MUS420/EE367A Lecture 7A
Woodwind Models

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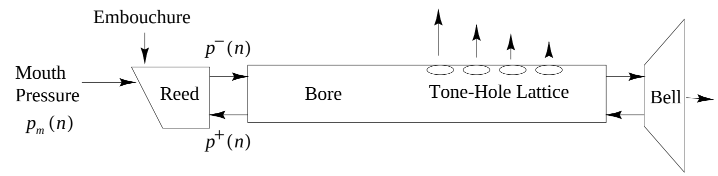
## Outline

- Single-Reed Instruments
  - Schematic Physical Model
  - Digital Waveguide Model
- Bernoulli Effect
- Single-Reed Theory
- Single-Reed Computation Models



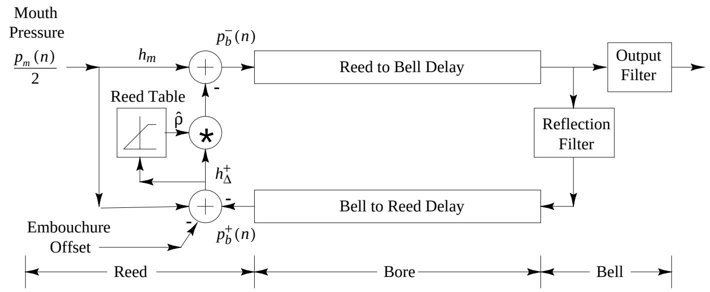
# Single-Reed Instruments

## Schematic Model




- Main control variable = air pressure applied to reed
- Secondary control variable = reed embouchure
- Pressure waves = natural choice for simulation
- Bell ≈ power-complementary "cross-over" filter:
  - Low frequencies reflect (inverted)
  - High frequencies transmit
  - Cross-over frequency ≈ 1500 Hz for clarinet (where wavelength ≈ bore diameter)
- Radiation ≈ "Omni" at LF, more directional at HF

## Single-Reed Digital Waveguide Model




- Bore = bidirectional delay line (losses lumped)
- Bore length = 1/4 wavelength in lowest register
  - Bell reflection ≈ -1 at low frequencies
  - Mouthpiece reflection ≈ +1
- Reflection filter depends on first few open toneholes
- In a simple implementation, the bore is "cut to a new length" for each pitch
- Reed = Nonlinear scattering junction
  - Reed mass neglected
  - Reed table interpolated (usually linear)
  - In software, simple "if statement" possible
  - Embouchure can be a simple address offset



## Qualitative Description of Single-Reed Oscillation

- Apply pressure at the mouthpiece
- Reed is "biased" in "negative-resistance" region
- High-pressure front travels to open tonehole or bell where it reflects with a sign inversion
- A "canceling wave" travels back toward mouthpiece
- The canceling wave reflects from the mouthpiece with no inversion
- A negative-pressure "wake" is left behind
- The reflected-canceling-wave travels back to the open end where it reflects with inversion
- The negative-pressure throughout bore is canceled by this wave as it travels to the mouthpiece
- Upon reaching mouthpiece, one period is finished
- One period = four trips across bore length

## Reed Causes Oscillation Growth

- Wave loses energy propagating in bore (mainly at the open-end reflection)
- The nonlinear reed action must restore this lost energy
- "Power supply" = "dc mouth pressure"
  $\Rightarrow$ Reed converts dc to ac
- Reed action "sharpens" pressure transitions
  - Reed closure increases reflection coefficient in bore
  - As pressure falls in bore, it is amplified by increasing $\rho$
  - As pressure falls in bore, it is further amplified by decreasing flow input from the mouth
  - As pressure rises in bore, it is amplified by increasing mouth flow input (although reflection coefficient decreases)
  - Reflection of a positive wave is boosted when the incoming wave is below a certain level and it is attenuated above that level
- When the oscillation reaches a very high amplitude, it is limited on the negative side by the shutting of the



reed, and on the positive side by the attenuation described above

## Bernoulli's Equation

In an ideal *inviscid, incompressible flow*, we have, by *conservation of energy*,

$$p + \frac{1}{2}\rho u^2 + \rho g h = \text{constant}$$

where

$$\begin{aligned} p &= \text{pressure (Newtons/m}^2 = \text{kg /(m s}^2\text{))} \\ u &= \text{particle velocity (m/s)} \\ \rho &= \text{volume density of air (kg/m}^3\text{)} \\ g &= \text{Newton's gravitational constant (m/s}^2\text{)} \\ h &= \text{Height of flow's center-of-mass axis (m)} \\ \text{“Inviscid”} &= \text{“Frictionless”, “Lossless”} \end{aligned}$$

