Appendix F

Notation, Definitions, Glossary

b	scalar or column vector (italic $abcdefghijklmnopqrstuvwxyz$)
b_i	i^{th} *entry of vector* $b=[b_i\,,\ i=1\ldots n]$ or i^{th} b *vector from a list* $\{b_j\,,\ j=1\ldots n\}$ or i^{th} *iterate of vector* b
$b_{i:j}$	or $b(i:j)$: truncated vector comprising i^{th} through j^{th} entry of vector b (294)
$b_k(i:j)$	or $b_{i:j,k}$: truncated vector comprising i^{th} through j^{th} entry of vector b_k
b^{T}	vector transpose or row vector
b^{H}	complex conjugate transpose $b^{*\mathrm{T}}$
A	matrix (italic $ABCDEFGHIJKLMNOPQRSTUVWXYZ$)
A^{T}	Matrix transpose $[A_{ij}] \leftarrow [A_{ji}]$ is a linear operator. Regarding A as a linear operator, A^{T} is its adjoint.
$A^{-\mathrm{T}}$	matrix transpose of inverse; and *vice versa,* $\left(A^{-1}\right)^{\mathrm{T}} = \left(A^{\mathrm{T}}\right)^{-1}$ (*confer* p.489 *no.*47)
A^{T_1}	first of various transpositions of a cubix or quartix A (p.555, p.559)
A^{-1}	inverse of matrix A
$A^{\dagger}$	Moore-Penrose pseudoinverse of matrix A (§E)
A_{ij}	or $A(i\,,j)$: ij^{th} entry of matrix $A = \begin{matrix} & {\scriptstyle 1} & {\scriptstyle 2} & {\scriptstyle 3} & \\ \left[\begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix}\right. & \begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix} & \begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix} & \left.\begin{matrix} \cdot \\ \cdot \\ \cdot \end{matrix}\right] & \begin{matrix} {\scriptstyle 1} \\ {\scriptstyle 2} \\ {\scriptstyle 3} \end{matrix} \end{matrix}$ or rank-1 matrix $a_i a_j^{\mathrm{T}}$ (§4.10)
$A(i\,,j)$	A is a function of i and j
A_i	i^{th} matrix from a set or i^{th} principal submatrix (1329) or i^{th} iterate of A
$A(i\,,:)$	or $A_{i:}$: i^{th} row of matrix A
$A(:\,,j)$	or $A_{:j}$: j^{th} column of matrix A [185, §1.1.8]
$A(i:j\,,k:\ell)$	or $A_{i:j,\,k:\ell}$: submatrix; i^{th} through j^{th} row and k^{th} through ℓ^{th} column of A
$\sqrt{\ }$	positive square root
$\sqrt[\circ]{x}$	entrywise positive square root of vector x

$\sqrt[\ell]{\ }$	positive ℓ^{th} root
$A^{1/2}$ and $\sqrt{A}$	$A^{1/2}$ is any matrix such that $A^{1/2}A^{1/2}=A$. For $A\in\mathbb{S}^n_+$, $\sqrt{A}\in\mathbb{S}^n_+$ is unique and $\sqrt{A}\sqrt{A}=A$. [59, §1.2] (§A.5.1.4)
$\sqrt[\circ]{D}$	$=[\sqrt{d_{ij}}\,]$ *absolute distance matrix* (1517) or *Hadamard positive square root*: $D=\sqrt[\circ]{D}\circ\sqrt[\circ]{D}$
thin	a skinny matrix; meaning, more rows than columns: $\begin{bmatrix} & \\ & \\ & \end{bmatrix}$ *When there are more equations than unknowns,* *we say that the system* $Ax=b$ *is overdetermined.* [185, §5.3]
wide	a fat matrix; meaning, more columns than rows: $\begin{bmatrix} \quad\quad\quad\quad \end{bmatrix}$ *underdetermined*
hollow	matrix having $\mathbf{0}$ main diagonal
$\mathcal{A}$	some set (calligraphic $\mathcal{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$)
$\mathbb{A}$	set of vectors or matrices (blackboard $\mathbb{ABCDEFGHIJKLMNOPQRSTUVWXYZ}$)
$\mathfrak{F}$	discrete Fourier transform (932) (Euler Fraktur $\mathfrak{A\,B\,C\,D\,E\,F\,G\,H\,I\,J\,K\,L\,M\,N\,O\,P\,Q\,R\,S\,T\,U\,V\,W\,X\,Y\,Z}$)
$\mathcal{F}(\mathcal{C}\ni A)$	smallest face (176) that contains element A of set $\mathcal{C}$
$\mathcal{G}(\mathcal{K})$	generators (§2.13.4.2.1) of set $\mathcal{K}$; any collection of points and directions whose hull constructs $\mathcal{K}$
$\mathcal{L}^\nu_\nu$	level set (571)
$\mathcal{L}_\nu$	sublevel set (575)
$\mathcal{L}^\nu$	superlevel set (669)
$\mathfrak{L}$	Lagrangian (516)
$\mathfrak{E}$	member of elliptope $\mathcal{E}_t$ (1258) parametrized by scalar t
$\mathcal{E}$	elliptope (1237)
E	elementary matrix
E_{ij}	member of standard orthonormal basis for symmetric (62) or symmetric hollow (78) matrices
id est	from the Latin meaning *that is*
e.g	*exempli gratia,* from the Latin meaning *for sake of example*
sic	from the Latin meaning *so* or *thus* or *in this manner*; something meant as written
videlicet	from the Latin meaning *it is permitted to see*
ibidem	from the Latin meaning *in the same place*
no.	*number,* from the Latin *numero*
vs.	*versus,* from the Latin

