Outline

- The Reverb Problem
- Reverb Perception
- Early Reflections
- Late Reverb
- Schroeder Reverbs
- Feedback Delay Network (FDN) Reverberators
- Waveguide Reverberators

Reverberation Transfer Function

\[ h_{11} \]
\[ h_{12} \]
\[ h_{13} \]
\[ h_{21} \]
\[ h_{22} \]
\[ h_{23} \]

\[ s_1(n) \]
\[ s_2(n) \]
\[ s_3(n) \]

\[ y_1(n) \]
\[ y_2(n) \]

- Three sources
- One listener (two ears)
- Filters should include pinnae filtering (spatialized reflections)
- Filters change if anything in the room changes

In principle, this is an exact computational model.
Implementation

Let $h_{ij}(n) = \text{impulse response from source } j \text{ to ear } i$. Then the output is given by six convolutions:

$y_1(n) = (s_1 \ast h_{11})(n) + (s_2 \ast h_{12})(n) + (s_3 \ast h_{13})(n)$

$y_2(n) = (s_1 \ast h_{21})(n) + (s_2 \ast h_{22})(n) + (s_3 \ast h_{23})(n)$

- For small $n$, filters $h_{ij}(n)$ are sparse
- Tapped Delay Line (TDL) a natural choice

Transfer-function matrix:

$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix} = \begin{bmatrix} H_{11}(z) & H_{12}(z) & H_{13}(z) \\ H_{21}(z) & H_{22}(z) & H_{23}(z) \end{bmatrix} \begin{bmatrix} S_1(z) \\ S_2(z) \\ S_3(z) \end{bmatrix}$$

Complexity of Exact Reverberation

Reverberation time is typically defined as $t_{60}$, the time, in seconds, to decay by 60 dB.

Example:

- Let $t_{60} = 2$ seconds
- $f_s = 50$ kHz
- Each filter $h_{ij}$ requires 100,000 multiplies and additions per sample, or 5 billion multiply-adds per second.
- Three sources and two listening points (ears) $\Rightarrow$ 60 billion operations per second
  - 20 dedicated CPUs clocked at 3 Gigahertz
  - multiply and addition initiated each clock cycle
  - no wait-states for parallel input, output, and filter coefficient accesses
- FFT convolution is faster, if throughput delay is tolerable (and there are low-latency algorithms)

Conclusion: Exact implementation of point-to-point transfer functions is generally too expensive for real-time computation.
Possibility of a Physical Reverb Model

In a complete physical model of a room,

- sources and listeners can be moved without affecting the room simulation itself,
- spatialized (in 3D) stereo output signals can be extracted using a “virtual dummy head”

How expensive is a room physical model?

- Audio bandwidth = 20 kHz \( \approx \) 1/2 inch wavelength
- Spatial samples every 1/4 inch or less
- A 12’x19’x8’ room requires \( > 200 \text{ million grid points} \)
- A lossless 3D finite difference model requires one multiply and 6 additions per grid point \( \Rightarrow 60 \text{ billion additions per second at } f_s = 50 \text{ kHz} \)
- A 100’x50’x20’ concert hall requires more than 3 quadrillion operations per second

Conclusion: Fine-grained physical models are too expensive for real-time computation, especially for large halls.

Perceptual Aspects of Reverberation

Artificial reverberation is an unusually interesting signal processing problem:

- “Obvious” methods based on physical modeling or input-output modeling are too expensive
- We do not perceive the full complexity of reverberation
- What is important perceptually?
- How can we simulate only what is audible?
Perception of Echo Density and Mode Density

- For typical rooms
  - Echo density increases as $t^2$
  - Mode density increases as $f^2$
- Beyond some time, the echo density is so great that a stochastic process results
- Above some frequency, the mode density is so great that a random frequency response results
- There is no need to simulate many echoes per sample
- There is no need to implement more resonances than the ear can hear

Proof that Echo Density Grows as Time Squared

Consider a single spherical wave produced from a point source in a rectangular room.

