MUS420/EE367A Lecture 3
Artificial Reverberation and Spatialization

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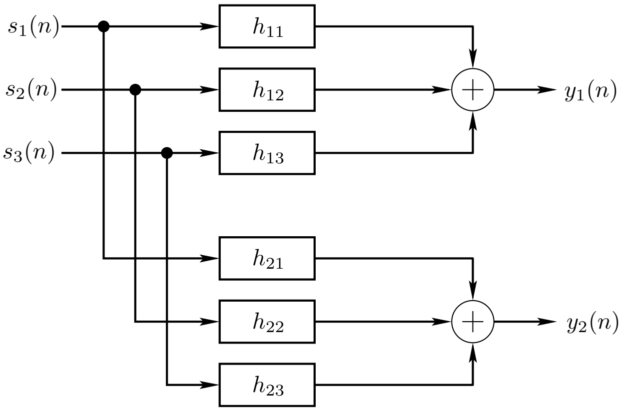
Outline

- The Reverb Problem
- Reverb Perception
- Early Reflections
- Late Reverb
- Schroeder Reverbs
- Feedback Delay Network (FDN) Reverberators
- Waveguide Reverberators

Reverberation Transfer Function



- Three sources
- One listener (two ears)
- Filters should include *pinnae filtering* (*spatialized* reflections)
- Filters change if *anything* in the room changes

In principle, this is an exact computational model.

Implementation

Let $h_{ij}(n) =$ impulse response from source j to ear i. Then the output is given by *six convolutions*:

$$\begin{aligned} y_1(n) &= (s_1 * h_{11})(n) + (s_2 * h_{12})(n) + (s_3 * h_{13})(n) \\ y_2(n) &= (s_1 * h_{21})(n) + (s_2 * h_{22})(n) + (s_3 * h_{23})(n) \end{aligned}$$

- For small n, filters $h_{ij}(n)$ are *sparse*
- Tapped Delay Line (TDL) a natural choice

Transfer-function matrix:

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix} = \begin{bmatrix} H_{11}(z) & H_{12}(z) & H_{13}(z) \\ H_{21}(z) & H_{22}(z) & H_{23}(z) \end{bmatrix} \begin{bmatrix} S_1(z) \\ S_2(z) \\ S_3(z) \end{bmatrix}$$

Complexity of Exact Reverberation

Reverberation time is typically defined as t_{60}, the time, in seconds, to decay by 60 dB.

Example:

- Let $t_{60} = 2$ seconds
- $f_s = 50$ kHz
- Each filter h_{ij} requires 100,000 multiplies and additions per sample, or 5 *billion* multiply-adds per second.
- Three sources and two listening points (ears) ⇒ 60 billion operations per second
  - 20 dedicated CPUs clocked at 3 Gigahertz
  - multiply and addition initiated each clock cycle
  - no wait-states for parallel input, output, and filter coefficient accesses
- FFT convolution is faster, if throughput delay is tolerable (and there are low-latency algorithms)

Conclusion: Exact implementation of point-to-point transfer functions is generally too expensive for real-time computation.

Possibility of a Physical Reverb Model

In a complete *physical model* of a room,

- sources and listeners can be moved without affecting the room simulation itself,
- *spatialized* (in 3D) stereo output signals can be extracted using a "virtual dummy head"

How expensive is a room physical model?

- Audio bandwidth = 20 kHz ≈ 1/2 inch wavelength
- Spatial samples every 1/4 inch or less
- A 12'x19'x8' room requires > 200 million grid points
- A lossless 3D finite difference model requires one multiply and 6 additions per grid point ⇒ 60 billion additions per second at $f_s = 50$ kHz
- A 100'x50'x20' concert hall requires more than *3 quadrillion operations per second*

Conclusion: Fine-grained physical models are too expensive for real-time computation, especially for large halls.

Perceptual Aspects of Reverberation

Artificial reverberation is an unusually interesting signal processing problem:

- "Obvious" methods based on physical modeling or input-output modeling are too expensive
- We do not perceive the full complexity of reverberation
- What is important perceptually?
- How can we simulate only what is audible?

Perception of Echo Density and Mode Density

- For typical rooms
  - Echo density increases as t^2
  - Mode density increases as f^2
- Beyond some time, the echo density is so great that a *stochastic process* results
- Above some frequency, the mode density is so great that a *random frequency response* results
- There is no need to simulate many echoes per sample
- There is no need to implement more resonances than the ear can hear

Proof that Echo Density Grows as Time Squared

Consider a single spherical wave produced from a point source in a rectangular room.

