FX Basics

Filtering

STOMPBOX DESIGN WORKSHOP

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FX Basics: **Filtering**

Filtering effects modify the frequency content of the audio signal, achieving **boosting or weakening** specific **frequency bands** or regions.

Although their broad application to processing sound signals dates back from the early days of recording, their use application to processing guitar electrical signal may have started in the 1950s.

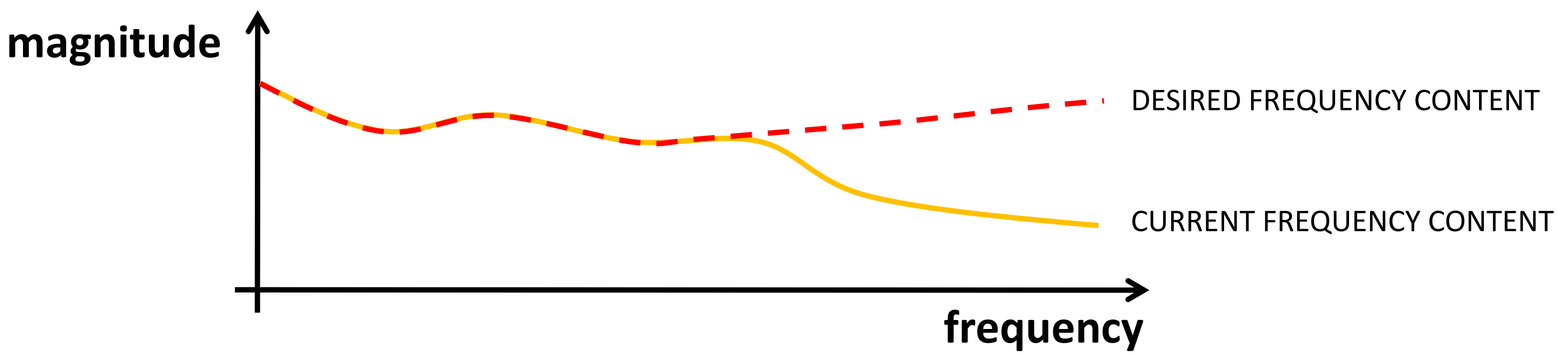
Filtering effects make use of **filters**, which are signal processors which **alter magnitude and phase** of signals by different amount s to different frequency components.

Ex:	equalization, wah-wah

Equalization

Original term coined from the task of ‘adjusting the balance between of (or *equalize*)’ different frequency components of a signal.

Equalization is commonly achieved by means of a device specifically designed for a **user-friendly control** of the parameters governing the **behavior of filters** used for its construction.

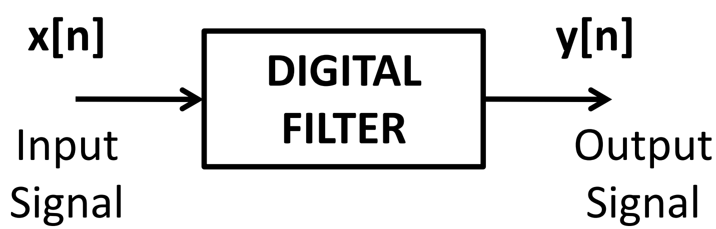


User-friendly interface to controlling filters so that a **desired alteration** is achieved…

DIGITAL FILTERS!