- Tessellate 3D space with copies of the original room
- Count rooms intersected by spherical wavefront

Proof that Echo Density Grows as Time Squared

Consider a single spherical wave produced from a point source in a rectangular room.

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- Count rooms intersected by spherical wavefront
Proof that Mode Density Grows as Freq. Squared

The resonant modes of a rectangular room are given by

\[ k^2(l, m, n) = k_x^2(l) + k_y^2(m) + k_z^2(n) \]

- \( k_x(l) = l\pi/L_x = l \)th harmonic of the fundamental standing wave in the \( x \)
- \( L_x = \) length of the room along \( x \)
- Similarly for \( y \) and \( z \)
- Mode frequencies map to a uniform 3D Cartesian grid indexed by \((l, m, n)\)
- Grid spacings are \( \pi/L_x, \pi/L_y, \) and \( \pi/L_z \) in \( x, y, \) and \( z \), respectively.
- Spatial frequency \( k \) of mode \((l, m, n)\) is the distance from the \( (0,0,0) \) to \((l, m, n)\)
- Therefore, the number of room modes having a given spatial frequency grows as \( k^2 \)

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Early Reflections and Late Reverb

Based on limits of perception, the impulse response of a reverberant room can be divided into two segments

- *Early reflections* = relatively sparse first echoes
- *Late reverberation*—so densely populated with echoes that it is best to characterize the response statistically.

Similarly, the frequency response of a reverberant room can be divided into two segments.

- Low-frequency sparse distribution of resonant modes
- Modes packed so densely that they merge to form a random frequency response with regular statistical properties

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footnote[For a tutorial on vector wavenumber, see Appendix E, section E.6.5, in the text: http://ccrma.stanford.edu/˜jos/pasp/VectorWavenumber.html]
Perceptual Metrics for Ideal Reverberation

Some desirable controls for an artificial reverberator include

- $t_{60}(f) =$ desired reverberation time at each frequency
- $G^2(f) =$ signal power gain at each frequency
- $C(f) =$ “clarity” $=$ ratio of impulse-response energy in early reflections to that in the late reverb
- $\rho(f) =$ inter-aural correlation coefficient at left and right ears

Perceptual studies indicate that reverberation time $t_{60}(f)$ should be independently adjustable in at least three frequency bands.

Energy Decay Curve (EDC)

For measuring and defining reverberation time $t_{60}$, Schroeder introduced the so-called energy decay curve (EDC) which is the tail integral of the squared impulse response at time $t$:

$$\text{EDC}(t) \triangleq \int_t^\infty h^2(\tau) d\tau$$

- EDC$(t) =$ total signal energy remaining in the reverberator impulse response at time $t$
- EDC decays more smoothly than the impulse response itself
- Better than ordinary amplitude envelopes for estimating $t_{60}$
**Energy Decay Relief (EDR)**

The *energy decay relief (EDR)* generalizes the EDC to multiple frequency bands:

\[
EDR(t_n, f_k) \triangleq \sum_{m=n}^{M} |H(m, k)|^2
\]

where \(H(m, k)\) denotes bin \(k\) of the short-time Fourier transform (STFT) at time-frame \(m\), and \(M\) is the number of frames.

- FFT window length \(\approx 30 - 40\) ms
- \(EDR(t_n, f_k) = \) total signal energy remaining at time \(t_n\) sec in frequency band centered at \(f_k\)

**Energy Decay Relief (EDR) of a Violin Body Impulse Response**

- Energy summed over frequency within each “critical band of hearing” (Bark band)
- Violin body = “small box reverberator”
Reverb = Early Reflections + Late Reverb

\[ x(n) \rightarrow \text{Tapped Delay Line} \rightarrow \text{Late Reverb} \rightarrow y(n) \]

- TDL taps may include lowpass filters (air absorption, lossy reflections)
- Several taps may be fed to late reverb unit, especially if it takes a while to reach full density
- Some or all early reflections can usually be worked into the delay lines of the late-reverberation simulation (transposed tapped delay line)

