A Physically Intuitive Haptic Drumstick

Edgar Berdahl, Bill Verplank, and Julius O. Smith III
Center for Computer Research in Music and Acoustics
—
Günter Niemeyer
Telerobotics Laboratory
—
Stanford University
—
International Computer Music Conference
Copenhagen, Denmark

August, 2007

Outline

Explain how drum rolls are played

Develop a physical model

Implement the model dynamics on a haptic display

Alter the dynamics to make it easier to play drum rolls

Drum Rolls

- Drummers can control drum rolls at rates up to 30Hz.

Drum Rolls

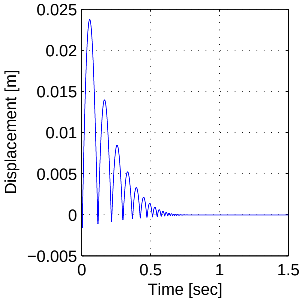
- Drummers can control drum rolls at rates up to 30Hz.
- The human neuromuscular system has a reaction time of over 100ms.

Drum Rolls

- Drummers can control drum rolls at rates up to 30Hz.
- The human neuromuscular system has a reaction time of over 100ms.
- How is this possible?!

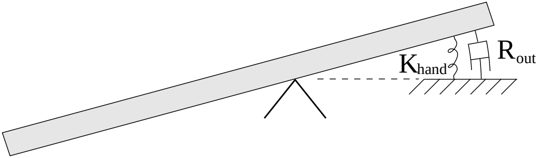
Drum Rolls

- Drummers can control drum rolls at rates up to 30Hz.
- The human neuromuscular system has a reaction time of over 100ms.
- How is this possible?!
  1. The bouncing effect: the drum membrane “throws” the stick back at the drummer.



Drum Rolls

- Drummers can control drum rolls at rates up to 30Hz.
- The human neuromuscular system has a reaction time of over 100ms.
- How is this possible?!
  1. The bouncing effect: the drum membrane “throws” the stick back at the drummer.
  2. Impedance modulation: the drummer can alter the impedance of his or her hand in real time.



Outline

Explain how drum rolls are played

Develop a physical model

Implement the model dynamics on a haptic display

Alter the dynamics to make it easier to play drum rolls

Above The Drum Membrane

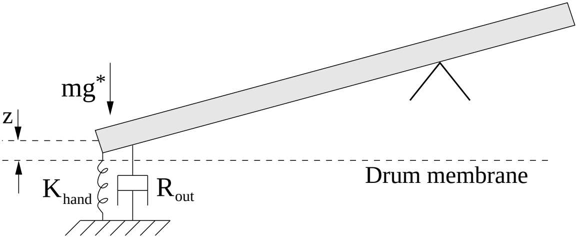


Figure: Drumstick dynamics for $z > 0$

- For the purposes of investigating drum rolls, we model the drumstick as a bouncing ball with mass m.
- The spring and dashpot are commuted to the end.
- Drummer can adjust rest position z_{h0} of the spring K_{hand}.
- Letting $z_{ss} = z_{h0} - mg^* / K_{hand}$, we have

$$m\ddot{z} + R_{out}\dot{z} + K_{hand}(z - z_{ss}) = 0.$$

“Inside” The Drum Membrane

- At DC, a drumstick being pressed into a drum membrane behaves like a linear spring K_{coll}.
- There is still damping R_{in} due to losses in the hand and collision.

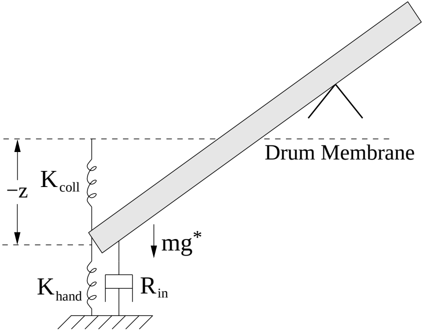


Figure: Drumstick dynamics for $z < 0$

$$m\ddot{z} + R_{in}\dot{z} + K_{hand}(z - z_{ss}) + K_{coll}z = 0$$

Coefficient Of Restitution

- Let β be the coefficient of restitution (COR) of drumstick/membrane collisions.

