Virtual Acoustic Musical Instruments: Review and Update

Julius O. Smith III

Center for Computer Research in Music and Acoustics (CCRMA)
Department of Music, Stanford University, Stanford, California 94305 USA

September 13, 2010

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Abstract

This article[1] gives an overview of selected developments in musical sound synthesis based on physical models of musical instruments—sometimes called "virtual acoustic" sound synthesis. Emphasis is placed on techniques which yield the highest *playability* and *sound quality* in real time at a reasonable computational expense.

1 Introduction

The "virtualization" of traditional musical instruments is well under way.[2] For many instrument families (such as plucked strings), it is possible to devise real-time computer algorithms which produce sound so realistic that most people cannot distinguish the synthesized sound from real recordings, under normal listening conditions.[3] This development enables a fork in the future evolution of traditional musical instruments. Down one fork is the continued refinement of traditional instrument making. Down the other lies a potentially large variety of new realizations in terms of sensitive physical *controllers* coupled to *sound rendering engines*. In "traditional mode," a sound rendering engine may implement a virtual acoustic sound synthesis algorithm; alternatively, there is no limit to the range of new sounds controllable by the player. A schematic depiction for the case of bowed strings is shown in Fig. 1. While controllers may continue to resemble traditional musical instruments, due to player preference, they will be freed of the responsibility for quality sound production. Instrument makers will be able to focus instead on the *playability* of the controller [201], without compromising ease of play for better sound. As an example, the body of the double-bass can be made closer to that of the cello, or eliminated entirely, leaving only the strings, fingerboard, and suitable bracing, as in Chris Chafe's *celletto*, shown in Fig. 2. In summary, it is now possible to *factor* traditional musical instruments into the ideal controller for that instrument, together with a *matched* synthesis engine.

Note that *matching* a controller to its synthesis algorithms in the context of a particular instrument family, as obvious as that may sound at first, is still a relatively novel subject. This is because most commercially available synthesizers are designed to be driven by keyboards and the Musical Instrument Digital Interface (MIDI) standard.[4] As a result, there is often an unavoidable delay, or *latency* between controller signals and the synthesizer response. MIDI itself introduces up to three milliseconds of delay for each musical note, and there is often much more delay than that associated with sound buffering. These delays are often unacceptable to serious performing musicians. For string controllers, it is typical to drive a synthesizer using the measured pitch of the string vibration. However, at least a period of vibration must pass before the synthesizer can

[1]This is the author's final draft version of an article submitted to the Journal of New Music Research. The published version should appear early in 2005.

[2]For purposes of this article, "virtualization" refers to the process of creating real-time, performable synthesis algorithms which are competitive with natural physical instruments such as pianos, guitars, clarinets, and so on.

[3]Sound examples: `http://ccrma.stanford.edu/~jos/waveguide/Sound_Examples.html`

[4]One exception is the Yamaha VL series of synthesizers [123], which provided a separate high-speed, low-latency input for the breath controller.

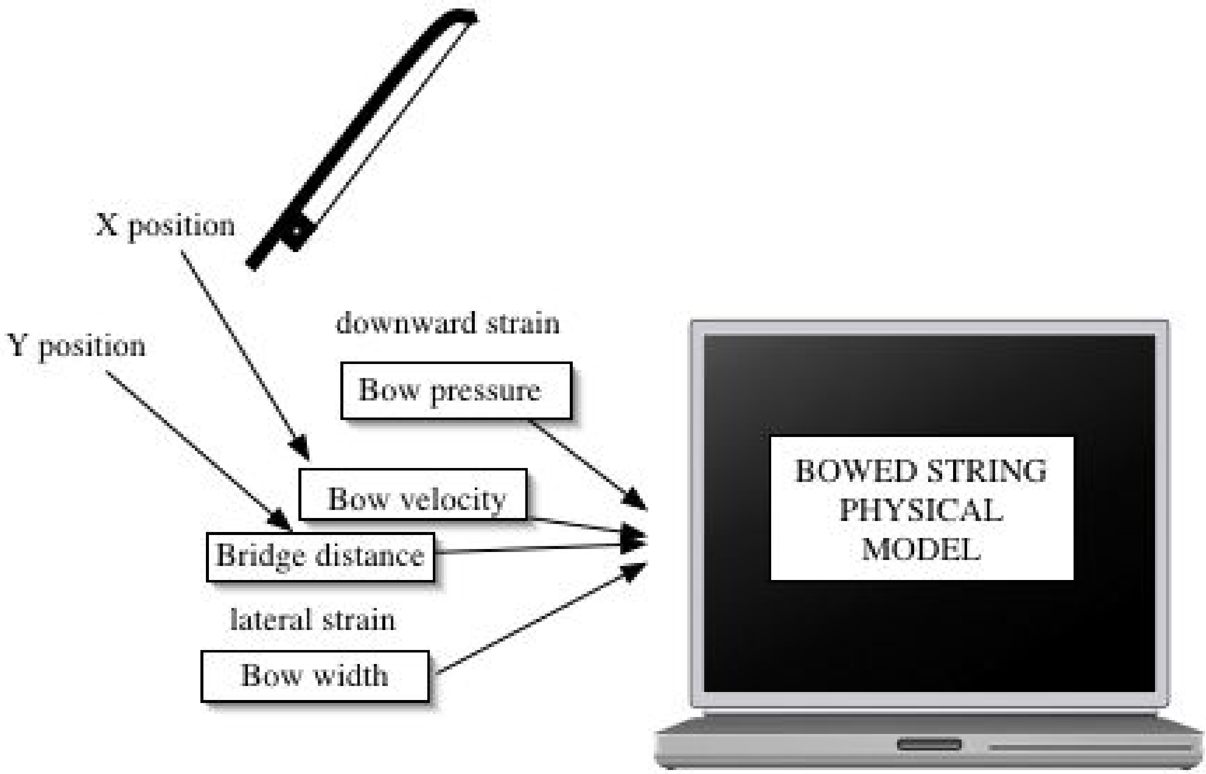

Figure 1: Virtual bowed string instrument (from [144]). A bow with embedded sensors (developed by Diana Young) is connected wirelessly to a real-time bowed-string physical model (written by Stefania Serafin). (From [164].)

Figure 2: Chris Chafe playing the *celletto*—a block of solid maple with a piezo-ceramic pick-up under each string (inlaid by Max Mathews). The pick-up signals are processed externally.

respond, and one period is too long to wait. For example, a note near the bottom range of the guitar has a period larger than 10 milliseconds. This delay is comparable to that of being more than 10 feet away, and few players can put up with it except within a narrow range of playing styles. When the controller and synthesis algorithm are matched, such delays are easily eliminated. As a simple example, when a guitar string is plucked, the initial sound is independent of the pitch of the note (since that is determined by later acoustic reflections on the string). A matched synthesis algorithm can emit the initial signal immediately, well before the pitch is known. In short, a virtual acoustic synthesis algorithm can simulate the physical response of a desired instrument with no perceivable latency whatsoever. Few musicians have yet experienced this degree of virtual instrument playability.

This article will focus on synthesis algorithms for virtual acoustic instruments, neglecting the important companion topics of controllers and controller-algorithm interfaces [196]. Moreover, we will restrict attention here to synthesis of *traditional* musical instruments, such as strings and winds, leaving aside the many new variations which become possible in this domain, such as "bowed pipes," "reed-driven strings," and impossibly large instruments, to name a few. We will also emphasize methods which are, in the opinion of the author, most cost-effective for delivering maximum sound quality and playability.

2 Synthesized Musical Instruments

Musical sound synthesis has a long history spanning more than half a century [125, 152, 156]. Since the 1960s [74, 130], various research threads have been aimed at developing sound synthesis algorithms based on the acoustics of traditional instruments. Since 1994, when the Yamaha "Virtual Lead" synthesizer was introduced [123], such synthesis techniques have been available commercially as well (principally from Yamaha and Korg). We begin with a general review of synthesis techniques, with emphasis on considerations pertaining to the synthesis of these instruments. Related overview publications include [26, 81, 152, 156, 170].