Digital Filters

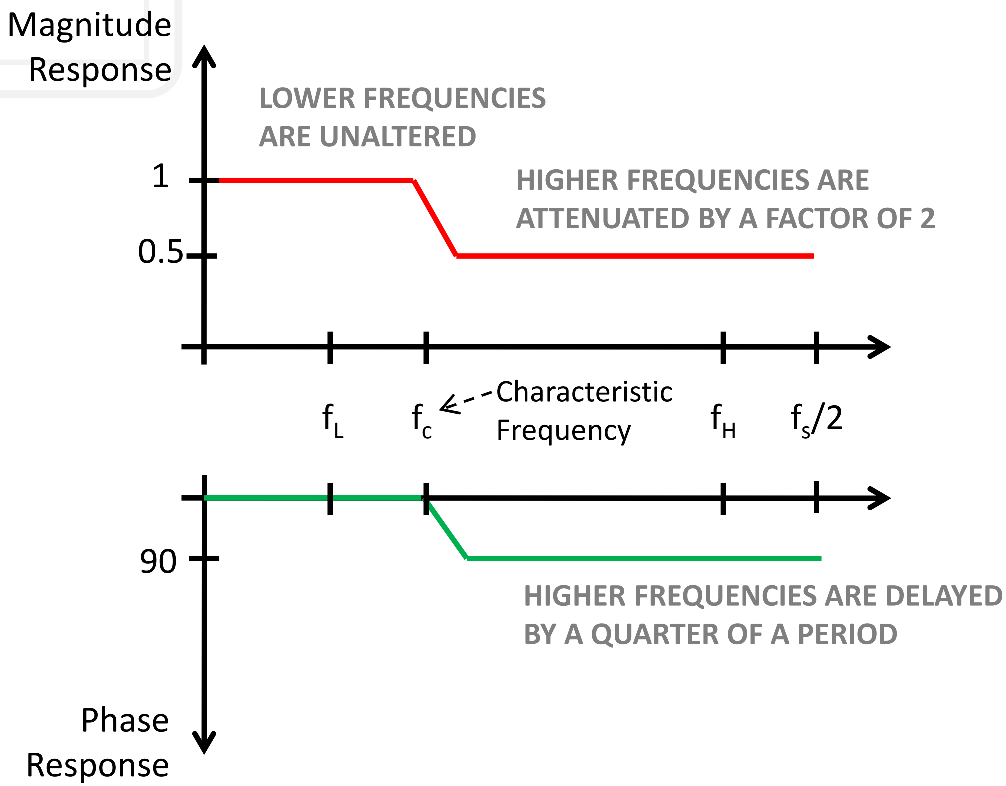
Systems that perform mathematical operations (multiplications and additions) to a **discrete input signal** x[n] to modify some of its characteristics and **obtain a discrete output signal** y[n].

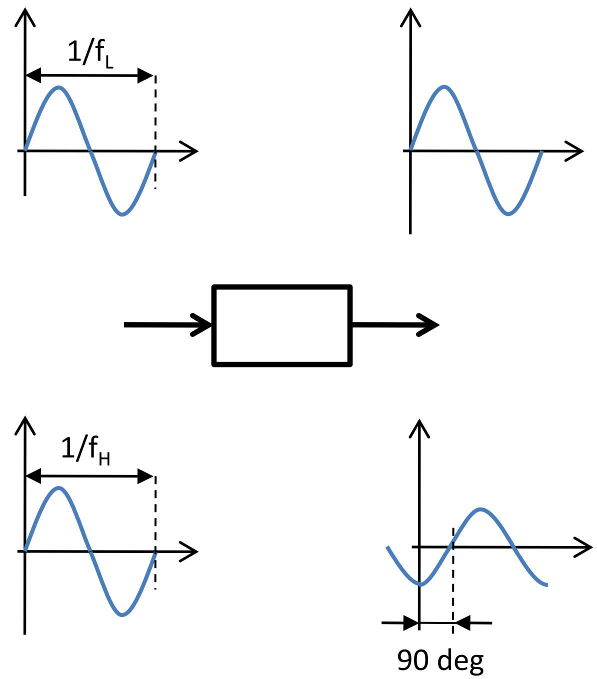


It is common to **describe a digital filter** in terms of **how it affects amplitude and phase** of **different frequency components** of a signal.

Ultimately, the design of digital filters is driven by such desired features. In general, **digital filter design** is not an easy task.

