Transient Response of Digital Filters

If you have a comparatively short run of data to analyze,
you have to be careful about using a recursive digital filter;
you may run out of data before you have gotten rid of the spurious initial conditions!
-R. W. Hamming[1]

The present state of the art in filter implementation can be completely described using linear constant-coefficient difference equations. Consider a simple example: a real causal first-order recursive filter described by the difference equation

$$y[n] = (1-a)\, x[n] + a\, y[n-1] \qquad (1)$$

The induced transient response in this case is of the form $k\, a^n\, u[n-n_0]$ which appears at the filter output, more or less, for almost any input $x[n]$. This transient term has been tacitly accepted throughout the evolution of DSP as a leftover from analog filter implementation. But if you ask engineers what they really want a digital filter to do, they rarely mention a transient term. [Paiss]

Let us be a bit more rigorous in our explanation of what we mean by the *transient* in regard to our simple example above. Consider now the homogeneous difference equation describing the same first order digital filter for some range of n, say $n = 0\ldots N-1$.

$$y[n] - a\, y[n-1] = 0$$

By assuming the existence of a nonzero initial state, we can determine all the autonomous modes of oscillation for the filtering system described by Eq.(1).

$$\begin{pmatrix} 1 & -a & & & \mathbf{0} \\ & 1 & -a & & \\ & & 1 & -a & \\ & & & \ddots & \ddots \\ \mathbf{0} & & & 1 & -a \end{pmatrix} \begin{pmatrix} y[N-1] \\ y[N-2] \\ \vdots \\ y[0] \end{pmatrix} = 0$$

$$H \qquad\qquad y \quad = 0$$

The solution vector y is in the nullspace $\mathcal{N}$ of the *homogeneous system matrix* H; more precisely in this case, $y = (a^{N-1}\; a^{N-2} \ldots 1)^T$ spans the nullspace $\mathcal{N}$ of the matrix H.

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[1] R. W. Hamming, *Digital Filters*, §13.12, 1989, Prentice-Hall

Definition

The nullspace $\mathcal{N}$ of the homogeneous system matrix H comprises the basic components of the transient response to any input. The number of transient components is equal to the dimension of the nullspace.

Having found the nullspace, then a problem is to determine the amplitude and onset of each transient component in response to any particular input.

Applications

1) Sinusoidal Modeling. The purpose of a DFT filter-bank analyzer [Vaidyanathan] is to separate an incoming signal into distinct frequency bands so that each band may be separately analyzed for its sinusoidal components. The foremost obstacle encountered using this approach is an artifact due to the transient response of each bandpass FIR filter.[2] That makes frequency estimation by such nonlinear techniques as Prony's method [Marple] very difficult.

2) Automobile Suspension. The wheel suspension of an automobile is a classical analog filtering system. Transient response must be controlled if the tires are to remain in contact with the road in the presence of, say, step changes in pavement height.

References

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[Paiss] Omry Paiss, 'Elimination of Exponential Interference from Finite-Length Discrete Signals', IEEE Trans. ASSP, vol.38, no.12, pp.2189-2191, Dec. 1990

[Vaidyanathan] P.P. Vaidyanathan, *Multirate Systems and Filter Banks*, 1993, Prentice-Hall

[Wang] Avery Li-Chun Wang, 'On Fast FIR Filters Implemented As Tail-Canceling IIR Filters', Stanford, Calif. : CCRMA, Dept. of Music, Stanford University, 1994

[2]The FIR filters are handled as equivalent impulse-response-truncated IIR systems, [Wang] thus giving rise to the concept of the *virtual pole.* [Dattorro2]