(\frac{Z_{num}sC_{32}}{Z_{num}sC_{23}+Z_{denom}(R_{31}sC_{31}+1)}\right) \\ &= \left(\frac{R_L}{R_{28}+R_L}\right)\left(\frac{(R_{28}+R_L)sC_{21}}{(R_{28}+R_L)s^2C_{21}C_{23}+((R_{28}+R_L)sC_{21}+1)(R_{31}sC_{31}+1)}\right) \\ &= R_L\left(\frac{sC_{21}}{(R_{28}+R_L)s^2C_{21}C_{23}+R_{31}(R_{28}+R_L)s^2C_{21}C_{23}+(R_{28}+R_L)sC_{21}+R_{31}sC_{23}+1}\right) \\ &= \frac{R_LsC_{21}}{(R_{31}+1)(R_{28}+R_L)C_{21}C_{23}s^2+((R_{28}+R_L)C_{21}+R_{31}C_{23})s+1} \end{aligned}$$

2.3 Coefficients

This transfer function is second-order in the numerator and second-order in the denominator (as would be expected for a circuit with two capacitors). The transfer function of a general second-order filter is:

$$H(s) = \frac{\beta_2 s^2+\beta_1 s+\beta_0}{\alpha_2 s^2+\alpha_1 s+\alpha_0}$$

The Casio SK-1 Bass Filter can be described as a second-order filter with analog filter coefficients:

$$
\begin{aligned}
\beta_2 &= 0 \\
\beta_1 &= R_L C_{21} \\
\beta_0 &= 0 \\
\alpha_2 &= (R_{31}+1)(R_{28}+R_L) C_{21} C_{23} \\
\alpha_1 &= (R_{28}+R_L) C_{21} + R_{31} C_{23} \\
\alpha_0 &= 1
\end{aligned}
$$

3 The Bilinear Transform

3.1 Definition

Much like a transfer function in continuous time, a transfer function in discrete time can be written in terms of digital filter coefficients.

$$
H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{a_0 + a_1 z^{-1} + a_2 z^{-2}}
$$

The bilinear transform is used to transform from a continuous-time representation of a system to a discrete-time representation by substituting in an approximation of the trapezoidal rule for integration (where T is the sampling period):

$$
s = \frac{2}{T} \frac{1 - z^{-1}}{1 + z^{-1}}
$$

Substituting this expression for each s in the continuous-time representation of the transfer function $H(s)$ will give the discrete-time representation of the transfer function $H(z)$. This can be done manually, or with a symbolic manipulator. However, in the case of commonly used filters (such as the biquad), expressions for the digital filter coeffcients in terms of the analog filter coefficients are already available.

$$
\begin{aligned}
b_0 &= 4\beta_2 + 2T\beta_1 + T^2\beta_0 \\
b_1 &= -8\beta_2 + 2\beta_0 T^2 \\
b_2 &= 4\beta_2 - 2T\beta_1 + T\beta_0 \\
a_0 &= 4\alpha_2 + 2T\alpha_1 + T^2\alpha_0 \\
a_1 &= -8\alpha_2 + 2T^2\alpha_0 \\
a_2 &= 4\alpha_2 - 2T\alpha_1 + T^2\alpha_0
\end{aligned}
$$

For most implementations, α_0 should equal 1, so the whole filter can be normalized by dividing by α_0:

$$
\begin{aligned}
b_0 &= \frac{4\beta_2 + 2T\beta_1 + T^2\beta_0}{4\alpha_2 + 2T\alpha_1 + T^2\alpha_0} \\
b_1 &= \frac{-8\beta_2 + 2T^2\beta_0}{4\alpha_2 + 2T\alpha_1 + T^2\alpha_0} \\
b_2 &= \frac{4\beta_2 - 2T\beta_1 + T\beta_0}{4\alpha_2 + 2T\alpha_1 + T^2\alpha_0} \\
a_0 &= 1 \\
a_1 &= \frac{-8\alpha_2 + 2T^2\alpha_0}{4\alpha_2 + 2T\alpha_1 + T^2\alpha_0} \\
a_2 &= \frac{4\alpha_2 - 2T\alpha_1 + T^2\alpha_0}{4\alpha_2 + 2T\alpha_1 + T^2\alpha_0}
\end{aligned}
$$

3.2 Alternatives

The bilinear transform is only a first-order approximation of the exact transformation form the s-plane to the z-plane. Higher-order approximations may also be employed, or other methods such as impulse invariance.