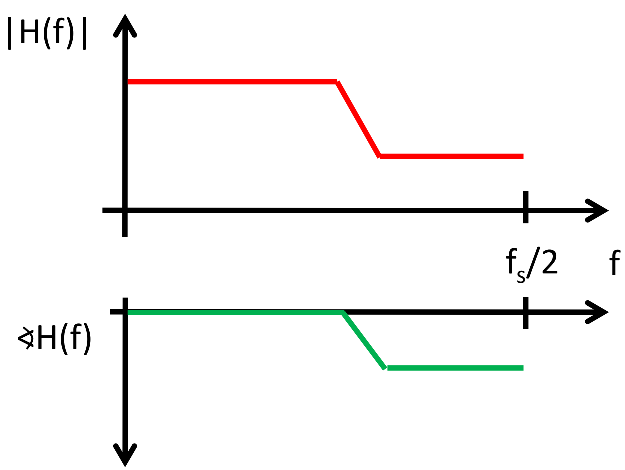
Digital Filters (ii)

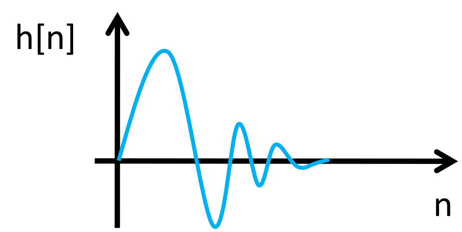


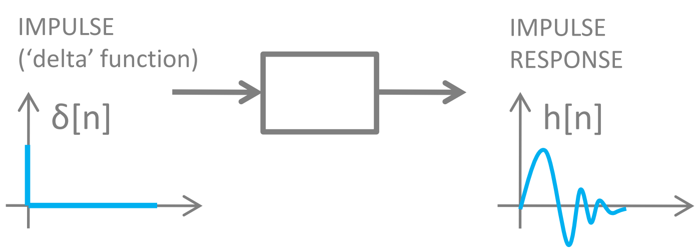


Digital Filters (iii)

FREQUENY DOMAIN	DFT	TIME DOMAIN
FREQUENCY RESPONSE		**IMPULSE RESPONSE**







PRODUCT

$$X[f] \rightarrow \boxed{H[f]} \rightarrow Y[f] = X[f]\cdot H[f]$$

CONVOLUTION

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] = x[n]*h[n]$$

Digital Filters (iv)

How to explore the frequency domain response of a given filter?
Among other options…

SINUSOIDAL ANALYSIS

- Generate a sinusoidal $x_i[n]$ for each frequency f_i to study
- Feed filter with each sinusoidal signal $x_i[n]$ and obtain a sinusoidal $y_i[n]$
- Obtain magnitude and phase responses for each frequency f_i :

$$|H(f_i)| = A(y_i)/A(x_i) \qquad \sphericalangle H[f_i] = \sphericalangle y_i - \sphericalangle x_i$$

IMPULSE RESPONSE

- Generate an impulse ‘delta’ signal $\delta[n]$
- Feed filter with signal $\delta[n]$ and obtain output signal $h[n]$
- Obtain $H(f)$ via DFT($h[n]$)
- Obtain magnitude and phase responses as:

$$|H[f]| = \text{abs}(\ H[\Omega]\ ) \qquad \sphericalangle H[\Omega] = \text{angle}(\ H[\Omega]\ )$$

Digital Filters (v)

Digital filters are commonly expressed by their **difference equation**:

$$
\begin{aligned}
y[n] = \; & b_0 \cdot x[n] + b_1 \cdot x[n-1] + \ldots + b_M \cdot x[n-M] \longleftarrow \text{CURRENT AND PREVIOUS INPUT SAMPLES} \\
& - a_1 \cdot y[n-1] - \ldots - a_N \cdot y[n-N] \longleftarrow \text{PREVIOUS OUTPUT SAMPLES} \\
= \; & \underbrace{\sum_{i=0}^{M} b_i \cdot x[n-i]}_{\text{NON-RECURSIVE PART}} - \underbrace{\sum_{j=1}^{N} a_j \cdot y[n-j]}_{\text{RECURSIVE PART}}
\end{aligned}
$$

b_i , a_j ⟶ FILTER COEFFICIENTS

max(M,N) ⟶ FILTER ORDER

…or by their **transfer function** (in the frequency domain, through the 'Z' transform):

$$
H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 \cdot z^{-1} + \ldots + b_N \cdot z^{-M}}{1 + a_1 \cdot z^{-1} + \ldots + a_N \cdot z^{-N}}
$$

z^{-M} denotes *M* samples of delay

Digital Filters (vi)