a.i.	affinely independent (§2.4.2.3)
c.i.	conically independent (§2.10)
l.i.	linearly independent
w.r.t	*with respect to*
`a.k.a`	*also known as*
re	real part
im	imaginary part
$\imath$ or $\jmath$	$\sqrt{-1}$
$\subseteq$ $\supseteq$	*subset, superset*
$\subset$ $\supset$	*proper subset, proper superset*
$\cap$ $\cup$	*intersection, union*
$\in$	membership, *element belongs to,* or *element is a member of*
$\ni$	membership, *contains* as in $\mathcal{C} \ni y$ ($\mathcal{C}$ contains element y)
$\backepsilon$	*such that*
$\exists$	*there exists*
$\therefore$	*therefore*
$\forall$	*for all,* or *over all*
&	(ampersand) *and*
&	(ampersand italic) *and*
$\propto$	*proportional to*
∞	infinity
$\equiv$	*equivalent to*
$\triangleq$	*defined equal to, equal by definition*
$\approx$	*approximately equal to*
$\simeq$	*isomorphic to* or *with*
$\cong$	*congruent to* or *with*
—	Hadamard quotient as in, for $x, y \in \mathbb{R}^n$, $\dfrac{x}{y} \triangleq [\, x_i/y_i \;,\; i=1 \ldots n \,] \in \mathbb{R}^n$
$\circ$	Hadamard product of matrices: $x \circ y \triangleq [\, x_i\, y_i \;,\; i=1 \ldots n \,] \in \mathbb{R}^n$ (§D.1.2.2, §A.1.1)
$\otimes$	Kronecker product of matrices (§D.1.2.1, §A.1.1)
$\oplus$	vector sum of sets $\mathcal{X} = \mathcal{Y} \oplus \mathcal{Z}$ where every element $x \in \mathcal{X}$ has unique expression $x = y + z$ where $y \in \mathcal{Y}$ and $z \in \mathcal{Z}$; [349, p.19] then summands are *algebraic complements.* $\mathcal{X} = \mathcal{Y} \oplus \mathcal{Z} \Rightarrow \mathcal{X} = \mathcal{Y} + \mathcal{Z}$. Now assume $\mathcal{Y}$ and $\mathcal{Z}$ are nontrivial subspaces. $\mathcal{X} = \mathcal{Y} + \mathcal{Z} \Rightarrow \mathcal{X} = \mathcal{Y} \oplus \mathcal{Z} \Leftrightarrow \mathcal{Y} \cap \mathcal{Z} = \mathbf{0}$ [350, §1.2] [123, §5.8]. Each element from a vector sum (+) of subspaces has unique expression ($\oplus$) when a basis from each subspace is linearly independent of bases from all the other subspaces.