- Tesselate 3D space with copies of the original room
- Count rooms intersected by spherical wavefront

Proof that Mode Density Grows as Freq. Squared

The resonant modes of a rectangular room are given by[1]

$$k^2(l,m,n) = k_x^2(l) + k_y^2(m) + k_z^2(n)$$

- $k_x(l) = l\pi/L_x = l$th harmonic of the fundamental standing wave in the x
- L_x = length of the room along x
- Similarly for y and z
- Mode frequencies map to a uniform 3D Cartesian grid indexed by (l,m,n)
- Grid spacings are π/L_x, π/L_y, and π/L_z in x,y, and z, respectively.
- Spatial frequency k of mode (l,m,n) = *distance* from the (0,0,0) to (l,m,n)
- Therefore, the number of room modes having a given spatial frequency grows as k^2

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[1] For a tutorial on *vector wavenumber*, see Appendix E, section E.6.5, in the text: http://ccrma.stanford.edu/˜jos/pasp/Vector_Wavenumber.html

Early Reflections and Late Reverb

Based on limits of perception, the impulse response of a reverberant room can be divided into two segments

- *Early reflections* = relatively sparse first echoes
- *Late reverberation*—so densely populated with echoes that it is best to characterize the response *statistically*.

Similarly, the *frequency response* of a reverberant room can be divided into two segments.

- Low-frequency sparse distribution of resonant modes
- Modes packed so densely that they merge to form a *random frequency response* with regular statistical properties

Perceptual Metrics for Ideal Reverberation

Some desirable controls for an artificial reverberator include

- $t_{60}(f)$ = desired reverberation time at each frequency
- $G^2(f)$ = signal power gain at each frequency
- $C(f)$ = "clarity" = ratio of impulse-response energy in early reflections to that in the late reverb
- $\rho(f)$ = *inter-aural correlation coefficient* at left and right ears

Perceptual studies indicate that reverberation time $t_{60}(f)$ should be independently adjustable in at least *three* frequency bands.

Energy Decay Curve (EDC)

For measuring and defining reverberation time t_{60}, Schroeder introduced the so-called *energy decay curve (EDC)* which is the *tail integral* of the squared impulse response at time t:

$$\text{EDC}(t) \overset{\Delta}{=} \int_t^\infty h^2(\tau)d\tau$$

- EDC(t) = total signal energy remaining in the reverberator impulse response at time t
- EDC decays more smoothly than the impulse response itself
- Better than ordinary amplitude envelopes for estimating t_{60}

Energy Decay Relief (EDR)

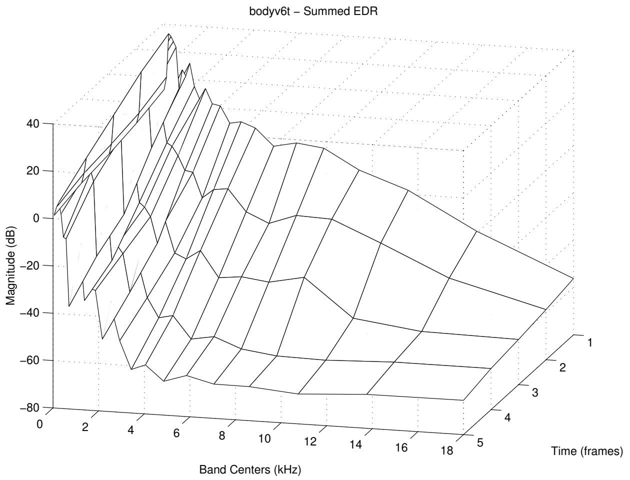
The *energy decay relief (EDR)* generalizes the EDC to multiple frequency bands:

$$\text{EDR}(t_n, f_k) \stackrel{\Delta}{=} \sum_{m=n}^{M} |H(m,k)|^2$$

where $H(m,k)$ denotes bin k of the short-time Fourier transform (STFT) at time-frame m, and M is the number of frames.