Two main types of digital filters:

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + \ldots + b_N \cdot z^{-M}}{1 + a_1 \cdot z^{-1} + \ldots + a_N \cdot z^{-N}}$$

Finite **I**mpulse **R**esponse (**FIR**)

- Presents only b_i coefficients being non-zero : **NON-RECURSIVE**
- Finite h[n]
- Phase response is linear

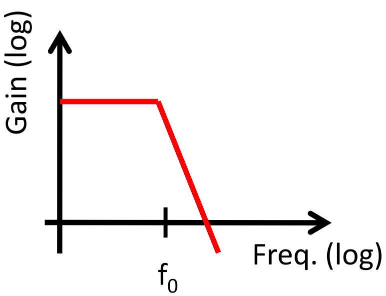
Infinite **I**mpulse **R**esponse (**IIR**)

- Presents both b_i and a_j coefficients being non-zero: **RECURSIVE**
- Infinite h[n]
- Phase response is non-linear
- Need less computations for similar desired characteristics
- May suffer from numerical problems due to feedback

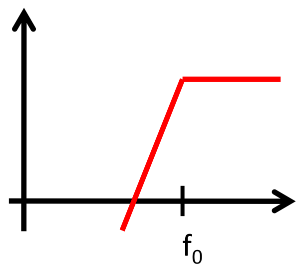
Digital Filters (vii)

Some **prototypical** basic filters (magnitude response):

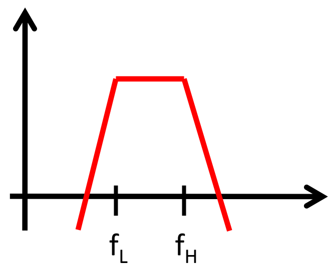
LOW-PASS (**LP**)



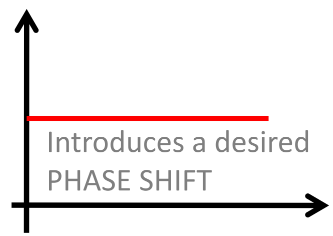
HIGH-PASS (**HP**)



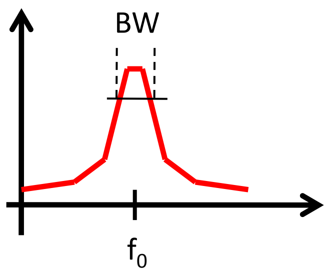
BAND-PASS (**BP**)



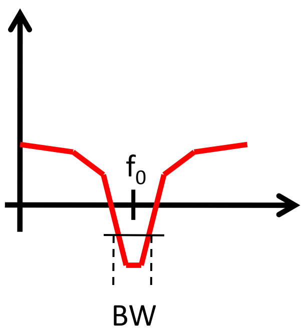
ALL-PASS (**AP**)



