MUS320A&B: Introduction to Digital Audio Signal Processing

Center for Computer Research in Music and Acoustics (CCRMA)
Department of Music, Stanford University

320A (spectra): Autumn Quarter
320B (filters): Winter Quarter

2020–2021

Contents

Music 320 A & B: Introduction to Digital Audio Signal Processing

1 Course Description

Music 320 is a two-quarter first-course in digital signal processing with applications in computer music and audio.

The lectures present fundamental elements of digital audio signal processing, such as sinusoids, spectra, the Discrete Fourier Transform (DFT), digital filters, z transforms, transfer-function analysis, and basic Fourier analysis in the discrete-time case. Matlab is used for in-class demonstrations and homework/lab assignments. The labs focus on practical applications of the theory, with emphasis on working with waveforms and spectra, "getting sound", and developing proficiency in the matlab language.

Prerequisites: High-school level algebra and trigonometry, some calculus, and prior exposure to complex numbers.

Time and Place

Term:	Autumn and Winter Quarters
Location:	CCRMA Classroom (Knoll 217)
Lectures:	Mondays and Wednesdays 2:30–3:50 PM
Units:	3
Instructors:	Aut (320A): Marina Bosi (mab@ccrma.stanford.edu) Win (320B): Jonathan Abel (abel@ccrma.stanford.edu) and Dave Berners (dpberner@ccrma.stanford.edu)
TA:	TBD
Office Hours:	See "Office Hours and Getting Help"[1] below

2 Administrative Information

2.1 Announcements

Class announcements are often made via *email*. For this we are presently using Piazza:

`https://piazza.com/stanford/fall2020/music320a/home`

If you signed up for the class in axess before the first day of classes, you should receive an invitation from Piazza to join the class (using the email address known to axess). Otherwise, please join by visiting the above URL and entering your preferred email address.

[1] `http://ccrma.stanford.edu/~jos/intro320/Office_Hours_Getting_Help.html`

2.2 Assignments

There are five homework/lab assignments, each covering roughly two weeks of the course. In each two-week "section", the first week is devoted primarily to theory while the second week is focused more on software and applications. Thus, each assignment contains both a theory and laboratory part. The lab portion typically requires programming in matlab.

Each assignment is typically announced on Thursday during the first week of the section. The theory part is normally due the following Thursday at class time, 3 pm, in the 320 mailbox at CCRMA. The lab part is normally due by midnight the following day, i.e., at the end of the two-week section. Available homework time after turning in the assignments should be devoted to viewing the lecture videos for the next section.

For lab assignments, we will be using the Canvas[2] website. To sign up, go to the Canvas website and find Music320B. Once you are enrolled in the class, you can upload your matlab files in the "drop box" on the left menu.

See §2.3 below regarding obtaining help with theory and lab assignments.

Regarding late homeworks, 7 free late days are allowed (with hours rounded up to the nearest day). Late homeworks beyond this will be penalized at 5% per day. When using late days, write the number of late days used at the top of the assignment (date and time).

Students are encouraged to discuss the homework assignments with each other. It is fine to learn from a classmate how to solve any of the homework problems, but each student is responsible for carrying out and writing up the assignments individually. It is an honor code violation to *copy* the work of others.

2.3 Office Hours and Getting Help

We will be using Piazza[3] for sharing answers to posted questions with the whole class. To sign up, see the 320 Piazza site.[4] It is free and allows you to view past questions from other students, and discuss questions together. Try it first for any homework questions you may have.

TA weekly office hours are TBD in the TBD.

3 Textbooks

Music 320A (fall) is based on assigned chapters of

Mathematics of the Discrete Fourier Transform (DFT),[5] by Julius O. Smith

Music 320B (winter) is based on assigned chapters of

Introduction to Digital Filters,[6] by Julius O. Smith

See §4 for the list of assigned chapters. Both books are fully available on-line. Softcover versions are available from Amazon.com.

[2] https://canvas.stanford.edu
[3] https://www.piazza.com
[4] https://piazza.com/stanford/fall2020/music320a/home
[5] http://ccrma.stanford.edu/~jos/mdft/
[6] http://ccrma.stanford.edu/~jos/filters/

4 Schedule and Pointers

Note: The online version[7] of this schedule contains hyperlinks to all reading and assignments.

To obtain printable versions of the assignments and solutions from off-campus locations, you can use commands such as

```
scp you@ccrma-gate.stanford.edu:/usr/ccrma/web/html/courses/320/hw/hw1x/hw1x.pdf .
scp you@ccrma-gate.stanford.edu:/usr/ccrma/web/html/courses/320/hw/hw1x/hw1xsol.pdf .
```

where `you` refers to your CCRMA login, and `x` is `a` for 320A and `b` for 320B.

You can alternatively use VPN[8] (Virtual Private Network) access.

For more info, see `https://ccrma.stanford.edu/guides/remoteaccess/`.

Music 320A

4.1 Section 1: Course Overview, Signal Math, Intro to Matlab

A "section" is typically two weeks in duration, with the first week devoted primarily to theory, and the second primarily to software and applications.

- **Reading**
  - This course overview[9]
  - Chapter 1 (DFT Intro)[10] of **Mathematics of the DFT**.[11]
  - If you are not comfortable with the decibel scale, read Appendix B (Logarithms and Decibels)[12]
  - **Assignment 1**[13] (complex number problems)
- Supplementary Demos, Reading, and Exercises

  Our main task is to approximate any signal $x(t)$ as a sum of sinusoids $s_\omega(t)$, where the term *sinusoid* refers to cosine (or sine) having any amplitude A and any phase offset ϕ:

  $$s_\omega(t) = A\cos(\omega t + \phi)$$

  We call ω the *radian frequency* (frequency in radians per second), and ϕ the *phase* of the sinusoid.