2.1 Sampling Synthesis

Far and away the most commonly used method today is "sampling synthesis" (also called "wavetable synthesis"). "Sampling" an instrument means simply to record it under a wide variety of useful playing conditions. For example, every note of a piano can be recorded at a wide variety of key velocities. Sounds corresponding to velocities in between the recorded velocities can be approximated by *interpolating* the two sounds recorded at the nearest lower and higher key velocities. Similarly, not every key needs to be recorded, and it is quite typical to use one recorded sample to cover several adjacent keys by modifying its playback rate to scale the pitch. Additionally, memory can be saved by *looping* the steady-state portion of the sound. Sampling synthesizers have been around for at least a quarter century (the Fairlight sampling synthesizer was available in the late 1970s [173]). The great majority of current synthesizers use this method with various refinements.

The great advantage of sampling synthesis is *static fidelity*. That is, the quality of the sound produced is limited only by the quality of the original recorded sound. Since any sound an instrument makes can be recorded, there is no fundamental lack of generality in sampling synthesis. Anything is possible at some price.

The two major drawbacks of sampling synthesis are

1. consumption of enormous quanitities of memory (when done completely), and
2. prohibitively expensive development for full playability.

The second disadvantage is the more serious for the performing musician. For many instrument families, it is simply too much work to capture the complete range of playing conditions [106]. Since pianos are controlled by a single control dimension—the key velocity—(pedals aside), they are relatively easy to capture in full (although pianos require more memory than most instruments due to their rich, inharmonic spectra [8]). On the other hand, bowed string instruments, such as the violin, have never to the author's knowledge been sampled very completely, nor coupled to a sufficiently complete and well calibrated controller in order to provide a virtual instrument comparable with the expressive playing range of a real violin.

2.2 Spectral Modeling

Spectral modeling can be viewed "sampling synthesis done right" [152]. That is, in spectral modeling synthesis, segments of the time-domain signal are replaced by their short-time Fourier transforms, thus providing a sound representation much closer to the *perception* of sound by the brain [66, 108, 204]. This yields two immediate benefits: (1) computational cost reductions based on perceptual modeling, and (2) more perceptually fundamental data structures. Cost reductions follow naturally from the observation [167] that roughly 90% of the information contained in a typical sound is not perceived by the brain. For example, the popular MP3 audio compression format [27, 28] can achieve an order of magnitude data reduction with little or no loss in perceived sound quality because it is based on the short-time Fourier transform, and because it prioritizes the information retained in each spectral frame based on psychoacoustic principles. To first order, MPEG audio coding eliminates all spectral components which are *masked* by nearby louder components.

The disadvantages of spectral modeling are the same as those of sampling synthesis, except that memory usage can be greatly reduced. Sampling the full playing range of a musical instrument is made more difficult, however, by the need to capture every detail in the form of spectral transformations. Sometimes this is relatively easy, such as when playing harder only affects brightness. In other cases, it can be difficult, such as when nonlinear noise effects begin to play a role.

An excellent recent example of spectral modeling synthesis is the so-called *Vocaloid* developed by Yamaha in collaboration with others [5]. In this method, the short-time spectrum is modeled as sinusoids plus a residual signal, together with higher level spectral features such as vocal formants. The model enables the creation of "vocal fonts" which effectively provide a "virtual singer" who can be given any material to sing at any pitch. Excellent results can be achieved with this approach (and some of the demos are very impressive), but it remains a significant amount of work to encode a particular singer into the form of a vocal font. Furthermore, while the sound quality is generally excellent, subtle "unnaturalness" cues may creep through from time to time, rendering the system most immediately effective for automatic back-up vocals, or choral synthesis, as opposed to highly exposed foreground lead-singer synthesis.

Zooming out, spectral modeling synthesis can be regarded as modeling sound inside the *inner ear*, enabling reductions and manipulations in terms of human *perception* of sound.

2.3 Physical Modeling

In contrast to spectral modeling, physical modeling synthesis can be regarded as modeling sound at its *source*, thereby enabling parsimonious representations and sonic manipulations following closely the *physics* of sound production. To the delight of musical instrument *performers*, physical modeling synthesis naturally provides the complete playability range for virtual acoustic instruments. While there is no fundamental reason why spectral modeling (or sampling synthesis) cannot provide the same range of playability—any effect can be recorded and indexed—it is simply impractical to do that much work for most instruments. The required effort is prohibitive even to obtain initial recordings which span all possible variations in tone and expression, let alone to subsequently model them as spectral components and transformations on spectral components.

In the following sections, we will review the topic of physical modeling synthesis of acoustic instruments, beginning with basic strings (the simplest case), and continuing on with overviews of current physical synthesis models for selected instrument types.

3 Physical Models of Musical Instruments

The field of *musical acoustics* [16, 61, **?**] is a branch of physics devoted to the improved understanding of musical instruments. This understanding typically takes the form of a mathematical model verified by experiment. For the most part, traditional musical instruments are pretty well understood, and their equations of motion can largely be formulated based on classical Newtonian mechanics [68, 129]. For wind instruments, however, there are many details that have yet to be pinned down with sufficient accuracy to enable a straightforward computational model to be based upon pure theory. Fortunately, musically motivated scientific investigations (such as [47]) are ongoing, and we can expect this situation to improve over time.

A *mathematical* model of a musical instrument normally takes the form of a set of *differential equations* describing the various components of the instrument, such as a vibrating string, air column, or resonator. The components are described by the equations in the sense that their motion is always observed to obey the differential equations. As a result, given the initial conditions (initial positions and velocities), boundary conditions (constraints describing connections to other components) and knowing all external forces over time ("time-varying boundary conditions"), it is possible to simulate mathematically the motion of the real system with great accuracy.

Computational models of musical instruments may be obtained by *discretizing* the differential equations to obtain *difference equations*. This approximation step in going from a differential equation to a difference equation is sometimes called a *finite difference approximation* (FDA), since derivatives are replaced by *finite differences* defined on a discrete space-time grid [168]. An example will be given for strings in the next section. The difference equations can be "integrated" numerically to obtain estimates of the acoustic field in response to external forces and boundary conditions. Numerical integration of FDAs is perhaps the most general path to finite difference schemes yielding accurate computational models of acoustic systems (see, e.g., [32, 49]).

While finite-difference methods are quite general, they are computationally more expensive than what is really necessary to achieve excellent real-time musical instruments based on physics. The following sections, organized by instrument family, will discuss various avenues of complexity reduction based on perceptual equivalents from psychoacoustics and computational equivalents from the field of signal processing.

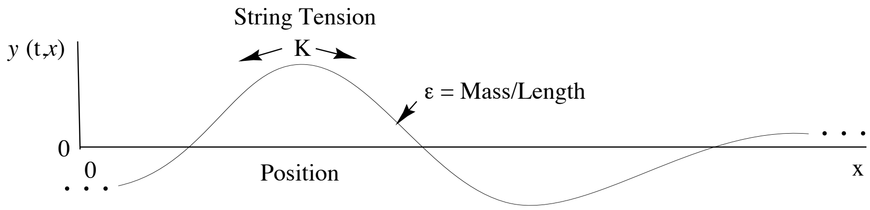

Figure 3: The ideal vibrating string.