Coefficient Of Restitution

- Let β be the coefficient of restitution (COR) of drumstick/membrane collisions.
- For a collision beginning at any time t_{in} and ending at any time t_{out}, if $\dot{z}(t_{in}) = v_0$, then $\dot{z}(t_{out}) = -\beta v_0$.

Coefficient Of Restitution

- Let β be the coefficient of restitution (COR) of drumstick/membrane collisions.
- For a collision beginning at any time t_{in} and ending at any time t_{out}, if $\dot{z}(t_{in}) = v_0$, then $\dot{z}(t_{out}) = -\beta v_0$.
- Perfectly elastic collisions: $\beta = 1$

Coefficient Of Restitution

- Let β be the coefficient of restitution (COR) of drumstick/membrane collisions.
- For a collision beginning at any time t_{in} and ending at any time t_{out}, if $\dot{z}(t_{in}) = v_0$, then $\dot{z}(t_{out}) = -\beta v_0$.
- Perfectly elastic collisions: $\beta = 1$
- Inelastic collisions: $0 < \beta < 1$

Coefficient Of Restitution

- Let β be the coefficient of restitution (COR) of drumstick/membrane collisions.
- For a collision beginning at any time t_{in} and ending at any time t_{out}, if $\dot{z}(t_{in}) = v_0$, then $\dot{z}(t_{out}) = -\beta v_0$.
- Perfectly elastic collisions: $\beta = 1$
- Inelastic collisions: $0 < \beta < 1$
- $\beta \approx \exp \frac{-R_{in}\pi}{\sqrt{4mK_{coll} - R_{in}^2}}$

Summary Of Model Dynamics

- Above membrane ($z > 0$, COR α):

$$m\ddot{z} + R_{out}\dot{z} + K_{hand}(z - z_{ss}) = 0$$

Summary Of Model Dynamics

- Above membrane ($z > 0$, COR α):

$$m\ddot{z} + R_{out}\dot{z} + K_{hand}(z - z_{ss}) = 0$$

- "Inside" membrane ($z < 0$, COR β):

$$m\ddot{z} + R_{in}\dot{z} + K_{hand}(z - z_{ss}) + K_{coll}z = 0$$

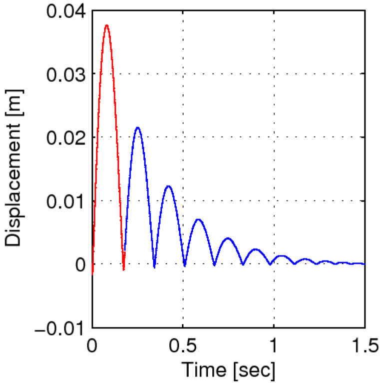
Summary Of Model Dynamics

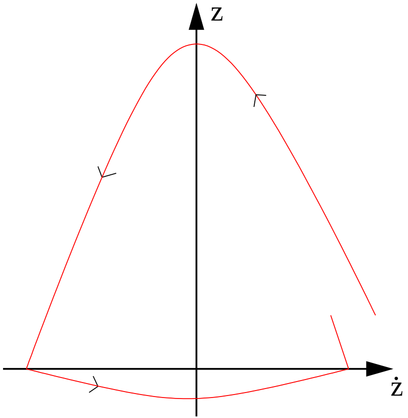
- Above membrane ($z > 0$, COR α):

$$m\ddot{z} + R_{out}\dot{z} + K_{hand}(z - z_{ss}) = 0$$

- "Inside" membrane ($z < 0$, COR β):