  The connection to complex numbers is via Euler's Identity, which can be used to show

  $$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

  The supplementary material below pertains to both sinusoids and complex numbers:

[7] `https://ccrma.stanford.edu/~jos/intro320/Lectures_Assignments.html`
[8] `https://uit.stanford.edu/service/vpn`
[9] `https://ccrma.stanford.edu/courses/320/`
[10] `https://ccrma.stanford.edu/~jos/mdft/Introduction_DFT.html`
[11] `https://ccrma.stanford.edu/~jos/mdft/mdft.html`
[12] `https://ccrma.stanford.edu/~jos/mdft/Logarithms_Decibels.html`
[13] `https://ccrma.stanford.edu/~jos/hw320/`

- But what is a Fourier series? From heat flow to circle drawings — DE4[14]
- Animation of the Ear doing Spectrum Analysis[15]
- Brilliant demo of *aliasing*[16]
- "The Acoustic Origins of Harmonic Analysis[17]" by Olivier Darrigol treatment (published in the **Archive for History of the Exact Sciences**, vol. 61, no. 4, July 2007)
- "History of Virtual Musical Instruments and Effects Based on Physical Modeling Principles[18]" by JOS, DAFx-2017
- Interactive Tutorial on Sound, Spectra, and Additive Synthesis[19]
- Discrete Fourier Transform Demo (Truncated Sinc Spectrum)[20]
- Building up a Spectrum Analyzer in WebGL[21]
- For an educational Matlab GUI on *sinusoids*, download sinedrill[22] from the Educational Matlab GUIs[23] collection at Georgia Tech. There are other nice Matlab-based exercises that can use later in the quarter and next quarter.
- There is another Matlab GUI illustrating *Fourier series* approximations, *i.e.*, using sums of sinusoids to approximate classic waveforms such as square wave, sawtooth, and triangle. Download fseriesdemo[24] from the Educational Matlab GUIs[25] collection at Georgia Tech.
- Interactive demos for Basic physics and signal-processing demos[26] by `java@falstad.com`
  - In particular, here is nice interactive applet for seeing and optionally hearing the *spectra of basic waveforms*[27] in your web browser
  - Interactive tool for seeing and hearing classic *digital filters*[28] (mostly the subject of winter quarter, but we start FIR filters this quarter)

- Chapter 2 (Complex Numbers)[29]
- If you need more practice with complex numbers, work some Khan Academy exercises[30]
- For a Matlab GUI providing complex-number drills, download zdrill[31] from the Educational Matlab GUIs[32] collection at Georgia Tech.

[14] https://www.youtube.com/watch?v=r6sGWTCMz2k
[15] https://youtu.be/dyenMluFaUw
[16] https://observablehq.com/@cscheid/aliasing
[17] http://ocw.nctu.edu.tw/course/fourier/supplement/harmonic_history.pdf
[18] https://ccrma.stanford.edu/~jos/pdf/DAFx17-keynote1-jos.pdf
[19] https://pudding.cool/2018/02/waveforms/
[20] http://madebyevan.com/dft/
[21] https://acko.net/files/gltalks/toolsforthought/#29
[22] http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/sindrill-v209.zip
[23] http://users.ece.gatech.edu/mcclella/matlabGUIs/
[24] http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/fseriesdemo-v130.zip
[25] http://users.ece.gatech.edu/mcclella/matlabGUIs/
[26] http://www.falstad.com/mathphysics.html
[27] http://www.falstad.com/fourier/
[28] https://www.falstad.com/dfilter/
[29] https://ccrma.stanford.edu/~jos/mdft/Complex_Numbers.html
[30] https://www.khanacademy.org/math/algebra2/complex-numbers-a2
[31] http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/zdrill-v210.zip
[32] http://users.ece.gatech.edu/mcclella/matlabGUIs/

- Here also are some prerequisite-level Khan Academy exercises on dealing with polynomials[33] (you can skip "Synthetic Division" and stop before "Partial Fraction Expansions" since we get to that in 320B next quarter).

- For more advanced math studies, beyond what's needed for this course, but needed for more advanced signal processing: Georgia Tech Online Mathematics Textbooks[34]

- **Lecture Videos:**

  IMPORTANT NOTICE: The videos are hosted on YouTube and they use *annotations* for corrections and supplementary information. These annotations are *not supported on mobile devices*. It is therefore unfortunately important to view these videos *in a Web browser on a desktop/laptop computer*.

  - Music 320A Overview (first class recording in fall 2014)[35]
    Introductory Demonstrations for 320A *and* 320B[36]
  - Administrative overview (this document[37]), discussed *only* in class (no video), so *definitely read it* if you missed the first class
  - Intro to the Fourier Transform (FT) and the Discrete Fourier Transform (DFT)[38] [30:52]
  - 3Blue1Brown S4 E4: "But what is a Fourier series? From heat flow to circle drawings"[39] [24:46]
  - Albert Michelson's Harmonic Analyzer[40] [First three: 3:30+5:00+3:30]
  - Euler's Identity, Complex Sinusoids[41] [9:43]
  - Complex Plane Intro[42] [4:53]
  - Euler's Identity Corollaries[43] [12:38]
  - Review of DFT and Euler Identity Intro presented 10/02/2014[44]
  - Introduction to Piazza, Coursework, and Matlab[45] [20:08]
    * Regarding the Matlab intro, if you do not know the rules of matrix multiplication, read Appendix H (Matrices).[46]
    * Matlab Documentation[47]

[33] https://www.khanacademy.org/math/algebra2/polynomial_and_rational
[34] http://people.math.gatech.edu/~cain/textbooks/onlinebooks.html
[35] https://www.youtube.com/watch?v=jo5qZk7zB3I&list=PLsBNrwwINMkYHSpb5pwq2rjzg3RyXysPY
[36] https://www.youtube.com/watch?v=hSkdaqNQH8M
[37] https://ccrma.stanford.edu/courses/320/
[38] https://www.youtube.com/watch?v=GW6654dYTD8
[39] https://www.youtube.com/watch?v=r6sGWTCMz2k
[40] https://www.youtube.com/watch?v=NAsM30MAHLg&list=PL0INsTTU1k2UYO9Mck-i5HNqGNW5AeEwq
[41] https://www.youtube.com/watch?v=ugrZAu6f6qQ
[42] https://www.youtube.com/watch?v=gC4L_rUWbuw
[43] https://www.youtube.com/watch?v=1zWE0-A8WHg
[44] https://www.youtube.com/watch?v=kF1BeOq-en0&list=PLsBNrwwINMkaCgKJD0rnx-_jPm8r5Cdpp
[45] https://www.youtube.com/watch?v=uLPCz2shnNM
[46] https://ccrma.stanford.edu/~jos/mdft/Matrices.html
[47] http://www.mathworks.com/help/matlab/index.html

- * Read the first two sections of Appendix J (Matlab Examples)[48] in **Mathematics of the DFT**
  * Do **Lab Assignment 0**[49] if you are new to Matlab.