4 The Discrete-Time Transfer Function

4.1 Bilinear Transform

Substituting the analog filter coefficients into the expressions for the digital filter coefficients, the digital filter coefficients are obtained in terms of the circuit's component values and the sampling period:

$$
\begin{aligned}
b_0 &= 4\beta_2 + 2T\beta_1 + \beta_0 T^2 \\
&= 2TR_L C_{21}
\end{aligned}
$$

$$
\begin{aligned}
b_1 &= -8\beta_2 + 2\beta_0 T^2 \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
b_2 &= 4\beta_2 - 2T\beta_1 + \beta_0 T^2 \\
&= -2TR_L C_{21}
\end{aligned}
$$

$$
\begin{aligned}
a_0 &= 4\alpha_2 + 2T\alpha_1 + T^2\alpha_0 \\
&= 4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2
\end{aligned}
$$

$$
\begin{aligned}
a_1 &= -8\alpha_2 + 2T^2\alpha_0 \\
&= -8\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T^2
\end{aligned}
$$

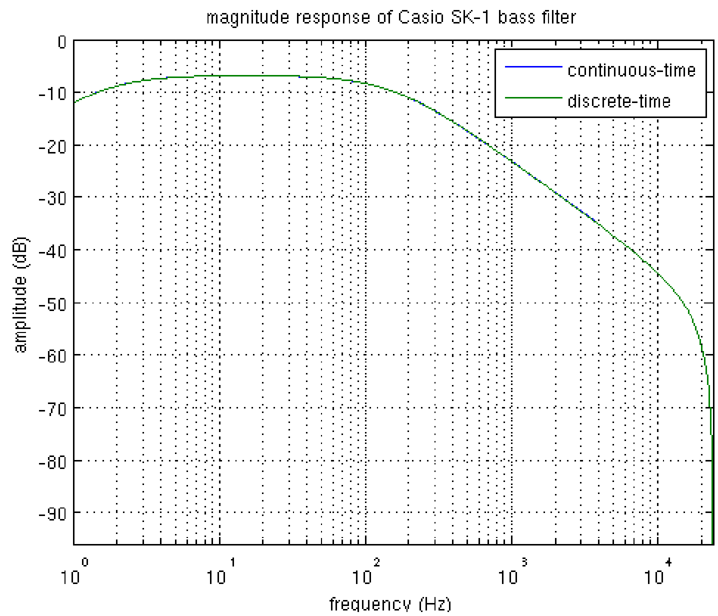
$$
\begin{aligned}
a_2 &= 4\alpha_2 - 2T\alpha_1 + T^2\alpha_0 \\
&= 4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} - 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2
\end{aligned}
$$

For most implementations, α_0 should equal 1, so the whole filter can be normalized by dividing by α_0:

$$
\begin{aligned}
b_0 &= \frac{2TR_LC_{21}}{4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2} \\
b_1 &= 0 \\
b_2 &= \frac{-2TR_LC_{21}}{4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2} \\
a_0 &= 1 \\
a_1 &= \frac{-8\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T^2}{4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2} \\
a_2 &= \frac{4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} - 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2}{4\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23} + 2T\left(\left(R_{28}+R_L\right)C_{21} + R_{31}C_{23}\right) + T^2}
\end{aligned}
$$

4.2 Magnitude Response

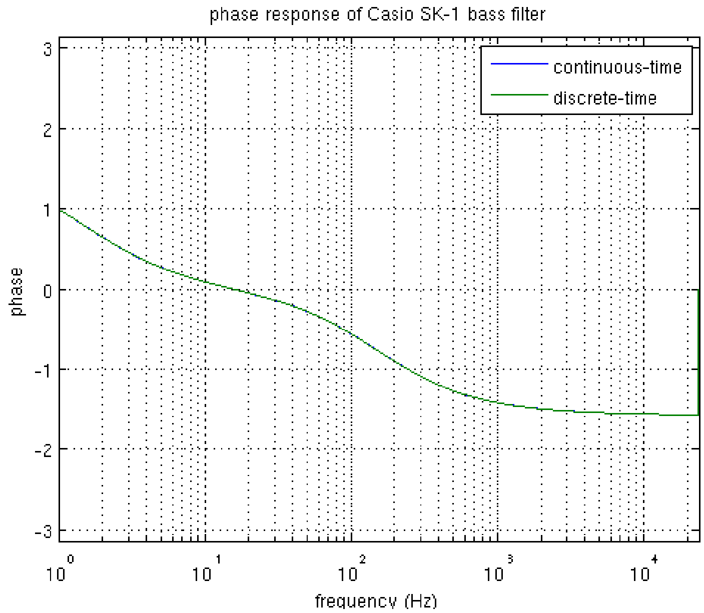
The magnitude response of the Bass Filter (with sampling rate $f_s = 48000\,Hz$) is plotted in both in continuous time and in discrete time.



The Bass Filter is clearly a band-pass filter. By inspection, the cutoff frequency is around $20\,Hz$ and the filter has a high-frequency roll-off of $6\,dB/octave$ or $20\,dB/decade$ (and, presumably, an identical low-frequency roll-off).

4.3 Phase Response

The phase response of the Bass Filter (with $f_s = 48000\,Hz$) is plotted in both in continuous time and in discrete time.



For both the magnitude and phase responses, there is no visible difference between the plots for the continuous-time and discrete-time filters, so the bilinear transform is assumed to be an approximation of high enough order to well-describe this system. Presumably, frequency warping is not an issue for low-order filters.