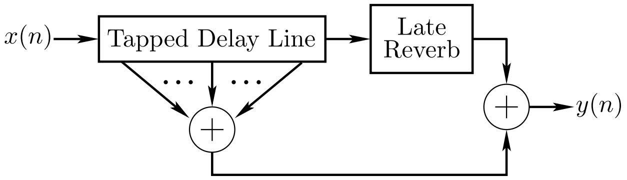
- FFT window length $\approx 30 - 40$ ms
- EDR$(t_n, f_k) =$ total signal energy remaining at time t_n sec in frequency band centered at f_k

Energy Decay Relief (EDR) of a Violin Body Impulse Response



- Energy summed over frequency within each "critical band of hearing" (Bark band)
- Violin body = "small box reverberator"

Reverb = Early Reflections + Late Reverb



- TDL taps may include lowpass filters (air absorption, lossy reflections)
- Several taps may be fed to late reverb unit, especially if it takes a while to reach full density
- Some or all early reflections can usually be worked into the delay lines of the late-reverberation simulation (transposed tapped delay line)

Early Reflections

The "early reflections" portion of the impulse response is defined as everything up to the point at which a statistical description of the late reverb becomes appropriate

- Often taken to be the first 100ms
- Better to test for *Gaussianness*
  - *Histogram* test for sample amplitudes in 10ms windows
  - *Exponential fit* (t_{60} match) to EDC (Prony's method, matrix pencil method)
  - *Crest factor* test (peak/rms)
- Typically implemented using *tapped delay lines* (TDL) (suggested by Schroeder in 1970 and implemented by Moorer in 1979)
- Early reflections should be *spatialized* (Kendall)
- Early reflections influence *spatial impression*

Late Reverberation

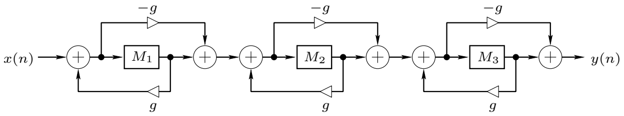
Desired Qualities:

1. a smooth (but not too smooth) decay, and
2. a smooth (but not too regular) frequency response.

- Exponential decay no problem
- Hard part is making it *smooth*
  - Must not have “flutter,” “beating,” or unnatural irregularities
  - Smooth decay generally results when the echo density is sufficiently high
  - Some short-term energy fluctuation is required for naturalness
- A smooth *frequency response* has no large “gaps” or “hills”
  - Generally provided when the mode density is sufficiently large
  - Modes should be spread out uniformly
  - Modes may not be too regularly spaced, since audible periodicity in the time-domain can result

- Moorer’s ideal late reverb: *exponentially decaying white noise*
  - Good smoothness in both time and frequency domains
  - High frequencies need to decay faster than low frequencies
- Schroeder’s rule of thumb for echo density in the late reverb is 1000 echoes per second or more
- For impulsive sounds, 10,000 echoes per second or more may be necessary for a smooth response

Schroeder Allpass Sections (Late Reverb)



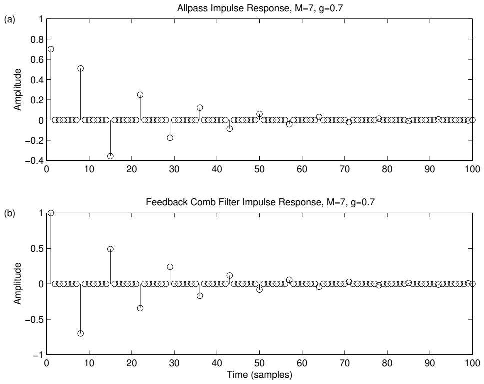
- Typically, $g = 0.7$
- Delay-line lengths M_i mutually prime, and span successive orders of magnitude e.g., $1051, 337, 113$
- Allpass filters in series are allpass
- Each allpass *expands* each nonzero input sample from the previous stage into an entire infinite allpass impulse response
- Allpass sections may be called “*impulse expanders*”, “impulse diffusers” or simply “diffusers”
- NOT a physical model of diffuse reflection, but single reflections are expanded into many reflections, which is qualitatively what is desired.

Why Allpass?

- Allpass filters do not occur in natural reverberation!
- “Colorless reverberation” is an idealization only possible in the “virtual world”
- **Perceptual factorization:**
  Coloration now orthogonal to decay time and echo density

Are Allpass Filters Really Colorless?