4 Strings

The classical physical model for transverse vibrations in an ideal string, as illustrated in Fig. 3, is the following *wave equation* [109, 110, 117, 179]:[5]

$$Ky'' = \epsilon \ddot{y}$$

where

$$\begin{array}{rcl@{\qquad\qquad}rcl} K & \stackrel{\Delta}{=} & \text{string tension} & y & \stackrel{\Delta}{=} & y(t,x) \\ \epsilon & \stackrel{\Delta}{=} & \text{linear mass density} & \dot{y} & \stackrel{\Delta}{=} & \frac{\partial}{\partial t} y(t,x) \\ y & \stackrel{\Delta}{=} & \text{string displacement} & y' & \stackrel{\Delta}{=} & \frac{\partial}{\partial x} y(t,x) \end{array}$$

(Note that "$\stackrel{\Delta}{=}$" means "is defined as".) Applying the finite difference approximation (FDA) means to replace each partial derivative by a finite difference [168], e.g.,

$$\dot{y}(t,x) \approx \frac{y(t,x) - y(t-T,x)}{T}$$

and

$$y'(t,x) \approx \frac{y(t,x) - y(t,x-X)}{X}$$

where T and X are the time and position sampling intervals to be used, respectively. Note that the finite-difference approximations become exact, in the limit, as T and X approach zero. In practice, more accurate simulation is obtained with increasing sampling rates. Applying a FDA to the ideal string wave equation above yields

$$K \frac{y(t,x+X) - 2y(t,x) + y(t,x-X)}{X^2} = \epsilon \frac{y(t+T,x) - 2y(t,x) + y(t-T,x)}{T^2}.$$

Normalizing T, X, and K/ϵ to 1 leads to the simplified recursion

$$y(n+1,m) = y(n,m+1) + y(n,m-1) - y(n-1,m).$$

Thus, for each time step n, the string position samples can be updated using all samples along the string from the previous two time steps. The FDA method for numerical string simulation was

[5]This section follows portions of [158, Appendix B]. Figures in this article not otherwise attributed are reprinted from [158].

used by Pierre Ruiz in his early work on vibrating-string simulation [74, 130], and it is still in use today [31, 32, 49].

Perhaps surprisingly, it can be shown that the above recursion is *exact* at the sample points, despite the apparent crudeness of the finite difference approximation at low sampling rates, provided the string initial conditions and excitations are *bandlimited* to less than half the sampling rate. An easy proof is based on showing its equivalence to the *digital waveguide model* for the ideal string [157, pp. 430–431].

4.1 Digital Waveguide Models

A more efficient alternative to the finite-difference approximation, when it applies, is the so-called "digital waveguide approach" [151, 158]. Its characteristics are well matched to the problem of real-time string simulation. Instead of digitizing the differential equation for the string, digital waveguide models modify the *solution* of the ideal wave equation by introducing digital filters which simulate amplitude attenuation and phase dispersion [154]. This two-step procedure yields finite difference schemes which are orders of magnitude less expensive computationally, relative to FDAs, without sacrificing sound quality in the acoustic simulation [155]. Since the variables in the resulting model retain precise physical interpretations, nonlinearities and time-varying parameters can be implemented in the same way as in the FDA method. Furthermore, FDAs and digital waveguides can be freely intermixed [85, 86, 95, 118, 119]. The digital waveguide approach is most effective for modeling physical systems that support traveling waves and for which losses, dispersion, and any nonlinearities can be well modeled psychoacoustically at spatially localized points. Fortunately, many acoustic musical instruments fit this description, including strings, winds, brasses, and tonal percussion.

As a general rule, linear elements consisting of uniform mass distributions primarily along one dimension, such as vibrating strings, woodwind bores, pipes, horns, and the like, are most efficiently modeled using the digital waveguide framework. (From a modal synthesis point of view, digital waveguides are highly efficient for simulating large numbers of quasi-harmonic resonances.) On the other hand, systems which are nonlinear, or which have only one important resonance (or anti-resonance) in the band of human hearing such as clarinet reeds, toneholes, lip valves, and piano hammers, are generally better modeled using the FDA approach. Other situations arise as well, some of which will be noted below.

4.2 Ideal String with a Moving Termination

Perhaps the simplest example of a digital waveguide string model arises for the case of an ideal string, rigidly terminated, wherein one of the terminations is set in motion at a uniform velocity, as shown in Fig. 4.

The left endpoint is moved at velocity v_0 by an external force $f_0 = Rv_0$, where $R = \sqrt{K\epsilon}$ is the *wave impedance* for transverse waves on the string [158].[6]

The moving-termination simulation is highly relevant to *bowed strings*, since, when the bow pulls the string, the string can be modeled, to first order, as two strings terminated by the (moving) bow.

Figure 5 illustrates two "waveguide equivalent circuits" for the uniformly moving rigid string termination, for velocity and force waves, respectively (typically chosen wave variables). The upper

[6] `http://ccrma.stanford.edu/~jos/waveguide/Force_Waves.html`

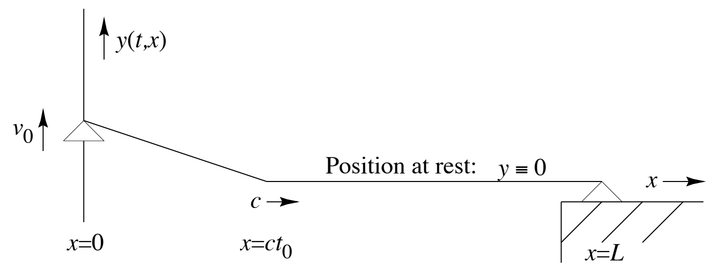

Figure 4: Moving rigid termination for an ideal string at time $0 < t_0 < L/c$.

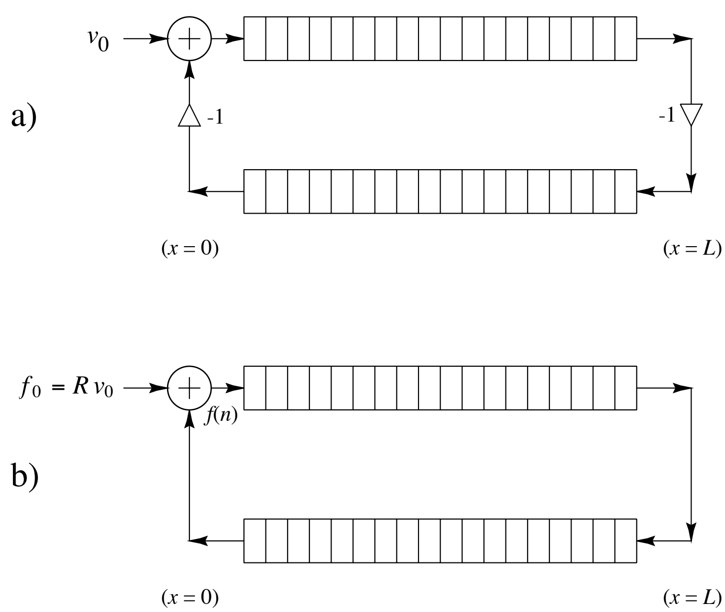

Figure 5: Digital waveguide simulation using a) velocity waves, b) force waves.

delay line holds samples of *traveling waves* propagating to the right (increasing x direction), while the lower delay line holds the *left-going* traveling-wave components of the string vibration. Because delay lines are very inexpensive to implement, this is where the digital waveguide method saves much computation relative to a FDA. The rigid string terminations reflect with a sign inversion for velocity waves, and no sign inversion for force waves. Note that the physical transverse velocity distribution along the string may be obtained by simply adding the contents of the upper and lower delay lines, and similarly for the force distribution [154]. (The string force at any point is defined as minus the tension times the slope of the string.) Also, a velocity wave may be converted to a force wave by multiplying it by the wave impedance $R = \sqrt{K\epsilon}$, and flipping the sign in the left-going case.

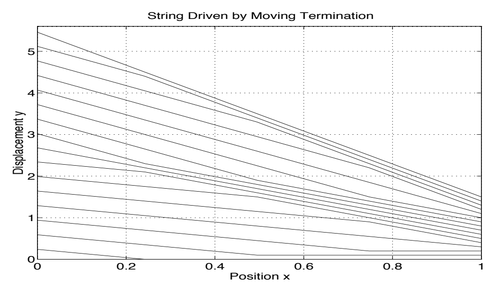

Figure 6: String displacement snapshots for a moving termination.

