MUS420 Lecture
Digital Waveguide Modeling of Bowed Strings

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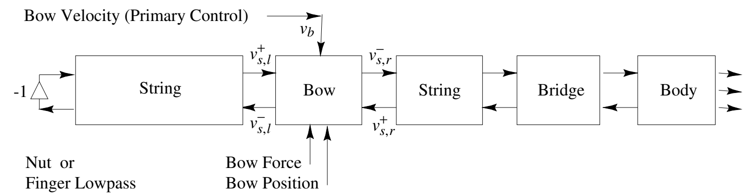
February 5, 2019

Outline

- Physical Model
- Digital Waveguide Model
- Bow-String Theory and Computational Model
- Commuted Synthesis of Bowed Strings

Bowed Strings

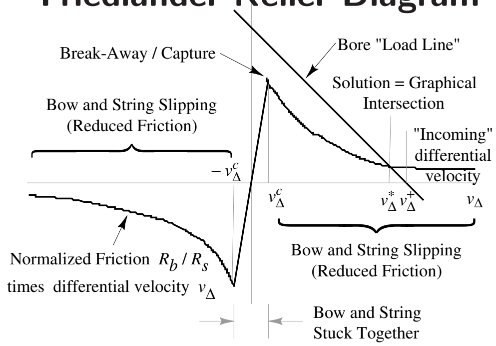
Schematic Model



A schematic model for bowed-string instruments.

- Bow divides string into two sections
- Bow junction = nonlinear *two-port*
- Primary control variable = bow velocity
  $\Rightarrow$ *velocity waves* = natural choice of wave variable
- Bow-string interface is analogous to the reed-bore interface: Find intersection of bow-string friction curve with the string wave impedance "load line"
- In other words, a velocity input (injected equally to left and right) must be found such that the transverse force of the bow against the string is balanced by the reaction force of the moving string

Friedlander-Keller Diagram



Overlay of normalized bow-string friction curve $R_b(v_\Delta)/R_s$ with the bore "load line" $v_\Delta^+ - v_\Delta$. The "capture" and "break-away" differential incoming velocity is denoted v_Δ^c. Note that increasing the bow force increases v_Δ^c as well as enlarging the maximum force applied (at the peaks of the curve).

$$\begin{aligned}
\text{Applied Force} &= \text{Friction Curve} \times \text{Differential Velocity} \\
\text{Reaction Force} &= \text{String Wave Impedance} \times \text{Velocity Change}
\end{aligned}$$

- Nominally, $R_b(v_\Delta)$ is constant for $|v_\Delta| \leq v_\Delta^c$, where v_Δ^c is both the capture and break-away differential velocity. (*static* coeff. of friction)
- For $|v_\Delta| > v_\Delta^c$, $R_b(v_\Delta)$ falls quickly to a low *dynamic* coefficient of friction
- Dynamic coefficient decreases with differential velocity

Bow-String Scattering Junction

Friedlander-Keller diagram is solved when

$$R_b(v_\Delta) \times v_\Delta = R_s \left[v_\Delta^+ - v_\Delta\right]$$

which implies (in a manner analogous to the single reed case)

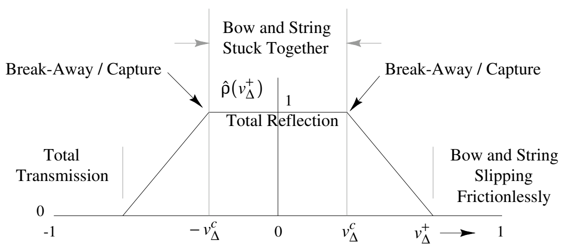
$$\begin{aligned}
v_{s,r}^- &= v_{s,l}^+ + \hat{\rho}(v_\Delta^+) \cdot v_\Delta^+ \\
v_{s,l}^- &= v_{s,r}^+ + \hat{\rho}(v_\Delta^+) \cdot v_\Delta^+, \quad \text{where}
\end{aligned}$$

$$\begin{aligned}
v_{s,r} &= \text{transverse string velocity on the } \textit{right} \text{ of the bow} \\
v_{s,l} &= \text{string velocity } \textit{left} \text{ of the bow } (v_{s,l} = v_{s,r}) \\
v_\Delta^+ &\triangleq v_b - (v_{s,r}^+ + v_{s,l}^+) = \text{“incoming differential velocity”} \\
v_b &= \text{bow velocity, and}
\end{aligned}$$

$$\boxed{\hat{\rho}(v_\Delta^+) = \frac{r\left(v_\Delta(v_\Delta^+)\right)}{1 + r\left(v_\Delta(v_\Delta^+)\right)}} \quad \text{where}$$

$$\begin{aligned}
r(v_\Delta) &= 0.25 R_b(v_\Delta)/R_s \\
v_\Delta &= v_b - v_s \qquad \text{bow velocity minus string velocity} \\
v_s &= v_{s,l}^+ + v_{s,l}^- = v_{s,r}^+ + v_{s,r}^- = \text{transverse string velocity} \\
R_s &= \text{wave impedance of string} \\
R_b(v_\Delta) &= \text{friction coefficient for the bow against the string, i.e.,} \\
&\quad F_b(v_\Delta) = R_b(v_\Delta) \cdot v_\Delta
\end{aligned}$$