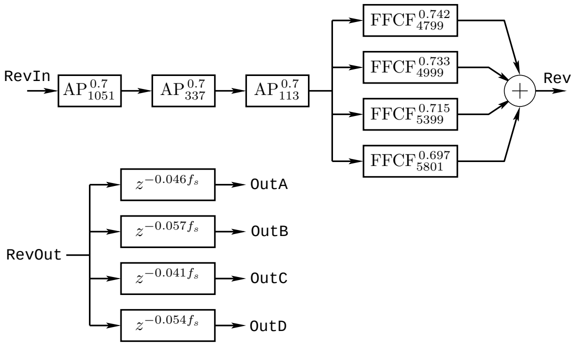
- Allpass impulse response only "colorless" when extremely short (less than 10 ms or so).
- Long allpass impulse responses sound like feedback comb-filters
- The difference between an allpass and feedback-comb-filter impulse response is *one echo*!



(a) $H(z) = \frac{0.7+z^{-7}}{1+0.7z^{-7}}$ (b) $H(z) = \frac{1}{1+0.7z^{-7}}$

- Steady-state tones (sinusoids) really do see the same gain at every frequency in an allpass, while a comb filter has widely varying gains.

A Schroeder Reverberator called JCRev



Classic Schroeder reverberator `JCRev`.

`JCRev` was developed by John Chowning and others at CCRMA based on the ideas of Schroeder.

- Three Schroeder allpass sections:

$$\mathrm{AP}_N^g \triangleq \frac{g + z^{-N}}{1 + gz^{-N}}$$

- Four feedforward comb-filters (STK uses FBCFs):

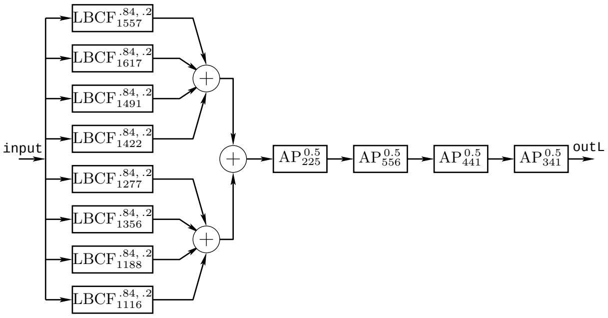
$$\mathrm{FFCF}_N^g \triangleq g + z^{-N}$$

- Schroeder suggests a progression of delays close to

$$M_i T \approx \frac{100 \text{ ms}}{3^i}, \quad i = 0, 1, 2, 3, 4.$$

- Comb filters impart distinctive coloration:
  - Early reflections
  - Room size
  - Could be one tapped delay line
- Usage: Instrument adds scaled output to `RevIn`
- Reverberator output `RevOut` goes to four *delay lines*
  - Four channels *decorrelated*
  - *Imaging* of reverberation between speakers avoided
- For stereo listening, Schroeder suggests a *mixing matrix* at the reverberator output, replacing the decorrelating delay lines
- A mixing matrix should produce maximally rich yet uncorrelated output signals
- `JCRev` is in the Synthesis Tool Kit (STK)
  - `JCRev.cpp`
  - `JCRev.h`.

Freeverb



- Four Schroeder "diffusion allpasses" in series
- Eight parallel Schroeder-Moorer lowpass-feedback-comb-filters:

$$\mathrm{LBCF}_N^{f,d} \triangleq \frac{1}{1 - f\frac{1-d}{1-d\,z^{-1}}\,z^{-N}}$$

- Second stereo channel: increase all 12 delay-line lengths by "stereo spread" (default = 23 samples)
- Used extensively in the free-software world

Freeverb Parameters

- d ("damping") default:

  $\texttt{damp} = \texttt{initialdamp * scaledamp} = 0.5 \cdot 0.4 = 0.2$

- f ("room size") default:

  $$\begin{aligned}\texttt{roomsize} &= \texttt{initialroom * scaleroom + offsetroom}\\ &= 0.5 \cdot 0.28 + 0.7 = 0.84\end{aligned}$$

- Feedback lowpass $(1-d)/(1-dz^{-1})$ causes reverberation time $t_{60}(\omega)$ to decrease with frequency ω, which is natural
- f mainly determines reverberation time at low-frequencies (where feedback lowpass has negligible effect)
- At very high frequencies, $t_{60}(\omega)$ is dominated by the diffusion allpass filters

T60 in Freeverb

- "Room size" f sets low-frequency t_{60}
- "damping" d controls how rapidly t_{60} shortens as frequency increases
- Diffusion allpasses set lower bound on t_{60}