[URL footnotes are mostly suppressed below, but links persist in the online version of this page.]

4.2 Section 2: Euler's Formula, Exponentials, Sinusoids, and Spectra

- **Reading:**
  - Chapter 3 (Proof of Euler's Theorem)[50] of **Mathematics of the DFT**
  - Chapter 4 (Sinusoids and Exponentials)[51]
  - Assignment 2
- **Lecture Videos (Total Viewing Time ≈ 4 Hours):**
  - Euler's Identity Proof[52] [30:21]
  - DFT Overview, Euler's Identity on the Complex Plane, Additive Synthesis of a Square Wave by Fourier's Theorem[53] [12:03]
  - Additive Synthesis of a Square Wave at Codepen
  - Sinusoidal Motion as Projection of Circular Motion[54] [2:08]
  - Sinusoidal Motion as Projection of Circular Motion, Continued[55] [0:53]
  - Mathematics of Sinusoids, Complex Sinusoids[56] [10:30]
  - Spectra of Sinusoids[57] [15:43]
  - Amplitude Modulation (AM) and its Spectral Effects, AM Demo, "Ring Modulation," Perception of AM, "Beats" demo in Matlab (beatcon.m), Hearing Mechanics, Critical Bands[58] [49:33]
  - Editable Online Javascript for Additive Synthesis Visualized[59]
  - Additive Synthesis Visualized with Audio and More Controls[60]
  - 3D Animation of Euler's Identity (GIF)

[48] https://ccrma.stanford.edu/~jos/mdft/Matlab_Octave_Examples.html
[49] https://ccrma.stanford.edu/~jos/hw320/
[50] https://ccrma.stanford.edu/~jos/mdft/Proof_Euler_s_Identity.html
[51] https://ccrma.stanford.edu/~jos/mdft/Sinusoids_Exponentials.html
[52] https://www.youtube.com/watch?v=WTbp2MpoF48
[53] https://www.youtube.com/watch?v=kR1CebAZy9M
[54] https://www.youtube.com/watch?v=StcvBa1solQ
[55] https://www.youtube.com/watch?v=obtGcTM2VwA
[56] https://www.youtube.com/watch?v=O9mivztKN3M
[57] https://www.youtube.com/watch?v=CxP_CuZfmDA
[58] https://www.youtube.com/watch?v=bhvSralHoWo
[59] http://codepen.io/anon/pen/jPGJMK
[60] http://bgrawi.com/Fourier-Visualizations/

- Circular Motion Projection and Superposition[61] [1:43 (Kitty)]
- Time Constant of Decay, Plotting Exponentials[62] [1:00]
- Euler's Identity in the Complex Plane[63] [5:10]
- Supplementary: Euler's Identity Viewed as "Actions"[64] [6:13]
- Generalized Complex Sinusoids[65] [6:09]
- Generalized Complex Sinusoids II[66] [6:56]
- Demo of Exponentially Decaying Sinusoids in Matlab, Spectrograms, DTMFs[67] [9:01 (Kitty)]
- Making a sinusoid in Pure Data (Pd)[68] [6:23]
- Drawing the word "minimum" in a spectrogram using various vocalizations[69] [5:21]
- Exponentials[70] [13:03]
- AM Review, Frequency Modulation (FM)[71] [12:28]
- FM Spectra, Bessel Functions[72] [18:09]
- Generalized Complex Sinusoids Review, The S-Plane[73] [11:03]
- Laplace Transforms[74] [5:27]
- Sampled Generalized Complex Sinusoids[75] [5:00]
- Sampled Generalized Complex Sinusoids II, Z Transforms[76] [7:39]
- Domain of Z Transforms, The Sampled Sinusoid j^n, Normalized Frequency[77] [5:57]
- Mapping s to z, Bilinear Transform Preview[78] [6:44]
- Overview and Demos Presented 10/9/2014: Sinusoidal AM Demos, Continuous/Discrete Fourier Transforms (Four Cases), Generalized Sinusoids, Laplace Transform, Z Transform[79] [10:40]
- You should now understand the twist in the complex-plane versus time display used in the WebGL Spectrum Analyzer[80]

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[61] https://www.youtube.com/watch?v=DQxg8xrJr5c
[62] https://www.youtube.com/watch?v=NrN2Sn1vEaA
[63] https://www.youtube.com/watch?v=_zf_hYgiR4U
[64] https://www.youtube.com/watch?v=F_0yfvm0UoU
[65] https://www.youtube.com/watch?v=gNtHF6aiNJ8
[66] https://www.youtube.com/watch?v=J0h0ml2sr7s
[67] https://www.youtube.com/watch?v=3hFzAGWwuDE
[68] https://www.youtube.com/watch?v=SLIOTncnqpU
[69] https://www.youtube.com/watch?v=PHIGke6Yzh8
[70] https://www.youtube.com/watch?v=wAkRVbOZZlg
[71] https://www.youtube.com/watch?v=WbLGL3bv1Lo
[72] https://www.youtube.com/watch?v=FMdUjLn3JCA
[73] https://www.youtube.com/watch?v=EksjJGldZ4M
[74] https://www.youtube.com/watch?v=XOKcnUFYT1E
[75] https://www.youtube.com/watch?v=xPTi4HVe300
[76] https://www.youtube.com/watch?v=BkDL5SDRiP4
[77] https://www.youtube.com/watch?v=Vq2ioQqcy7k
[78] https://www.youtube.com/watch?v=109KPuzIA-k
[79] https://www.youtube.com/watch?v=uW9nXIZ4FPM&list=PLsBNrwwINMkYoShkyGEdMc3r7af_Trjo2
[80] https://acko.net/files/gltalks/toolsforthought/#29

- Supplementary flipped-class lecture on the meanings of points in the s and z planes as generators of generalized complex sinusoids, and derivation of the Laplace and Z Transforms via projection onto them:
  1. Part 1/2[81] [45:26]
  2. Part 2/2[82] [47:05]