Figure 6 shows successive snapshots of the string displacement, plotted with an increasing vertical offset for clarity.[7] String displacement can be computed from Fig. 5a by summing the delay lines to get physical string velocity, and then adding that to an "accumulator" buffer each time instant to approximate an integral with respect to time. Note that the string shape is always piecewise linear, with only one or two straight segments in existence at any time. A "Helmholtz corner" (slope discontinuity) shuttles back and forth at speed c. While the string velocity takes on only two values (v_0 or 0) at each point, the string slope increases without bound as the left termination proceeds to $y = \infty$ with the right termination fixed. The magnitude of the applied force at the termination, which is proportional to string slope, similarly steps to infinity as time increases.[8] As a result, a velocity-wave simulation is better behaved numerically than a force-wave simulation in this instance.

4.3 Driven Terminated Strings

Figure 7 depicts a digital waveguide model for a rigidly terminated ideal string excited at an interior point $x = mX$. It is easy to show using elementary block-diagram manipulations [159] that the system of Fig. 8 is equivalent. The advantage of the second form is that it can be implemented as

[7] A GIF89A animation of Fig. 6 is available on-line at http://ccrma.stanford.edu/~jos/swgt/movet.html .

[8] Note that the force goes to infinity for the wrong reason in this model. The basic approximation of force as tension times slope is only valid for slope magnitudes much less than unity, as any derivation of the ideal string wave equation shows [158, Appendix B]. Compensating for this approximation would yield a maximum force equal to the string tension. However, that force would go to infinity in a model incorporating tension modulation (which is neglected here), because the string is being stretched to infinite length. Of course, in a truly complete model, the string should break at some point.

a cascade of standard *comb filter* sections.[9]

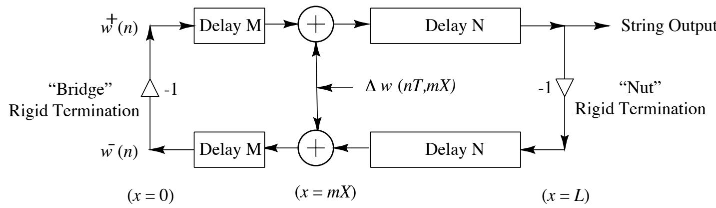

Figure 7: External string excitation at a point.

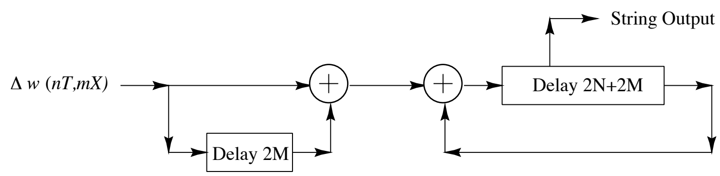

Figure 8: System equivalent to that in Fig. 7.

When damping is introduced, the string model becomes as shown in Fig. 9 [154], shown for the case of displacement waves (changing to velocity or force waves would only change the signal name from y to either v or f, respectively in this simple example). Also shown are two "pick-up" points at $x = 0$ and $x = 2X$ which read out the physical string displacements $y(t_n, x_0)$ and $y(t_n, x_2)$.[10] The symbol z^{-1} means a one-sample delay. Because delay-elements and gains g *commute* (i.e., can be reordered arbitrarily in cascade), the diagram in Fig. 10 is equivalent, when round-off error can be ignored. (In the presence of round-off error after multiplications, the commuted form is more accurate because it implements fewer multiplies.) This example illustrates how internal string losses may be *consolidated sparsely* within a waveguide in order to simplify computations (by as much as three orders of magnitude in practical simulations, since there are typically hundreds of (delay-element,gain) pairs which can be replaced by one long delay line and single gain). The same results hold for *dispersion* in the string: the gains g simply become digital filters $G(z)$ having any desired gain (damping) versus frequency, and any desired delay (dispersion) at each frequency [158].[11]

4.4 Body Resonators

A common feature of stringed musical instruments is the presence of a *resonator*, usually made of wood. For the violin family and guitars, this resonator is called the *body* of the instrument. For pianos, harpsichords, and the like, the principal resonator is a large wooden plate called the *sound board*. The number of resonant modes in the range of human hearing depends strongly on the size of the resonator.

[9] `http://ccrma.stanford.edu/~jos/waveguide/Comb_Filters.html`

[10] If this were instead a velocity-wave simulation, we could say we were simulating *magnetic pick-ups* on an electric-guitar string.

[11] `http://ccrma.stanford.edu/~jos/waveguide/Digital_Waveguide_Theory.html`

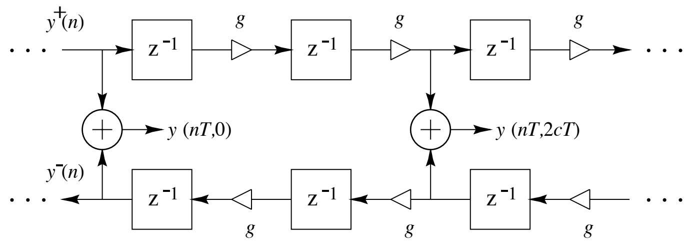

Figure 9: Digital waveguide string simulation with damping.

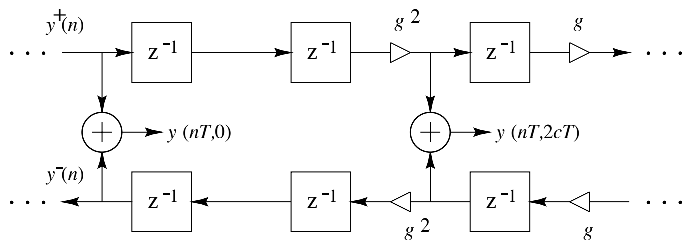

Figure 10: Damped string simulation with commuted losses.

Signal-processing models of physical resonators typically employ a two-pole digital filter to model each resonant mode. Such a second-order filter is commonly called a *biquad*, because its transfer function is a ratio of quadratic polynomials in z. The resonance frequency, bandwidth, phase, and gain can all be adjusted to match a resonant peak in measured frequency-response data [87]. A general biquad section requires five multiplications for each sample of sound computed. Even with today's high-speed computers, it is prohibitively expensive to model resonators mode for mode. Modal synthesis [4], [159, Appendix E][12] refers to models of resonators consisting of a sum of second-order resonators. Modal synthesis is effective when there is a small number of long-ringing modes, such as in the marimba or xylophone.

For larger systems, such as stringed instrument bodies, the number of resonant modes in the range of human hearing is prohibitively large. In volumes, the number of resonances below some frequency limit increases, asymptotically, as frequency cubed, while in membranes (and thin plates) it grows as frequency squared [117, p. 293].

It is not always necessary to preserve the precise tuning and bandwidth of all the modes in a body resonator model. As an example, it is known that between 2 and 25 sinusoids ("undamped resonances") having equal amplitude and random phases can sound equivalent to noise in approximately one critical band of hearing [63, 108, 204]. (Below 5 kHz, around 15 suffice.) Also, the required number of sinusoids decreases in the presence of reverberation [63]. Applying this finding to resonant modes, we expect to be able to model the modes *statistically* when there are many per critical band of hearing. Moreover, when the bandwidths are relatively large, we may revert to a statistically matched model when there are several modes per bandwidth.

[12] http://ccrma.stanford.edu/~jos/filters/Modal_Representation.html

One should note, however, that psychoacoustic results for a specific type of stimulus do not often generalize to every type of stimulus. Thus, for example, the specifications for statistically matched high-frequency modes in a body-resonator may be one thing for *impulse responses*, and quite another for *periodic excitations with vibrato*. As J. Woodhouse puts it [200]:

> "... somewhere in the subtleties of resonance patterns must be found the answer to the whole mysterious question of what makes one violin more expensive than another. ... when it comes to ultimate judgements of musical quality there may be years of research still to be done. It is an interesting question whether virtual violins will become good enough that players start making 'strad-level' judgements about them."

There are several ways to potentially take advantage of the ear's relative insensitivity to exact modal frequencies and bandwidths at high frequencies. One is to superimpose several different harmonic sums of modes, implemented efficiently using 1D digital waveguides. Another is to use the *digital waveguide mesh* for high-frequency statistical mode distribution models; see §9.2 for some pointers into the waveguide mesh literature.

