Pitch Drift in Choral Music

Hiroko Terasawa

19 March 2004
Abstract

1 Introduction

Pitch drift phenomenon is an effect that a choir sings a piece in tune during the performance, but ends with a pitch lower or higher which it is supposed to be. Many choristers experience this phenomenon in their performance. The pitch drift is dependent on many factors in the music performance, such as the skill of the performers, the difficulty of the piece or the psychological stresses. However, I would like to introduce a story about the pitch drift by Stanford Chamber chorale.

Stanford chamber chorale, which is the highest level choral group at Stanford University, experienced the pitch drift in their performance of “Os Justi” by Bruckner. Whenever they sang the piece, it ended with a pitch exactly semitone flat. The conductor Stephen Sano told that they were usually able to figure out which part of a music, for example a chromatic movement or a particular leading tone, had caused the pitch drift. But particularly with “Os Justi”, they could not find the reason for the pitch drift. Even in their recording session, they still sang the piece ending with the pitch a semitone flat.

What this story suggests is, that one reason of the pitch drift could be music itself. Stanford Chamber Chorale was usually able to fix the pitch drift except “Os Justi.” It means that the pitch drift problem is unique to the piece and the reason might lie in the composition.

Mark Lindley, in his article “just intonation” in New Grove [1] provides a good summary of historical discussions on the relation between pitch drift and just intonation. Zarlino, in 1558, argued that “good singers when unaccompanied would adhere to the pure intervals.” Zarlino eventually realized the sour fifth caused by the just intonation, but he believed that “singers’ capacity to intone in a flexible manner would enable them to avoid such problems.” In 1581, Vincenzo Galilei, a
former pupil of Zarlino “denied that just intonation was used in vocal music.” In 1650s, Benedetti, a mathematician and physicist, wrote letters to Cipriano Rore and pointed out that if a progression of four fifths was sung in the just intonation, it would cause a pitch fall of a syntonic comma. However, almost forty years later, Printz praised a fifth tempered by a quarter-comma as “pleasant beating.” Rameau in 1737, stated that “an accompanied singer is guided by the temperament of the instruments only for the fundamental sounds, and automatically modifies... Everything contrary to the just rapport of the fundamental sounds. “ However, Lindley himself mentions that “Melodic and harmonic considerations pull intonation in opposite directions: in melody, toward the brightness of Pythagorean tuning... and in harmony, toward the just tuning of vertical sonorities.”

This duality appears in the interview with British choral conductors as well. [2] Ralf Allwod, the conductor of Eton College Chapel Choir talks about “slightly flat major thirds” in the just intonation and the mean-tone temperament, while Simon Halsey, the conductor of City of Birmingham Symphony Chorus mentions that “major intervals are very bright.”

Barbour, in his book [3] shows his interest in this issue. “With modal progressions, as in Palestrina, it is more likely to remain stationary... If the music contains much the chromaticism and remote modulations, even the best-trained choir would flounder,” referring Malenzio’s madrigal “O voi che sospirate a miglior note” which has a modulation around the circle of fifths, although he does not clearly answer his own question “Could Marenzio’s madrigal have been sung in just intonation?”

Howard, in his most recent study[4], did a measurement of choir singing. He measured the glottal vibration of each person by attaching the sensors to the neck. It showed the pitch deviation from the equal temperament.

I would like to investigate this issue via the psychoacoustic tests with synthesis, and the rule based analysis of “Os Justi” from the viewpoint of pitch stability.
2 Pitch drift model

Although “pitch” is a purely psychoacoustic value, the perception of the pitch is closely related to the fundamental frequency of a sound in general. In this paper, the pitch drift is discussed in terms of the fundamental frequency \( f \). The frequency in the equal temperament is use as the reference to the realistic fundamental frequency, because many people use piano to check their notes.

The fundamental frequency ratio of the equal temperament to the performance intonation (EPR) is defined as

\[
EPR = \frac{f_{\text{real}}}{f_{\text{eq.temp}}} \tag{1}
\]

Note that the EPR cannot be explained in terms of the syntonic comma, because the syntonic comma is essentially the fundamental frequency ratio of Pythagorean third to the just intonation third, and not the ratio between the equal temperament and the just intonation.

The frequency ratio of an interval is \( \alpha \). For example, \( \alpha \) for the major third is

\[
\alpha_{\text{just}} = \frac{5}{4} \quad \text{(just intonation)} \tag{2}
\]

\[
\alpha_{\text{eq.temp}} = 2^{4/12} \quad \text{(equal temperament)} \tag{3}
\]

In practice, the final note of a piece is the multiplication of the starting fundamental frequency \( f_{\text{start}} \) and the cumulative product of alpha. Therefore, the EPR of the final note is as follows.

\[
EPR_{\text{final}} = \frac{f_{\text{start}} \cdot \prod \alpha_{\text{real}}}{f_{\text{start}} \cdot \prod \alpha_{\text{eq.temp}}} \tag{4}
\]

In a real performance, the interval ratio \( \alpha_{\text{real}} \) is supposed to be dependent on many factors. The following is a list of the possible reasons for the pitch deviation.

- **When:** room (acoustic environment.)
• **Where:** time of the day, position in the music.

• **Who:** choristers, conductor.