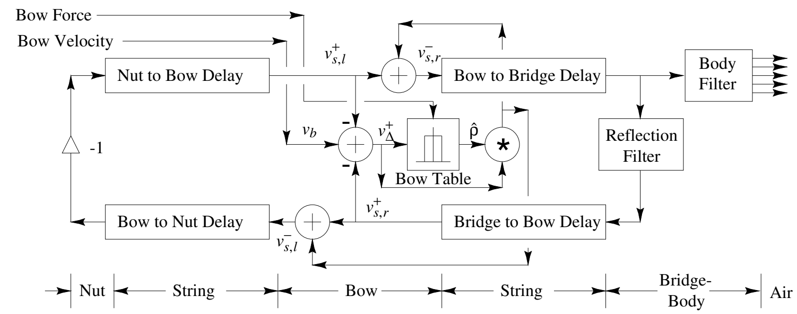
Simplified, Piecewise Linear Bow Table



Simple, qualitatively chosen bow table for the digital waveguide violin.

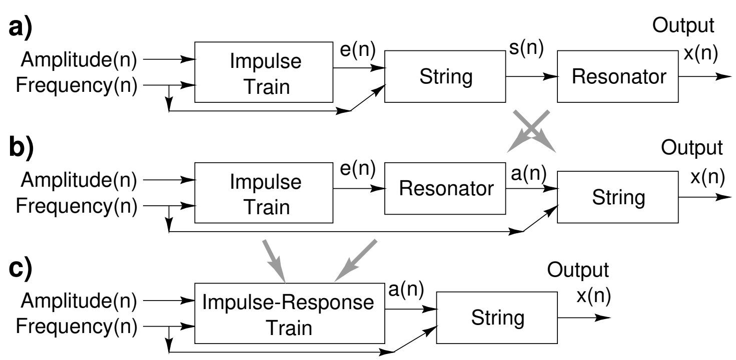
- Flat center portion corresponds to a fixed reflection coefficient "seen" by a traveling wave encountering the bow stuck against the string
- Outer sections give a smaller reflection coefficient corresponding to the reduced bow-string interaction force while the string is slipping under the bow
- The notation v_Δ^c at the corner point denotes the capture or break-away differential velocity
- Hysteresis is neglected

Complete Digital Waveguide Model



- Reflection filter implements all losses (bridge, bow, finger, and the round-trip attenuation & dispersion)
- Nut $\approx$ inverting reflection
- Neglecting bow-hair dynamics, bow-string interaction is simulated using a *memoryless* lookup table (or segmented polynomial) like we had for woodwinds (where we neglected the mass of the reed)
- The bow-string interface is driven by *differential velocity* $v_\Delta^+ =$ bow velocity minus the total incoming string velocity
- Secondary controls are *bow force* and *angle* (changed by modifying the bow table)

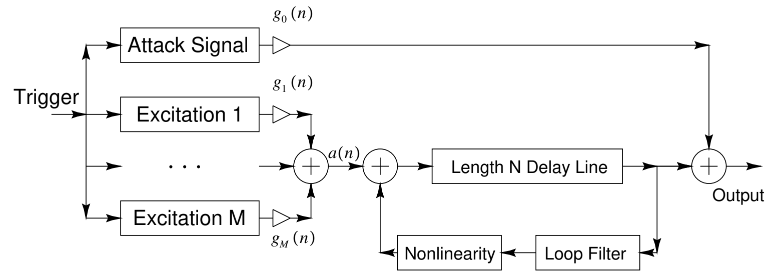
Linear Commuted Violin Synthesis



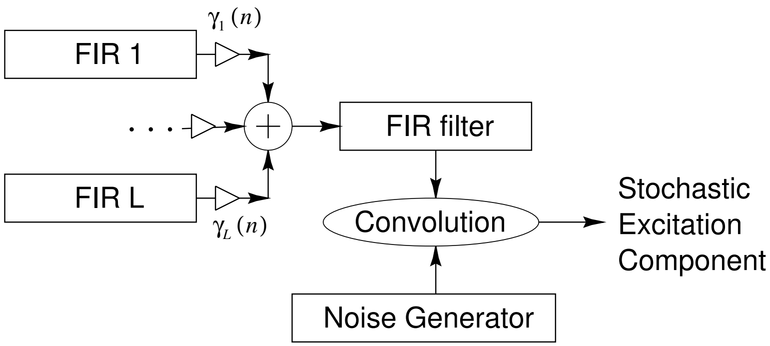
- Assumes *ideal Helmholtz motion*
- Sound examples:

`http://ccrma.stanford.edu/~jos/wav/vln-lin-cs.wav`

Multiple-Excitation Commuted Synthesis



Filtered-Noise Excitation Synthesis



Commuted Synthesis of the Linearized Violin