Bowed Strings

Schematic Model

A schematic model for bowed-string instruments.

- Bow divides string into two sections
- Bow junction = nonlinear two-port
- Primary control variable = bow velocity
  ⇒ velocity waves = natural choice of wave variable
- Bow-string interface is analogous to the reed-bore interface: Find intersection of bow-string friction curve with the string wave impedance “load line”
- In other words, a velocity input (injected equally to left and right) must be found such that the transverse force of the bow against the string is balanced by the reaction force of the moving string
Overlay of normalized bow-string friction curve $R_b(v_\Delta)/R_s$ with the bore “load line” $v^+ - v_\Delta$. The “capture” and “break-away” differential incoming velocity is denoted $v^c_\Delta$. Note that increasing the bow force increases $v^c_\Delta$ as well as enlarging the maximum force applied (at the peaks of the curve).

Applied Force = Friction Curve $\times$ Differential Velocity

Reaction Force = String Wave Impedance $\times$ Velocity Change

- Nominally, $R_b(v_\Delta)$ is constant for $|v_\Delta| \leq v^c_\Delta$, where $v^c_\Delta$ is both the capture and break-away differential velocity. (static coeff. of friction)
- For $|v_\Delta| > v^c_\Delta$, $R_b(v_\Delta)$ falls quickly to a low dynamic coefficient of friction
- Dynamic coefficient decreases with differential velocity

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**Bow-String Scattering Junction**

Friedlander-Keller diagram is solved when

$$R_b(v_\Delta) \times v_\Delta = R_s \left[ v^+ - v_\Delta \right]$$

which implies (in a manner analogous to the single reed case)

$$v^+_{sr} = v^+_s + \hat{\rho}(v^+_{\Delta}) \cdot v^+_{\Delta}$$

$$v^-_{sl} = v^+_s + \hat{\rho}(v^-_{\Delta}) \cdot v^-_{\Delta}$$

where

$v_{sr}$ = transverse string velocity on the right of the bow
$v_{sl}$ = string velocity left of the bow ($v_{sl} = v_{sr}$)
$v^\Delta_{\Delta} = v_b - (v^+_s + v^-_s) = \text{“incoming differential velocity”}$
$v_b$ = bow velocity, and

$$\hat{\rho}(v^\Delta_{\Delta}) = \frac{r(v^\Delta_{\Delta})}{1 + r(v^\Delta_{\Delta})}$$

where

$r(v_\Delta) = 0.25 R_b(v_\Delta)/R_s$
$\Delta_v = v_b - v_s$ = bow velocity minus string velocity
$v_s = v^+_s + v^-_s = v^+_s + v^-_s$ = transverse string velocity
$R_s$ = wave impedance of string
$R_b(v_\Delta)$ = friction coefficient for the bow against the string, i.e.,

$$F_b(v_\Delta) = R_b(v_\Delta) \cdot v_\Delta$$
Simplified, Piecewise Linear Bow Table

Simple, qualitatively chosen bow table for the digital waveguide violin.

- Flat center portion corresponds to a fixed reflection coefficient “seen” by a traveling wave encountering the bow stuck against the string
- Outer sections give a smaller reflection coefficient corresponding to the reduced bow-string interaction force while the string is slipping under the bow
- The notation $v^c_\Delta$ at the corner point denotes the capture or break-away differential velocity
- Hysteresis is neglected

Complete Digital Waveguide Model

- Reflection filter implements all losses (bridge, bow, finger, and the round-trip attenuation & dispersion)
- Nut $\approx$ inverting reflection
- Neglecting bow-hair dynamics, bow-string interaction is simulated using a memoryless lookup table (or segmented polynomial) like we had for woodwinds (where we neglected the mass of the reed)
- The bow-string interface is driven by differential velocity $v^+_\Delta = \text{bow velocity} - \text{total incoming string velocity}$
- Secondary controls are bow force and angle (changed by modifying the bow table)
Linear Commuted Violin Synthesis

- Assumes ideal Helmholtz motion
- Sound examples:


Multiple-Excitation Commuted Synthesis
Filtered-Noise Excitation Synthesis

FIR 1

\[ \gamma_1(n) \]

\[ \ldots \]

FIR filter

\[ \gamma_L(n) \]

FIR L

\[ x(n) \]

Noise Generator

\[ \text{Convolution} \]

Stochastic Excitation Component

Commuted Synthesis of the Linearized Violin

a)

Amplitude(n)
Frequency(n)

Impulse Train

\[ e(n) \]

String

\[ s(n) \]

Resonator

\[ x(n) \]

b)

Amplitude(n)
Frequency(n)

Impulse Train

\[ e(n) \]

Resonator

\[ a(n) \]

String

\[ x(n) \]

c)

Amplitude(n)
Frequency(n)

Impulse-Response Train

\[ a(n) \]

String

\[ x(n) \]

Output