MUS420/EE367A Lecture 7A
Woodwind Models

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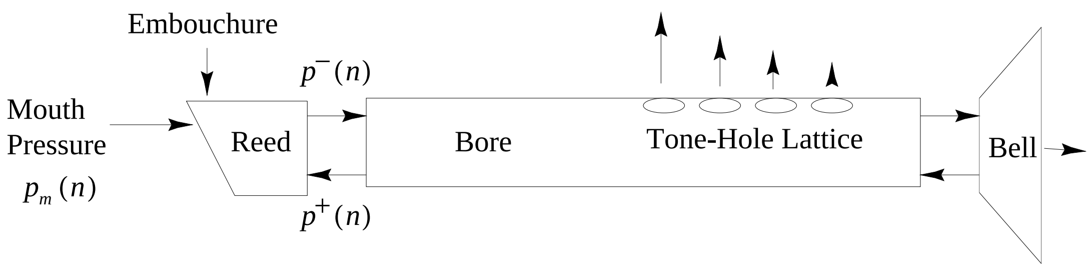
Outline

- Single-Reed Instruments
  - Schematic Physical Model
  - Digital Waveguide Model
- Bernoulli Effect
- Single-Reed Theory
- Single-Reed Computation Models

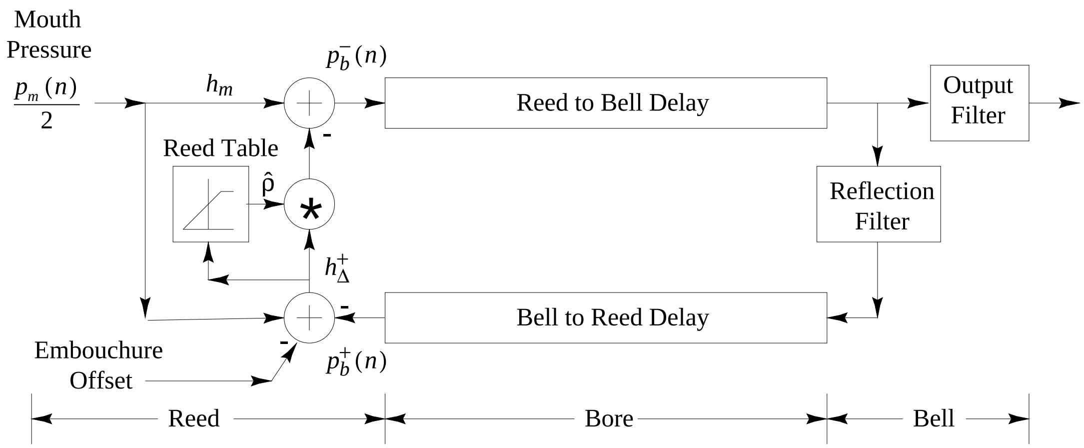
Single-Reed Instruments

Schematic Model



- Main control variable = air pressure applied to reed
- Secondary control variable = reed embouchure
- Pressure waves = natural choice for simulation
- Bell ≈ power-complementary "cross-over" filter:
  - Low frequencies reflect (inverted)
  - High frequencies transmit
  - Cross-over frequency ≈ 1500 Hz for clarinet (where wavelength ≈ bore diameter)
- Radiation ≈ "Omni" at LF, more directional at HF

Single-Reed Digital Waveguide Model



- Bore = bidirectional delay line (losses lumped)
- Bore length = 1/4 wavelength in lowest register
  - Bell reflection ≈ -1 at low frequencies
  - Mouthpiece reflection ≈ +1
- Reflection filter depends on first few open toneholes
- In a simple implementation, the bore is "cut to a new length" for each pitch
- Reed = Nonlinear scattering junction
  - Reed mass neglected
  - Reed table interpolated (usually linear)
  - In software, simple "if statement" possible
  - Embouchure can be a simple address offset

Qualitative Description of Single-Reed Oscillation

- Apply pressure at the mouthpiece
- Reed is "biased" in "negative-resistance" region
- High-pressure front travels to open tonehole or bell where it reflects with a sign inversion
- A "canceling wave" travels back toward mouthpiece
- The canceling wave reflects from the mouthpiece with no inversion
- A negative-pressure "wake" is left behind
- The reflected-canceling-wave travels back to the open end where it reflects with inversion
- The negative-pressure throughout bore is canceled by this wave as it travels to the mouthpiece
- Upon reaching mouthpiece, one period is finished
- One period = four trips across bore length