$\ominus$ likewise, unique vector difference of sets

$\boxplus$ orthogonal vector sum of sets $\mathcal{X}=\mathcal{Y}\boxplus\mathcal{Z}$ where every element $x\in\mathcal{X}$ has unique orthogonal expression $x=y+z$ where $y\in\mathcal{Y}$, $z\in\mathcal{Z}$, and $y\perp z$. [372, p.51] $\mathcal{X}=\mathcal{Y}\boxplus\mathcal{Z}\Rightarrow\mathcal{X}=\mathcal{Y}+\mathcal{Z}$. If $\mathcal{Z}\subseteq\mathcal{Y}^\perp$ then $\mathcal{X}=\mathcal{Y}\boxplus\mathcal{Z}\Leftrightarrow\mathcal{X}=\mathcal{Y}\oplus\mathcal{Z}$. [123, §5.8] If $\mathcal{Z}=\mathcal{Y}^\perp$ then summands are *orthogonal complements.*

$\pm$ *plus or minus* or *plus and minus*

$\backslash$ as in $\backslash\mathcal{A}$ means logical *not* $\mathcal{A}$, or *relative complement of set* $\mathcal{A}$; *id est,* $\backslash\mathcal{A}=\{x\notin\mathcal{A}\}$; *e.g,* $\mathcal{B}\backslash\mathcal{A}\triangleq\{x\in\mathcal{B}\mid x\notin\mathcal{A}\}\equiv\mathcal{B}\cap\backslash\mathcal{A}$

$\Rightarrow$ and $\Leftarrow$ sufficient and necessary, *implies* and *is implied by*; *e.g,*

A is sufficient: $A\Rightarrow B$,	A is necessary: $A\Leftarrow B$,
$A\Rightarrow B\ \Leftrightarrow\ \backslash A\Leftarrow\backslash B$,	$A\Leftarrow B\ \Leftrightarrow\ \backslash A\Rightarrow\backslash B$,
if A *then* B,	*if* B *then* A,
A *only if* B,	B *only if* A.

$\Leftrightarrow$ *if and only if* (iff) or *corresponds with* or *necessary and sufficient* or *logical equivalence*

is plural *are,* as in A *is* B means $A\Rightarrow B$; conventional usage of English language imposed by logicians

$\nRightarrow$ and $\nLeftarrow$ insufficient and unnecessary, *does not imply* and *is not implied by*; *e.g,*

A is insufficient: $A\nRightarrow B$,	A is unnecessary: $A\nLeftarrow B$,
$A\nRightarrow B\ \Leftrightarrow\ \backslash A\nLeftarrow\backslash B$,	$A\nLeftarrow B\ \Leftrightarrow\ \backslash A\nRightarrow\backslash B$.

$\leftarrow$ *is replaced with*; substitution, assignment

$\rightarrow$ *goes to,* or *approaches,* or *maps to*

$t\rightarrow 0^+$ t *goes to* 0 *from above*; meaning, *from the positive* [230, p.2]

$\vdots\ \ddots\ \cdots$ as in $1\cdots 1$ means ones in a row or $[\,s_1\cdots s_N\,]$ means *continuation*; a matrix whose columns are s_i for $i=1\ldots N$ or as in $n(n-1)(n-2)\cdots 1$ means continuation of a product

$\ldots$ as in $i=1\ldots N$ meaning, i is a *sequence* of successive integers beginning with 1 and ending with N; *id est,* $1\ldots N=1\!:\!N$

$:$ as in $f:\mathbb{R}^n\rightarrow\mathbb{R}^m$ meaning f *is a mapping,* or sequence of successive integers specified by bounds as in $i\!:\!j=i\ldots j$ (if $j<i$ then sequence is descending)

f real function or multidimensional function `a.k.a` *operator*

$f:\mathcal{M}\rightarrow\mathcal{R}$ meaning f *is a mapping from ambient space* $\mathcal{M}$ *to ambient* $\mathcal{R}$, not necessarily denoting either domain or range

$|$ as in $f(x)\,|\,x\in\mathcal{C}$ means *with the condition(s)* or *such that* or *evaluated for,* or as in $\{f(x)\,|\,x\in\mathcal{C}\}$ means *evaluated for each and every* x *belonging to set* $\mathcal{C}$

$g|_{x_{\rm p}}$ *expression* g *evaluated at* $x_{\rm p}$

$\|$ *parallel*

$A\,,B$ as in, for example, $A\,,B\in\mathbb{S}^N$ means $A\in\mathbb{S}^N$ and $B\in\mathbb{S}^N$

