MUS320A&B: Introduction to Digital Audio Signal Processing

Center for Computer Research in Music and Acoustics (CCRMA)
Department of Music | Stanford University

320A (spectra): Autumn Quarter
320B (filters): Winter Quarter
2017–2018

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Music 320 A & B: Introduction to Digital Audio Signal Processing

1 Course Description

Music 320 is a two-quarter first-course in digital signal processing with applications in computer music and audio.

The lectures present fundamental elements of digital audio signal processing, such as sinusoids, spectra, the Discrete Fourier Transform (DFT), digital filters, $z$ transforms, transfer-function analysis, and basic Fourier analysis in the discrete-time case. Matlab is used for in-class demonstrations and homework/lab assignments. The labs focus on practical applications of the theory, with emphasis on working with waveforms and spectra, "getting sound", and developing proficiency in the Matlab language.

Prerequisites: High-school level algebra and trigonometry, some calculus, and prior exposure to complex numbers.

Time and Place

Term: Autumn and Winter Quarters
Location: CCRMA Classroom (Knoll 217)
Lectures: Tuesdays and Thursdays 3:00–4:50 PM
Units: 2–4
Instructor: Julius O. Smith (jos@ccrma.stanford.edu)
TA: Orchi Das (orchi@ccrma.stanford.edu)
Office Hours: See “Office Hours and Getting Help” below
Schedule: See “Schedule and Pointers” below

2 Administrative Information

2.1 Announcements

Class announcements are often made via email. For this we are presently using Piazza:

https://piazza.com/stanford/fall2017/music320a/home

If you signed up for the class in axess before the first day of classes, you should receive an invitation from Piazza to join the class (using the email address known to axess). Otherwise, please join by visiting the above URL and entering your preferred email address.

http://ccrma.stanford.edu/~jos/intro320/Office_Hours_Getting_Help.html
http://ccrma.stanford.edu/~jos/intro320/
2.2 Assignments

There are five homework/lab assignments, each covering roughly two weeks of the course. In each two-week “section”, the first week is devoted primarily to theory while the second week is focused more on software and applications. Thus, each assignment contains both a theory and laboratory part. The lab portion typically requires programming in matlab.

Each assignment is typically announced on Tuesday in the first week of the section. The theory part is normally due the following Tuesday at 3:15 pm in the 320 mailbox (located in the Knoll, central wing, second floor, facing the printer). The lab part is normally due by midnight the following Friday, i.e., at the end of the two-week section.

For lab assignments, we will be using the Canvas website. To sign up go to the Canvas website and find Music320A. Once you are enrolled in the class, you can upload your matlab files in the “drop box” on the left menu.

See 2.5 below regarding obtaining help with theory and lab assignments.

Regarding late homeworks, 7 free late days are allowed (with hours rounded up to the nearest day). Late homeworks beyond this will be penalized at 5% per day. When using late days, write the number of late days used at the top of the assignment (date and time).

Students are encouraged to discuss the homework assignments with each other. It is fine to learn from a classmate how to solve any of the homework problems, but each student is responsible for carrying out and writing up the assignments individually. It is an honor code violation to copy the work of others.

2.3 Exams

The final examination will be held in the CCRMA Classroom (Knoll 217) on the University-assigned date, also listed for convenience in the class schedule (§6 on page 4).

2.4 Grading

Grades are based on the homeworks/labs (60%), and the final exam (40%). There are also bonus points available based on general participation. The weightings may be changed as we see fit.

2.5 Office Hours and Getting Help

We will be using Piazza for sharing answers to posted questions with the whole class. To sign up, see the 320 Piazza site. It is free and allows you to view past questions from other students, and discuss questions together. Try it first for any homework questions you may have. You are also welcome, of course, to catch us whenever you see us at CCRMA, such as during office hours, etc.

TA weekly office hours are Wednesday evenings 7:30-9:30 PM in the CCRMA Ballroom (2nd floor). Meetings with JOS are arranged via email for half-hour slots after class, or other times when necessary.

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3 https://canvas.stanford.edu
4 https://www.piazza.com
5 https://piazza.com/stanford/fall2017/music320a/home
2.6 Computer Usage

Lab exercises will be computer based. All students may obtain a computer account at CCRMA in order to use the computer facilities. It is also possible to work entirely on your own computer, as long as you have the necessary software. However, note that some course materials are restricted to on-campus access, so you should have at least one Stanford computer account from which you access those.