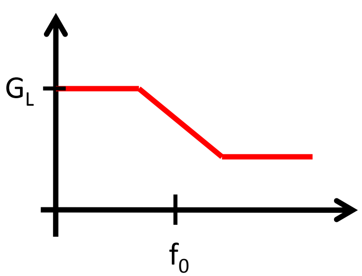
PEAK



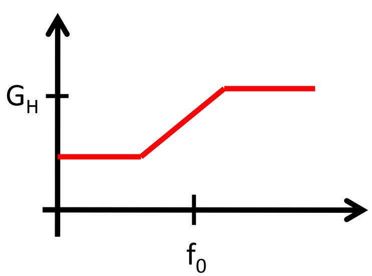
NOTCH



LOW-SHELF

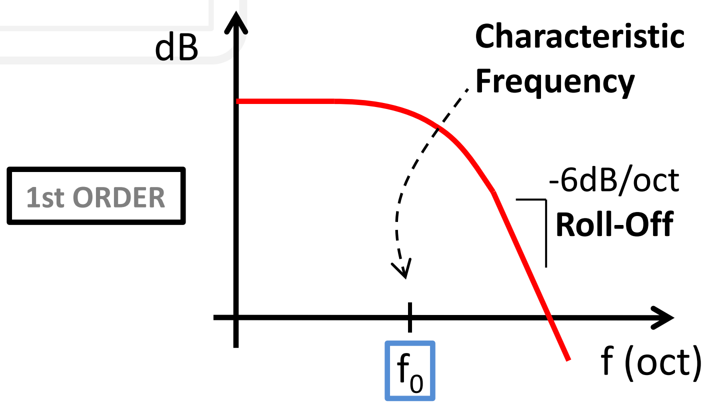


HIGH-SHELF



Digital Filters (viii)

LPF (Butterworth) design parameters/constraints:



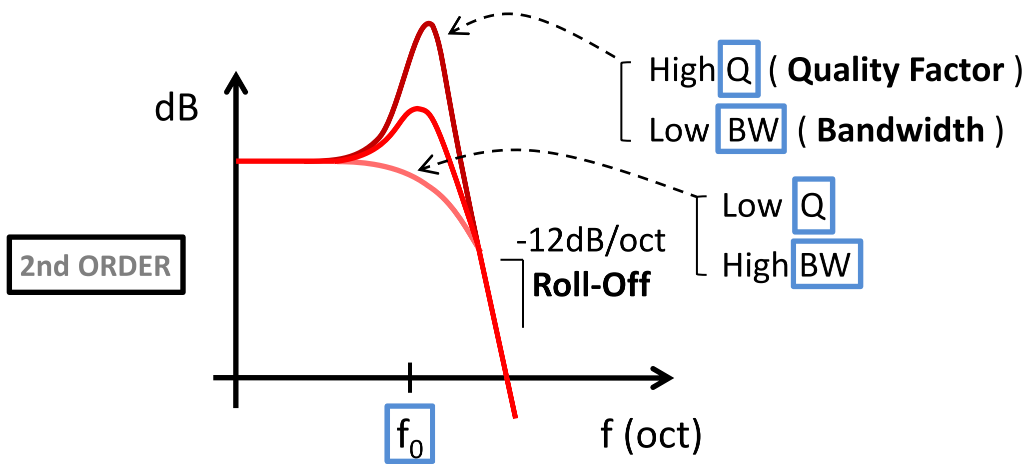
Desired $\{f_0\}$

↓

DESIGN PROCEDURE

↓

Filter Coefficients $\{b_i, a_j\}$



Desired $\{f_0, Q\}$ or $\{f_0, BW\}$

↓

DESIGN PROCEDURE

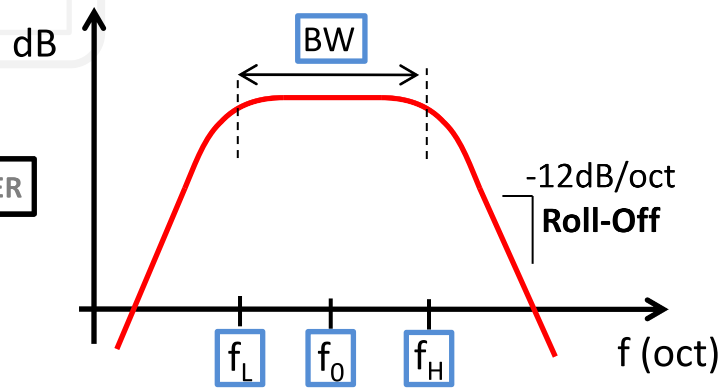
↓

Filter Coefficients $\{b_i, a_j\}$

Digital Filters (ix)