Interpreting "Room Size" Parameter

- Low-frequency reflection-coefficient for two plane-wave wall bounces
- Could be called `liveness` or `reflectivity`
- Changing roomsize normally requires changing delay-line lengths

Freeverb Allpass Approximation

Schroeder Diffusion Allpass

$$\text{AP}_N^g \stackrel{\Delta}{=} \frac{-g + z^{-N}}{1 - gz^{-N}}$$

Freeverb implements

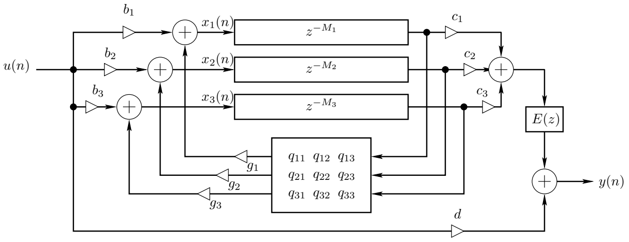
$$\text{AP}_N^g \approx \frac{-1 + (1+g)z^{-N}}{1 - gz^{-N}}$$

- Each Freeverb "allpass" is more precisely a feedback comb-filter FBCF_N^g in series with a feedforward comb-filter $\text{FFCF}_N^{-1,1+g}$, where

$$\begin{aligned}\text{FBCF}_N^g &\stackrel{\Delta}{=} \frac{1}{1 - g\,z^{-N}}\\ \text{FFCF}_N^{-1,1+g} &\stackrel{\Delta}{=} -1 + (1+g)z^{-N}.\end{aligned}$$

- A true allpass is obtained at $g = (\sqrt{5}-1)/2 \approx 0.618$ (reciprocal of "golden ratio")
- Freeverb default is $g = 0.5$

FDN Late Reverberation



Jot (1991) FDN Reverberator for $N = 3$

- Generalized state-space model (unit delays replaced by arbitrary delays)
- Note direct path weighted by d
- The "tonal correction" filter $E(z)$ equalizes mode energy independent of reverberation time (perceptual orthogonalization)
- Gerzon 1971: "orthogonal matrix feedback reverb" cross-coupled feedback comb filters (see below)

Choice of Orthogonal Feedback Matrix Q

Late reverberation should resemble exponentially decaying noise. This suggests the following two-step procedure for reverberator design:

1. Set $t_{60} = \infty$ and make a *good white-noise generator*
2. Establish desired reverberation times in each frequency band by *introducing losses*

The white-noise generator is the *lossless prototype* reverberator.

Hadamard Feedback Matrix

A second-order *Hadamard matrix*:

$$\mathbf{H}_2 \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},$$

Higher order Hadamard matrices defined by recursive embedding:

$$\mathbf{H}_4 \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} \mathbf{H}_2 & \mathbf{H}_2 \\ -\mathbf{H}_2 & \mathbf{H}_2 \end{bmatrix}.$$

- Since H_3 does not exist, the FDN example figure above can be redrawn for $N = 4$, say, (instead of $N = 3$), so that we can set $Q = H_4$
- The *Hadamard conjecture* posits the existence of Hadamard matrices H_N of order $N = 4k$ for all positive integers k.
- "As of 2008, there are 13 multiples of 4 less than or equal to 2000 for which no Hadamard matrix of that order is known. They are: 668, 716, 892, 1004, 1132, 1244, 1388, 1436, 1676, 1772, 1916, 1948, 1964." [`http://en.wikipedia.org/wiki/Hadamard_matrix`]

Choice of Delay Lengths M_i

- Delay line lengths M_i are typically *mutually prime* (Schroeder)
- For *sufficiently high mode density*, $\sum_i M_i$ must be sufficiently large.
  - No "ringing tones" in the late impulse response
  - No "flutter"

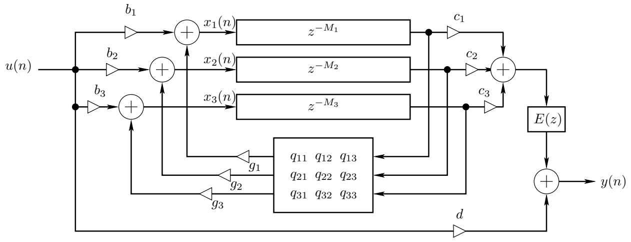
Mode Density Requirement

FDN order = sum of delay lengths:

$$M \triangleq \sum_{i=1}^{N} M_i \qquad \text{(FDN order)}$$

- Order = number of poles
- All M poles are on the unit circle in the lossless prototype
- If uniformly distributed, mode density =
$$\frac{M}{f_s} = MT \quad \text{modes per Hz}$$
- Schroeder suggests 0.15 modes per Hz (when $t_{60} = 1$ second)
- Generalizing:
$$M \geq 0.15\, t_{60}\, f_s$$
- Example: For $f_s = 50$ kHz and $t_{60} = 1$ second, $M \geq 7500$
- Note that $M = t_{60}\, f_s$ is the length of the FIR filter giving a *perceptually exact* implementation. Thus, *recursive filtering is about 7 times more efficient* by this rule of thumb.

Choice of Loss Gains g_i



Jot (1991) FDN Reverberator for $N = 3$

- To set the reverberation time t_{60}, we need to move the poles of the lossless prototype slightly *inside* the unit circle
- The scaling coefficients g_i can accomplish this for $0 < g_i < 1$
- Since high-frequencies decay faster in propagation through air, we want to move the high-frequency poles farther in than low-frequency poles
- Therefore, we need to generalize g_i above to $G_i(z)$, with $|G_i(e^{j\omega T})| \leq 1$ imposed to ensure stability

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Damping Filter Design

The *damping filter* $G_i(z)$ associated with the delay line of length M_i in the FDN can be written in principle as

$$G_i(z) = G_T^{M_i}(z) L_i(z)$$

where $G_T(z)$ is the lowpass filter corresponding to *one sample* of wave propagation through air, and $L_i(z)$ is a lowpass corresponding to absorbing/scattering boundary reflections along the (hypothetical) ith propagation path.

Define

$$\begin{aligned} t_{60}(\omega) &= \text{desired reverberation time at frequency } \omega \\ p_k &= e^{j\omega_k T} = k\text{th pole of the lossless prototype} \end{aligned}$$

We can introduce *frequency-independent* damping with the (conformal map) substitution

$$z^{-1} \leftarrow g\, z^{-1}$$

- This z-plane mapping pulls all poles in the z plane from the unit circle to the circle of radius g
- Pole $p_k = e^{j\omega_k T}$ moves to $\tilde{p}_k = g\, e^{j\omega_k T}$

Example

- Start with a pole at dc (digital integrator):

$$H(z) = \frac{1}{1 - z^{-1}} \;\leftrightarrow\; [1, 1, 1, \ldots]$$

- Move it from radius 1 to radius 0.9 using $z^{-1} \leftarrow 0.9z^{-1}$:

$$H(z) = \frac{1}{1 - 0.9z^{-1}} \;\leftrightarrow\; [1, 0.9, 0.81, \ldots]$$

Frequency-Dependent Damping

For *frequency-dependent* damping, consider the mapping

$$\boxed{z^{-1} \leftarrow G(z)\, z^{-1}}$$

where $G(z)$ is a lowpass filter satisfying $\left|G(e^{j\omega T})\right| \leq 1$, $\forall\omega$

- Neglecting phase in the loss filter $G(z)$, the substitution $z^{-1} \leftarrow G(z)\, z^{-1}$ only affects the pole radius, not angle
- $G(z)$ = *per-sample filter* in the propagation medium
- Schroeder (1961):

  > The reverberation times of the individual modes must be equal or nearly equal so that different frequency components of the sound decay with equal rates ⇒

  - All pole radii in the reverberator should vary smoothly with frequency
  - Otherwise, late decay will be dominated by largest pole(s)

Lossy Mapping

Let's in more detail look at the z-plane mapping

$$z^{-1} \leftarrow G(z)\, z^{-1}$$

- Pole k contributes the term

  $$H_k(z) = \frac{r_k}{1 - p_k z^{-1}} = r_k \cdot \left(1 + p_k z^{-1} + p_k^2 z^{-2} + \cdots\right)$$

  to the partial fraction expansion of the transfer function
- This term maps to

  $$\begin{aligned} \tilde{H}_k(z) &= \frac{r_k}{1 - p_k[G(z)z^{-1}]} \\ &= r_k \cdot \left(1 + [G(z)p_k]z^{-1} + [G(z)p_k]^2 z^{-2} + \cdots\right) \end{aligned}$$

- Thus, pole k moves from $z = p_k = e^{j\omega_k T}$ to

  $$\tilde{p}_k = R_k\, e^{j\omega_k T}$$

  where

  $$R_k = G\left(R_k\, e^{j\omega_k T}\right) \approx G\left(e^{j\omega_k T}\right)$$

  which is a good approximation here since R_k is nearly 1 for reverberators.