4.3 Section 3: Vectors, Geometric Signal Theory, Orthogonal Projection

- **Reading:**
  - Chapter 5 (Geometric Signal Theory) of **Mathematics of the DFT**
  - Chapter 6 (The DFT Derived)
  - Assignment 3
- **Lecture Videos (Total Viewing Time $\approx$ 2 Hours):**
  - Geometric Signal Theory[83] [25:28]
  - Preview of Orthogonal Projection[84] [3:12]
  - DFT Review, Orthogonal Projection[85] [15:36]
  - Orthogonal Projection Example[86] [4:12]
  - Orthogonal Projection onto Coordinate Axes[87] [3:38]
  - Reconstruction by Summing Orthogonal Projections, Change of Coordinates in Signal Space, Spanning N-Space with N Orthogonal Vectors, Linear Independence, Length 2 DFT[88] [22:34]
  - You should now understand how the twisted signal plot "pancakes" to produce the DFT displayed on the complex-plane at time zero in the WebGL Spectrum Analyzer[89]
  - Inverse DFT as a Sum of Projections, Normalized DFT, Gram-Schmidt Orthogonalization[90] [3:34]
  - Nth Roots of Unity W_N^k, Twiddle Factors[91] [2:21]
  - Nth Roots of Unity for $N = 8$[92] [12:01]
  - DFT Sinusoids[93] [4:55]
  - Review of DFT Sinusoid Orthogonality, Normalization, DFT as Inner Product, IDFT as Sum of Projections[94] [4:01]

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[81] https://www.youtube.com/watch?v=6QNhOXj2mj0
[82] https://www.youtube.com/watch?v=x8wszWHDlec
[83] https://www.youtube.com/watch?v=nUNroez07y4
[84] https://www.youtube.com/watch?v=2JnpxzFPCO0
[85] https://www.youtube.com/watch?v=QYy4FZo9wiw
[86] https://www.youtube.com/watch?v=J-06LS2wggQ
[87] https://www.youtube.com/watch?v=yEggett4Mx4
[88] https://www.youtube.com/watch?v=pqF6RUdcGQw
[89] https://acko.net/files/gltalks/toolsforthought/#29
[90] https://www.youtube.com/watch?v=IgQvn_GxRqk
[91] https://www.youtube.com/watch?v=v3jMHmRnlYM
[92] https://www.youtube.com/watch?v=-pvMt1YGntA
[93] https://www.youtube.com/watch?v=8IU-HBNXYLM
[94] https://www.youtube.com/watch?v=3yL55HQevkg

- Inverse DFT Derived[95] [1:34]
- DFT of DFT-Sinusoids, DFT of Non-DFT-Sinusoids, Geometric Series Closed Form, Magnitude Spectrum, Phase Spectrum, Sinc Function, Aliased Sinc Function, Bins, Zero Padding, Windowing[96] [19:13]
- You should now understand how the phase-rotated signal in the complex-plane versus time display is summed to the green front complex-plane in the WebGL Spectrum Analyzer[97]
- Supplementary flipped-class lecture on the inner product and its applications:
  1. Part 1/2[98] [49:45]
  2. Part 2/2[99] [47:22]
  3. Image Credits:
     (a) Deep Neural Net: `https://i.stack.imgur.com/OH3gI.png`
     (b) FFT Butterfly: `http://www.cmlab.csie.ntu.edu.tw/cml/dsp/training/coding/transform`

[95] https://www.youtube.com/watch?v=cqVcSb608z0
[96] https://www.youtube.com/watch?v=LlXeZWKgu6Y
[97] https://acko.net/files/gltalks/toolsforthought/#29
[98] https://www.youtube.com/watch?v=V1VqvTEGgf8
[99] https://www.youtube.com/watch?v=rSfX5h72x38

4.4 Section 4: Fourier Theorems, Convolution and Correlation

- **Reading:**
  - Chapter 7 (Fourier Theorems for the DFT) of **Mathematics of the DFT**
  - Appendix B (Fourier Transforms for Continuous/Discrete Time/Frequency)
  - Filters and Convolution
  - Assignment 4

- **Lecture Videos (Total Viewing Time $\approx$ 4 Hours):**
  - Signal and Spectra Notation for the DFT Theorems[100] [5:14]
  - Review of DFT as a change of coordinates, Periodic Extension versus Time Limited Signal Windows, DFT interpolation between spectral samples, "spectral splatter" in DFT of non-DFT-sinusoids[101] [20:44]
  - Windowed Signal Segment Gives One Time Sample in the Time-Frequency Distribution[102] [1:28]
  - DFT Linearity, Flip Operator (index reversal), Flip Theorem, Real Signals have Hermitian (conjugate symmetric) Spectra, Fourier Duality[103] [23:31]
  - DFT Symmetry Theorems, Even and Odd Functions, DCT & DST, "Zero-Phase" Spectra[104] [34:18]
  - Shift Operator, Shift Theorem, Linear Phase Terms, Convolution Thm Preview[105] [11:16]
  - Derivation of Convolution from Linearity and Time-Invariance (LTI) (Superposition) [2015][106] [29:08]
  - Circular Convolution, Commutativity of Convolution, Graphical Convolution, Convolution Reverb, Impulse Response, Convolution Representation of Linear Time-Invariant (LTI) Filters, Convolution Theorem Stated[107] [36:59]
  - Aliasing Demo[108] [4:13]
  - *Continuous* Graphical Convolution Demo[109] [11:15]
  - Convolution Theorem Proof, FFT Convolution, Filter Frequency Response[110] [21:41]
  - Dual of Convolution Theorem, Application to Time-Domain Windowing[111] [6:00]

[100] https://www.youtube.com/watch?v=vkVOGizEX-c
[101] https://www.youtube.com/watch?v=uAXA5NCQk3Q
[102] https://www.youtube.com/watch?v=mi8DTSMO7g8
[103] https://www.youtube.com/watch?v=_ohQ-osR9iI
[104] https://www.youtube.com/watch?v=3nx0pk0JFf4
[105] https://www.youtube.com/watch?v=aqKbcnmLIY4
[106] https://www.youtube.com/watch?v=KWhqV95fKRw
[107] https://www.youtube.com/watch?v=IE3JSnQUr7s
[108] https://www.youtube.com/watch?v=sBjMrt6Iiw0
[109] https://www.youtube.com/watch?v=zoRJZDiPGds
[110] https://www.youtube.com/watch?v=iXcedI7302c
[111] https://www.youtube.com/watch?v=mkOh1ssPMtA