5 Parameterizing

5.1 Filter Parameters

It will be helpful to understand the Bass Filter's behavior in terms of filter parameters (Q, ω_c). This is easier done in continuous-time, so the analog filter coefficients are revisited:

$$\begin{aligned}
\beta_2 &= 0 \\
\beta_1 &= R_L C_{21} \\
\beta_0 &= 0 \\
\alpha_2 &= (R_{31}+1)(R_{28}+R_L)\,C_{21}C_{23} \\
\alpha_1 &= (R_{28}+R_L)\,C_{21}+R_{31}C_{23} \\
\alpha_0 &= 1
\end{aligned}$$

The Bass Filter closely resembles the case of a resonant peaking filter, as defined in the course notes for *MUS424/EE367D: Signal Processing Techniques for Digital Audio Effects* (Abel/Berners, 2011), whose transfer function is given as:

$$H\left(s\right) = \frac{\frac{s}{\omega_0}}{\left(\frac{s}{\omega_0}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_0}\right) + 1}$$

And whose filter coefficients are:

$$\begin{aligned}
\beta_2 &= 0 \\
\beta_1 &= \frac{s}{\omega_0} \\
\beta_0 &= 0 \\
\alpha_2 &= \left(\frac{1}{\omega_0}\right)^2 \\
\alpha_1 &= \frac{1}{Q\omega_0} \\
\alpha_0 &= 1
\end{aligned}$$

In this case, the denominator coefficients will be used to find the values for Q and ω_0 in terms of the component values.

5.2 Center Frequency

First, solving for ω_0 using the analog filter coefficient α_1:

$$\begin{aligned}
\frac{1}{\omega_0} &= R_L C_{21} \\
\omega_0 &= \frac{1}{\sqrt{R_L C_{21}}}
\end{aligned}$$

ω_c is in radians, to get the cutoff frequency in Hertz (f_0), ω_0 is multiplied by 2π:

$$f_0 = \frac{2\pi}{\sqrt{R_L C_{21}}}$$

Since the center frequency is sensitive to the circuit load R_L, it is important to obtain an accurate value for R_L if an accurate Bass Filter is expected! Using the "reasonable" value of $1\,M\Omega$ for R_L, the cutoff frequency of the Bass Filter is $28.98\,Hz$.

5.3 Bandwidth (Q)

Now, solving for Q using the analog filter coefficient α_1 and the expression for ω_c:

$$
\begin{aligned}
\frac{1}{Q\omega_0} &= \left(R_{28}+R_L\right)C_{21}+R_{31}C_{23} \\
Q &= \frac{1}{\left(\left(R_{28}+R_L\right)C_{21}+R_{31}C_{23}\right)\omega_0} \\
&= \frac{\sqrt{R_L C_{21}}}{\left(R_{28}+R_L\right)C_{21}+R_{31}C_{23}}
\end{aligned}
$$

Since the Q is somewhat sensitive to the circuit load R_L, it could be important to obtain an accurate value for R_L is an accurate Bass Filter is expected! Using the "reasonable" value of $1\,M\Omega$ for R_L, the value of Q is 4.3442.

5.4 Gain at Center Frequency

The gain at center frequency is found by evaluating the transfer function $H\left(s\right)$ at $s=\omega_0$:

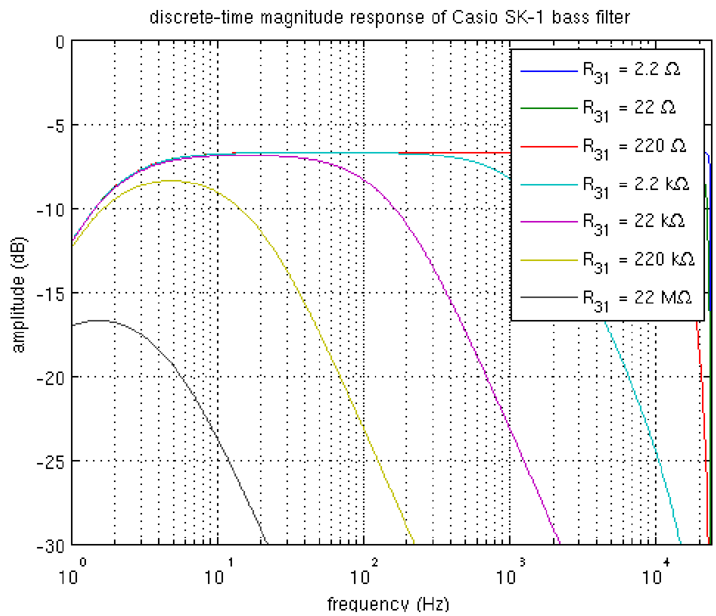
$$
H\left(w_0\right) = \frac{R_L C_{21}\omega_0}{\left(R_{31}+1\right)\left(R_{28}+R_L\right)C_{21}C_{23}\omega_0{}^2+\left(\left(R_{28}+R_L\right)C_{21}+R_{31}C_{23}\right)\omega_0+1}
$$

Using the "reasonable" value of $1\,M\Omega$ for R_L, the gain at center frequency of the Bass Filter is 0.5391.