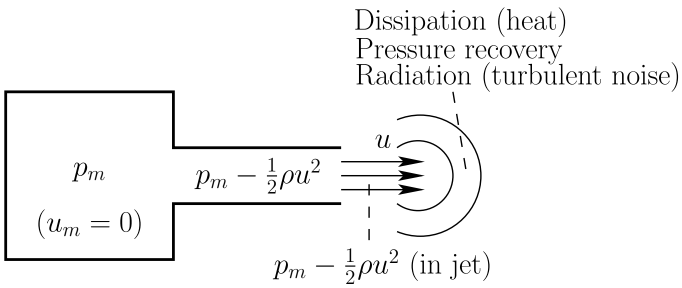
### Pressure is Proportional to Kinetic Energy

$$p = \frac{1}{3}\rho \left\langle u^2 \right\rangle$$

- *Average momentum transfer* / area / time
- Caused by collisions of gas molecules with boundary
- Proof: See *Kinetic Theory of Gases*

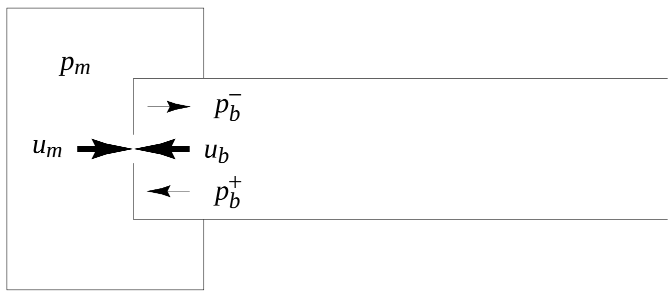


### Bernoulli Effect




- $p_m =$ “mouth pressure”
- Flow inside “mouth” neglected
- Pressure kinetic energy converts to flow kinetic energy within channel
- Jet “carries its own pressure” until it dissipates
- Jet dissipation can go to
  - heat (now allowing “friction” into the model)
  - vortices (angular momentum)
  - radiation (sound waves)
  - pressure recovery:
    (flow kinetic energy $\longrightarrow$ pressure kinetic energy)

## Single-Reed Theory




$$\begin{aligned}
p_m &\triangleq \text{mouth pressure (constant)}\\
p_b &\triangleq \text{bore pressure (dynamic)}\\
p_\Delta &\triangleq p_m - p_b \triangleq \text{pressure drop across mouthpiece}\\
u_m &\triangleq \text{resulting flow into mouthpiece}\\
R_m(p_\Delta) &\triangleq \text{reed-aperture impedance (measured)}\\
&\phantom{\triangleq}\ \text{where}\\
u_b + u_m &= 0 \qquad \text{(by continuity of volume velocity)}\\
u_m(p_\Delta) &\triangleq \frac{p_\Delta}{R_m(p_\Delta)} \qquad \text{(``Ohm's law'' for the reed)}\\
u_b &\triangleq u_b^+ + u_b^- = \frac{p_b^+ - p_b^-}{R_b}, \quad R_b \triangleq \text{bore impedance}
\end{aligned}$$



## Toward a Computational Model

**Given:**

$$\begin{aligned}
p_m &= \text{Mouth pressure}\\
p_b^+ &= \text{Incoming traveling bore pressure}
\end{aligned}$$