Essl's 2002 thesis [56] introduces the topic of "banded waveguides." The basic modeling idea is to use a filtered delay-line loop to model a "closed wave-train path" corresponding to an individual mode in extended objects such as cymbals and bars. The loop contains a bandpass filter which eliminates energy at frequencies other than the desired mode. The result is a modeling element somewhere in between a digital waveguide loop (yielding many quasi-harmonic modes) and a second-order resonator which most efficiently models a single mode. For certain classes of physical objects, improved transient responses are obtained relative to ordinary modal synthesis. Banded waveguides efficiently implement a large number of quickly decaying modes for each long-ringing mode retained in the model.

In summary, there are many ways to avoid the expense of modeling each resonant mode using a second-order digital filter, and this section has mentioned some of them.

4.5 Recent Developments for Strings

The work of Laurson et al. [99, 98] represents perhaps the best results to date in the area of classical guitar synthesis. A key factor in the quality of these results is the great attention to detail in the area of musical control when driving the model from a written score. Most of the sound examples in [158],[13] in contrast, were played in real time using physical controllers driving algorithms running in SynthBuilder on a NeXT Computer [120]. In general, it is much easier to "voice" a real-time algorithm driven by a physical controller than it is to synthesize a convincing performance from a written score. However, Laurson et al. show that the latter can be done very well.

In [171], Tolonen et al. developed a simplified extension to digital waveguide string models to incorporate nonlinear *tension modulation*. The first-order audible effect of nonlinearity in strings is an increase in pitch at high vibration amplitudes.[14] This occurs because the average tension is increased when the vibration amplitude is large (because the string must stretch to traverse the high-amplitude wave shape). Since tension waves travel much faster than transverse waves (more than ten times faster in piano strings [8][15]), the tension increase is generally modeled as an instantaneous global effect as a function of instantaneous string length.

[13] `http://ccrma.stanford.edu/~jos/waveguide/Sound_Examples.html`
[14] Another measurable effect in nonlinear musical string vibration is *combination tones* [34].
[15] http://www.speech.kth.se/music/5_lectures/conklin/longitudinal.html

The 2002 thesis of Erkut [55] comprises nine publications covering analysis and synthesis of acoustic guitar, as well as other plucked-string instruments (tanbur, kantele, lute, and ud).

In [95], Krishnaswamy et al. proposed new, more physically accurate algorithms for simulating vibrating strings which strike against physical objects (for simulation of slap bass, tambura, sitar, etc.). Both a pure "waveguide approach" and a hybrid approach involving a "mass-spring" string model inside a digital waveguide are considered. The hybrid waveguide/finite-difference model enables maximally efficient simulation of freely vibrating string sections ("waveguide sections"), while also providing finite-difference sections for nonlinear interactions.[16] A similar hybrid model was used by Pitteroff and Woodhouse to model a string bowed by a bow with finite with [118, 119]. In [85], Karjalainen presented a systematic technique for mixing finite difference and digital waveguide simulations. More specifically, adaptors are given for converting modularly between traveling-wave components (used in digital waveguide simulations) and physical acoustic variables such as force and velocity (typical in finite difference schemes derived from differential equations).

Recent efforts to calibrate waveguide string models using a genetic algorithm are described by Riionheimo and Välimäki in [124]. The genetic algorithm was found to successfully automate parameter tunings that were formerly done by hand. The error measure is based on human perception of short-time spectra, similar to what is done in perceptual audio coding [27].

5 Pianos

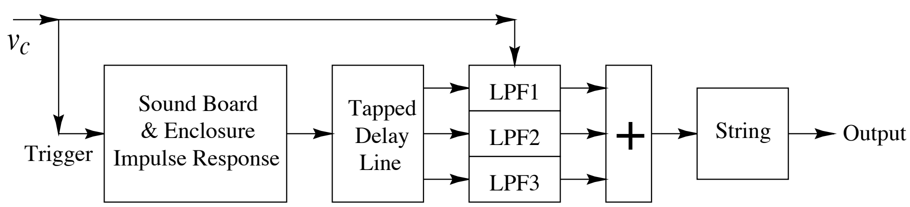

Figure 11: Block diagram of piano synthesis using commuted waveguide synthesis.

Figure 11 shows a block diagram of a piano synthesis algorithm based on commuted waveguide synthesis [165, 158, 181].[17] Commuted synthesis introduces an enormous computational simplification based on the commutativity of linear, time-invariant systems [159]. When cascaded systems are commutative, their order can be interchanged. This is why the soundboard model appears before the string model in Fig. 11. The "trigger" signal occurs when a piano key is pressed, and it starts a playback of the soundboard impulse response from a table. The tapped delay line feeds three lowpass filters which depend on the key velocity v_c. The filter outputs are summed and fed to a digital waveguide string model (including coupled-string effects [13, 195, 158]). The sum of the filter impulse responses models the interaction-force between the string and the piano hammer, as shown in Fig. 12. Sound examples from the commuted piano synthesis model can be heard at the

[16]While both the digital waveguide and second-order finite difference models are *exact* simulations in the case of bandlimited ideal strings [157], they become different models in the presence of nonlinear interactions.

[17]`http://ccrma.stanford.edu/~jos/waveguide/Commuted_Piano_Synthesis.html`

on-line version of [158].[18]

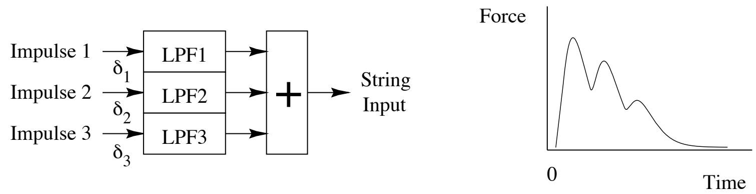

Figure 12: Illustration of the building of a hammer-string interaction force as the superposition of three filter impulse response.

5.1 Recent Developments

Much recent experience in piano modeling is represented by the 2003 paper of Bank et al. [14], including good summaries of prior research on efficient computational string models, loss-filter design, dispersion filter design, and soundboard models. A careful computational model of the piano, partially based on physical modeling and suitable for high-quality real-time sound synthesis, has been recently developed by Bensa in the context of his thesis work [20]. Related publications include [21, 22]. An excellent simulation of the *clavichord*, also using the commuted waveguide synthesis technique, is described in [174]. A detailed simulation of the *harpsichord*, along similar lines, but with some interesting new variations, is presented in [178].

A software example of commuted synthesis for the mandolin appears in the Synthesis Tool Kit[19] (STK) [40] as `Mandolin.cpp`. This software is written in the `C++` programming language, and runs in real time on typical personal computers. It can be controlled in real time from a MIDI device, a Tcl/Tk graphical user interface (GUI), or driving software.

6 Bowed Strings

Figure 13 shows a high-level schematic for a bowed-string instrument, such as the violin, and Fig. 14 shows a digital waveguide synthesis model [150, 158].

The bow divides string into two sections. The primary control variable is bow velocity, which implies that *velocity waves* are a natural choice of wave variable for the simulation. The computation must find a velocity input to the string (injected equally to the left and right) so that the *friction force* is equal at all times to the string's *reaction force* [43, 107]. In this model, bow-hair dynamics [67], and the width of the bow [118, 119] are neglected.

The reflection filter in Fig. 14 summarizes all losses per period (due to bridge, bow, finger, etc., again by commutativity, and nut-side losses may be moved to the bridge side as an additional approximation step). The bow-string junction is typically a *memoryless* lookup table (or segmented polynomial). More elaborate models employ a *thermodynamic model* of bow friction in which the bow rosin has a time-varying viscosity due to its temperature variations within one period of sound [202]. It is well known by bowed-string players that rosin is sensitive to temperature. In [148], J.H.

[18] `http://ccrma.stanford.edu/~jos/waveguide/Sound_Examples.html`

[19] `http://ccrma.stanford.edu/CCRMA/Software/STK/` — version 4.1.2 was current when this article was written.