Here is how to obtain a CCRMA computer account:

https://cm-knoll.stanford.edu/usersignup

Note: This link only works at CCRMA.

Once you have your account, please log in at CCRMA and take a look at the User’s guides tab in the left-frame menu of the main CCRMA website to learn more about computer usage and other facilities at CCRMA.

3 Textbooks

Music 320A (fall) is based on assigned chapters of

Mathematics of the Discrete Fourier Transform (DFT) by Julius O. Smith

Music 320B (winter) is based on assigned chapters of

Introduction to Digital Filters by Julius O. Smith

See § for the list of assigned chapters. Both books are fully available on-line. Softcover versions are available from Amazon.com.

4 The Partially Flipped Classroom

With the lectures recorded, class time is freed up for other activities. Here is how a typical “partially flipped class” is organized:

- Q&A session on the reading/video content
- Review of main points in the reading/videos
- Demos in support of the reading/videos
- Presentation of the homework/lab assignment
- Worked problems similar to those in the homework
- Matlab session on theory/lab-related topics
- Live coding in matlab

Additional available time may be devoted to

- More demos
- More discussion

^http://ccrma.stanford.edu/guides/
^http://ccrma.stanford.edu/~jos/mdft/
^http://ccrma.stanford.edu/~jos/filters/
• “Backwards learning” examples:
  • Plugins using spectral techniques
  • Faust language and some of its examples
• More on applications and why all this is useful
• Preview material coming up
• General in-class discussion
• Getting to know your fellow class-members better

5  A Recipe for Learning

Learning something new requires multiple passes on the material. For example:

1. Do the assigned reading at a fixed pace to get a picture of what’s covered
2. Watch the lecture videos, pausing and taking notes on anything newly learned
3. Make a first pass on the homework, flagging and skipping when stuck on a problem
4. Discuss nonobvious homework problems with other students, the TA, and/or JOS
5. Write up the homework problems, everything now understood
6. Exam prep: Reread the text for full comprehension
7. Exam prep: Reread your notes
8. Prepare your one-page summary of the course allowed in the exam
9. Exam experience: Exercise in problem solving using the material

These multiple engagements result in a good amount of learning.

6  320A Schedule and Pointers

Note: The online version of this schedule contains hyperlinks to all reading, lecture videos, and assignments.

To obtain printable versions of the assignments and solutions from off-campus locations, you can use commands such as

```
scp you@ccrma-gate.stanford.edu:/usr/ccrma/web/html/courses/320/hw/hw1/hw1sol.pdf .
```

For more info, see [https://ccrma.stanford.edu/guides/remoteaccess/](https://ccrma.stanford.edu/guides/remoteaccess/).
You can alternatively use VPN (Virtual Private Network) access.

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9 [https://ccrma.stanford.edu/~jos/intro320/Lectures_Assignments.html](https://ccrma.stanford.edu/~jos/intro320/Lectures_Assignments.html)

10 [https://uit.stanford.edu/service/vpn](https://uit.stanford.edu/service/vpn)
6.1 Section 1: Course Overview, Signal Math, Intro to Matlab

A “section” is typically two weeks in duration, with the first week devoted primarily to theory, and the second primarily to software and applications.

- **Reading**
  - This course overview
  - Chapter 1 (DFT Intro) of *Mathematics of the DFT*
  - If you are not comfortable with the decibel scale, read Appendix B (Logarithms and Decibels)
  - **Assignment** (complex number problems)

- **Supplementary Demos, Reading, and Exercises**
  - Our main task is to approximate any signal \( x(t) \) as a sum of sinusoids \( s_\omega(t) \), where the term *sinusoid* refers to cosine (or sine) having any amplitude \( A \) and any phase offset \( \phi \):
    \[
    s_\omega(t) = A \cos(\omega t + \phi)
    \]
  - We call \( \omega \) the **radian frequency** (frequency in radians per second), and \( \phi \) the **phase** of the sinusoid.
  - The connection to complex numbers is via Euler’s Identity, which can be used to show
    \[
    \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}
    \]
  - The supplementary reading below pertains to both sinusoids and complex numbers:
    - “The Acoustic Origins of Harmonic Analysis” by Olivier Darrigol treatment (published in the *Archive for History of the Exact Sciences*, vol. 61, no. 4, July 2007)
    - “History of Virtual Musical Instruments and Effects Based on Physical Modeling Principles” by JOS, DAFx-2017
    - Discrete Fourier Transform Demo (Truncated Sinc Spectrum)
    - Building up a Spectrum Analyzer in WebGL
    - For an educational Matlab GUI on *sinusoids*, download sindedrill from the Educational Matlab GUIs collection at Georgia Tech. There are other nice Matlab-based exercises that can use later in the quarter and next quarter.