- Review/FAQ Presented 11/11/2014: Why Vector Spaces, Signal Energy as Length Squared, Power, RMS, Shift Theorem, Convolution Theorem[112] [26:34]
- Correlation, Lagged Product, Correlation Thm, Autocorrelation ⇔ Power Spectrum[113] [6:42]
- Power Thm, Parseval's Thm[114] [7:50]
- Normalized DFT (NDFT) [2015][115] [7:04]
- Review of Linearity, Flip, Symmetry, Shift, Convolution, Correlation, and Power Theorems[116] [19:51]
- [Optional] Intuitive Explanation of the Sampling Theorem[117] [14:57]
- Scaling Theorem (continuous time), Stretch Operator, Stretch Thm, Filter Guard Bands, Discrete-Time Stretch Thm, Downsampling, Aliasing, Downsampling Theorem[118] [31:36]
- FFT Bandlimited Interpolation[119] [9:39]

4.5 Section 5: DFT Applications, Spectrograms, and Examples

- **Reading:**
  - Chapter 8 (DFT Applications)
  - Windows (up through Blackman-Harris Window Family)[120]
  - Short Time Fourier Transform (STF) (up through Classic Spectrograms)[121]
  - Optional but you should know it: Appendix D (Sampling Theory)[122]
  - Assignment 5
- **Spectrogram Demos**
  - Music Lab
  - Music Lab Spectrogram
  - `https://borismus.github.io/spectrogram/`
- **Lecture Videos (Total Viewing Time ≈ 2 Hours):**
  - Spectrum-Analyzing Frequencies in the "Cracks" between DFT Sinusoids[123] [30:02]
  - FFT Examples in Matlab: DFT Sinusoid, Non-DFT Sinusoid, and Zero-Padded Windowed Sinusoid for Rectangular, Blackman, and Hann Windows, Spectral Interpolation, Main Lobe, Side Lobes, FFT of Complex Sinusoid[124] [36:36]

[112] https://www.youtube.com/watch?v=uIin-Ny6peE&list=PLsBNrwwINMkZxF2PaJUg_XWoB9RM3C-6G
[113] https://www.youtube.com/watch?v=q4fgEvOW5Zs
[114] https://www.youtube.com/watch?v=eWS3s6WtnfM
[115] https://www.youtube.com/watch?v=6bIYiiQQgEU
[116] https://www.youtube.com/watch?v=a4WF-KTmqFs
[117] https://www.youtube.com/watch?v=2NCYis0Jdhk
[118] https://www.youtube.com/watch?v=8v475fo-uK8
[119] https://www.youtube.com/watch?v=IY9J7hBhPcI
[120] https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Windows.html
[121] https://ccrma.stanford.edu/~jos/sasp/Dual_Views_Short_Time.html
[122] https://ccrma.stanford.edu/~jos/mdft/Sampling_Theory.html
[123] https://www.youtube.com/watch?v=N8-CBmjKAuM
[124] https://www.youtube.com/watch?v=rWiXMDeXTBo

- Spectrograms, Matlab/Octave Implementation, Voice Spectrogram, Formants[125] [11:24]
- Plots of Blackman Window and its DFT Revisited[126] [6:12]
- Linear Time-Invariant (LTI) Filters, Convolution, Ideal Lowpass, Guard Band, Transition Band, Simplest Lowpass, Impulse Response, DTFT, Frequency Response, Linear Phase, Sinewave Analysis[127] [38:36]
- Application: Automatic Chord Recognition (ACE)[128] [8:10]

4.6 Final Exam

Final Exam: Tuesday, December 12, 2017, 3:30-6:30 PM, CCRMA Classroom.

- The exam will cover readings, homework problems, and laboratory assignments.
- The exam will be *closed book*, except that you may bring an 8.5" by 11" sheet of paper, covered front and back with handwritten notes.
- No calculators allowed (you shouldn't need one).

Music 320B

4.1 Section 1: Linearity and Time Invariance; Time-Domain Representations

- **Demos:**
  - Interactive display and hearing of classic *digital filters*[129] by `java@falstad.com`
- **Reading:**
  - Chapters 1 and 2 of **Introduction to Digital Filters**[130]
  - Chapter 4 (Linearity and Time Invariance) and
    Chapter 5 (Time Domain Filter Representations) of **Introduction to Digital Filters**
  - First section of Chapter 9 (Implementation Structures) on the Four Direct Forms
  - Matrix Filter Representations
  - Optionally peruse the Music 421 overheads pertaining to acyclic convolution
  - Supplementary: Audio Signal Processing in Faust
  - Assignment 1

[125] https://www.youtube.com/watch?v=OHN7K-fpFWY
[126] https://www.youtube.com/watch?v=yNkkdFf2mgo
[127] https://www.youtube.com/watch?v=p19QzBxnhvg
[128] https://www.youtube.com/watch?v=6auORRL5z3s
[129] https://www.falstad.com/dfilter/
[130] https://ccrma.stanford.edu/~jos/filters/filters.html

- **Lecture Videos (Total Viewing Time ≈ 2 Hours):**

  IMPORTANT NOTICE: The videos are hosted on YouTube and they use *annotations* for corrections and supplementary information. These annotations are *not supported on mobile devices.* It is therefore unfortunately important to view these videos *in a Web browser on a desktop/laptop computer.*

  - Linear Time-Invariant (LTI) Filters, Convolution, Linearity, Nonlinear Example, Time-Varying Example, Ideal Lowpass Filter, Pass Band, Cutoff Frequency, Guard Band, Stop Band, Transition Band, Simplest Lowpass Filter, Impulse Response, DTFT, Frequency Response, Amplitude Response, Phase Response, Linear Phase, Sinewave Analysis[131] [38:36]
  - Derivation of Convolution from Linearity and Time-Invariance (LTI) (Superposition) [2015][132] [29:08]
  - General Linear [Causal] [Time-Invariant] Filters — Matrix Representations[133] [19:03]
  - Recursive Filters, Simplest Lowpass, Phase Delay, Group Delay[134] [28:01]
  - Supplementary: Faust in the Classroom[135] [41:00]
  - Supplementary: Faust Intro[136] [26:00]
  - Supplementary: Faust Implementation of the Simplest Lowpass Filter[137] [18:22]
  - Simplest RECURSIVE LPF, Pole Gain, PFE, Time-Constant of a Pole, Stability Pole, Bandwidth, Laplace Transform, s-plane poles and zeros, s-plane pole corresponds to exponential[138] [38:37]
  - What does the Laplace Transform really tell us? A visual explanation (plus applications)[139] [20:24]
    Nice 3D plot of two poles at t=512[140] []
  - Direct Form Digital Filters, Transposing a Flow Graph, Transposed Direct Forms 1 and 2, Direct Form 1 Biquad, Direct Form 2 Biquad, Transposed Direct Form 2 Biquad, Interpolated Delay-Line Read, Interpolated Delay-Line Write = Transpose of Read[141] [14:35]
  - Simplest Mechanical LPF: Ideal Mass on Frictionless Surface, Newton's law of motion f=ma, Analog Transfer Function for Driving-Force Input, Velocity Output, Admittance (Mobility) of a Mass[142] [5:31]