Reed Causes Oscillation Growth

- Wave loses energy propagating in bore (mainly at the open-end reflection)
- The nonlinear reed action must restore this lost energy
- "Power supply" = "dc mouth pressure"
  $\Rightarrow$ Reed converts dc to ac
- Reed action "sharpens" pressure transitions
  - Reed closure increases reflection coefficient in bore
  - As pressure falls in bore, it is amplified by increasing ρ
  - As pressure falls in bore, it is further amplified by decreasing flow input from the mouth
  - As pressure rises in bore, it is amplified by increasing mouth flow input (although reflection coefficient decreases)
  - Reflection of a positive wave is boosted when the incoming wave is below a certain level and it is attenuated above that level
- When the oscillation reaches a very high amplitude, it is limited on the negative side by the shutting of the

reed, and on the positive side by the attenuation described above

Bernoulli's Equation

In an ideal *inviscid, incompressible flow*, we have, by *conservation of energy*,

$$p + \frac{1}{2}\rho u^2 + \rho g h = \text{constant}$$

where

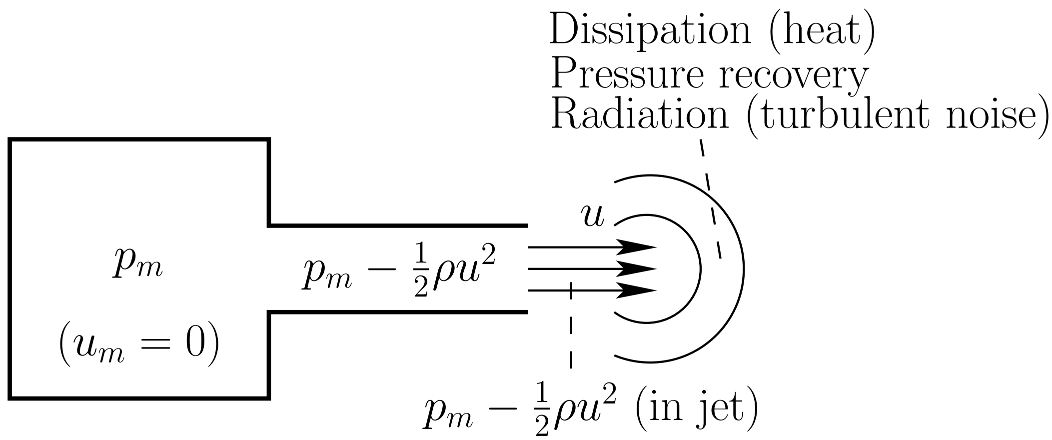
$$\begin{aligned} p &= \text{pressure (Newtons/m}^2 = \text{kg /(m s}^2)) \\ u &= \text{particle velocity (m/s)} \\ \rho &= \text{volume density of air (kg/m}^3) \\ g &= \text{Newton's gravitational constant (m/s}^2) \\ h &= \text{Height of flow's center-of-mass axis (m)} \\ \text{“Inviscid”} &= \text{“Frictionless”, “Lossless”} \end{aligned}$$

Pressure is Proportional to Kinetic Energy

$$p = \frac{1}{3}\rho \left\langle u^2 \right\rangle$$

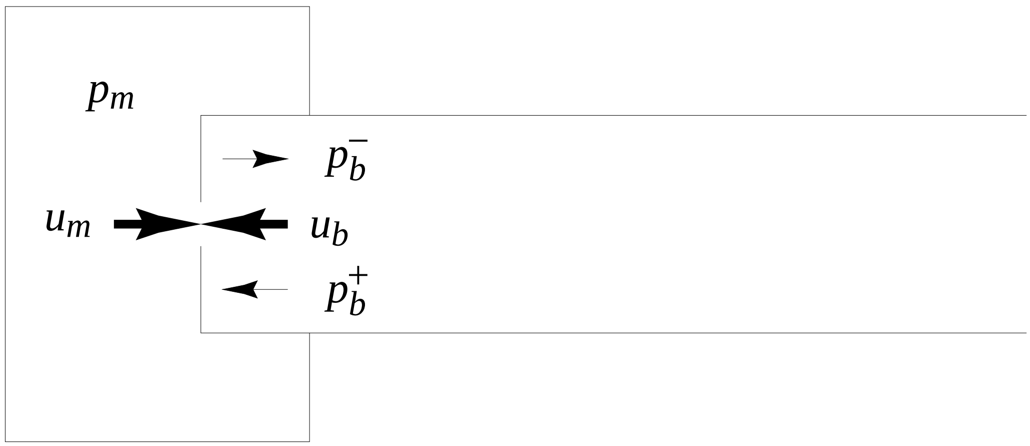
- *Average momentum transfer* / area / time
- Caused by collisions of gas molecules with boundary
- Proof: See *Kinetic Theory of Gases*