**Find:**

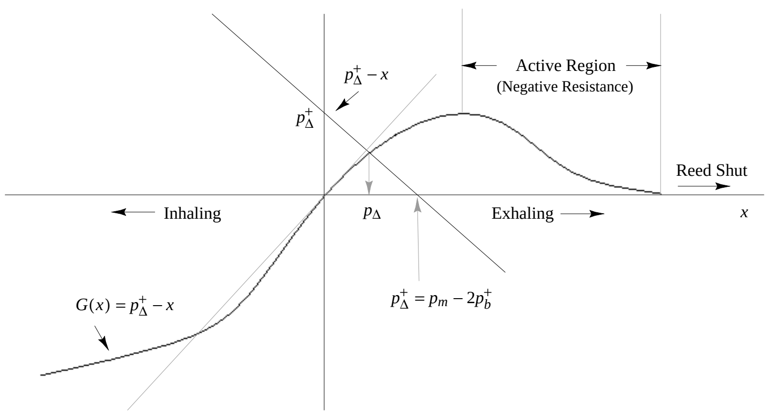
$$p_b^- = \text{Outgoing traveling bore pressure}$$

**such that:**

$$\begin{aligned}
0 &= u_m + u_b = \frac{p_\Delta}{R_m(p_\Delta)} + \frac{p_b^+ - p_b^-}{R_b},\\
p_\Delta &\triangleq p_m - p_b = p_m - (p_b^+ + p_b^-)
\end{aligned}$$

Solving for $p_b^-$ is not immediate because $R_m$ depends on $p_\Delta$ which depends on $p_b^-$.

## Graphical Solution Technique




### Graphically solve:

$$G(p_\Delta) = p_\Delta^+ - p_\Delta, \qquad p_\Delta^+ \stackrel{\Delta}{=} p_m - 2p_b^+$$

where

$$G(p_\Delta) \stackrel{\Delta}{=} R_b u_m(p_\Delta) = R_b p_\Delta / R_m(p_\Delta)$$

- Introduced by Friedlander and Keller (1953)
- Analogous to finding the "operating point" of a transistor by intersecting its "operating curve" with the "load line" determined by the load resistance.
- Outgoing wave is then $p_b^- = p_m - p_b^+ - p_\Delta(p_\Delta^+)$



## Scattering Formulation

Solve for $p_b^-$ to obtain

$$\begin{aligned} p_b^- &= \frac{1-r}{1+r} p_b^+ + \frac{r}{1+r} p_m \\ &= \rho p_b^+ + \frac{1-\rho}{2} p_m \\ &= \frac{p_m}{2} - \rho \frac{p_\Delta^+}{2} \end{aligned}$$

where

$$\rho(p_\Delta) \stackrel{\Delta}{=} \frac{1 - r(p_\Delta)}{1 + r(p_\Delta)}, \qquad r(p_\Delta) \stackrel{\Delta}{=} \frac{R_b}{R_m(p_\Delta)}$$

$$\rho(p_\Delta) = \textit{signal-dependent reflection coefficient.}$$

In practice, $R_m \gg R_b \;\Rightarrow\; r \approx 0, \rho \approx 1$
(mouthpiece looks largely like a closed end)

## For Faster Real-Time Computation

*Pre-solve* the graphical intersection and store the result in a *look-up table*

Let $h$ denote *half-pressure* $p/2$. Then

$$p_b^- = -\rho(p_\Delta) \cdot h_\Delta^+$$

Subtracting both sides from $p_b^+$ and solving for $\rho$ gives

$$\rho(p_\Delta) = \frac{p_\Delta}{h_\Delta^+} - 1$$

Now, for each $h_\Delta^+ = -p_b^+$, find $p_\Delta$ graphically, and store the resulting reflection coefficient $\rho(p_\Delta)$ as a function of $h_\Delta^+$:

$$\hat{\rho}(h_\Delta^+) = \rho(p_\Delta(h_\Delta^+))$$

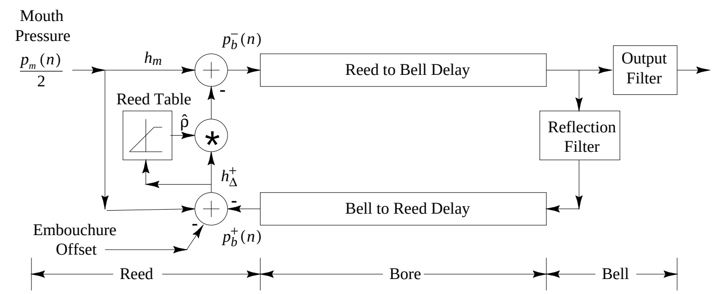
Then the real-time reed computation reduces simply to