$(a\,,b)$	*open interval* between a and b in $\mathbb{R}$ or *variable pair* perhaps of disparate dimension
$[\,a\,,b\,]$	*closed interval* or *line segment* between a and b in $\mathbb{R}$
$(\ \ )$	*hierarchal, parenthetical, optional*
$\binom{n}{k}$	$\triangleq \begin{cases} 1\,, & k=0 \\ \left.\begin{array}{ll} -1^{k}\frac{(-n+k-1)!}{k!(-n-1)!}\,, & k>0 \\ -1^{n-k}\frac{(-k-1)!}{(n-k)!(-n-1)!}\,, & k\le n \\ 0\,, & n<k<0 \end{array}\right\} & n<0 \\ \left.\begin{array}{ll} 0\,, & k<0 \ \textbf{or}\ \ k>n \\ \frac{n!}{k!(n-k)!}\,, & \text{otherwise} \end{array}\right\} & n\ge 0 \end{cases}$ [260] *binomial coefficient* on $\mathbb{Z}^{\mathbf{2}}$
!	factorial; *id est*, for integer $n>0$, $n!\triangleq n(n-1)(n-2)\cdots 1$, $(-n)!\triangleq\infty$, $0!\triangleq 1$
$\{\ \}$	curly braces denote a *set* or *list; e.g*, $\{Xa \mid a\succeq 0\}$ *the set comprising* Xa *evaluated for each and every* $a\succeq 0$ where membership of a to some space is implicit, a *union*; or $\{0,1\}^n$ represents a binary vector of dimension n
$\langle\ \rangle$	angle brackets denote *vector inner-product* (35) (40)
$[\ \ ]$	matrix or vector, or quote insertion, or citation
$[d_{ij}]$	matrix whose ij^{th} entry is d_{ij}
$[x_i]$	vector whose i^{th} entry is x_i
x_{p}	particular value of x
x_0	particular instance of x, or initial value of a sequence x_i
x_1	first entry of vector x, or first element of a list $\{x_i\}$
x_ε	*extreme point*
x_+	vector x whose negative entries are replaced with 0: $x_+ = \frac{1}{2}(x+\lvert x\rvert)$ (546) *nonnegative part of* x or *clipped vector* x
x_-	$x_-\triangleq\frac{1}{2}(x-\lvert x\rvert)$: *nonpositive part of* $x = x_+ + x_-$
$\check{x}$	known data
$x^\star$	optimal value of variable x. optimal $\Rightarrow$ feasible
x^*	*complex conjugate* or *dual variable* or *extreme direction of dual cone*
f^*	*convex conjugate function* $f^*(s)=\sup\{\langle s\,,x\rangle - f(x) \mid x\in\operatorname{dom} f\}$
$P_{\mathcal{C}}x$ or Px	projection of point x on set $\mathcal{C}$, P is operator or idempotent matrix
$P_k x$	projection of point x on set $\mathcal{C}_k$ or on range of implicit vector
$\delta(A)$	(`a.k.a` $\operatorname{diag}(A)$, §A.1) *vector made from main diagonal of* A if A is a matrix; otherwise, *diagonal matrix made from vector* A
$\delta^2(A)$	$\equiv\delta(\delta(A))$. For vector or diagonal matrix Λ, $\delta^2(\Lambda)=\Lambda$
$\delta(A)^2$	$=\delta(A)\delta(A)$ where A is a vector

$\lambda_i(X)$ i^{th} entry of vector λ is function of X

$\lambda(X)_i$ i^{th} entry of vector-valued function of X

$\lambda(A)$ *vector of eigenvalues of matrix* A (§A.5) (a nonlinear vector-valued function)

$\lambda(\mathbb{A})$ spectral cone $\mathcal{K}_\lambda$ for matrix set $\mathbb{A}$ (§5.11.2.3)

$\sigma(A)$ *vector of singular values of matrix* A (§A.6) (a nonlinear vector-valued function always arranging in nonincreasing order), or *support function in direction* A

Σ diagonal matrix of singular values, not necessarily square

$\sum$ sum. Empty sum equals, conventionally, 0 or $\mathbf{0}$

$\pi(\gamma)$ nonlinear *permutation operator* (or *presorting function*) arranges vector γ into nonincreasing order (§7.1.3). π_i is a permutation matrix; *e.g*, (961).

Ξ permutation matrix

Π doublet or permutation operator or matrix or set of all permutation matrices

$\prod$ product. Empty product equals, conventionally, 1 or I

$\psi(Z)$ signum-like *step function* that returns a scalar for matrix argument (767), it returns a vector for vector argument (1752)

D symmetric hollow matrix of distance-square or *Euclidean distance matrix*

$\mathbf{D}$ Euclidean distance matrix operator

$\mathbf{D}^{\mathrm{T}}(X)$ adjoint operator

$\mathbf{D}(X)^{\mathrm{T}}$ transpose of $\mathbf{D}(X)$

$\mathbf{D}^{-1}(X)$ inverse operator

$\mathbf{D}(X)^{-1}$ inverse of $\mathbf{D}(X)$

$D^\star$ optimal value of variable D

D^* dual to variable D

$\mathbf{V}$ geometric centering operator, $\mathbf{V}(D) = -VDV\frac{1}{2}$ (1157)