Bernoulli Effect



- $p_m =$ "mouth pressure"
- Flow inside "mouth" neglected
- Pressure kinetic energy converts to flow kinetic energy within channel
- Jet "carries its own pressure" until it dissipates
- Jet dissipation can go to
  - heat (now allowing "friction" into the model)
  - vortices (angular momentum)
  - radiation (sound waves)
  - pressure recovery: (flow kinetic energy $\longrightarrow$ pressure kinetic energy)

Single-Reed Theory



$$p_m \stackrel{\Delta}{=} \text{mouth pressure (constant)}$$

$$p_b \stackrel{\Delta}{=} \text{bore pressure (dynamic)}$$

$$p_\Delta \stackrel{\Delta}{=} p_m - p_b \stackrel{\Delta}{=} \text{pressure drop across mouthpiece}$$

$$u_m \stackrel{\Delta}{=} \text{resulting flow into mouthpiece}$$

$$R_m(p_\Delta) \stackrel{\Delta}{=} \text{reed-aperture impedance (measured)}$$

where

$$u_b + u_m = 0 \qquad \text{(by continuity of volume velocity)}$$

$$u_m(p_\Delta) \stackrel{\Delta}{=} \frac{p_\Delta}{R_m(p_\Delta)} \qquad \text{(``Ohm's law'' for the reed)}$$

$$u_b \stackrel{\Delta}{=} u_b^+ + u_b^- = \frac{p_b^+ - p_b^-}{R_b}, \quad R_b \stackrel{\Delta}{=} \text{bore impedance}$$

Toward a Computational Model

Given:

$$\begin{aligned} p_m &= \text{Mouth pressure} \\ p_b^+ &= \text{Incoming traveling bore pressure} \end{aligned}$$

Find:

$$p_b^- = \text{Outgoing traveling bore pressure}$$

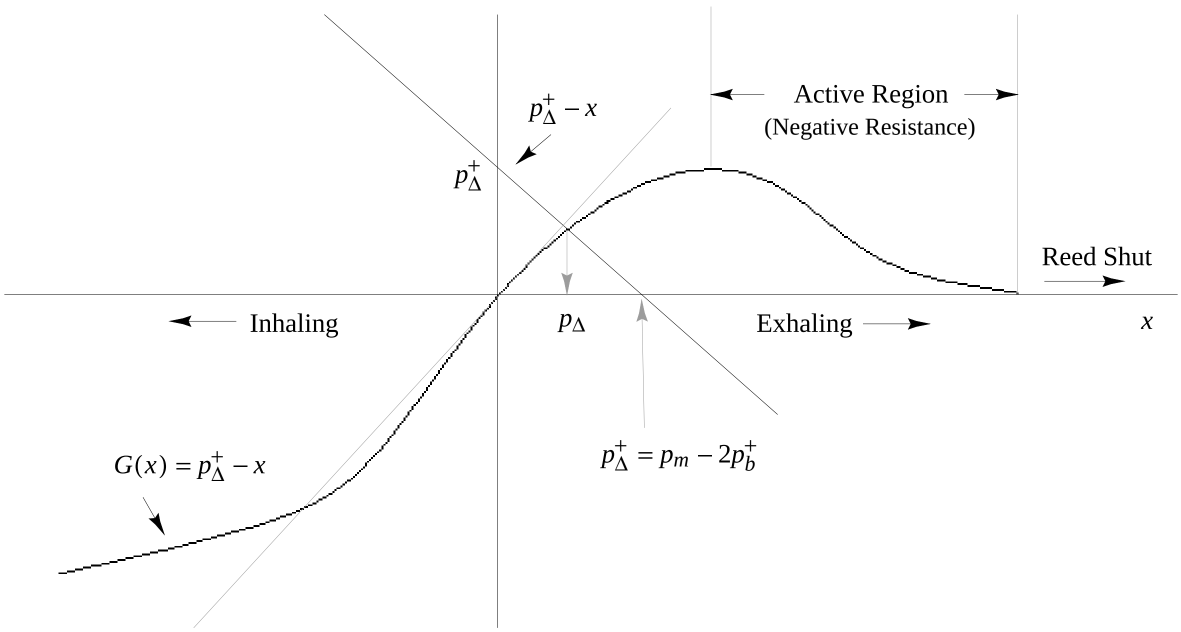
such that:

$$0 = u_m + u_b = \frac{p_\Delta}{R_m(p_\Delta)} + \frac{p_b^+ - p_b^-}{R_b},$$

$$p_\Delta \stackrel{\Delta}{=} p_m - p_b = p_m - (p_b^+ + p_b^-)$$

Solving for p_b^- is not immediate because R_m depends on p_Δ which depends on p_b^-.