$$m\ddot{z} + R_{in}\dot{z} + K_{hand}(z - z_{ss}) + K_{coll}z = 0$$





Outline

Explain how drum rolls are played

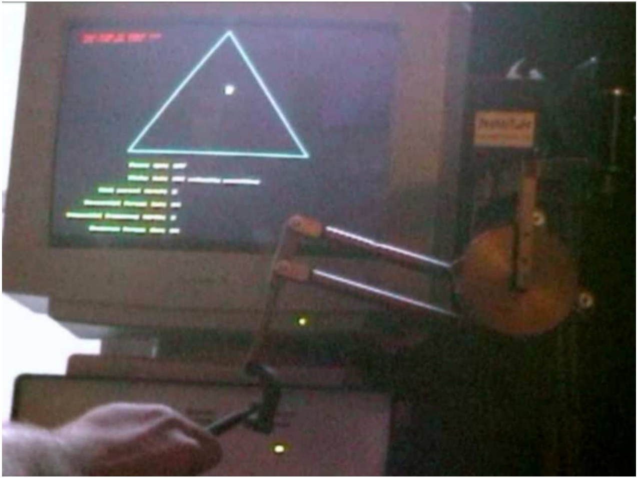
Develop a physical model

Implement the model dynamics on a haptic display

Alter the dynamics to make it easier to play drum rolls

Haptic Drumstick

- We use the three DOF Model T PHANTOM robotic arm[1].



[1] From SensAble Technologies, see http://www.sensable.com.

Sound Synthesis

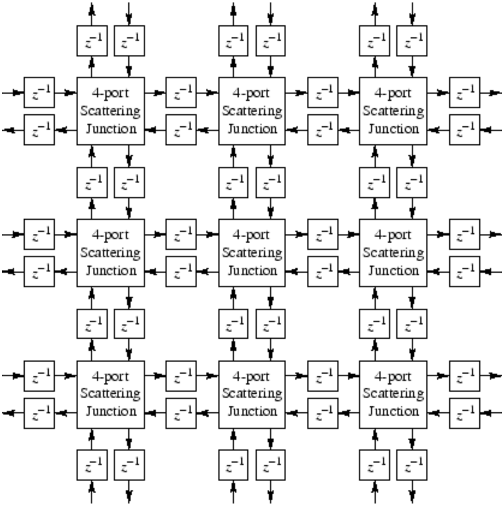


Figure: Rectilinear 2D Mesh

Sound Synthesis

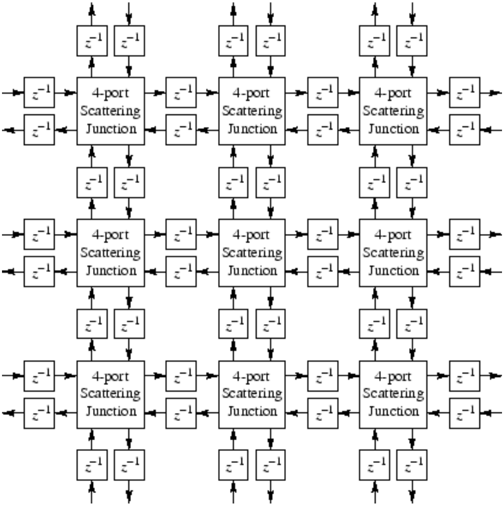


Figure: Rectilinear 2D Mesh

Outline

Explain how drum rolls are played

Develop a physical model

Implement the model dynamics on a haptic display

Alter the dynamics to make it easier to play drum rolls

Altering The Drumstick Dynamics

- Noise and deterministic chaos are not physically intuitive.

Altering The Drumstick Dynamics

- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.

Altering The Drumstick Dynamics

- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.
- Limit cycles also manifest themselves in bowed strings, vibrating reeds, and drum rolls.

Altering The Drumstick Dynamics

- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.
- Limit cycles also manifest themselves in bowed strings, vibrating reeds, and drum rolls.
- Goal: Assist the drummer in playing drum rolls.