• **What:** composition of the repertoire.

• **How:** rehearsal experience, atmosphere of the music.

However, in the mind of a conductor or a chorister, there must be an ideal image of intonation which is appropriate for the music, although this image would be very individual. The interval ratio of the ideal image should be called $c_{\text{ideal}}$, which is dependent only on the music itself under the perfect condition for the rest of pitch deviation factors.

### 3 Psychoacoustic experiments

A series of psychoacoustic experiments was organized in order to study individual perceptions about the pitch drift. It consists of test 1 and 2.

#### 3.1 Method

The first test is to listen to the chord progression of I - V - I - IV -I as shown in Fig. 1, with different intonations. Two kinds of synthesis were prepared: the just intonation and the equal temperament. The subjects listen to the first stimuli in just intonation, then the second stimuli in the equal temperament. The subjects are asked to answer to a couple of questions.

1. Do you hear the difference between the two stimuli? Choose an answer from Yes or no.

2. Which one is close to your ideal choir harmony? Choose an answer from The first or the second.
The second test is to listen to a sequence of major chords repeated four times, as shown in Fig. 2. The sequence is designed to have the maximum pitch fall when it is performed in the just intonation. The synthesis is prepared using the just intonation. At every transient from one chord to another, a Major third comma between the equal temperament and the just intonation (M3 comma) is cumulated. Since it has three major chords in a sequence, and is repeated for four times, the EPR of the last C is the M3 comma powered by twelve, resulting almost 9% fall of the fundamental frequency from the reference tone, such as:

\[
\text{M3 comma} = \frac{5/4}{2^{1/12}} = \frac{1.25}{1.2599} = 0.992 \quad (5)
\]

\[
EPR_{\text{final}} = (\text{M3 comma})^{12} = 0.9096 \quad (6)
\]

The subjects are asked to specify the point where the pitch is different from their expectation. Another question is if they recognize the pitch fall. The questions posed to the subjects are as following:

1. Please mark the score whenever you feel the pitch which you just have listened to is different from the pitch which you would have sung if you were involved in the performance.

2. How do you perceive the pitch drift? Choose an answer from fall, stable (no drift) or rise.
Five randomly chosen CCRMA people participated in the psychoacoustic experiments.

3.2 Results

3.2.1 Test 1

For question 1, five of five subjects discriminated the synthesis in the just intonation and that in the equal temperament. For question 2, two of five subjects chose the just intonation as closer to his ideal choir harmony, while three of them chose the equal temperament.

3.2.2 Test 2

In Table 1, the number of subject who marked the score at each note is shown. The perception of the pitch drift is as following.

- pitch falling — three people of five
- pitch stable (no drift) — one of five
- pitch rising — one of five
Table 1: The number of subject who marked the score at each note

<table>
<thead>
<tr>
<th>Repeat</th>
<th>C</th>
<th>G</th>
<th>E</th>
<th>B</th>
<th>G#</th>
<th>D#</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

3.2.3 Comments

For the test 1, the preferences of the intonation were varied. Some people preferred the beating by the equal temperament, and some did not. If these five people sing in a group, the intonation will not be either of the just intonation or the equal temperament, which is very realistic.

For the test 2, the sensation of pitch drift was various too. While most of people perceived the pitch falling, but some of them did not. However, the score tend to be marked often with major thirds. It means that major thirds in the just intonation are more or less different from what people might sing. Since the marks means the point of the pitch adjustment, it is possible that the pitch fall in a choral performance is smaller than the pitch drift in the just intonation synthesis.

3.3 Discussion

The main difference between example 1 and 2 is the design of the fundamental bass. If the fundamental bass has the pair of ascent and descent for an interval as presented in example 1, the bass return to the same fundamental frequency easily. However, the second example does not have any descending major thirds in the fundamental bass, while ascending major thirds appear often. The second example is more fragile in the pitch, because of the cumulative commas. Technically, this is the cause of the fundamental frequency deviation. The just intonation is applicable for some
Table 2: Fundamental bass of “Os Justi”

<table>
<thead>
<tr>
<th>Interval</th>
<th>Ascent</th>
<th>Descent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major 3rd</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>minor 3rd</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Fifth</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

type of music like example 1, but not applicable for the other type of music such as example 2. Therefore, when the fundamental bass is designed with complimentary pairs of ascent and descent of the intervals, the structure of the music is more robust from the pitch drift point of view.

4 Analysis of “Os Justi”

According to the observation with the synthesis, the fundamental bass of “Os Justi” was analyzed by the complimentary interval ascent and descent. The table 2 shows the number of occurrence of each interval leaps. The nonequivalence of the interval exists, which might confuse the pitch accuracy.

5 Conclusion

The fundamental bass design causes the pitch drift when a piece is synthesized in non-equal temperament intonation. However, the individual perception of the synthesis varies, which suggests that the expectation to the pitch does not stay in a particular temperament. The fundamental bass of “Os Justi” by Bruckner was analyzed and the result shows that the fundamental bass is not composed in a manner of complimentary intervals, which may lead to the pitch instability.
Acknowledgments

I would like to thank Jonathan Barger, Bill Mahrt, Herb Meyers, Stephen Sano and Greg Wait for the discussion and the help for finding literatures.

References