Early Reflections

The “early reflections” portion of the impulse response is defined as everything up to the point at which a statistical description of the late reverb becomes appropriate

- Often taken to be the first 100ms
- Better to test for *Gaussian*ness
  - *Histogram* test for sample amplitudes in 10ms windows
  - *Exponential fit* (\(t_{60}\) match) to EDC (Prony’s method, matrix pencil method)
  - *Crest factor* test (peak/rms)
- Typically implemented using *tapped delay lines* (TDL) (suggested by Schroeder in 1970 and implemented by Moorer in 1979)
- Early reflections should be *spatialized* (Kendall)
- Early reflections influence *spatial impression*
Late Reverberation

Desired Qualities:

1. a smooth (but not too smooth) decay, and
2. a smooth (but not too regular) frequency response.

- Exponential decay no problem
- Hard part is making it smooth
  - Must not have “flutter,” “beating,” or unnatural irregularities
  - Smooth decay generally results when the echo density is sufficiently high
  - Some short-term energy fluctuation is required for naturalness
- A smooth frequency response has no large “gaps” or “hills”
  - Generally provided when the mode density is sufficiently large
  - Modes should be spread out uniformly
  - Modes may not be too regularly spaced, since audible periodicity in the time-domain can result

- Moorer’s ideal late reverb: exponentially decaying white noise
  - Good smoothness in both time and frequency domains
  - High frequencies need to decay faster than low frequencies
- Schroeder’s rule of thumb for echo density in the late reverb is 1000 echoes per second or more
- For impulsive sounds, 10,000 echoes per second or more may be necessary for a smooth response
Schroeder Allpass Sections (Late Reverb)

\[ x(n) \rightarrow M_1 \rightarrow M_2 \rightarrow M_3 \rightarrow y(n) \]

- Typically, \( g = 0.7 \)
- Delay-line lengths \( M_i \) mutually prime, and span successive orders of magnitude e.g., 1051, 337, 113
- Allpass filters in series are allpass
- Each allpass expands each nonzero input sample from the previous stage into an entire infinite allpass impulse response
- Allpass sections may be called “impulse expanders”, “impulse diffusers” or simply “diffusers”
- NOT a physical model of diffuse reflection, but single reflections are expanded into many reflections, which is qualitatively what is desired.

Why Allpass?

- Allpass filters do not occur in natural reverberation!
- “Colorless reverberation” is an idealization only possible in the “virtual world”
- **Perceptual factorization:**
  Coloration now orthogonal to decay time and echo density
Are Allpass Filters Really Colorless?

- Allpass impulse response only “colorless” when extremely short (less than 10 ms or so).
- Long allpass impulse responses sound like feedback comb-filters
- The difference between an allpass and feedback-comb-filter impulse response is one echo!

(a) $H(z) = \frac{0.7 + z^{-7}}{1 + 0.7 z^{-7}}$ (b) $H(z) = \frac{1}{1 + 0.7 z^{-7}}$

- Steady-state tones (sinusoids) really do see the same gain at every frequency in an allpass, while a comb filter has widely varying gains.

A Schroeder Reverberator called JCRev

JCRev was developed by John Chowning and others at CCRMA based on the ideas of Schroeder.

- Three Schroeder allpass sections:
  \[ \text{AP}_N^g \triangleq g + z^{-N} \]
  \[ \frac{1}{1 + g z^{-N}} \]

- Four feedforward comb-filters (STK uses FBCFs):
  \[ \text{FFCF}_N^g \triangleq g + z^{-N} \]
• Schroeder suggests a progression of delays close to
  \[ M_i T \approx \frac{100 \text{ ms}}{3^i}, \quad i = 0, 1, 2, 3, 4. \]

• Comb filters impart distinctive coloration:
  • Early reflections
  • Room size
  • Could be one tapped delay line

• Usage: Instrument adds scaled output to RevIn

• Reverberator output RevOut goes to four delay lines
  • Four channels decorrelated
  • Imaging of reverberation between speakers avoided

• For stereo listening, Schroeder suggests a mixing matrix at the reverberator output, replacing the decorrelating delay lines