5.5 "Circuit Bending"

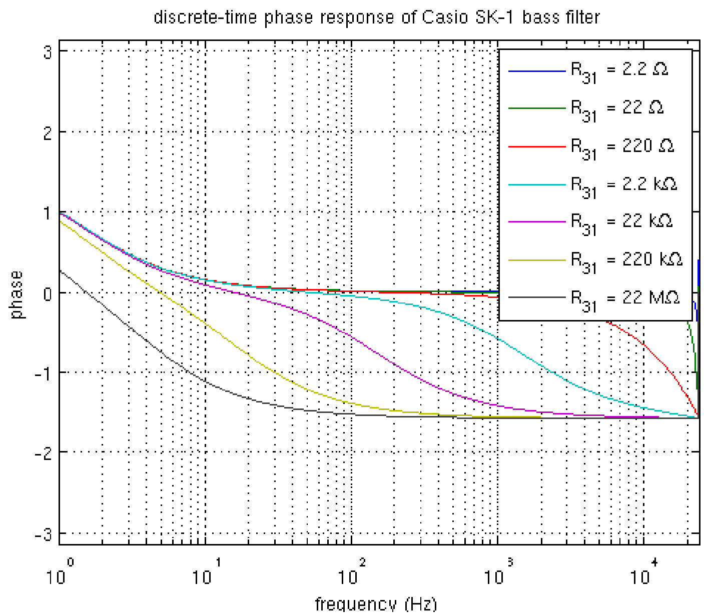
The Casio SK-1 Bass Filter cirucit has static component values. However, it may be desirable to modify the circuit to allow some of the parameters (for instance ω_c, Q) to be controlled (essentially, a digital implementation of circuit-bending). By experimentation, it is found that changing any of the component values of the filter will affect the magnitude and phase response of the filter, but that changing the value of Resistor R_{31} in particular has the result of changing the Q of the filter.

The magnitude response of the filter (with $f_s = 48000\,Hz$) is plotted in discrete time, with the resistance of R_{31} varying between $22\,\Omega$ and $22\,M\Omega$.



By inspection, a ten-fold increase in the resistance of R_{31} generally corresponds to a ten-fold decrease of the bandwidth of the filter, and a ten-fold decrease in the resistance of R_{31} generally corresponds to a ten-fold increase of the bandwidth. This behavior is somewhat different around DC and the Nyquist frequency.

The phase response of the filter (with $f_s = 48000\,Hz$) is plotted in discrete time, with the resistance of R_{31} varying between $22\,\Omega$ and $22\,M\Omega$.

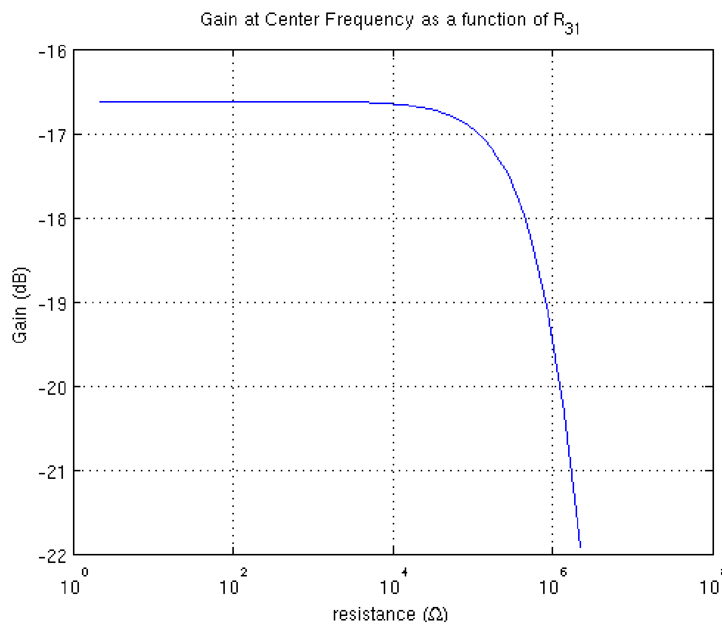


It will be useful to find the value of resistor R_{31} that is required to produce a filter with a desired Q. The equation for Q is revisited, solving for R_{31} in terms of the desired Q and component values:

$$\begin{aligned} Q &= \frac{\sqrt{R_L C_{21}}}{(R_{28}+R_L)\,C_{21}+R_{31}C_{23}} \\ R_{31}C_{23} &= \frac{\sqrt{R_L C_{21}}}{Q}-(R_{28}+R_L)\,C_{21} \\ R_{31} &= \frac{\frac{\sqrt{R_L C_{21}}}{Q}-(R_{28}+R_L)\,C_{21}}{C_{23}} \end{aligned}$$

This allows the filter model to include a control for the Q of the filter, which is analagous to replacing resistor R_{31} in the physical circuit with a potentiometer - *a digital implementation of circuit-bending*! Since the center frequency is always going to be so low, it might even make sense (from an interface standpoint) to parameterize this sort of filter in terms of a "cutoff frequency" corresponding to the point where the high-freuqency roll-off hits an arbitrary attenuation ($-3\,dB$, $-6\,dB$, etc.).