Example

- Start with a pole at dc (digital integrator):

  $$H(z) = \frac{1}{1 - z^{-1}} \;\leftrightarrow\; [1, 1, 1, \ldots]$$

- Move it from radius 1 to radius 0.9 using $z^{-1} \leftarrow 0.9\, z^{-1}$:

  $$H(z) = \frac{1}{1 - 0.9\, z^{-1}} \;\leftrightarrow\; [1,\ 0.9,\ 0.81,\ \ldots]$$

- Now progress from radius 0.9 to 0.8 using

  $$z^{-1} \leftarrow 0.9 \frac{1 + \alpha z^{-1}}{1 + \alpha} z^{-1}$$

  with $0.8 = (1 - \alpha)/(1 + \alpha)$
  $\Rightarrow \alpha = (1 - 0.8)/(1 + 0.8) = 1/0.9$:

  $$\begin{aligned} H(z) &= \frac{1}{1 - 0.9\, \frac{1 + \alpha z^{-1}}{1 + \alpha} z^{-1}} = \frac{1}{1 - 0.9\, \frac{0.9 + z^{-1}}{0.9 + 1} z^{-1}} \\ &= \frac{1}{1 - \frac{0.81}{1.9} z^{-1} + \frac{1}{1.9} z^{-2}} \end{aligned}$$

Desired Pole Radius

Pole radius R_k and t_{60} are related by

$$R_k^{t_{60}(\omega_k)/T} = 0.001$$

The ideal loss filter $G(z)$ therefore satisfies

$$|G(\omega)|^{t_{60}(\omega)/T} = 0.001$$

The desired delay-line filters are therefore

$$G_i(z) = G^{M_i}(z)$$

$\Rightarrow$

$$\left|G_i(e^{j\omega T})\right|^{\frac{t_{60}(\omega)}{M_i T}} = 0.001.$$

or

$$\boxed{20 \log_{10} \left|G_i(e^{j\omega T})\right| = -60 \frac{M_i T}{t_{60}(\omega)}.}$$

Now use `invfreqz` or `stmcb`, etc., in Matlab to design low-order filters $G_i(z)$ for each delay line.

First-Order Delay-Filter Design

Jot used first-order loss filters for each delay line:

$$G_i(z) = g_i \frac{1 - a_i}{1 - a_i z^{-1}}$$

- g_i gives desired reverberation time at dc
- a_i sets reverberation time at high frequencies

Design formulas:

$$\begin{aligned} g_i &= 10^{-3 M_i T / t_{60}(0)} \\ a_i &= \frac{\ln(10)}{4} \log_{10}(g_i) \left(1 - \frac{1}{\alpha^2}\right) \end{aligned}$$

where

$$\alpha \triangleq \frac{t_{60}(\pi/T)}{t_{60}(0)}$$

Tonal Correction Filter

Let $h_k(n)$ = impulse response of kth system pole. Then

$$\mathcal{E}_k = \sum_{n=0}^{\infty} |h_k(n)|^2 = \text{total energy}$$

Thus, *total energy is proportional to decay time*.

To compensate, Jot proposes a *tonal correction filter* $E(z)$ for the late reverb (not the direct signal).

First-order case:

$$E(z) = \frac{1 - bz^{-1}}{1 - b}$$

where

$$b = \frac{1-\alpha}{1+\alpha}$$

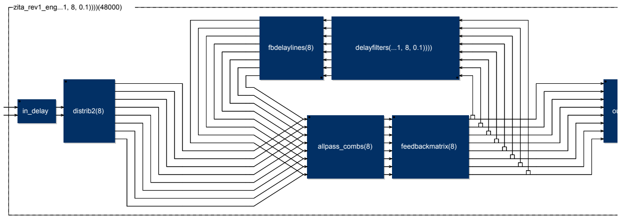
and

$$\alpha \triangleq \frac{t_{60}(\pi/T)}{t_{60}(0)}$$

as before.

Zita-Rev1 Reverberator

- FDN+Schroeder reverberator
- Free open-source C++ for Linux by Fons Adriaensen
- Faust example `zita_rev1.dsp`



`faust2firefox examples/zita_rev1.dsp`

Feedback Delay Network + Schroeder Allpass Comb Filters:

- Allpass coefficients ± 0.6
- Inspect Faust block diagram for delay-line lengths, etc.