Graphical Solution Technique



Graphically solve:

$$G(p_\Delta) = p_\Delta^+ - p_\Delta, \qquad p_\Delta^+ \stackrel{\Delta}{=} p_m - 2p_b^+$$

where

$$G(p_\Delta) \stackrel{\Delta}{=} R_b u_m(p_\Delta) = R_b p_\Delta / R_m(p_\Delta)$$

- Introduced by Friedlander and Keller (1953)
- Analogous to finding the "operating point" of a transistor by intersecting its "operating curve" with the "load line" determined by the load resistance.
- Outgoing wave is then $p_b^- = p_m - p_b^+ - p_\Delta(p_\Delta^+)$

Scattering Formulation

Solve for p_b^- to obtain

$$\begin{aligned} p_b^- &= \frac{1-r}{1+r} p_b^+ + \frac{r}{1+r} p_m \\ &= \rho p_b^+ + \frac{1-\rho}{2} p_m \\ &= \frac{p_m}{2} - \rho \frac{p_\Delta^+}{2} \end{aligned}$$

where

$$\rho(p_\Delta) \stackrel{\Delta}{=} \frac{1-r(p_\Delta)}{1+r(p_\Delta)}, \qquad r(p_\Delta) \stackrel{\Delta}{=} \frac{R_b}{R_m(p_\Delta)}$$

$\rho(p_\Delta) =$ *signal-dependent reflection coefficient.*

In practice, $R_m \gg R_b \;\Rightarrow\; r \approx 0, \rho \approx 1$
(mouthpiece looks largely like a closed end)

For Faster Real-Time Computation

Pre-solve the graphical intersection and store the result in a *look-up table*

Let h denote *half-pressure* $p/2$. Then

$$p_b^- = -\rho(p_\Delta) \cdot h_\Delta^+$$

Subtracting both sides from p_b^+ and solving for ρ gives

$$\rho(p_\Delta) = \frac{p_\Delta}{h_\Delta^+} - 1$$

Now, for each $h_\Delta^+ = -p_b^+$, find p_Δ graphically, and store the resulting reflection coefficient $\rho(p_\Delta)$ as a function of h_Δ^+:

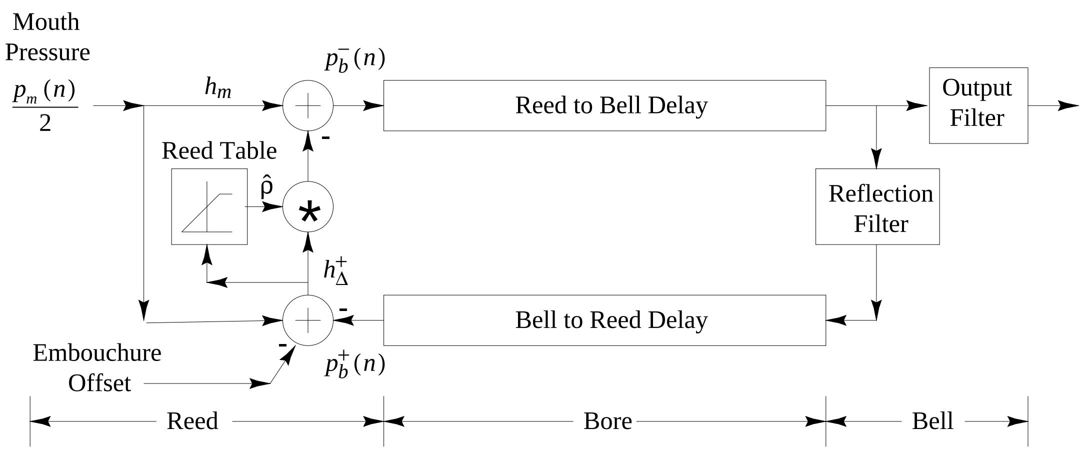
$$\hat{\rho}(h_\Delta^+) = \rho(p_\Delta(h_\Delta^+))$$