[131] https://www.youtube.com/watch?v=p19QzBxnhvg
[132] https://www.youtube.com/watch?v=KWhqV95fKRw
[133] https://www.youtube.com/watch?v=S7ye_HIA_hc
[134] https://www.youtube.com/watch?v=r0fg8eZAKGs
[135] https://www.youtube.com/watch?v=2lEt7dszi00
[136] https://www.youtube.com/watch?v=qE1_UzQZnnM
[137] https://www.youtube.com/watch?v=jNcKGlMHE9A
[138] https://www.youtube.com/watch?v=iJ7mnqhVBfk
[139] https://www.youtube.com/watch?v=n2y7n6jw5d0
[140] https://www.youtube.com/watch?v=n2y7n6jw5d0?t=490
[141] https://www.youtube.com/watch?v=qZUcTsHkHBQ
[142] https://www.youtube.com/watch?v=BULkMAst6_U

– Simplest Mechanical LPF: Ideal Mass on Frictionless Surface, Differentiation Theorem for Laplace Transforms, Transfer Function of the Force-Driven Mass: Frequency Response, Poles and Zeros,, Amplitude Response, 6dB per octave roll off, Bode Plot, Harald Bode, Phase Response[143] [27:29]

4.2 Section 2: Transfer-Function and Pole-Zero Analysis of Digital Filters, Analog and State Variable Filters, Digitizing Filters

- **Reading:**
  - Appendix D (Laplace Transform Analysis)
  - Chapter 6 (Z-transform),
  - Chapter 6 (Transfer Function Analysis)
  - Appendix E (Analog Filters)
  - Laplace Analysis of a Force-Driven Mass
  - Appendix I.3 (Bilinear Transform)
  - Supplementary: Digital State-Variable Filters
  - Supplementary: Interactive Möbius Transformation
- Assignment 2
- **Lecture Videos (Total Viewing Time $\approx$ 3 Hours):**
  - Transfer Functions, Partial Fraction Expansion, Repeated Poles[144] [44:52]
  - Transfer Functions[145] [50:43]
  - State Variable Analog Filters and Digitization[146] [47:50]
  - Repeated Poles at $s = 0$
    - One Pole at DC in the s Plane[147] [14:17]
    - Mechanical Integrator using a Mass[148] [3:46]
    - Integrator made by a Spring or Inductor[149] [5:46]
    - One Pole at DC in the s Plane, Continued[150] [1:31]
    - Frequency Response of an Integrator[151] [5:05]
    - Repeated Poles at DC[152] [4:16]

[143] https://www.youtube.com/watch?v=YWJPqHhjf8c
[144] https://www.youtube.com/watch?v=fRLfliem52M
[145] https://www.youtube.com/watch?v=3C3K7P5xCwg
[146] https://www.youtube.com/watch?v=CBpVm9Bn7Hs
[147] https://www.youtube.com/watch?v=DIhH2JIWQeE
[148] https://www.youtube.com/watch?v=c1TIX2Ybn3U
[149] https://www.youtube.com/watch?v=qPLyMge71F4
[150] https://www.youtube.com/watch?v=lqXYMuna3yw
[151] https://www.youtube.com/watch?v=VLtpw4eD4Uc
[152] https://www.youtube.com/watch?v=5s6umF7I4T4

    * General Transfer Function of a Pile of Poles at DC[153] [3:38]
    * Impulse Response of a Pile of Poles at DC[154] [3:22]
  - Simplest Electrical LPF: RC lowpass; RLC Circuits: Resistor Equation V = IR, Capacitor Equation Q = CV, Inductor Equation V = L dI/dt; Kirchhoff Node and Loop Analysis: Kirchhoff Loop Constraint (Sum of voltages around a loop is zero), Kirchhoff Node Constraint (Sum of currents into a node is zero); Voltage Transfer Circuits, Laplace Transform Circuit Analysis, Transfer Function of RC LPF: Pole-Zero Analysis, Impulse Response, Time Constant of Decay, Bode Plot[155] [21:39]
  - Bilinear Transform Frequency Mapping, Analog Computers, State Space Formulation, Physical Derivation of Bilinear Transform, State Variable[156] [38:29]
  - Bilinear Transform = special case of Moebius Transformation [DON'T MISS THIS ONE!][157] [2:34]

4.3 Section 3: Frequency-Response Analysis, Quality Factor Q, Allpass Filters

- **Reading:**
  - Chapter 7 (Frequency Response Analysis)
  - First three sections of Chapter 8 (Pole-Zero Analysis)
  - Second section of Chapter 9 (Implementation Structures) on parallel/series filter sections
  - Appendix B (Elementary Audio Digital Filters) on one/two pole/zero sections, allpass filters, dc blockers, low and high shelf, peaking equalizers
  - Appendix C (Allpass Filters), through the first subsection (*i.e.*, the rest is "supplemental" starting at Paraunitary Filters)
  - Supplementary: Robust Design of Very High-Order Allpass Dispersion Filters[158]
  - Review: Complex Resonators (PDF)[159]
  - Review: Comparing Analog and Digital Complex Planes[160] from last quarter
- Assignment 3
- **Lecture Videos**
  - Simplest Electrical LPF: RC lowpass, continued; Bode Plot; 3dB Bandwidth[161] [7:45]

[153] https://www.youtube.com/watch?v=6_VRX5Fvdig
[154] https://www.youtube.com/watch?v=d6uQYhQteaw
[155] https://www.youtube.com/watch?v=dEmmtsN-ka4
[156] https://www.youtube.com/watch?v=GRpAqeVbUWs
[157] https://www.youtube.com/watch?v=JX3VmDgiFnY
[158] http://www.dafx.ca/proceedings/papers/p_013.pdf
[159] https://ccrma.stanford.edu/~jos/intro320/Music320B-2018-02-13-ComplexResonators.pdf
[160] https://ccrma.stanford.edu/~jos/mdft/Comparing_Analog_Digital_Complex.html
[161] https://www.youtube.com/watch?v=MOBH66RyXZw