BPF design parameters/constraints:

2nd ORDER



Desired $\{\mathbf{f_L},\mathbf{f_H}\}$ or $\{\mathbf{f_0},\mathbf{BW}\}$

↓

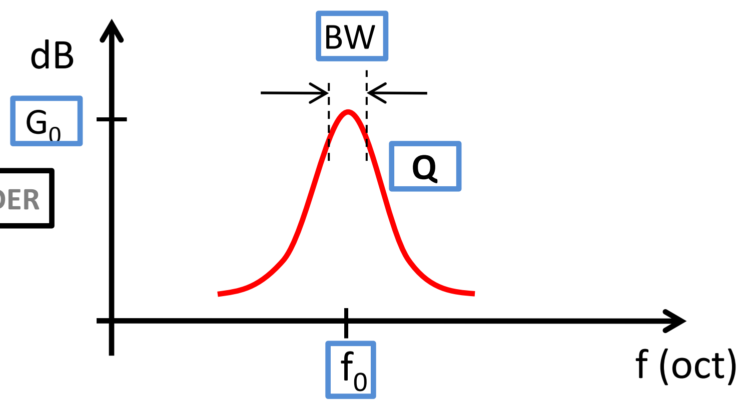
DESIGN PROCEDURE

↓

Filter Coefficients $\{\mathbf{b_i},\mathbf{a_j}\}$

PEAK design parameters/constraints:

2nd ORDER



Desired $\{\mathbf{f_0},\mathbf{G_0},\mathbf{BW}\}$ or $\{\mathbf{f_0},\mathbf{G_0},\mathbf{Q}\}$

↓

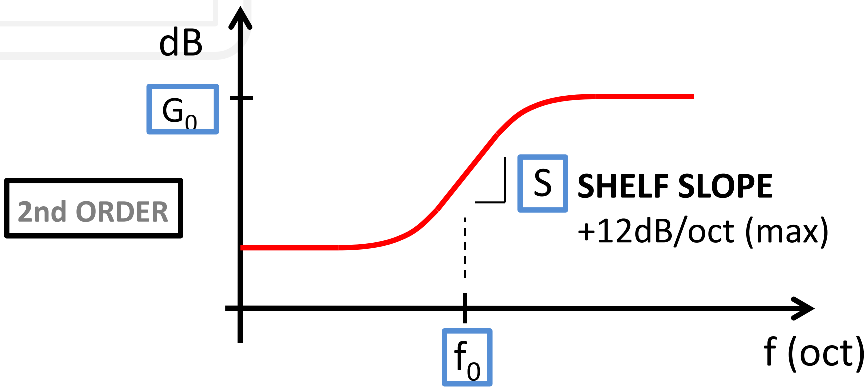
DESIGN PROCEDURE

↓

Filter Coefficients $\{\mathbf{b_i},\mathbf{a_j}\}$

Digital Filters (x)

HIGH-SHELF design parameters/constraints:



Desired $\{\mathbf{f_0}, \mathbf{G_0}, \mathbf{S}\}$

↓

DESIGN PROCEDURE

↓

Filter Coefficients $\{\mathbf{b_i}, \mathbf{a_j}\}$

All these filters functions can be implemented by means of the 2nd order '**BIQUAD**' section:

$$H(z) = \frac{b_0 + b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{a_0 + a_1 \cdot z^{-1} + a_N \cdot z^{-2}}$$