$\mathbf{V}_{\mathcal{N}}$ $\mathbf{V}_{\mathcal{N}}(D) = -V_{\mathcal{N}}^{\mathrm{T}} D V_{\mathcal{N}}$ (1171)

V $N\times N$ symmetric elementary, auxiliary, projector, geometric centering matrix, $\mathcal{R}(V) = \mathcal{N}(\mathbf{1}^{\mathrm{T}})$, $\mathcal{N}(V) = \mathcal{R}(\mathbf{1})$, $V^2 = V$ (§B.4.1)

$V_{\mathcal{N}}$ $N\times N-1$ Schoenberg auxiliary matrix $\mathcal{R}(V_{\mathcal{N}}) = \mathcal{N}(\mathbf{1}^{\mathrm{T}})$, $\mathcal{N}(V_{\mathcal{N}}^{\mathrm{T}}) = \mathcal{R}(\mathbf{1})$ (§B.4.2)

$V_{\mathcal{X}}$ $V_{\mathcal{X}} V_{\mathcal{X}}^{\mathrm{T}} \equiv V^{\mathrm{T}} X^{\mathrm{T}} X V$ (1349)

X point list ((79) having cardinality N) arranged columnar in $\mathbb{R}^{n\times N}$, or set of generators, or extreme directions, or matrix variable

G Gram matrix $X^{\mathrm{T}} X$ (1058)

r affine dimension (1203)

α_c	geometric center (1134)
ρ	rank of matrix or bound on affine dimension
$\mathbf{k}$	number of conically independent generators
$\Bbbk$	raw-data domain of Magnetic Resonance Imaging (MRI) machine, as in $\Bbbk$-*space*
n	Euclidean (ambient spatial) dimension of list $X \in \mathbb{R}^{n\times N}$, or integer
η	noise factor or noise signal or normal vector or $\min\{m\,,\,n\}$
N	cardinality of list $X \in \mathbb{R}^{n\times N}$, or integer
epi	function epigraph
dom	function domain
$\mathcal{R}f$	function range
$\mathcal{R}(A^{\mathrm{T}})$	the subspace: *rowspace of* A (145) or $\operatorname{span}\operatorname{basis}\mathcal{R}(A^{\mathrm{T}})\,;\quad \mathcal{R}(A^{\mathrm{T}}) \perp \mathcal{N}(A)$
$\mathcal{R}(A)$	the subspace: *range of* A (146) or $\operatorname{span}\operatorname{basis}\mathcal{R}(A)\,;\quad \mathcal{R}(A) \perp \mathcal{N}(A^{\mathrm{T}})$
span	as in $\operatorname{span} A = \mathcal{R}(A) = \{Ax \mid x \in \mathbb{R}^n\}$ when A is a matrix
$\operatorname{basis}\mathcal{R}(A)$	*overcomplete columnar basis for range of* A or *minimal set constituting generators for vertex-description of* $\mathcal{R}(A)$ or *linearly independent set of vectors spanning* $\mathcal{R}(A)$
$\mathcal{N}(A)$	the subspace: *nullspace of* A (147) `a.k.a` *kernel of* A; $\mathcal{N}(A) \perp \mathcal{R}(A^{\mathrm{T}})$
$\mathbb{R}^n$	Euclidean n-dimensional real vector space (nonnegative integer n); a subspace, conventionally, but not a proper subspace. [259, §2.1] $\mathbb{R}^{\mathbf{0}} = \mathbf{0}$. $\mathbb{R} = \mathbb{R}^{\mathbf{1}}$ or vector space of unspecified dimension. [450]
$\mathbb{R}^{m\times n}$	Euclidean vector space of m by n dimensional real matrices
$\times$	Cartesian product. $\mathbb{R}^{m\times n-m} \triangleq \mathbb{R}^{m\times(n-m)}$. $\mathcal{K}_1 \times \mathcal{K}_2 = \begin{bmatrix} \mathcal{K}_1 \\ \mathcal{K}_2 \end{bmatrix}$
$\begin{bmatrix} \mathbb{R}^m \\ \mathbb{R}^n \end{bmatrix}$	$\mathbb{R}^m \times \mathbb{R}^n = \mathbb{R}^{m+n}$
$\mathbb{Z}$	the real integers
$\mathbb{N}$	the nonnegative natural numbers; *id est*, $\mathbb{Z}_+$
$\mathbb{B}^n,\ \mathbb{B}^{n\times n}$	$\{0,1\}^n$ and $\{0,1\}^{n\times n}$ binary vectors of respective dimension n and $n\times n$
$\mathbb{B}^n_\pm\,,\ \mathbb{B}^{n\times n}_\pm$	$\{-1,1\}^n$ and $\{-1,1\}^{n\times n}$ bipolar binary vectors of dimension n and $n\times n$
$\mathbb{C}^n,\ \mathbb{C}^{n\times n}$	Euclidean complex vector space of respective dimension n and $n\times n$
$\mathbb{R}^n_+\,,\ \mathbb{R}^{n\times n}_+$	nonnegative orthant in Euclidean vector space of respective dimension n and $n\times n$
$\mathbb{R}^n_-\,,\ \mathbb{R}^{n\times n}_-$	nonpositive orthant in Euclidean vector space of respective dimension n and $n\times n$
$\mathbb{S}^n$	subspace of real symmetric $n\times n$ matrices; the *symmetric matrix subspace.* $\mathbb{S}^{\mathbf{0}} = \mathbf{0}$. $\mathbb{S} = \mathbb{S}^{\mathbf{1}}$ or symmetric subspace of unspecified dimension.
$\mathbb{S}^{n\perp}$	orthogonal complement of $\mathbb{S}^n$ in $\mathbb{R}^{n\times n}$, the antisymmetric matrices (54)