• A mixing matrix should produce maximally rich yet uncorrelated output signals

• JCRev is in the Synthesis Tool Kit (STK)
  • JCRev.cpp
  • JCRev.h

• Four Schroeder “diffusion allpasses” in series

• Eight parallel Schroeder-Moorer lowpass-feedback-comb-filters:
  \[
  \text{LBCF}_{N}^{f,d} \triangleq \frac{1}{1 - f^\frac{1-d}{1-dz^{-1}} z^{-N}}
  \]

• Second stereo channel: increase all 12 delay-line lengths by “stereo spread” (default = 23 samples)

• Used extensively in the free-software world
Freeverb Parameters

• \(d\) ("damping") default:
  \[
d = \text{initialdamp} \times \text{scaledamp} = 0.5 \times 0.4 = 0.2
\]

• \(f\) ("room size") default:
  \[
  \text{roomsize} = \text{initialroom} \times \text{scaleroom} + \text{offsetroom}
  = 0.5 \times 0.28 + 0.7 = 0.84
\]

• Feedback lowpass \((1 - d) / (1 - dz^{-1})\) causes reverberation time \(t_{60}(\omega)\) to decrease with frequency \(\omega\), which is natural

• \(f\) mainly determines reverberation time at low-frequencies (where feedback lowpass has negligible effect)

• At very high frequencies, \(t_{60}(\omega)\) is dominated by the diffusion allpass filters

T60 in Freeverb

• "Room size" \(f\) sets low-frequency \(t_{60}\)

• "damping" \(d\) controls how rapidly \(t_{60}\) shortens as frequency increases

• Diffusion allpasses set lower bound on \(t_{60}\)

Interpreting “Room Size” Parameter

• Low-frequency reflection-coefficient for two plane-wave wall bounces

• Could be called liveness or reflectivity

• Changing roomsize normally requires changing delay-line lengths
Freeverb Allpass Approximation

Schroeder Diffusion Allpass

\[ \text{AP}_N^g \triangleq \frac{-g + z^{-N}}{1 - gz^{-N}} \]

Freeverb implements

\[ \text{AP}_N^g \approx \frac{-1 + (1 + g)z^{-N}}{1 - gz^{-N}} \]

- Each Freeverb “allpass” is more precisely a feedback comb-filter \( \text{FBCF}_N^g \) in series with a feedforward comb-filter \( \text{FFCF}_N^{-1,1+g} \), where
  \[ \text{FBCF}_N^g \triangleq \frac{1}{1 - gz^{-N}} \]
  \[ \text{FFCF}_N^{-1,1+g} \triangleq -1 + (1 + g)z^{-N}. \]

- A true allpass is obtained at \( g = \left( \sqrt{5} - 1 \right)/2 \approx 0.618 \) (reciprocal of “golden ratio”)
- Freeverb default is \( g = 0.5 \)

FDN Late Reverberation

- Generalized state-space model (unit delays replaced by arbitrary delays)
- Note direct path weighted by \( d \)
- The “tonal correction” filter \( E(z) \) equalizes mode energy independent of reverberation time (perceptual orthogonalization)
- Gerzon 1971: “orthogonal matrix feedback reverb” cross-coupled feedback comb filters (see below)
Choice of Orthogonal Feedback Matrix $Q$

Late reverberation should resemble exponentially decaying noise. This suggests the following two-step procedure for reverberator design:

1. Set $t_{60} = \infty$ and make a good white-noise generator
2. Establish desired reverberation times in each frequency band by introducing losses

The white-noise generator is the lossless prototype reverberator.