This will also affect the gain at the center frequency, (keep in mind, the original gain at center frequency is 0.5391, corresponding to $R_{31} = 22\,k\Omega$).



The gain is not affected very much, unless R_{31} gets above $100k\,\Omega$ (corresponding to a very low Q). This can be seen on the graph of the magnitude response as well.

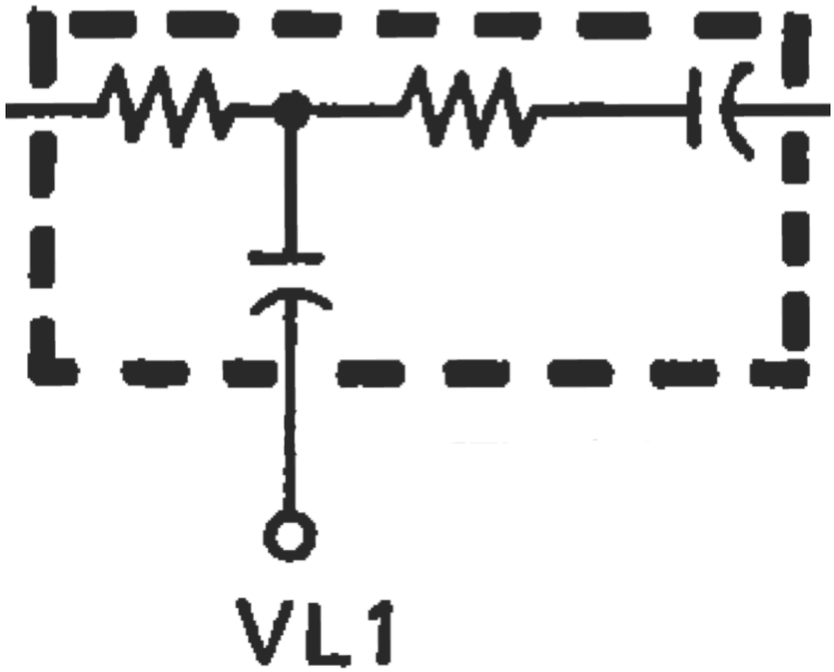
6 Implementation

A test implementation of the Casio SK-1 Bass Filter is made in "BassFilter.vst," a Steinberg Virtual Studio Technology plugin. The plugin has a control for Q that will select the proper value of R_{31} for the desired bandwidth (defaulting to 4.3442, corresponding to $R_{31} = 22\,k\Omega$). The transfer function $H(z)$ is implemented mostly as it is described in this write-up, with the biquad filter arranged in a Direct Form II Transposed topology. The only difference is that smoothing (via an RMS level estimator with a very short time constant, $\tau = 10\,milliseconds$) is done on the cutoff frequency to avoid "pops" when the parameter is changed.

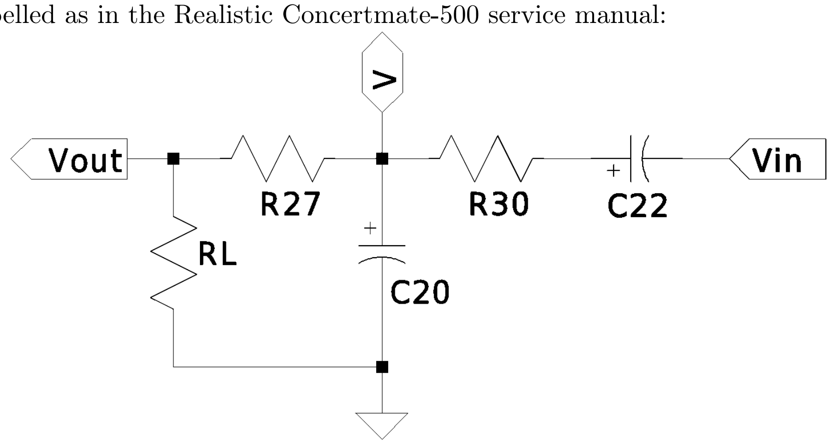
Part II
Chord Filter

7 Overview

The schematic diagram of the Chord Filter circuit is identical to that of the Bass Filter:



To analyze the circuit, a load resistor R_L will be added from V_{out} to ground. This represents the load that the rest of the circuit will contribute to the Chord Filter. For now, a "reasonable" value of $1M\,\Omega$ will be used. This can be replaced by a more accurate value once it is obtained. A node V is defined to make algebraic manipulations simpler (it will cancel out early on). Components are labelled as in the Realistic Concertmate-500 service manual:



The only difference from the Bass Filter is that the components are named differently, and one has a different value. Component values as given in the Realistic Concertmate-500 service manual:

$$R_{27} = 6.6\,k\Omega$$

$$
\begin{aligned}
R_{30} &= 22\,k\Omega \\
C_{20} &= 0.047\,\mu F \\
C_{22} &= 0.1\,\mu F
\end{aligned}
$$