Altering The Drumstick Dynamics

- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.
- Limit cycles also manifest themselves in bowed strings, vibrating reeds, and drum rolls.
- Goal: Assist the drummer in playing drum rolls.
- We can help bring about self-oscillations with
  1. system delay

Altering The Drumstick Dynamics

- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.
- Limit cycles also manifest themselves in bowed strings, vibrating reeds, and drum rolls.
- Goal: Assist the drummer in playing drum rolls.
- We can help bring about self-oscillations with
  1. system delay
  2. a hysteretic spring K_{coll}

Altering The Drumstick Dynamics

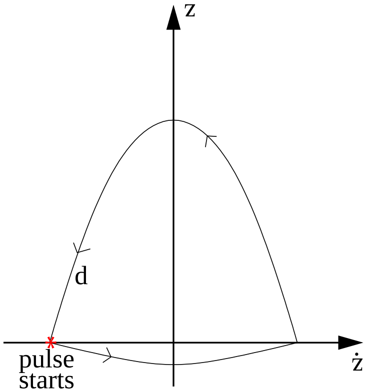
- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.
- Limit cycles also manifest themselves in bowed strings, vibrating reeds, and drum rolls.
- Goal: Assist the drummer in playing drum rolls.
- We can help bring about self-oscillations with
  1. system delay
  2. a hysteretic spring K_{coll}
  3. negative damping R_{out}

Altering The Drumstick Dynamics

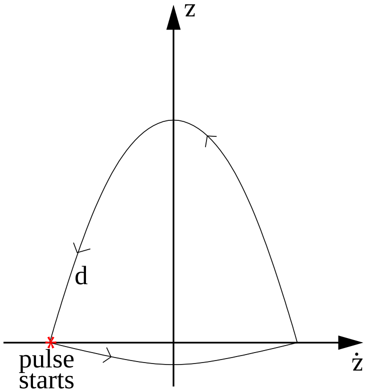
- Noise and deterministic chaos are not physically intuitive.
- Stable limit cycles, or self-sustaining attractive oscillations, describe biological oscillations such as the heart beating.
- Limit cycles also manifest themselves in bowed strings, vibrating reeds, and drum rolls.
- Goal: Assist the drummer in playing drum rolls.
- We can help bring about self-oscillations with
  1. system delay
  2. a hysteretic spring K_{coll}
  3. negative damping R_{out}
  4. forcing the drumstick in the z-dimension every time that the stick enters the simulated membrane

$$h(t) = \frac{m\Delta v_{pulse}}{\tau} e^{-t/\tau}$$

Forced Pulses Can Induce Limit Cycles

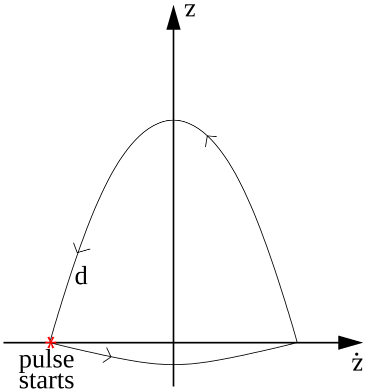


Forced Pulses Can Induce Limit Cycles



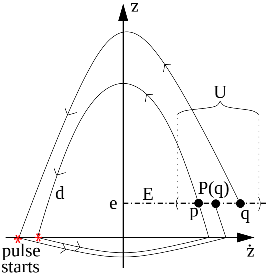
- Here we choose each pulse to be of constant magnitude.

Forced Pulses Can Induce Limit Cycles

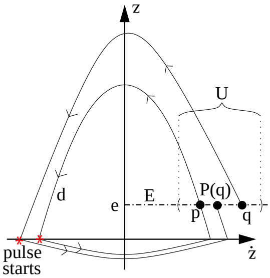


- Here we choose each pulse to be of constant magnitude.
- But is *d* stable?