How to design them? Extensive theory & literature!!
→ Quick method: R. Bristow-Johnson's cookbook:

http://www.musicdsp.org/files/Audio-EQ-Cookbook.txt

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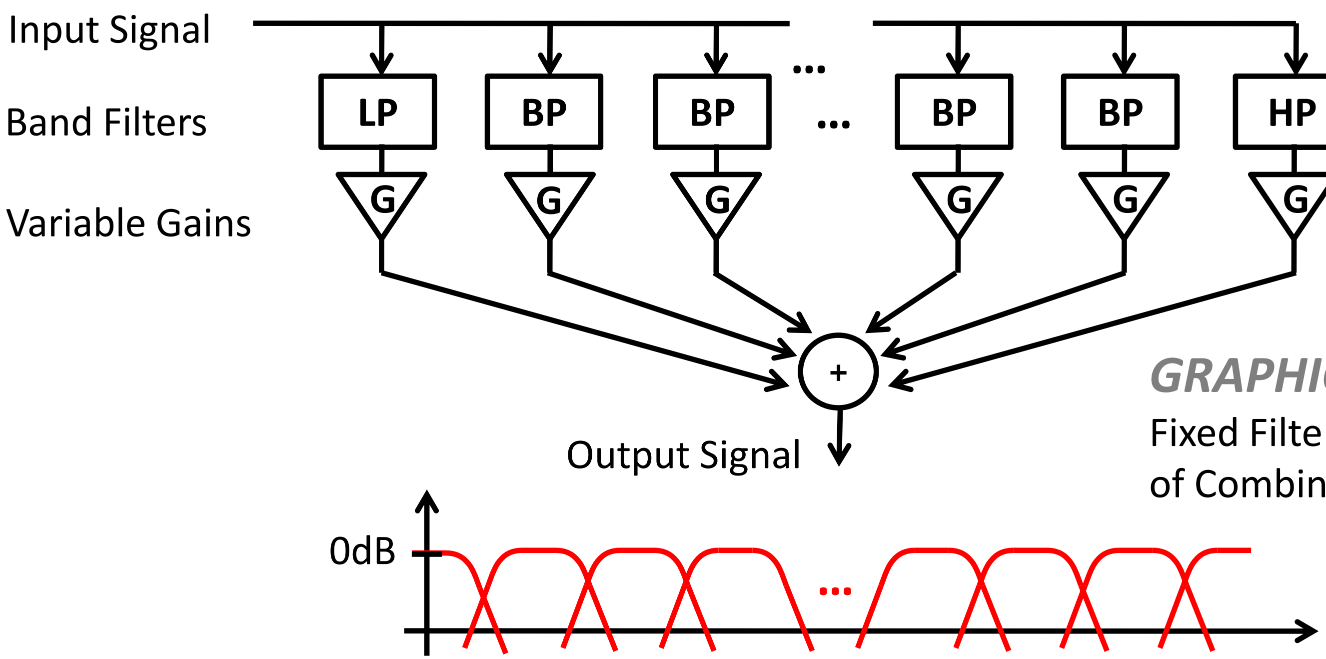
Equalization (ii)