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11 [https://ccrma.stanford.edu/courses/320/](https://ccrma.stanford.edu/courses/320/)
14 [https://ccrma.stanford.edu/~jos/mdft/Logarithms_Decibels.html](https://ccrma.stanford.edu/~jos/mdft/Logarithms_Decibels.html)
15 [https://ccrma.stanford.edu/~jos/hw320/](https://ccrma.stanford.edu/~jos/hw320/)
17 [https://ccrma.stanford.edu/~jos/pdf/DAFx17-keynote1-jos.pdf](https://ccrma.stanford.edu/~jos/pdf/DAFx17-keynote1-jos.pdf)
19 [https://acko.net/files/gltalks/toolsforthought/#29](https://acko.net/files/gltalks/toolsforthought/#29)
20 [http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/sindrill-v209.zip](http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/sindrill-v209.zip)
21 [http://users.ece.gatech.edu/mcclella/matlabGUIs/](http://users.ece.gatech.edu/mcclella/matlabGUIs/)
There is another Matlab GUI illustrating Fourier series approximations, i.e., using sums of sinusoids to approximate classic waveforms such as square wave, sawtooth, and triangle. Download fseriesdemo from the Educational Matlab GUIs collection at Georgia Tech.

Chapter 2 (Complex Numbers)

If you need more practice with complex numbers, work some Khan Academy exercises.

For a Matlab GUI providing complex-number drills, download zdrill from the Educational Matlab GUIs collection at Georgia Tech.

Here also are some prerequisite-level Khan Academy exercises on dealing with polynomials (you can skip “Synthetic Division” and stop before “Partial Fraction Expansions” since we get to that in 320B next quarter).

For more advanced math studies, beyond what’s needed for this course, but needed for more advanced signal processing: Georgia Tech Online Mathematics Textbook.

- Lecture Videos:
  
  IMPORTANT NOTICE: The videos are hosted on YouTube and they use annotations for corrections and supplementary information. These annotations are not supported on mobile devices. It is therefore unfortunately important to view these videos in a Web browser on a desktop/laptop computer.

- Music 320A Overview (first class recording in fall 2014)
- Introductory Demonstrations for 320A and 320B
- Administrative overview (this document), discussed only in class (no video), so definitely read it if you missed the first class
- Intro to the Fourier Transform (FT) and the Discrete Fourier Transform (DFT) [30:52]
- Albert Michelson’s Harmonic Analyzer [First three: 3:30+5:00+3:30]
- Euler’s Identity, Complex Sinusoids [9:43]
- Complex Plane Intro [4:53]
- Euler’s Identity Corollaries [12:38]
- Review of DFT and Euler Identity Intro presented 10/02/2014

22 http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/fseriesdemo-v130.zip
23 http://users.ece.gatech.edu/mcclella/matlabGUIs/
24 https://ccrma.stanford.edu/~jos/mdft/Complex_Numbers.html
26 http://users.ece.gatech.edu/mcclella/matlabGUIs/ZipFiles/zdrill-v210.zip
27 http://users.ece.gatech.edu/mcclella/matlabGUIs/
28 https://www.khanacademy.org/math/algebra2/polynomial_and_rational
29 http://people.math.gatech.edu/~cain/textbooks/onlinebooks.html
30 https://www.youtube.com/watch?v=joSvZkZtB3I&list=PLsBNrwwiNMkYNSpb5pwq2rjzg3RyXysPY
31 https://www.youtube.com/watch?v=hSkdaqNQHBM
32 https://ccrma.stanford.edu/courses/320/
33 https://www.youtube.com/watch?v=Gw6654d7TD8
34 https://www.youtube.com/watch?v=NA6sM30MAHLg&list=PL0INsTTU1k2U0Y9Mck-i5HNqGNW5AEwoq
35 https://www.youtube.com/watch?v=ugrZluU6f6oQ
36 https://www.youtube.com/watch?v=gC4L_rUWbuw
37 https://www.youtube.com/watch?v=1zWE0-AW8Ng
38 https://www.youtube.com/watch?v=kF1BeOq-enO&list=PLsBNrwI1MkaGxKJD0rnx--jP8rS5Cdpp
– Introduction to Piazza, Coursework, and Matlab [20:08]
  * Regarding the Matlab intro, if you do not know the rules of matrix multiplication, read Appendix H (Matrices) [10]
  * Matlab Documentation [41]
  * Read the first two sections of Appendix J (Matlab Examples) in Mathematics of the DFT
  * Do Lab Assignment [43] if you are new to Matlab.