Zita-Rev1 Damping Filters

FDN reverberators employ a *damping filter* for each delay line

Zita-Rev1 *three-band* damping filter:

$$H_d(z) = H_l(z) H_h(z)$$

where

$$\begin{aligned} H_l(z) &= g_m + (g_0 - g_m)\frac{1-p_l}{2}\frac{1+z^{-1}}{1-p_l z^{-1}} = \textit{low-shelf} \\ H_h(z) &= \frac{1-p_h}{1-p_h z^{-1}} = \textit{low-pass} \end{aligned}$$

g_0 = Desired gain at dc

g_m = Desired gain across "middle frequencies"

p_l = Low-shelf pole controlling low-to-mid crossover: $\triangleq \dfrac{1-\pi f_1 T}{1+\pi f_1 T}$

p_h = Low-pass pole controlling high-frequency damping: Gives *half* middle-band t_{60} at start of "high" band

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High-Frequency-Damping Lowpass

High-Frequency Damping Lowpass:

$$H_h(z) = \frac{1-p_h}{1-p_h z^{-1}}$$

For t_{60} at "HF Damping" frequency f_h to be half of middle-band t_{60} (gain g_m), we require

$$\left|H_h\left(e^{j2\pi f_h T}\right)\right| = \left|\frac{1-p_h}{1-p_h e^{-j2\pi f_h T}}\right| = g_m$$

Squaring and normalizing yields a quadratic equation:

$$p_h^2 + b\, p_h + 1 = 0$$

Solving for p_h using the quadratic formula yields

$$p_h = -\frac{b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - 1},$$

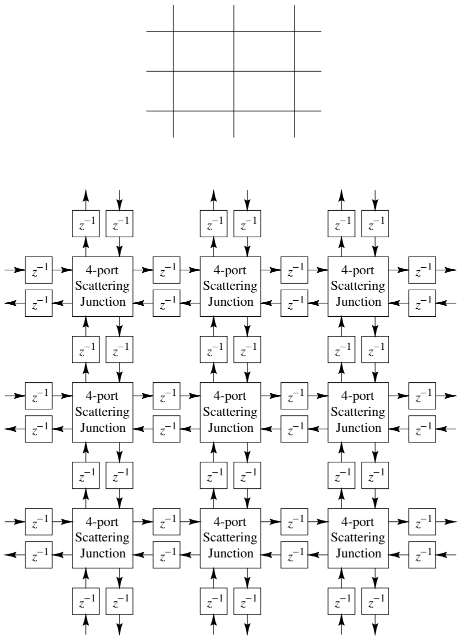
where

$$-\frac{b}{2} = \frac{1 - g_m^2 \cos(2\pi f_h T)}{1 - g_m^2} > 1,$$

Discard unstable solution $-b/2 + \sqrt{(b/2)^2 - 1} > 1$

To ensure $|g_m| < 1$, GUI keeps middle-band t_{60} finite

Rectilinear Digital Waveguide Mesh



Waveguide Mesh Features

- A *mesh* of such waveguides in 2D or 3D can simulate waves traveling in *any* direction in the space.
- Analogy: tennis racket = rectilinear mesh of strings = pseudo-membrane
- Wavefronts are explicitly simulated in *all directions*
- True *diffuse field* in late reverb
- Spatialized reflections are “free”
- Echo density grows naturally with time
- Mode density grows naturally with frequency
- Low-frequency modes very accurately simulated
- High-frequency modes mistuned due to *dispersion* (can be corrected) (often not heard)
- Multiply free almost everywhere
- Coarse mesh captures most perceptual details

Reverb Resources on the Web

- Book chapter from our texthttp://interactiveachttps://ccrma.stanford.edu/ jos/pasp/Arti
- Room Acoustics Modeling with Interactive Visualizations by Lauri Saviojahttp://interactiveacoustics.info/
- Room Mode Calculator[2]
- Valhalla reverb plugins[3]
- Harmony Central article[4] (with sound examples)
- William Gardner’s MIT Master’s thesis[5]

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[2] https://amcoustics.com/tools/amroc

[3] https://valhalladsp.com/

[4] http://www.harmony-central.com/Effects/Articles/Reverb/

[5] http://www.harmony-central.com/Computer/Programming/virtual-acoustic-room.ps.gz