N-BAND EQUALIZER by **PARALLEL**
BAND-DEDICATED, FIXED FILTERS



GRAPHIC EQUALIZER

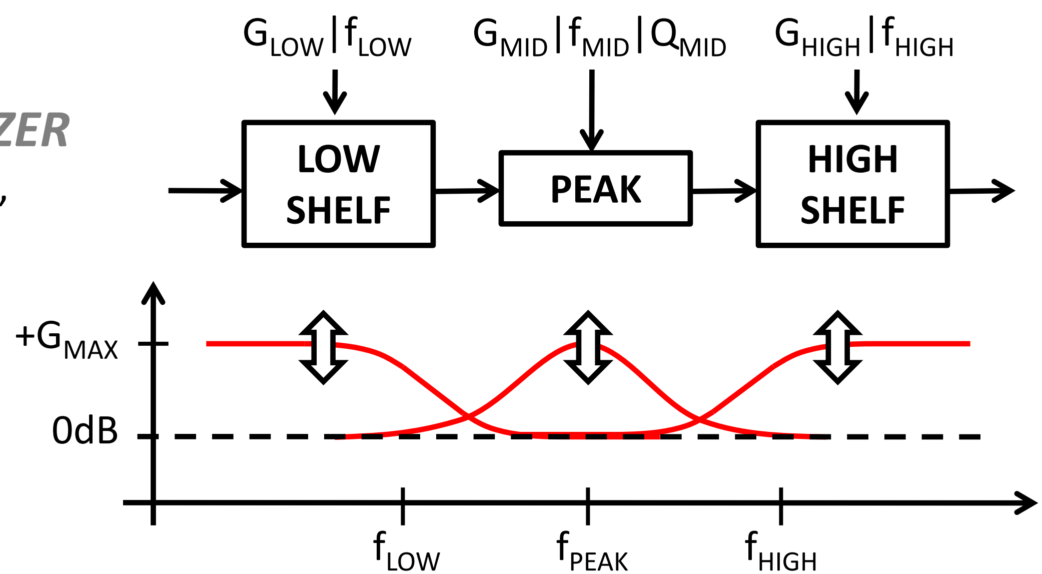
Fixed Filters: only Control of Combination (Gains)

Equalization (iii)

N-BAND EQUALIZER by **CASCADE** of
BAND-DEDICATED, CONTROLLABLE SECTIONS

Input Signal → **Section 1** → ··· → **Section N** → Output Signal

PARAMETRIC EQUALIZER
Variable Gains, Frequencies, and Bandwidths.



ISSUE: Changes in Gain/Frequency lead to Q/BW variation…

Constant-Q filters!

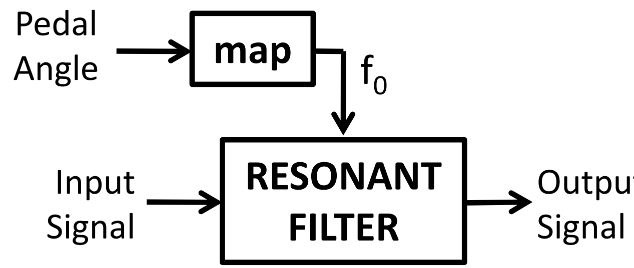
http://www.rane.com/note101.html

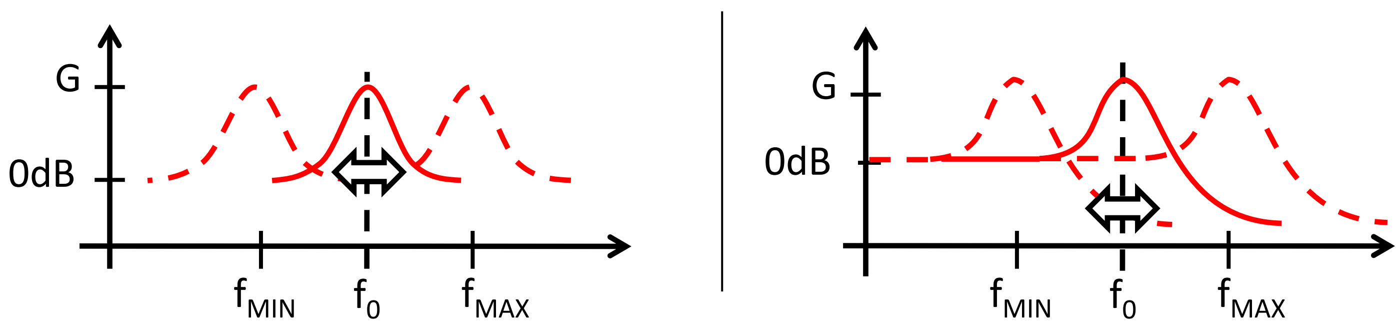
Wah-wah

Dating back from the 60s, its name was given after voice tone modulation (*formant* shift) caused by transition between vowels.

http://www.geofex.com/article_folders/wahpedl/voicewah.htm

In its most basic form, it consists on shifting the center frequency of a resonant filter
(Peak BP or LP)





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