Then the real-time reed computation reduces simply to

$$p_b^- = -\hat{\rho}(h_\Delta^+) \cdot h_\Delta^+$$

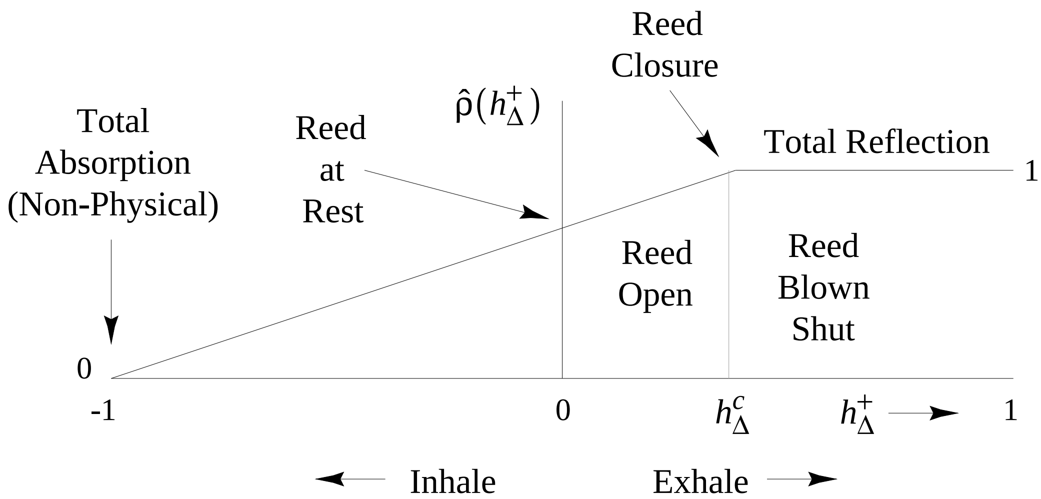
This is the form chosen for implementation above

Table-Reduced Reed Reflection Coefficient



- Control variable = mouth half-pressure
- $h_\Delta^+ = -p_b^+$ computed from incoming bore pressure by a subtraction
- Table is indexed by h_Δ^+
- Result of lookup is multiplied by h_Δ^+
- Result of multiplication is subtracted from
- Total reed cost = two subtractions, one multiply, and one table lookup per sample

Simple Piecewise-Linear Reed Table



$$\hat{\rho}(h_\Delta^+) = \begin{cases} 1 - m(h_\Delta^c - h_\Delta^+), & -1 \leq h_\Delta^+ < h_\Delta^c \\ 1, & h_\Delta^c \leq h_\Delta^+ \leq 1 \end{cases}$$

- Corner point h_Δ^c = smallest pressure difference giving reed closure
- In fixed-point, $h_\Delta^+ \triangleq p_m/2 - p_b^+$ is confined to $[-1, 1)$
- Embouchure and reed-stiffness set by h_Δ^c and slope m [$m = 1/(h_\Delta^c + 1)$ in the figure]
- Zero at maximum negative pressure $h_\Delta^+ = -1$ is not physical but is practical for inhibiting overflow
- A brighter tone is obtained by increasing the *curvature* as the reed begins to open [E.g., $\hat{\rho}^k(h_\Delta^+)$, for $k \gg 1$]

Further Details

- Input mouth pressure is summed with a small amount of white noise corresponding to turbulence, e.g., 0.1% plus more during attacks
- Turbulence *level* and *spectral shape* should be computed *automatically* as a function of pressure drop p_Δ and reed opening geometry (research topic)
- Simple reflection filter:

$$H(z) = \frac{1 + a_1(t)}{1 + a_1(t)z^{-1}}$$

  where $a_1(t) = v(t) - 0.642$, $v(t) = A_v \sin(2\pi f_v t)$,
  $A_v =$ vibrato amplitude (e.g., 0.03), and
  $f_v =$ vibrato frequency (e.g., 5 Hz)
- Further loop filtering occurs as a result of using a linearly interpolated delay line for the bore
- Only one (double length) delay line is really used in typical implementations
- To avoid finger-hole models, legato note transitions can be managed using two delay line taps and cross-fading from one to the other during a transition

Alternative Reed Models

- A direct signal lookup, though requiring much higher resolution, would eliminate the multiply associated with the scattering coefficient
- Coefficient tables can be quantized more heavily in address and word length than direct lookup of a signal value such as $p_\Delta(p_\Delta^+)$
- Piecewise polynomial approximations are also used

Software

`clarinet.cpp` in the Synthesis Toolkit (STK):

`http://ccrma.stanford.edu/CCRMA/Software/STK/`