- Bandwidth of a Pole, Continuous-Time Complex Resonator and Allpass Filter, Magnitude and Phase Response from Factored Transfer Function[162] [22:16]
- Analog Low-Shelf Filters, High Shelf, Peaking Equalizer, Mapping s to z, Bilinear Transform (BLT), BLT Doesn't Alias, BLT Frequency Warping[163] [12:30]
- Bilinear Transform Frequency Scaling, Resonance Preservation; Digitizing an Integrator (Mass), RC Filter, Low Shelf; BLT Stability Preservation, DC Blocker[164] [8:51]
- Supplementary: Shelf Filters in Faust[165] [22:25]
- Analog Filters Reviewed: Transfer Function, Frequency Response, Power Response; Analog Lowpass Design, Maximally Flat Passband, Butterworth Filters[166] [30:46]
- Quality Factor (Q) of a Resonator[167] [7:12]
- Complex One-Pole Resonator and its Q; Canonical Form of a Biquad (s-plane); Mechanical and Electrical Resonators; Limiters, Compressors, Expanders[168] [39:30]
- Filter Decay Time is about Q Periods[169] [3:16]
- Supplementary: Introduction to Functional Audio Stream (FAUST): Simplest Lowpass, Utilities in Faust's filter.lib[170] [37:30]
- Supplementary: More FAUST: Testing filters using faust2octave[171] [27:32]

4.4 Section 4: Linear and Minimum Phase Filters, Recursive Digital Filter Design, Butterworth Filters, More on Allpass Filters, More on State Space

- **Reading:**
  - Chapter 11 (Filters Preserving Phase)
  - Chapter 12 (Minimum-Phase Filters)
  - Appendix F (Matrix Filter Representations)
  - Appendix G (State Space Filters)
  - Supplementary: The Digital Waveguide Oscillator
  - State-Space Formulation of Multi-Input, Multi-Output (MIMO) Linear Filters
  - Digitizing the State-Variable Filter
  - Appendix I (Recursive Digital Filter Design)
  - Appendix K (Digital Filtering in FAUST and Pd)
    Supplementary: Audio Signal Processing in FAUST

[162] https://www.youtube.com/watch?v=m4zCmvvKFso
[163] https://www.youtube.com/watch?v=NqYGdcDW3dY
[164] https://www.youtube.com/watch?v=aaGdgf65PsY
[165] https://www.youtube.com/watch?v=9RDC4ylap7E
[166] https://www.youtube.com/watch?v=doDMmZfEfbg
[167] https://www.youtube.com/watch?v=V04yxnqBuYk
[168] https://www.youtube.com/watch?v=zeClzrKUfQU
[169] https://www.youtube.com/watch?v=kPKZQ16EdcU
[170] https://www.youtube.com/watch?v=FEf7dpApd6I
[171] https://www.youtube.com/watch?v=Ao1ZriZi8nY

- **Supplementary:** State-Space Canonical Forms
- **Supplementary:** State Variable Filter used in the ARP2500 analog synthesizer
- Assignment 4

- **Lecture Videos**
  - Butterworth Filter Properties: Maximum Flatness at Infinity, Low Ringing, Mild Phase Response, Poles on a Circle; Spectral Factorization, Series Biquad Realization, Elliptic Function Filters, Chebyshev Optimality, Remez Exchange (Parks-McLellan), firpm in matlab, cvx for Convex Optimization[172] [35:19]
  - Butterworth Power Response, Analytic Continuation, Butterworth Poles, Matlab `butter()` Function[173] [7:02]
  - Example Butterworth Filter of Order 2, Digitization via Bilinear Transform, Frequency Prewarping[174] [18:49]
  - Digital Filter Design and Implementation in Matlab: Noise Removal via Lowpass Filtering, Create Sinusoid and Noise, Matlab's `butter()`, `filter()`, `fdatool` (Filter Design and Analysis tool), Simulating a Telephone Channel Bandwidth[175] [10:30]
  - Matlab: `freqz()`, `freqs()`; Continuous Butterworth Filter Analysis; Converting to Second-Order Sections in Matlab using `tf2sos()`; Viewing Butterworth Poles in Matlab using `zplane()`; Excess Delay at Filter Cutoff; `grpdelay()`; Elliptic Filters using `ellip()`; Ripple; `impz()`; Zero Phase versus Minimum Phase (Pre-Ring versus Post-Ring); Minimum-Delay Property of Minimum-Phase Filters; Partial Fraction Expansion in Matlab using `residue()`, `residuez()`, or `residued()`[176] [31:10]
  - Supplementary Note on Repeated Poles[177] [4:40]
  - Supplementary: MicroModeler Digital Filter Design Tools[178] [5:23]
  - Minimum Phase Filters and Signals; Allpass Filters: Poles and Zeros, Graphical Amplitude and Phase Response, Biquad Realization, Phasing; Allpass-Minimum-Phase Decomposition[179] [28:44]
  - Allpass Filters in z and s Planes; Instability as Noncausality; Laurent Series; Bilateral DTFT; Cepstrum; Converting Arbitrary Spectra to Minimum-Phase Form[180] [39:13]
  - The *State Space* Formulation of Linear Systems [2016 flipped-class review]
    * Adding Feedback around the Integrator Chain, Derivation of the State Space Formulation[181] [10:46]

[172] https://www.youtube.com/watch?v=pUtUrzVHF3Q
[173] https://www.youtube.com/watch?v=nhhuAxBUleU
[174] https://www.youtube.com/watch?v=UcTHnf4B5tU
[175] https://www.youtube.com/watch?v=I0g5r0_BgRM
[176] https://www.youtube.com/watch?v=MxxDS01Ea5o
[177] https://www.youtube.com/watch?v=D6_AK7mfQnQ
[178] https://www.youtube.com/watch?v=FaG5lPmBY_o
[179] https://www.youtube.com/watch?v=Cj6Vjp6k7NM
[180] https://www.youtube.com/watch?v=nCwj1VeXQ44
[181] https://www.youtube.com/watch?v=a__oM8rYPHc