Hadamard Feedback Matrix

A second-order Hadamard matrix:

$$H_2 \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix},$$

Higher order Hadamard matrices defined by recursive embedding:

$$H_4 \triangleq \frac{1}{\sqrt{2}} \begin{bmatrix} H_2 & H_2 \\ -H_2 & H_2 \end{bmatrix}.$$  

- Since $H_3$ does not exist, the FDN example figure above can be redrawn for $N = 4$, say, (instead of $N = 3$), so that we can set $Q = H_4$
- The Hadamard conjecture posits the existence of Hadamard matrices $H_N$ of order $N = 4k$ for all positive integers $k$.
- “As of 2008, there are 13 multiples of 4 less than or equal to 2000 for which no Hadamard matrix of that order is known. They are: 668, 716, 892, 1004, 1132, 1244, 1388, 1436, 1676, 1772, 1916, 1948, 1964.” [http://en.wikipedia.org/wiki/Hadamard_matrix]
Choice of Delay Lengths $M_i$

- Delay line lengths $M_i$ are typically mutually prime (Schroeder)
- For sufficiently high mode density, $\sum_i M_i$ must be sufficiently large.
  - No “ringing tones” in the late impulse response
  - No “flutter”

Mode Density Requirement

FDN order = sum of delay lengths:

$$M \Delta \sum_{i=1}^{N} M_i$$  \hspace{1cm} (FDN order)

- Order = number of poles
- All $M$ poles are on the unit circle in the lossless prototype
- If uniformly distributed, mode density =

$$\frac{M}{f_s} = MT \text{ modes per Hz}$$

- Schroeder suggests 0.15 modes per Hz (when $t_{60} = 1$ second)
- Generalizing:

$$M \geq 0.15 t_{60} f_s$$

- Example: For $f_s = 50$ kHz and $t_{60} = 1$ second, $M \geq 7500$

- Note that $M = t_{60} f_s$ is the length of the FIR filter giving a perceptually exact implementation. Thus, recursive filtering is about 7 times more efficient by this rule of thumb.
Choice of Loss Gains $g_i$

- To set the reverberation time $t_{60}$, we need to move the poles of the lossless prototype slightly inside the unit circle.
- The scaling coefficients $g_i$ can accomplish this for $0 < g_i < 1$.
- Since high-frequencies decay faster in propagation through air, we want to move the high-frequency poles farther in than low-frequency poles.
- Therefore, we need to generalize $g_i$ above to $G_i(z)$, with $|G_i(e^{j\omega T})| \leq 1$ imposed to ensure stability.

Jot (1991) FDN Reverberator for $N = 3$

Damping Filter Design

The damping filter $G_i(z)$ associated with the delay line of length $M_i$ in the FDN can be written in principle as

$$G_i(z) = G_T^{M_i}(z)L_i(z)$$

where $G_T(z)$ is the lowpass filter corresponding to one sample of wave propagation through air, and $L_i(z)$ is a lowpass corresponding to absorbing/scattering boundary reflections along the (hypothetical) $i$th propagation path.

Define

$$t_{60}(\omega) = \text{desired reverberation time at frequency } \omega$$

$$p_k = e^{j\omega_k T} = k\text{th pole of the lossless prototype}$$

We can introduce frequency-independent damping with the (conformal map) substitution

$$z^{-1} \leftarrow g z^{-1}$$

- This $z$-plane mapping pulls all poles in the $z$ plane from the unit circle to the circle of radius $g$.
- Pole $p_k = e^{j\omega_k T}$ moves to $\tilde{p}_k = g e^{j\omega_k T}$.
Example

- Start with a pole at dc (digital integrator):
  \[ H(z) = \frac{1}{1 - z^{-1}} \leftrightarrow [1, 1, 1, \ldots] \]
- Move it from radius 1 to radius 0.9 using 
  \[ z^{-1} \mapsto 0.9 z^{-1} \]:
  \[ H(z) = \frac{1}{1 - 0.9 z^{-1}} \leftrightarrow [1, 0.9, 0.81, \ldots] \]

Frequency-Dependent Damping

For \textit{frequency-dependent} damping, consider the mapping

\[ z^{-1} \leftarrow G(z) z^{-1} \]

where \( G(z) \) is a lowpass filter satisfying \( |G(e^{j\omega T})| \leq 1 \), \( \forall \omega \)