$$p_b^- = -\hat{\rho}(h_\Delta^+) \cdot h_\Delta^+$$

This is the form chosen for implementation above

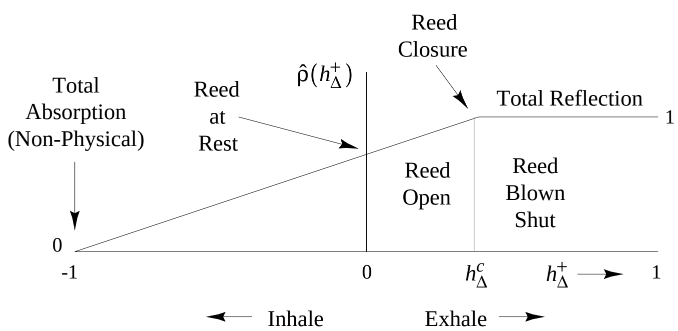


## Table-Reduced Reed Reflection Coefficient




- Control variable = mouth half-pressure
- $h_\Delta^+ = -p_b^+$ computed from incoming bore pressure by a subtraction
- Table is indexed by $h_\Delta^+$
- Result of lookup is multiplied by $h_\Delta^+$
- Result of multiplication is subtracted from
- Total reed cost = two subtractions, one multiply, and one table lookup per sample

## Simple Piecewise-Linear Reed Table




$$\hat{\rho}(h_\Delta^+) = \begin{cases} 1 - m(h_\Delta^c - h_\Delta^+), & -1 \leq h_\Delta^+ < h_\Delta^c \\ 1, & h_\Delta^c \leq h_\Delta^+ \leq 1 \end{cases}$$

- Corner point $h_\Delta^c$ = smallest pressure difference giving reed closure
- In fixed-point, $h_\Delta^+ \stackrel{\Delta}{=} p_m/2 - p_b^+$ is confined to $[-1, 1)$
- Embouchure and reed-stiffness set by $h_\Delta^c$ and slope $m$ [$m = 1/(h_\Delta^c + 1)$ in the figure]
- Zero at maximum negative pressure $h_\Delta^+ = -1$ is not physical but is practical for inhibiting overflow
- A brighter tone is obtained by increasing the *curvature* as the reed begins to open [E.g., $\hat{\rho}^k(h_\Delta^+)$, for $k \gg 1$]



## Further Details

- Input mouth pressure is summed with a small amount of white noise corresponding to turbulence, e.g., 0.1% plus more during attacks
- Turbulence *level* and *spectral shape* should be computed *automatically* as a function of pressure drop $p_\Delta$ and reed opening geometry (research topic)
- Simple reflection filter:

$$H(z) = \frac{1 + a_1(t)}{1 + a_1(t)z^{-1}}$$

  where $a_1(t) = v(t) - 0.642$, $v(t) = A_v \sin(2\pi f_v t)$,
  $A_v$ = vibrato amplitude (e.g., $0.03$), and
  $f_v$ = vibrato frequency (e.g., $5$ Hz)
- Further loop filtering occurs as a result of using a linearly interpolated delay line for the bore
- Only one (double length) delay line is really used in typical implementations
- To avoid finger-hole models, legato note transitions can be managed using two delay line taps and cross-fading from one to the other during a transition

## Alternative Reed Models

- A direct signal lookup, though requiring much higher resolution, would eliminate the multiply associated with the scattering coefficient
- Coefficient tables can be quantized more heavily in address and word length than direct lookup of a signal value such as $p_\Delta(p_\Delta^+)$
- Piecewise polynomial approximations are also used

## Software

`clarinet.cpp` in the Synthesis Toolkit (STK):

`http://ccrma.stanford.edu/CCRMA/Software/STK/`