$\mathbb{S}_+^n$ convex cone comprising all (real) symmetric positive semidefinite $n\times n$ matrices, the *positive semidefinite cone*

$\operatorname{intr}\mathbb{S}_+^n$ interior of convex cone comprising all (real) symmetric positive semidefinite $n\times n$ matrices; *id est*, positive definite matrices

$\mathbb{S}_+^n(\rho)$ $=\{X\in\mathbb{S}_+^n \mid \operatorname{rank} X \geq \rho\}$ (269) convex set of all positive semidefinite $n\times n$ symmetric matrices whose rank equals or exceeds ρ

$\mathbb{EDM}^N$ cone of $N\times N$ Euclidean distance matrices in the symmetric hollow subspace

$\sqrt{\mathbb{EDM}^N}$ nonconvex cone of $N\times N$ Euclidean absolute distance matrices in the symmetric hollow subspace (§6.3)

$\mathbb{S}_{\mathbf{0}}^n$ subspace comprising all symmetric $n\times n$ matrices having all zeros in first row and column (2220) (§5.4.2.1)

$\mathbb{S}_h^n$ subspace comprising all symmetric hollow $n\times n$ matrices ($\mathbf{0}$ main diagonal), the *symmetric hollow subspace* (69)

$\mathbb{S}_h^{n\perp}$ orthogonal complement of $\mathbb{S}_h^n$ in $\mathbb{S}^n$, the set of all diagonal matrices (70)

$\mathbb{S}_c^n$ subspace comprising all geometrically centered symmetric $n\times n$ matrices; *geometric center subspace* $\mathbb{S}_c^N=\{Y\in\mathbb{S}^N \mid Y\mathbf{1}=\mathbf{0}\}$ (2216)

$\mathbb{S}_c^{n\perp}$ orthogonal complement of $\mathbb{S}_c^n$ in $\mathbb{S}^n$ (2218); *translation-invariant subspace*

$\mathbb{R}_c^{m\times n}$ subspace comprising all geometrically centered $m\times n$ matrices (2215)

$\mathbb{R}_h^{n\times n}$ subspace of symmetric [*sic*] matrices having $\mathbf{0}$ main diagonal; `a.k.a.` *real hollow subspace* (66)

$\mathbb{R}_h^{n\times n\perp}$ subspace of antisymmetric antihollow matrices (67)

$X^\perp$ basis $\mathcal{N}(X^{\mathrm{T}})$ (§2.13.10, §E.3.4)

$x^\perp$ $\mathcal{N}(x^{\mathrm{T}})$; $\{y\in\mathbb{R}^n \mid x^{\mathrm{T}}y=0\}$ (§2.13.11.1.1)

$\perp$ as in $A\perp B$ meaning A *is orthogonal to* B (and *vice versa*), where A and B are sets, vectors, or matrices. When A and B are vectors (35) (36) (or matrices (40) under Frobenius' norm), $A\perp B \Leftrightarrow \langle A\,,B\rangle=0 \Leftrightarrow \|A+B\|^2=\|A\|^2+\|B\|^2$

$\mathcal{R}(P)^\perp$ $\mathcal{N}(P^{\mathrm{T}})$; orthogonal complement of $\mathcal{R}(P)$ (fundamental subspace relations (141))