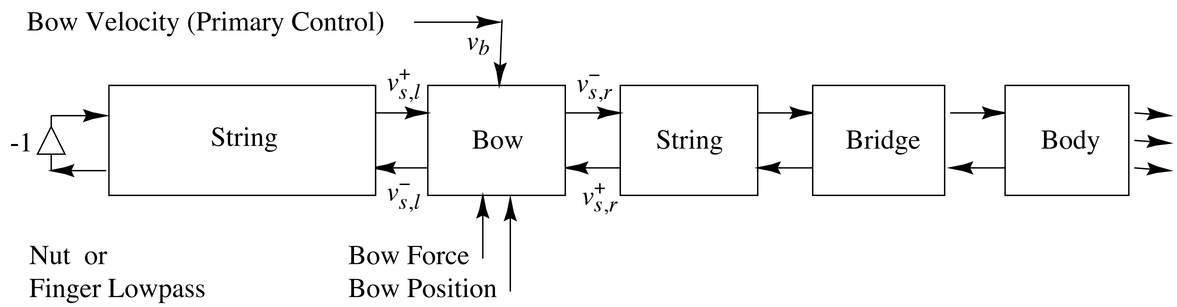

Figure 13: A schematic model for bowed-string instruments.

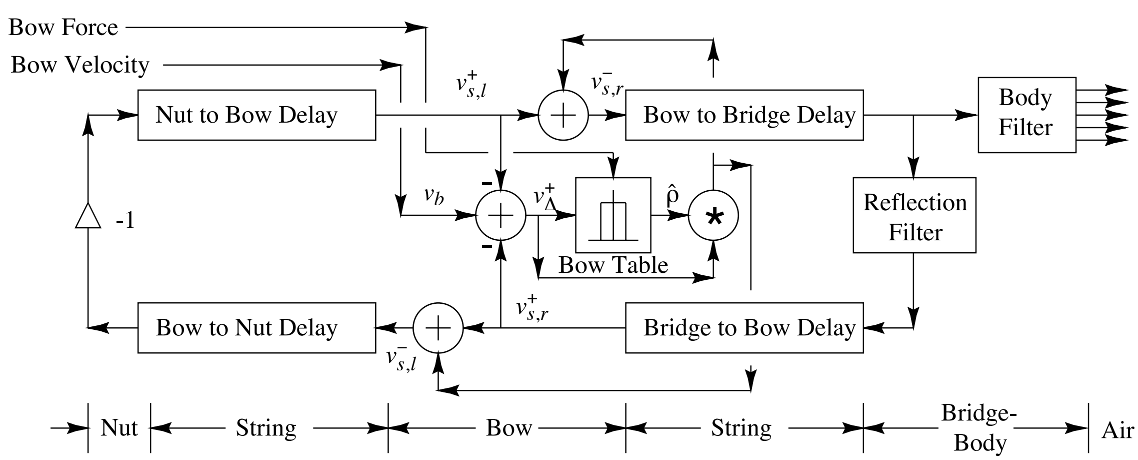

Figure 14: Digital waveguide bowed-string model.

Smith and Woodhouse present research on thermal models of dynamic friction in bowed strings. This major development has been incorporated into synthesis models by Serafin [143, 146, 10].

A real-time software implementation of a bowed-string model similar to that shown in Fig. 14 is available in the Synthesis Tool Kit (STK) distribution as `Bowed.cpp`.

7 Single-Reed Instruments

Figure 15 illustrates an overall schematic for a single-reed woodwind instrument, such as the clarinet, and Fig. 16 shows a particular digital waveguide synthesis model [150, 158].

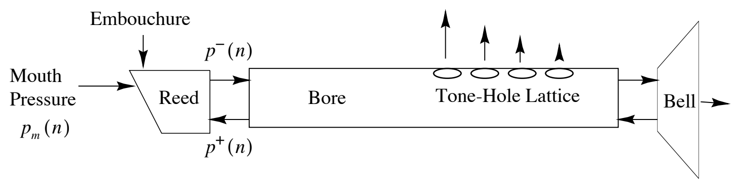

Figure 15: A schematic model for woodwind instruments.

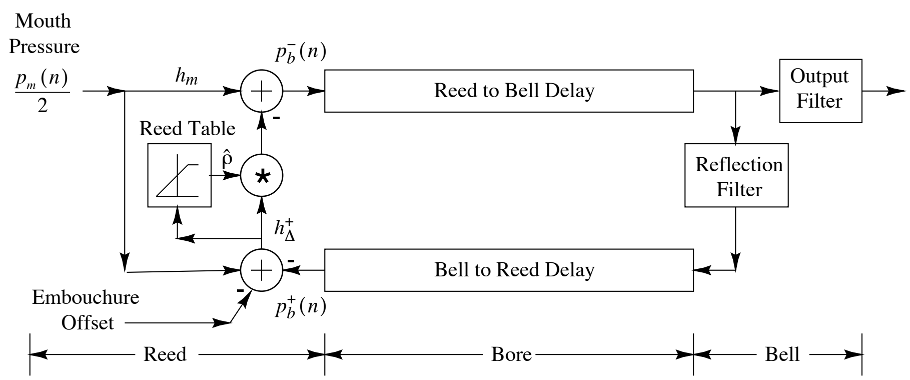

Figure 16: Waveguide model of a single-reed, cylindrical-bore woodwind, such as a clarinet.

The control variable is naturally chosen to be mouth pressure p_m (or more specifically in this case, half of the mouth pressure, $= p_m/2$). A kind of "incoming pressure difference" across the mouthpiece is computed as $h_\Delta^+ = -p_b^+$, and this is used to index a look-up table. (Specifically, h_Δ^+ is half of the pressure drop seen across the reed when the reed turns out to be closed.) The table look-up can also be implemented as a nonlinear function that switches between a third-order polynomial (reed open) and a constant (reed closed). The result of the reed-table look-up is multiplied by h_Δ^+ and subtracted from to form the outgoing traveling-wave component into the bore. The cost of a table look-up implementation such as this is only two subtractions, one multiplication, and the look-up itself, per sample of sound. It is possible to eliminate the multiplication by appropriately reformulating the table, but this requires additional resolution in the table for a given sound quality. Further details of this particular synthesis model for the clarinet are given in [158]. STK software implementing a model as in Fig. 16 can be found in the file `Clarinet.cpp`.

7.1 Bell Models

The bell of a woodwind instrument is generally modeled as a "cross-over filter" in which the reflectance (back down the bore) is a lowpass filter (rolling off above 1500 Hz for a clarinet [16]), and the transmittance (into the surrounding air) is a complementary high-pass filter. Typically, the reflection filter must be designed based on an empirical impulse response measured using, e.g., pulse reflectometry [147], but one can use theoretical approximations as well [100, 141]. See §8 below for discussion of a method for reducing the computational complexity of bell filters.

7.2 Recent Developments

A model of woodwind toneholes for real-time musical performance was developed by Scavone and Cook [139]. See [139, 141, 186] for related developments. A model of interaction between the conical bore and the vocal tract is given in [135].

Recent scientific observation of the clarinet reed [47] has largely verified the simple linear, Bernoulli-flow model wherein reed mass is neglected. (See, e.g., [92] for a review of the acoustics of the clarinet reed.) However, effects of viscosity become important near reed closure, and these have not yet been fully formulated theoretically as a function of normal playing conditions (embouchure, mouth pressure), let alone fully verified by experiment, although much has been measured and described [76, 75, 77, 197].

Van Walstijn and Campbell have published a recent paper on woodwind modeling [185] using a hybrid approach which combines a digital waveguide model of the bore with a "wave digital" model [60] of the mouthpiece. In more recent work [9], Avanzini and van Walstijn developed a model for the clarinet mouthpiece with particular attention to how the reed closes along the mouthpiece lay; these dynamic details have a large effect on the spectrum of the tone produced, and a sequel to this paper is concerned specifically with this aspect [184].