Related Discrete-Time System

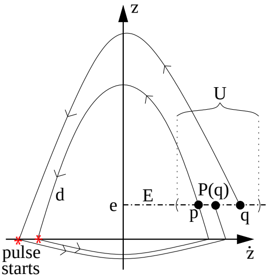


Related Discrete-Time System



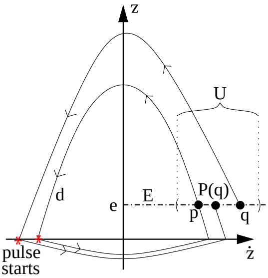
- $P(\cdot)$ is a Poincaré map.

Related Discrete-Time System



- $P(\cdot)$ is a Poincaré map.
- Let v_i is the velocity of the drumstick tip at the end of the ith collision with the drum membrane.

Related Discrete-Time System



- $P(\cdot)$ is a Poincaré map.
- Let v_i is the velocity of the drumstick tip at the end of the ith collision with the drum membrane.
- We analyze the stability of the closed orbit d by analyzing the stability of:

$$v_{i+1} = P(v_i) = \alpha\beta v_i + \beta\Delta v_{pulse}$$

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k$

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$
- At small amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) > 1$.

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$
- At small amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) > 1$.
- At large amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) \approx \beta$.

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$
- At small amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) > 1$.
- At large amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) \approx \beta$.
- $\alpha < 1$ and $\beta < 1 \Rightarrow \alpha\beta < 1$

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$
- At small amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) > 1$.
- At large amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) \approx \beta$.
- $\alpha < 1$ and $\beta < 1 \Rightarrow \alpha\beta < 1$
- $\Rightarrow \lim_{i\to\infty} v_i = \frac{k}{1-\alpha\beta} \triangleq v_{lc}$ is positive and finite.

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$
- At small amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) > 1$.
- At large amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) \approx \beta$.
- $\alpha < 1$ and $\beta < 1 \Rightarrow \alpha\beta < 1$
- $\Rightarrow \lim_{i\to\infty} v_i = \frac{k}{1-\alpha\beta} \triangleq v_{lc}$ is positive and finite.
- The discrete-time system is stable.

Applying Pulses With Constant Magnitude

- Choose $\Delta v_{pulse} = k/\beta$, where k is constant
- $v_{i+1} = P(v_i) = \alpha\beta v_i + k = \alpha(\beta + \frac{k}{\alpha v_i})v_i$
- At small amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) > 1$.
- At large amplitudes, the effective COR $\hat{\beta} = (\beta + \frac{k}{\alpha v_i}) \approx \beta$.
- $\alpha < 1$ and $\beta < 1 \Rightarrow \alpha\beta < 1$
- $\Rightarrow \lim_{i\to\infty} v_i = \frac{k}{1-\alpha\beta} \triangleq v_{lc}$ is positive and finite.
- The discrete-time system is stable.
- $P(\cdot)$ is a Poincaré map $\Rightarrow$ d is a *stable limit cycle*

Conclusions

- Informal testing suggests that the altered dynamics make it easier to play drum rolls.

Conclusions

- Informal testing suggests that the altered dynamics make it easier to play drum rolls.
- Drum roll limit cycles are guaranteed to be stable.

Conclusions

- Informal testing suggests that the altered dynamics make it easier to play drum rolls.
- Drum roll limit cycles are guaranteed to be stable.
- Drummers can increase the drum roll rate by increasing K_{hand} or decreasing z_{h0} as in traditional drum roll playing.

Conclusions

- Informal testing suggests that the altered dynamics make it easier to play drum rolls.
- Drum roll limit cycles are guaranteed to be stable.
- Drummers can increase the drum roll rate by increasing K_{hand} or decreasing z_{h0} as in traditional drum roll playing.
- The previous point suggests that the new musical instrument is *physically intuitive*–i.e., the new instrument supports physical interactions that are familiar to traditional performers of traditional drums.

Thank You!

Questions?