$\mathcal{N}(P)^\perp$ $\mathcal{R}(P^{\mathrm{T}})$

$\mathcal{R}^\perp$ $=\{y\in\mathbb{R}^n \mid \langle x,y\rangle=0\ \forall\, x\in\mathcal{R}\}$ (381). *Orthogonal complement of* $\mathcal{R}$ *in* $\mathbb{R}^n$ when $\mathcal{R}$ is a subspace

$\mathcal{K}^\perp$ normal cone (458)

$\mathcal{A}^\perp$ normal cone to affine subset $\mathcal{A}$ (§3.1.1.2.2)

$\mathcal{K}$ cone

$\mathcal{K}^*$ dual cone $-\mathcal{K}^\circ$

$\mathcal{K}^\circ$ polar cone $-\mathcal{K}^*$

D° polar variable D

$360°$	angular *degree*; *e.g*, $360° \Leftrightarrow 2\pi$ radians
$\mathcal{K}_{\mathcal{M}+}$	monotone nonnegative cone
$\mathcal{K}_{\mathcal{M}}$	monotone cone
$\mathcal{K}_{\lambda}$	spectral cone
$\mathcal{K}^*_{\lambda\delta}$	cone of majorization
$\mathcal{H}$	halfspace
$\mathcal{H}_-$	halfspace described using an outward-normal (110) to the hyperplane partially bounding it
$\mathcal{H}_+$	halfspace described using an inward-normal (111) to the hyperplane partially bounding it
$\partial\mathcal{H}$	hyperplane; *id est*, partial boundary of halfspace
$\underline{\partial\mathcal{H}}$	supporting hyperplane
$\underline{\partial\mathcal{H}}_-$	a supporting hyperplane having outward-normal with respect to set it supports
$\underline{\partial\mathcal{H}}_+$	a supporting hyperplane having inward-normal with respect to set it supports
∂	*partial derivative* or *partial differential* or *matrix of distance-square squared* (1557) or *boundary* of set $\mathcal{K}$ as in $\partial\mathcal{K}$ (18) (25)
d	*derivative* or *differential*
$\sqrt{d_{ij}}$	(absolute) distance scalar
d_{ij}	distance-square scalar, EDM entry
$\underline{d}$	vector of distance-square
$\underline{d_{ij}}$	lower bound on distance-square d_{ij}
$\overline{d_{ij}}$	upper bound on distance-square d_{ij}
$\overline{\mathrm{AB}}$	closed line segment between points A and B
AB	matrix multiplication of A and B
$\overline{\mathcal{C}}$	*closure of set* $\mathcal{C}$
g'	first derivative of possibly multidimensional function with respect to real argument
g''	second derivative with respect to real argument
$\overset{\rightarrow Y}{dg}$	first directional derivative of possibly multidimensional function g in direction $Y \in \mathbb{R}^{K\times L}$ (maintains dimensions of g)
$\overset{\rightarrow Y}{dg^2}$	second directional derivative of g in direction Y
∇	*gradient* from calculus, ∇f is shorthand for $\nabla_x f(x)$. $\nabla f(y)$ means $\nabla_y f(y)$ or *gradient* $\nabla_x f(y)$ *of* $f(x)$ *with respect to* x *evaluated at* y
∇^2	*second-order gradient*

Δ — *difference* or discrete differential or distance scalar (Figure **28**) or first-order difference matrix (945) or infinitesimal difference operator (§D.1.4)

$\triangle_{ijk}$ — triangle made by vertices i, j, and k

$\ddot{v}$ — coefficient vector for two spectral factors, Figure **112** `level` 2

$\dddot{v}$ — coefficient vector corresponding to four spectral factors, Figure **112** `level` 3

$\ddddot{v}$ — vector containing numerator or denominator coefficients of eight spectral factors; `level` 4 in a bifurcation tree like Figure **112**

CPU — central processing unit

dB — decibel

3D — three-dimensional or *three dimensions*

DC — direct current (0 Hz)

DCT — discrete cosine transform

DFT — discrete Fourier transform $\mathfrak{F}$ (932)

Hz — hertz (cycles per second), kHz means *kilohertz,* MHz *megahertz,* GHz *gigahertz*