[URL footnotes are mostly suppressed below, but links persist in the online version of this page.]

6.2 Section 2: Euler’s Formula, Exponentials, Sinusoids, and Spectra

- Reading:
  - Chapter 3 (Proof of Euler’s Theorem) of Mathematics of the DFT
  - Chapter 4 (Sinusoids and Exponentials)
  - Assignment 2

- Lecture Videos (Total Viewing Time ≈ 4 Hours):
  - Euler’s Identity Proof [30:21]
  - DFT Overview, Euler’s Identity on the Complex Plane, Additive Synthesis of a Square Wave by Fourier’s Theorem [12:03]
  - Additive Synthesis of a Square Wave at Codepen
  - Sinusoidal Motion as Projection of Circular Motion [2:08]
  - Sinusoidal Motion as Projection of Circular Motion, Continued [0:53]
  - Mathematics of Sinusoids, Complex Sinusoids [10:30]
  - Spectra of Sinusoids [15:43]
  - Amplitude Modulation (AM) and its Spectral Effects, AM Demo, “Ring Modulation,” Perception of AM, “Beats” demo in Matlab (beatcon.m), Hearing Mechanics, Critical Bands

[Links to videos and resources suppressed; see online version for details.]
– Editable Online Javascript for Additive Synthesis Visualized
– Additive Synthesis Visualized with Audio and More Controls
– 3D Animation of Euler’s Identity (GIF)
– Circular Motion Projection and Superposition
– Time Constant of Decay, Plotting Exponentials
– Euler’s Identity in the Complex Plane
– Supplementary: Euler’s Identity Viewed as “Actions”
– Generalized Complex Sinusoids
– Generalized Complex Sinusoids II
– Demo of Exponentially Decaying Sinusoids in Matlab, Spectrograms, DTMFs [9:01 (Kitty)]
– Making a sinusoid in Pure Data (Pd)
– Drawing the word “minimum” in a spectrogram using various vocalizations
– Exponentials
– AM Review, Frequency Modulation (FM)
– FM Spectra, Bessel Functions
– Generalized Complex Sinusoids Review, The S-Plane
– Laplace Transforms
– Sampled Generalized Complex Sinusoids
– Sampled Generalized Complex Sinusoids II, Z Transform
– Domain of Z Transforms, The Sampled Sinusoid \( j^n \), Normalized Frequency
– Mapping \( s \) to \( z \), Bilinear Transform Preview
– Overview and Demos Presented 10/9/2014: Sinusoidal AM Demos, Continuous/Discrete Fourier Transforms (Four Cases), Generalized Sinusoids, Laplace Transform, Z Transform
– You should now understand the twist in the complex-plane versus time display used in the WebGL Spectrum Analyzer.

– Supplementary flipped-class lecture on the meanings of points in the $s$ and $z$ planes as generators of generalized complex sinusoids, and derivation of the Laplace and $Z$ Transforms via projection onto them:
  1. Part 1/2
  2. Part 2/2

6.3 Section 3: Vectors, Geometric Signal Theory, Orthogonal Projection

• Reading:
  – Chapter 5 (Geometric Signal Theory) of Mathematics of the DFT
  – Chapter 6 (The DFT Derived)
  – Assignment 3