In the commercial world, a recent offering for "virtual wind players" is the "Patchman Turbo VL" upgrade chip for the Yamaha VL70-m.[20] The VL70-m is a rack-mount descendent of the Yamaha VL-1, introduced in 1994, that has accumulated a modest but apparently devoted user base among wind players. The Patchman upgrade is aimed specifically at wind players utilizing wind controllers such as the Yamaha WX7 et al.

8 Brasses

Brass instruments consist of a mouthpiece enclosing the lip-valve, an acoustic tube, and finally a rapidly flaring bell. Additionally, most brass instruments have valves which switch among different tube lengths.

The mouthpiece defines the precise geometry of the lip opening as well as providing partial terminations against which the lips work. For lip-valve modeling, one highly regarded model is that of S. Adachi [2, 3, 61] in which a mass modeling a lip follows an orbit in 2D, with different eccentricities at different frequencies. Such motion of brass players' lips has been observed by Copley and Strong [42] and others. One- and two-mass models were also developed by Rodet and Vergez [127, 190]. An early one-mass, "swinging door" model that works surprisingly well is in Cook's `HosePlayer` [38] and and `TBone` [37]. It is also used in the `Brass.cpp` patch in the Synthesis

[20] `http://www.patchmanmusic.com/turbovl.html`

Tool Kit [40]; in this simplified model, the lip-valve is modeled as a second-order resonator whose output is squared and hard-clipped to a maximum magnitude.

The acoustic tube can be modeled as a simple digital waveguide [11, 15, 153, 175, 187], whether it is cylindrical or conical. More complicated, flaring horn sections may be modeled as two-port digital filters [17, 23, 30, 53, 141, 187]. It is known, however, that nonlinear shock waves develop in the bore for large amplitude vibrations [78, 116]. A simple nonlinear waveguide simulation of this effect can be implemented by shortening the delay elements slightly when the instantaneous amplitude is large. (The first-order effect of air nonlinearity in large-amplitude wave propagation is an increase in sound speed at very high pressures [24].)

It is also known that the bores of "bent horns," such as the trumpet, do not behave as ideal waveguides [147]. This is because sharp bends in the metal tube cause some scattering, and mode conversion at high frequencies. Such points along the horn can be modeled using high-frequency loss and reflection.

As in the case of woodwinds, the flaring bell of a horn cannot be accurately modeled as a sparse digital waveguide, because traveling pressure waves only propagate without reflection in conical bores (which include cylindrical bores as a special case) [121].[21] Digital waveguides are "sparse" (free of internal scattering) only when there are long sections at a constant wave impedance.

Since the flare of most brass instruments occurs over a significant distance, the reflection impulse response of the bell is typically quite long. For example, the impulse response of a trumpet bell is hundreds of samples long at typical audio sampling rates [187]. Perhaps the most straightforward approach is to model the bell as a finite-impulse-response (FIR) digital filter. However, such an FIR filter requires hundreds of "taps" (coefficients), and is thus prohibitively expensive compared with everything else in a good synthesis model. It is well known that infinite-impulse response (IIR) filters can be much less expensive [114, 122], since they use poles and zeros instead of only zeros, but bell impulse responses turn out to pose extremely difficult IIR filter design problems [187]. The main source of difficulty is that the initial impulse response, corresponding to building "back-scatter" as the horn curvature progressively increases, appears to be *rising* quasi-exponentially over hundreds of samples. The natural solution for this provided by many filter design programs is an *unstable* digital filter. Methods that guarantee stability have been observed to suffer from poor numerical conditioning. The most cost-effective solution to date appears to be the use of *truncated IIR* (TIIR) digital filters [194]. These filters use an unstable pole to produce exponentially rising components in the impulse response, but the response is cut off after a finite time, as is needed in the case of a bell impulse response. By fitting a piecewise polynomial/exponential approximation to the reflection impulse response of the trumpet bell, very good approximations can be had for the computational equivalent of approximately a 10th order IIR filter (but using more memory in the form of a delay line, which costs very little computation).

In more detail, the most efficient computational model for flaring bells in brass instruments seems to be one that consists of one or more sections having an impulse response given by the sum of a growing exponential and a constant, i.e.,

$$y(n) = \begin{cases} ae^{cn} + b, & n = 0, 1, 2, \dots, N-1 \\ 0, & \text{otherwise.} \end{cases}$$

The truncated constant b can also be generated using a one-pole TIIR filter, with its pole set to $z = 1$. The remaining reflection impulse response has a decaying trend, and can therefore be

[21] For velocity waves, the flare may be *hyperbolic* [23].

modeled accurately using one of many conventional filter design techniques. In [187, 188], the Steiglitz-McBride IIR filter design algorithm [101] yielded good results, as shown in Fig. 18.

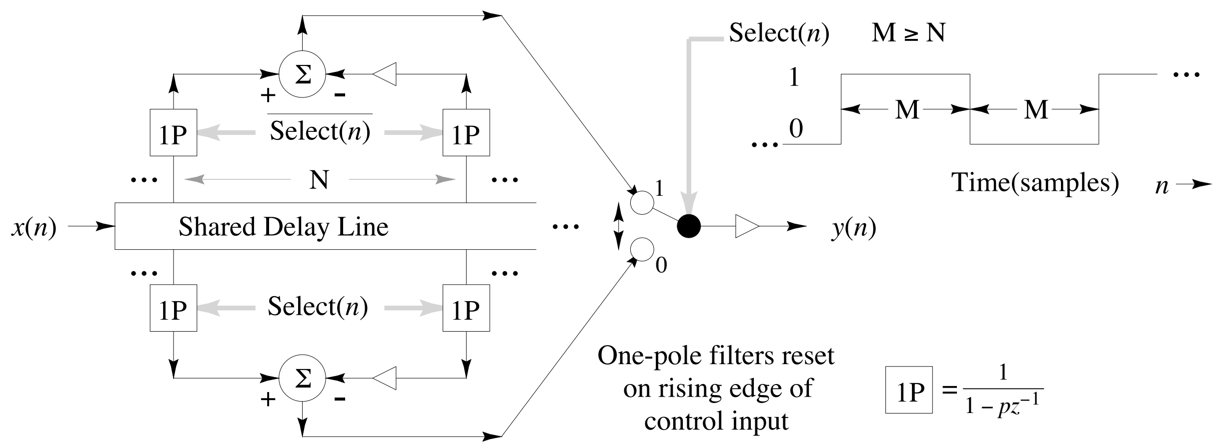

Figure 17: Example of a TIIR filter for generating a growing exponential or constant segment (from [187]).

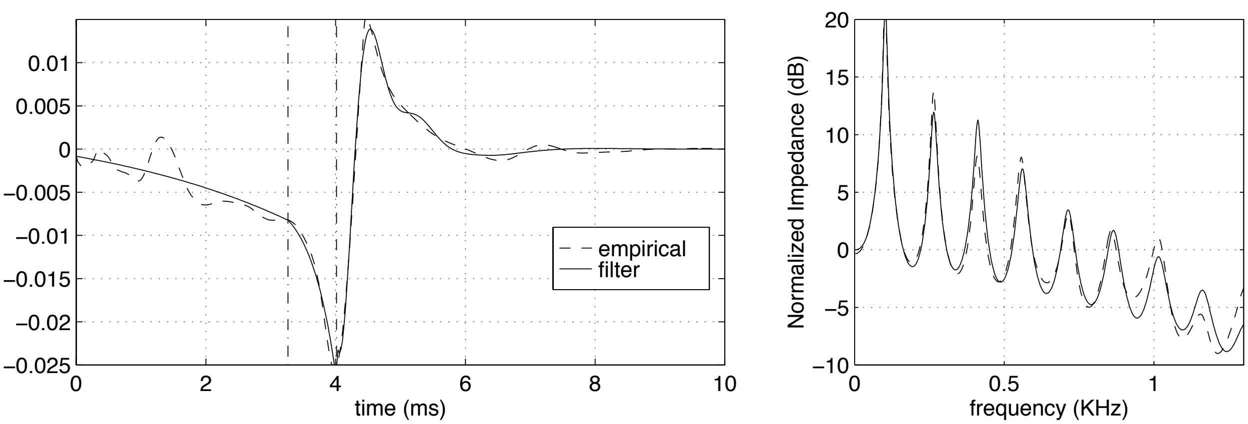

Figure 18: Impulse-response and driving-point-impedance fit for the trumpet bell using two offset exponentials and two biquads designed as a 4th-order IIR filter using the Steiglitz-McBride algorithm (from [187]). The dot-dashed lines show the model segment boundaries.