- Neglecting phase in the loss filter \( G(z) \), the substitution \( z^{-1} \leftarrow G(z) z^{-1} \) only affects the pole radius, not angle.
- \( G(z) = \text{per-sample filter} \) in the propagation medium.
- Schroeder (1961):
  The reverberation times of the individual modes must be equal or nearly equal so that different frequency components of the sound decay with equal rates ⇒
  - All pole radii in the reverberator should vary smoothly with frequency.
  - Otherwise, late decay will be dominated by largest pole(s).
Lossy Mapping

Let's in more detail look at the $z$-plane mapping

$$z^{-1} \leftarrow G(z) \cdot z^{-1}$$

• Pole $k$ contributes the term

$$H_k(z) = \frac{r_k}{1 - p_k z^{-1}} = r_k \cdot \left(1 + p_k z^{-1} + p_k^2 z^{-2} + \cdots\right)$$

to the partial fraction expansion of the transfer function

• This term maps to

$$\tilde{H}_k(z) = \frac{r_k}{1 - p_k [G(z) \cdot z^{-1}]}$$

$$= r_k \cdot \left(1 + [G(z) p_k] z^{-1} + [G(z) p_k]^2 z^{-2} + \cdots\right)$$

• Thus, pole $k$ moves from $z = p_k = e^{j\omega_k T}$ to

$$\tilde{p}_k = R_k e^{j\omega_k T}$$

where

$$R_k = G \left(R_k e^{j\omega_k T}\right) \approx G \left(e^{j\omega_k T}\right)$$

which is a good approximation here since $R_k$ is nearly 1 for reverberators.

Example

• Start with a pole at dc (digital integrator):

$$H(z) = \frac{1}{1 - z^{-1}} \leftrightarrow [1, 1, 1, \ldots]$$

• Move it from radius 1 to radius 0.9 using

$$z^{-1} \leftarrow 0.9 \cdot z^{-1}:$$

$$H(z) = \frac{1}{1 - 0.9 z^{-1}} \leftrightarrow [1, 0.9, 0.81, \ldots]$$

• Now progress from radius 0.9 to 0.8 using

$$z^{-1} \leftarrow 0.9 + \alpha z^{-1}$$

with $0.8 = (1 - \alpha)/(1 + \alpha)$

$$\Rightarrow \alpha = (1 - 0.8)/(1 + 0.8) = 1/0.9:$$

$$H(z) = \frac{1}{1 - 0.9 \frac{1 + \alpha z^{-1}}{1 + \alpha} z^{-1}} = \frac{1}{1 - 0.9 \frac{0.9 + z^{-1}}{0.9 + 1} z^{-1}}$$

$$= \frac{1}{1 - 0.81 \cdot z^{-1} + \frac{1}{1.9} z^{-2}}$$
Desired Pole Radius

Pole radius $R_k$ and $t_{60}$ are related by

$$R_k^{t_{60}(\omega_k)/T} = 0.001$$

The ideal loss filter $G(z)$ therefore satisfies

$$|G(\omega)|^{t_{60}(\omega)/T} = 0.001$$

The desired delay-line filters are therefore

$$G_i(z) = G_i^{M_i(z)}$$

Then

$$|G_i(e^{j\omega T})|^{-M_i T} = 0.001.$$  

or

$$20 \log_{10} \left| G_i(e^{j\omega T}) \right| = -60 \frac{M_i T}{t_{60}(\omega)}.$$  

Now use `invfreqz` or `stmcb`, etc., in Matlab to design low-order filters $G_i(z)$ for each delay line.

First-Order Delay-Filter Design

Jot used first-order loss filters for each delay line:

$$G_i(z) = g_i \frac{1 - a_i}{1 - a_i z^{-1}}$$

- $g_i$ gives desired reverberation time at dc
- $a_i$ sets reverberation time at high frequencies

Design formulas:

$$g_i = 10^{-3M_i T/t_{60}(0)}$$

$$a_i = \frac{\ln(10)}{4} \log_{10}(g_i) \left(1 - \frac{1}{\alpha^2}\right)$$

where

$$\alpha \Delta t_{60}(\pi/T) \frac{t_{60}(0)}{t_{60}(0)}$$
Tonal Correction Filter

Let \( h_k(n) \) = impulse response of \( k \)th system pole. Then

\[
E_k = \sum_{n=0}^{\infty} |h_k(n)|^2 = \text{total energy}
\]

Thus, total energy is proportional to decay time.