• Lecture Videos (Total Viewing Time ≈ 2 Hours):
  – Geometric Signal Theory [25:28]
  – Preview of Orthogonal Projection [3:12]
  – DFT Review, Orthogonal Projection [15:36]
  – Orthogonal Projection Example [4:12]
  – Orthogonal Projection onto Coordinate Axes [3:38]
  – You should now understand how the twisted signal plot “pancakes” to produce the DFT displayed on the complex-plane at time zero in the WebGL Spectrum Analyzer.
  – Inverse DFT as a Sum of Projections, Normalized DFT, Gram-Schmidt Orthogonalization [3:34]
  – Nth Roots of Unity $W_N^k$, Twiddle Factors [2:21]
  – Nth Roots of Unity for $N = 8$ [12:01]
  – DFT Sinusoids [4:55]
– Review of DFT Sinusoid Orthogonality, Normalization, DFT as Inner Product, IDFT as Sum of Projections [4:01]
– Inverse DFT Derived [1:34]
– You should now understand how the phase-rotated signal in the complex-plane versus time display is summed to the green front complex-plane in the WebGL Spectrum Analyzer [91]
– Supplementary flipped-class lecture on the inner product and its applications:
  1. Part 1 [50:??]
  2. Part 2 [48:00]

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88 https://www.youtube.com/watch?v=3yL55HQevkg
89 https://www.youtube.com/watch?v=cqVcSb608z0
90 https://www.youtube.com/watch?v=L1keZWKgu6Y
91 https://acko.net/files/gltalks/toolsforthought/#29
92 https://www.youtube.com/watch?v=V1VqTE0gf8
93 https://www.youtube.com/watch?v=rSfX5h72x38
6.4 Section 4: Fourier Theorems, Convolution and Correlation

- **Reading:**
  - Chapter 7 (Fourier Theorems for the DFT) of *Mathematics of the DFT*
  - Appendix B (Fourier Transforms for Continuous/Discrete Time/Frequency)
  - Filters and Convolution
  - Assignment 4

- **Lecture Videos (Total Viewing Time ≈ 4 Hours):**
  - Signal and Spectra Notation for the DFT Theorems [94] [5:14]
  - Review of DFT as a change of coordinates, Periodic Extension versus Time Limited Signal Windows, DFT interpolation between spectral samples, “spectral splatter” in DFT of non-DFT-sinusoids [95] [20:44]
  - Windowed Signal Segment Gives One Time Sample in the Time-Frequency Distribution [96] [1:28]
  - DFT Linearity, Flip Operator (index reversal), Flip Theorem, Real Signals have Hermitean (conjugate symmetric) Spectra, Fourier Duality [97] [23:31]
  - DFT Symmetry Theorems, Even and Odd Functions, DCT & DST, “Zero-Phase” Spectra [98] [34:18]
  - Shift Operator, Shift Theorem, Linear Phase Terms, Convolution Thm Preview [99] [11:16]
  - Derivation of Convolution from Linearity and Time-Invariance (LTI) (Superposition) [2015] [100] [29:08]
  - Circular Convolution, Commutativity of Convolution, Graphical Convolution, Convolution Reverb, Impulse Response, Convolution Representation of Linear Time-Invariant (LTI) Filters, Convolution Theorem Stated [101] [36:59]
  - Aliasing Demo [102] [4:13]
  - *Continuous* Graphical Convolution Demo [103] [11:15]
  - Convolution Theorem Proof, FFT Convolution, Filter Frequency Response [104] [21:41]
  - Dual of Convolution Theorem, Application to Time-Domain Windowing [105] [6:00]

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94 https://www.youtube.com/watch?v=vkVOGizEX-c
95 https://www.youtube.com/watch?v=uaXAxNQk3Q
96 https://www.youtube.com/watch?v=miBDDTSM07g8
97 https://www.youtube.com/watch?v=ohQ-oSR9iI
98 https://www.youtube.com/watch?v=3nxO9k0JFf4
99 https://www.youtube.com/watch?v=a9KbcmLIY4
100 https://www.youtube.com/watch?v=KWhqV95fKRW
101 https://www.youtube.com/watch?v=IE3JSnQu7s
102 https://www.youtube.com/watch?v=sB0Mrt6fJw0
103 https://www.youtube.com/watch?v=zoRJzDiPQds
104 https://www.youtube.com/watch?v=IxcedI7302c
105 https://www.youtube.com/watch?v=mkOhs9sPMtA
– Correlation, Lagged Product, Correlation Thm, Autocorrelation ⇔ Power Spectrum[107][6:42]
– Power Thm, Parseval’s Thm[108][7:50]
– Normalized DFT (NDFT) [2015][109][7:04]
– [Optional] Intuitive Explanation of the Sampling Theorem[111][14:57]
– Scaling Theorem (continuous time), Stretch Operator, Stretch Thm, Filter Guard Bands, Discrete-Time Stretch Thm, Downsampling, Aliasing, Downsampling Theorem[112][31:36]