8 The Continuous-Time Tranfer Function

8.1 Finding the Transfer Function

The continuous-time transfer function of the Chord Filter, which is identical to that of the Bass Filter, except with different component names substituted in.

$$
H\left(s\right) = \frac{R_L s C_{20}}{\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}s^2 + \left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)s+1}
$$

8.2 Coefficients

The Casio SK-1 Chord Filter can be described as a second-order filter with analog filter coefficients:

$$
\begin{aligned}
\beta_2 &= 0 \\
\beta_1 &= R_L C_{20} \\
\beta_0 &= 0 \\
\alpha_2 &= \left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22} \\
\alpha_1 &= \left(R_{27}+R_L\right)C_{20}+R_{30}C_{22} \\
\alpha_0 &= 1
\end{aligned}
$$

9 The Discrete-Time Transfer Function

9.1 Bilinear Transform

The discrete-time transfer function of the Chord Filter, which is identical to that of the Bass Filter, except with different component names substituted in:

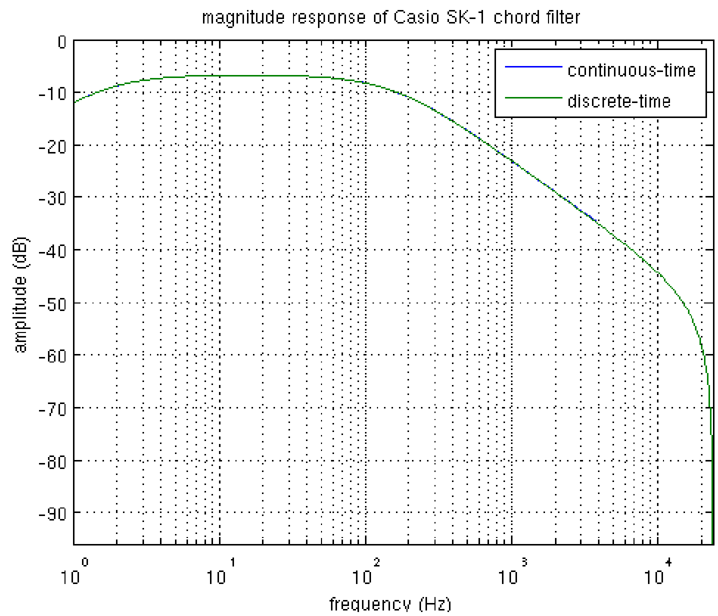
$$
\begin{aligned}
b_0 &= 2TR_L C_{20} \\
b_1 &= 0 \\
b_2 &= -2TR_L C_{20} \\
a_0 &= 4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2 \\
a_1 &= -8\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T^2 \\
a_2 &= 4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}-2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2
\end{aligned}
$$

For most implementations, α_0 should equal 1, so the whole filter can be normalized by dividing by α_0:

$$
\begin{aligned}
b_0 &= \frac{2TR_LC_{20}}{4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2} \\
b_1 &= 0 \\
b_2 &= \frac{-2TR_LC_{20}}{4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2} \\
a_0 &= 1 \\
a_1 &= \frac{-8\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T^2}{4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2} \\
a_2 &= \frac{4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}-2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2}{4\left(R_{30}+1\right)\left(R_{27}+R_L\right)C_{20}C_{22}+2T\left(\left(R_{27}+R_L\right)C_{20}+R_{30}C_{22}\right)+T^2}
\end{aligned}
$$

9.2 Magnitude Response

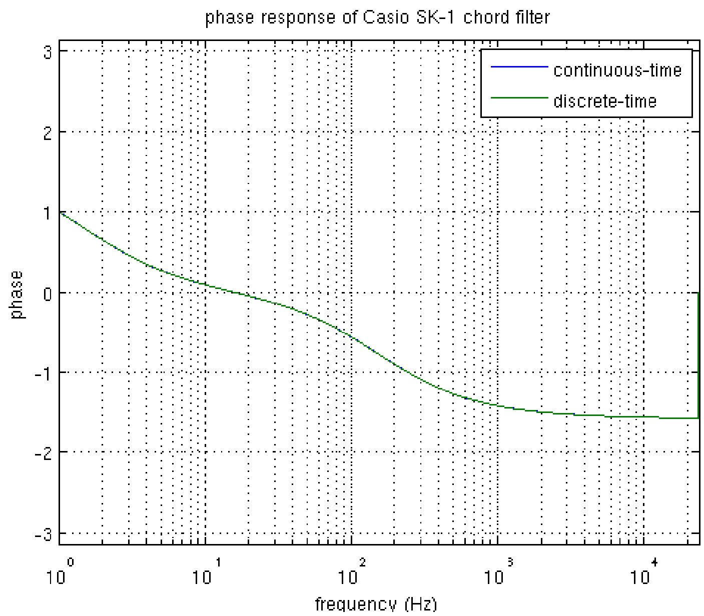
The magnitude response of the Chord Filter (with sampling rate $f_s = 48000\,Hz$) is plotted in both in continuous time and in discrete time.