In summary, a cost-effective synthesis model for brasses includes a careful finite-difference lip-valve model, a digital waveguide bore, and a TIIR bell filter.

8.1 Recent Developments

A summary of our state of knowledge regarding the musical acoustics of brass instruments was recently given by Campbell [29].

Recent research by Cullen et al. [44] and Gilbert et al. [64, 65] has been concerned with artificial mouth experiments to verify a theoretical model of vibrating lips, and to calibrate the model to realistic playing conditions. These data can be used in the construction of improved lip-valve models for virtual brass instruments.

Additional recent literature relevant to brass instruments includes [71, 70, 72, 91, 188, 111, 113, 115, 191, 192, 193].

9 Related Topics

This section lists some recommended starting points in the literature on instruments omitted due to space limitations.

9.1 Flutes, Recorders, and Pipe Organs

A chapter on the fundamental aero-acoustics of wind instruments has been written by Hirschberg [75], and a comprehensive treatment of the acoustics of air jets in recorder-like instruments is given in the 1995 thesis of Verge [189].

A comprehensive review article on lumped models for flue instruments was contributed by Fabre and Hirschberg [58] in a special issue (July/August 2000) of the *Acustica* journal on "musical wind instrument acoustics." An overview of research on woodwinds and organs was recently presented by Fabre [57]. Follow-up publications by this research group include papers concerning the influence of mouth geometry on sound production in flue instruments [48, 142].

9.2 Percussion Instruments

While sample-playback synthesis is especially effective for percussion instruments that are supposed to play "in the background," some form of parametrization is needed for the more expressive performances, or for highly variable percussion instruments such as the Indian tabla. Highly efficient computational models of 2D membranes and 3D volumes may be built using the *digital waveguide mesh* [62, 182, 180, 183]. More recently, Lauri and Välimäki have developed a frequency warping approach to compensating for dispersive wave propagation in a variety of mesh types [132, 133, 134]. The 2001 thesis of Bilbao [25] provides a unified view of the digital waveguide mesh and *wave digital filters* [60] as particular classes of energy invariant finite difference schemes [168]. The problem of modeling diffusion at a mesh boundary was addressed by Laird et al. [96].

The waveguide mesh is also useful for *artificial reverberation* [131] and modeling body resonances in stringed musical instruments [79, 145].

9.3 Voice Synthesis

Hui-Ling Lu, in her 2002 thesis [103], developed a model for the singing voice in which the driving glottal pulse train is estimated jointly with filter parameters describing the shape of the vocal tract (the complete airway from the base of the throat to the lip opening). The model can be seen as an improvement over linear-predictive coding (LPC) of voice in the direction of a more accurate physical model of voice production, while maintaining a low computational cost relative to more complex articulatory models of voice production. In particular, the parameter estimation involves only convex optimization plus a one-dimensional (possibly non-convex) line search over a compact interval. The line search determines the so-called "open quotient" which is fraction of the time there is glottal flow within each period. The glottal pulse parameters are based on the derivative-glottal-wave models of Liljencrants, Fant, and Klatt [59, 93]. Portions of this research have been published in the ICMC-00 [104] and WASPAA-01 [105] proceedings.

Earlier work in voice synthesis includes [18, 33, 35, 39, 80, 93, 128, 169]; see also the KTH "Research Topics" home page.[22]

10 Model Parameter Estimation

The important problem of *parameter estimation* for computational models can be considered to intersect with the field of *system identification* [102, 149]. Some references pertaining to model parameter identification from measured acoustic data are summarized below, roughly in chronological order.

Applications of methods for digital filter design and system identification to violin modeling were developed in [149], and an expanded paper regarding filter design over a Bark scale is given in [161]. A review paper on frequency-warped signal processing for audio applications is given in [69].

An interesting combined linear and nonlinear approach to modeling woodwinds by Cook and Scavone [36, 136] still appears promising today.

A method for estimating traveling-wave components from multiple non-uniformly spaced measurements of physical pressure in a bore was described in [166].

Methods for designing piano-string dispersion filters were reviewed and extended in [126], and recent relevant work includes [7, 20, 19, 22, 83, 97].

In his 1996 thesis [147], Sharp describes his system for measuring horn reflectance via acoustic pulse reflectometry. This technique has been used to obtain data, *e.g.*, for calibrating trumpet models [187].

Computational models of woodwind toneholes have been developed Scavone et al. [137, 138, 139, 186, 140, 163] based on models from musical acoustics by Keefe [89, 90]. More recent work on the musical acoustics of woodwind tone holes has been carried out by Dubos et al. [52] and Nederveen et al. [112], and experimental verifications have been conducted by Dalmont et al. [45, 46]. A novel filter architecture and associated design procedure for acoustic tube loss modeling is described by Abel et al. [1].

Methods for the identification of control parameters for a trumpet model from the acoustic waveform ("model inversion") have been developed by Hélie, D'haes, and Rodet, et al. [73, 50, 51].

In the Laboratory of Acoustics and Audio Signal Processing at the Helsinki University of Technology, many techniques have been developed for identifying the parameters of physical models of musical instruments. Most of these are available on their website (`http://www.acoustics.hut.fi`). Publications from this lab include [177, 176] (calibration of a guitar synthesizer), [54] (acoustical analysis and model-based sound synthesis of the kantele), [12, 13] (piano modeling, loss-filter design), [84] (bell-like sounds), [82] (audibility of string partial overtone tuning), and [174] (clavichord modeling).

It is generally very difficult to measure the dynamic frictional contact between a bow and string. However, it has recently been done indirectly by Woodhouse, Schumacher, and Garoff [203]. In this research, a digital waveguide model is *inverted* to estimate the friction force on the string by the bow, as well as the string velocity at the bowing point.

[22] `http://www.speech.kth.se/music/music\protect\unhbox\voidb@x\kern.06em\vbox{\hrulewidth.3em}research\protect\unh`

CCRMA Ph.D. student Krishnaswamy has recently developed an effective method for identifying selected control parameters in violin performance from measured audio spectra [94]. The idea of the method is to develop a forced classification scheme based on short-time power spectra indexed by pitch class. In other words, for each pitch class, a small linear-discriminant-analysis database is developed which returns the estimated playing parameters (bow position, which string was played, whether plucked or bowed) as a function of the measured power spectrum. Related work on estimating the plucking point along a guitar string based on recorded acoustic data was carried out by Traube et al. [172].

Derveaux and Chaigne have developed detailed time-domain simulations of the acoustic guitar [49], including finite element models of the guitar body as well as the surrounding air. Such detailed numerical simulations, while computationally demanding, can enable "virtual acoustic experiments" that can be used to calibrate simpler real-time methods along the lines discussed above. See also [198, 199].

11 Conclusions and Further Reading

This article provided an introductory overview, summary of recent advances, and a number of pointers into the research literature relevant to computational modeling of acoustic musical instruments for purposes of musical performance in real time. We close with some pointers to recently published books in this area.

A recent book by Cook [41] is devoted to sound synthesis related to physical models. It is especially useful for getting going quickly with effective and practical synthesis models. While Cook is also the principal author of the Synthesis Tool Kit (STK), the book is written independently of the STK.

A set of three interlinked books on physical modeling synthesis (primarily digital waveguide synthesis) and associated signal processing has been published to the Web in HTML format [158, 159, 160]. Much of this article can be viewed as a summary of portions of [158].

11.1 Acknowledgements

Thanks to the reviewers for many helpful comments on earlier drafts of this paper.

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