    * State Space Formulation, Continued[182] [1:33]
    * State Space Overview[183] [11:50]
    * Force-Driven Mass Revisited[184] [4:49]
    * General Discussion of State Space[185] [8:48]
    * Defining State Variables[186] [4:51]
    * State Variable Choice Summary[187] [5:14]
    * General State Space Model and Digitization via Backward Euler[188] [5:16]
    * State Space Converts Nth-Order to Vector First-Order[189] [2:08]
  - Supplementary Review (because we already did this in Section 2): State Variable Lowpass, Bandpass, and Highpass
    * Normalized Biquad Lowpass Filter, Continuous Time[190] [19:29]
    * State Variable Realization of Normalized Biquad Lowpass[191] [31:22]
    * State Variable Filter Lowpass/Bandpass/Highpass[192] [3:00]
    * State Variable Filter LP/BP/HP Frequency Scaling and Digitization[193] [11:35]

4.5 Section 5: Cross-Overs, Moog VCF, Voice Synthesis, F0 Estimation, Cepstra, Converting to Minimum Phase

- **Reading:**
  - *Review all assigned reading to date*, slowing down where needed for full understanding
  - **Supplementary:**
    Linkwitz–Riley filter (aka "Butterworth-squared filter")
    Linkwitz-Riley Crossovers: A Primer
    Active Crossover Networks for Noncoincident Drivers in JAES Volume 24, Issue 1, pp. 2-8, February 1976
  - **Supplementary:** Spectral Envelopes via Cepstrum or LPC [from Music 421 overheads]

- **Lecture Videos:**
  - Voice Vowel Synthesis in Faust[194] [11:26]
  - Moog VCF[195] [23:29]

182 https://www.youtube.com/watch?v=ZNO2687Nch0
183 https://www.youtube.com/watch?v=Ygq66m9NrDk
184 https://www.youtube.com/watch?v=zUibjGb6QQ8
185 https://www.youtube.com/watch?v=F0ku7LwHrI0
186 https://www.youtube.com/watch?v=45QwWhbJMvU
187 https://www.youtube.com/watch?v=LgggKFhh0uo
188 https://www.youtube.com/watch?v=rbrJnknh_dU
189 https://www.youtube.com/watch?v=zdO2nQKeaAY
190 https://www.youtube.com/watch?v=hjHPMbDa3yk
191 https://www.youtube.com/watch?v=9cXaEIOeWuU
192 https://www.youtube.com/watch?v=o-Zpot5TKoE
193 https://www.youtube.com/watch?v=LGzSnwTFB-0
194 https://www.youtube.com/watch?v=GR97SMvS4Fw
195 https://www.youtube.com/watch?v=KxBBcNWbZHY

- Moog VCF Live-Coded in Faust
  * Moog VCF Live-Coded in Faust[196] [30:49]
  * Moog VCF in Faust, Review[197] [5:02]
  * Moog VCF in Faust, Frequency Responses[198] [7:39]
  * **Supplementary:** Q-Correction and Gain-Correction Tables[199] [3:38]

All remaining videos are supplementary, either moved beyond the scope of the class, or redundantly covering material already presented.

- Voder[200] [0:43]
- Phonem for iPad by Wolfgang Palm
- Minimum-Phase Spectra Play-List[201]
  * Complex and Real Cepstrum, Quefrency[202] [7:54]
  * Mel Frequency Cepstral Coefficients (MFCC); Bark and Equivalent Rectangular Bandwidth (ERB) Psychoacoustic Frequency Scales based on Critical Bands of hearing[203] [7:54]
  * Complex Cepstrum Derived; Converting Mixed-Phase Signals to Minimum Phase[204] [5:39]
  * Series Expansion of Log of 1/(1-x)[205] [7:57]
  * Series Expansion of Log Transfer Function in Factored Form[206] [5:26]
  * Contribution of Zeros to the Complex Cepstrum[207] [3:15]
  * Contribution of Poles and Zeros (Inside and Outside the Unit Circle) to the Complex Cepstrum; Nonparametric Cepstral Folding Method for Converting Mixed Phase to Minumum Phase using the FFT; Testing for Time Aliasing[208] [9:14]
  * Nonparametric Cepstral Folding Method in Matlab: `minphasespec()`, `fold()`, `invfreqz()`[209] [6:11]
  * Review of another Cepstral Folding Example[210] [19:08]
  * Minimum Phase Conversion by Spectral Factorization or Cepstral Method[211] [5:45]
  * Minimum Phase Conversion by the Cepstral Method, Continued[212] [12:10]

[196] https://www.youtube.com/watch?v=WLvpGN_UNlA
[197] https://www.youtube.com/watch?v=VV2eOChjTrc
[198] https://www.youtube.com/watch?v=cGuuG1bp1JI
[199] https://www.youtube.com/watch?v=QOmOnjDsQYY
[200] https://www.youtube.com/watch?v=e5gQBei-z-c
[201] https://www.youtube.com/watch?v=Tlk6CLclPPU&list=PLsBNrwwINMkYNaFpsqn2BVQRZvmbdWu-I&index=0
[202] https://www.youtube.com/watch?v=Tlk6CLclPPU
[203] https://www.youtube.com/watch?v=2O1BBFeQGOE
[204] https://www.youtube.com/watch?v=OH1z4OW-7UA
[205] https://www.youtube.com/watch?v=wrslb9U7HaI
[206] https://www.youtube.com/watch?v=x3gCA_rj13k
[207] https://www.youtube.com/watch?v=niY8EA4-peA
[208] https://www.youtube.com/watch?v=-_yAOlUTCko
[209] https://www.youtube.com/watch?v=PTxOcapBmHU
[210] https://www.youtube.com/watch?v=V7K4rmT94PE
[211] https://www.youtube.com/watch?v=7GcCkMcqVao
[212] https://www.youtube.com/watch?v=5RNeaaFxfVg

* Cepstral Method Code, then CollideFX Demo by Chet Gnegy[213] [55:51]

4.6 Final Exam

Final Exam: Tuesday, March 20, 2018, 3:30-6:30 PM, in the CCRMA Classroom

- The exam will cover readings, homework problems, and laboratory assignments.
- The exam will be *closed book*, except that you may bring an 8.5" by 11" sheet of paper, covered front and back with handwritten notes.
- No calculators allowed (you shouldn't need one).

[213] https://www.youtube.com/watch?v=nkt3bRO4tI4