The Chord Filter is clearly a band-pass filter. By inspection, the cutoff frequency is around 20 Hz and the filter has a high-frequency roll-off of 6 $dB/octave$ or 20 $dB/decade$ (and, presumably, an identical low-frequency roll-off).

9.3 Phase Response

The phase response of the Chord Filter (with $f_s = 48000\,Hz$) is plotted in both in continuous time and in discrete time.



For both the magnitude and phase responses, there is no visible difference between the plots for the continuous-time and discrete-time filters, so the bilinear transform is assumed to be an approximation of high enough order to well-describe this system. Presumably, frequency warping is not an issue for low-order filters.

10 Parameterizing

10.1 Center Frequency

First, solving for ω_0 using the analog filter coefficient α_1:

$$\begin{aligned} \frac{1}{\omega_0} &= R_L C_{20} \\ \omega_0 &= \frac{1}{\sqrt{R_L C_{20}}} \end{aligned}$$

ω_c is in radians, to get the cutoff frequency in Hertz (f_0), ω_0 is multiplied by 2π:

$$f_0 = \frac{2\pi}{\sqrt{R_L C_{20}}}$$

Since the center frequency is sensitive to the circuit load R_L, it is important to obtain an accurate value for R_L if an accurate Chord Filter is expected! Using the “reasonable” value of $1\,M\Omega$ for R_L, the cutoff frequency of the Chord Filter is $28.98\,Hz$.

10.2 Bandwidth (Q)

Now, solving for Q using the analog filter coefficient α_1 and the expression for ω_c:

$$\begin{aligned}\frac{1}{Q\omega_0} &= (R_{27}+R_L)\,C_{20}+R_{30}C_{22}\\ Q &= \frac{1}{((R_{27}+R_L)\,C_{20}+R_{30}C_{22})\,\omega_0}\\ &= \frac{\sqrt{R_LC_{20}}}{(R_{27}+R_L)\,C_{20}+R_{30}C_{22}}\end{aligned}$$

Since the Q is somewhat sensitive to the circuit load R_L, it could be important to obtain an accurate value for R_L is an accurate Chord Filter is expected! Using the "reasonable" value of $1\,M\Omega$ for R_L, the value of Q is 4.3788. Notice, this is only very slightly different from the Bass Filter.

10.3 Gain at Center Frequency

The gain at center frequency is found by evaluating the transfer function $H(s)$ at $s = \omega_0$:

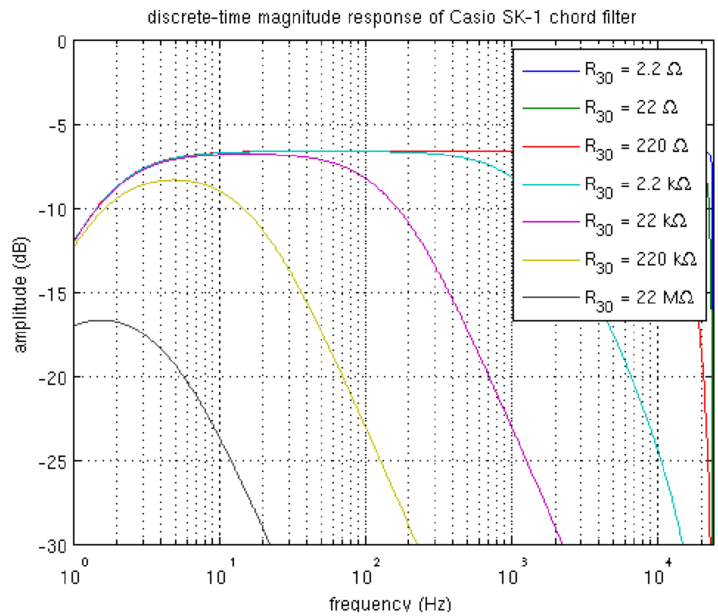
$$H(w_0) = \frac{R_LC_{20}\omega_0}{(R_{30}+1)\,(R_{27}+R_L)\,C_{20}C_{22}\omega_{0^2}+((R_{27}+R_L)\,C_{20}+R_{30}C_{22})\,\omega_0+1}$$

Using the "reasonable" value of $1\,M\Omega$ for R_L, the gain at center frequency of the Chord Filter is 0.6305. Notice, this is only very slightly different from the Bass Filter.