To compensate, Jot proposes a tonal correction filter \( E(z) \) for the late reverb (not the direct signal).

First-order case:

\[
E(z) = \frac{1 - bz^{-1}}{1 - b}
\]

where

\[
b = \frac{1 - \alpha}{1 + \alpha}
\]

and

\[
\alpha = \frac{t_{60}(\pi/T)}{t_{60}(0)}
\]

as before.

Zita-Rev1 Reverberator

- FDN+Schroeder reverberator
- Free open-source C++ for Linux by Fons Adriaensen
- Faust example zita_rev1.dsp

Feedback Delay Network + Schroeder Allpass Comb Filters:

- Allpass coefficients \( \pm 0.6 \)
- Inspect Faust block diagram for delay-line lengths, etc.
Zita-Rev1 Damping Filters

FDN reverberators employ a damping filter for each delay line.

Zita-Rev1 three-band damping filter:

\[ H_d(z) = H_l(z)H_h(z) \]

where

\[ H_l(z) = g_m + (g_0 - g_m) \frac{1 - p_l}{2} \frac{1 + z^{-1}}{1 - p_lz^{-1}} = \text{low-shelf} \]

\[ H_h(z) = \frac{1 - p_h}{1 - p_hz^{-1}} = \text{low-pass} \]

\[ g_0 = \text{Desired gain at dc} \]

\[ g_m = \text{Desired gain across “middle frequencies”} \]

\[ p_l = \text{Low-shelf pole controlling low-to-mid crossover:} \]

\[ \Delta = \frac{1 - \pi f_1 T}{1 + \pi f_1 T} \]

\[ p_h = \text{Low-pass pole controlling high-frequency damping:} \]

Gives half middle-band \( t_{60} \) at start of “high” band.

High-Frequency-Damping Lowpass

High-Frequency Damping Lowpass:

\[ H_h(z) = \frac{1 - p_h}{1 - p_hz^{-1}} \]

For \( t_{60} \) at “HF Damping” frequency \( f_h \) to be half of middle-band \( t_{60} \) (gain \( g_m \)), we require

\[ |H_h(e^{j2\pi f_h T})| = \left| \frac{1 - p_h}{1 - p_he^{-j2\pi f_h T}} \right| = g_m \]

Squaring and normalizing yields a quadratic equation:

\[ p_h^2 + b p_h + 1 = 0 \]

Solving for \( p_h \) using the quadratic formula yields

\[ p_h = \frac{-b}{2} - \sqrt{\left(\frac{b}{2}\right)^2 - 1}, \]

where

\[ \frac{b}{2} = \frac{1 - g_m^2 \cos(2\pi f_h T)}{1 - g_m^2} > 1, \]

Discard unstable solution \(-b/2 + \sqrt{(b/2)^2 - 1} > 1\).

To ensure \(|g_m| < 1\), GUI keeps middle-band \( t_{60} \) finite.
Rectilinear Digital Waveguide Mesh

Waveguide Mesh Features

- A mesh of such waveguides in 2D or 3D can simulate waves traveling in any direction in the space.
- Analogy: tennis racket = rectilinear mesh of strings = pseudo-membrane
- Wavefronts are explicitly simulated in all directions
- True diffuse field in late reverb
- Spatialized reflections are “free”
- Echo density grows naturally with time
- Mode density grows naturally with frequency
- Low-frequency modes very accurately simulated
- High-frequency modes mistuned due to dispersion (can be corrected) (often not heard)
- Multiply free almost everywhere
- Coarse mesh captures most perceptual details
Reverb Resources on the Web

- Harmony Central article (with sound examples)
- William Gardner’s MIT Master’s thesis