10.4 "Circuit Bending"

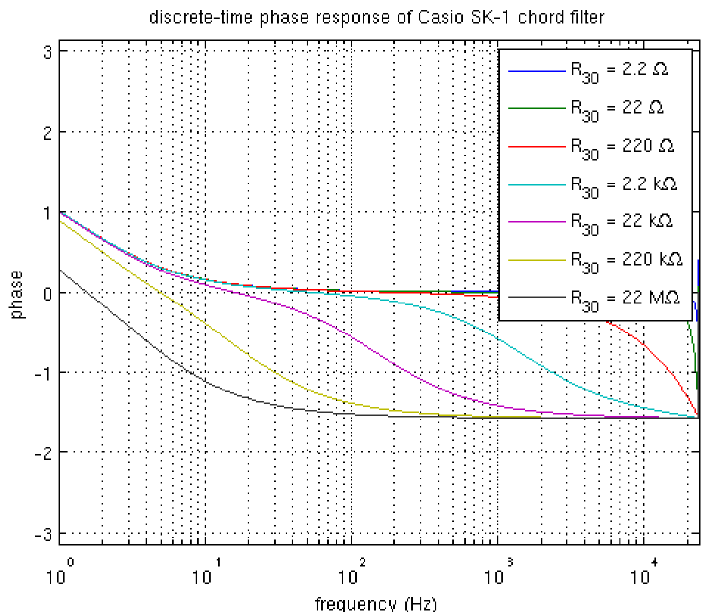
The Casio SK-1 Chord Filter cirucit has static component values. However, it may be desirable to modify the circuit to allow some of the parameters (for instance ω_c, Q) to be controlled (essentially, a digital implementation of circuit-bending). By experimentation, it is found that changing any of the component values of the filter will affect the magnitude and phase response of the filter, but that changing the value of Resistor R_{30} in particular has the result of changing the Q of the filter.

The magnitude response of the filter (with $f_s = 48000\,Hz$) is plotted in discrete time, with the resistance of R_{30} varying between $22\,\Omega$ and $22\,M\Omega$.



By inspection, a ten-fold increase in the resistance of R_{30} generally corresponds to a ten-fold decrease of the bandwidth of the filter, and a ten-fold decrease in the resistance of R_{30} generally corresponds to a ten-fold increase of the bandwidth. This behavior is somewhat different around DC and the Nyquist frequency.

The phase response of the filter (with $f_s = 48000\,Hz$) is plotted in discrete time, with the resistance of R_{30} varying between $22\,\Omega$ and $22\,M\Omega$.

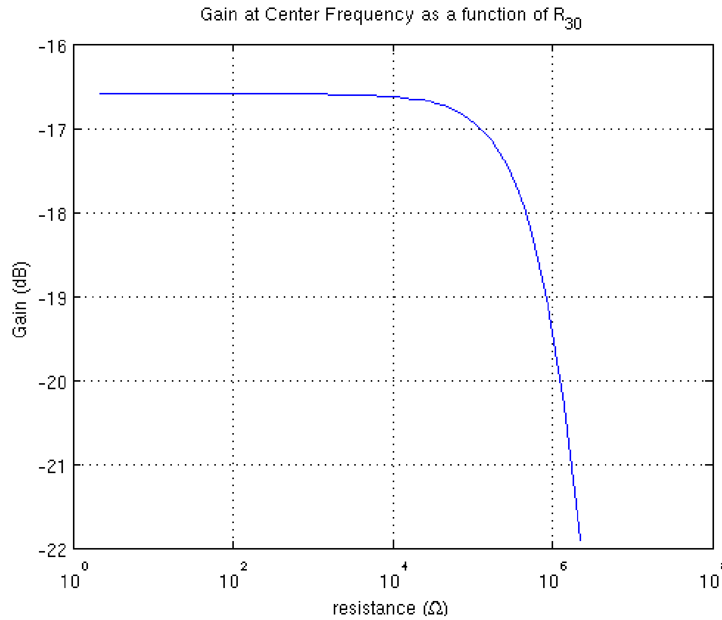


It will be useful to find the value of resistor R_{30} that is required to produce a filter with a desired Q. The equation for Q is revisited, solving for R_{30} in terms of the desired Q and component values:

$$R_{30} = \frac{\frac{\sqrt{R_L C_{20}}}{Q} - (R_{27} + R_L)\, C_{20}}{C_{22}}$$

This allows the filter model to include a control for the Q of the filter, which is analagous to replacing resistor R_{30} in the physical circuit with a potentiometer - *a digital implementation of circuit-bending*! Since the center frequency is always going to be so low, it might even make sense (from an interface standpoint) to parameterize this sort of filter in terms of a "cutoff frequency" corresponding to the point where the high-freuqency roll-off hits an arbitrary attenuation ($-3\,dB$, $-6\,dB$, etc.).

This will also affect the gain at the center frequency, (keep in mind, the original gain at center frequency is 0.6305, corresponding to $R_{30} = 22\,k\Omega$).



The gain is not affected very much, unless R_{30} gets above $100k\,\Omega$ (corresponding to a very low Q). This can be seen on the graph of the magnitude response as well.

11 Implementation

A test implementation of the Casio SK-1 Chord Filter is made in "ChordFilter.vst," a Steinberg Virtual Studio Technology plugin. The plugin has a control for Q that will select the proper value of R_{30} for the desired bandwidth (defaulting to 4.3788, corresponding to $R_{30} = 22\,k\Omega$). The transfer function $H(z)$ is implemented mostly as it is described in this write-up, with the biquad filter arranged in a Direct Form II Transposed topology. The only difference is that smoothing (via an RMS level estimator with a very short time constant, $\tau = 10\,milliseconds$) is done on the cutoff frequency to avoid "pops" when the parameter is changed.