Fs — $=1/T$, sample rate in Hz

EDM — Euclidean distance matrix

PSD — positive semidefinite

SDP — semidefinite program

SOCP — second-order cone program

LP — linear program

QUBO — quadratic unconstrained binary optimization

PCA — principal component analysis

SVD — singular value decomposition

SNR — signal to noise ratio

USA — United States of America

in — *function* f *in* x means x as argument to f
or x *in* $\mathcal{C}$ means element x is a member of set $\mathcal{C}$

on — *function* $f(x)$ *on* $\mathcal{A}$ means $\mathcal{A}$ is $\operatorname{dom} f$
or *relation* $\preceq$ *on* $\mathcal{A}$ means $\mathcal{A}$ is set whose elements are subject to $\preceq$
or *projection of* x *on* $\mathcal{A}$ means $\mathcal{A}$ is body on which projection is made
or *operating on vector* identifies argument type to f as "vector"

over — *function* $f(x)$ *over* $\mathcal{C}$ means f evaluated at each and every element of set $\mathcal{C}$

one-to-one — injective map or unique correspondence between sets

onto — *function* $f(x)$ *maps onto* $\mathcal{M}$ means f over its domain is a surjection w.r.t $\mathcal{M}$

injection	f that is *one-to-one*
surjection	f that is *onto*
bijection	f that is *one-to-one* and *onto*
orthant	generalization of two-dimensional *quadrant* $\llcorner$ to higher dimension
orthogonality	generalization of two-dimensional *perpendicularity* $\perp$ to higher dimension
decomposition	*orthonormal* (2117, p.582), *biorthogonal* (2092, p.577)
expansion	*orthogonal* (2127, p.584), *biorthogonal* (413, p.149)
vector	*column vector* in $\mathbb{R}^n$; identifiable by Cartesian coordinates of point at its head
entry	*scalar element or real variable constituting a vector or matrix*
cubix	member of $\mathbb{R}^{M\times N\times L}$
quartix	member of $\mathbb{R}^{M\times N\times L\times K}$
feasible	as in *feasible solution,* means satisfies the ("subject to") constraints of an optimization problem, may or may not be optimal
feasible set	most simply, *the set of all variable values satisfying all constraints of an optimization problem*
active set	an inequality constraint is termed *active* when it is met with equality; the set of all active constraints
solution set	most simply, *the set of all optimal solutions to an optimization problem*; a subset of the feasible set and not necessarily a single point
set	collection of elements in which order and multiplicity are ignored. A set member is called *element* [444]
list	ordered set retaining multiplicity
optimal	as in *optimal solution,* means a solution to an optimization problem. An optimal solution is not necessarily unique, but there is no better solution. optimal $\Rightarrow$ feasible
optimum	optimal value, usually the objective. Can be unique
same	as in *same problem,* means optimal solution set for one problem is identical to optimal solution set of another (without transformation)
equivalent	as in *equivalent problem,* means optimal solution to one problem can be derived from optimal solution to another via suitable transformation
convex	as in *convex problem,* essentially means a convex objective function optimized over a convex set (§4)
objective	the three objectives of Optimization are *minimize* (not min), *maximize* (not max), and *find*

program	*Semidefinite program* is any convex minimization, maximization, or feasibility problem constraining a variable to a subset of a positive semidefinite cone. *Prototypical semidefinite program* conventionally means: a semidefinite program having linear objective, affine equality constraints, but no inequality constraints except for cone membership. (§4.1.1) *Linear program* is any feasibility problem, or minimization or maximization of a linear objective, constraining the variable to some polyhedron. (§2.13.1.1.2) *Prototypical linear program* conventionally means: a linear program having linear objective, affine equality constraints, but no inequality constraints except for membership to a nonnegative orthant. (§4.1)
natural order	with reference to stacking columns in a vectorization means *a vector made from superposing column* 1 *on top of column* 2 *then superposing the result on column* 3 and so on; as in a vector made from entries of the main diagonal $\delta(A)$ means *taken from left to right and top to bottom*
partial order	relation $\preceq$ is a partial order, on a set, if it possesses reflexivity, antisymmetry, and transitivity (§2.7.2.2)
operator	mapping to a vector space (a multidimensional function)
projector	short for *projection operator*; not necessarily minimum-distance nor representable by matrix
sparsity	ratio of number of nonzero entries to matrix-dimension product
tight	with reference to a bound means *a bound that can be met,* with reference to an inequality means *equality is achievable*
trivial	with reference to $\mathbf{0}$ matrix, function, solution, or $\{\mathbf{0}\}$ subspace
$\emptyset$	*empty set,* an implicit member of every set
0	real zero
$\mathbf{0}$	*origin* or vector or matrix of zeros
O	*sort-index matrix*
O	*order of magnitude* or *polynomial order* or *computational intensity*: $\mathrm{O}(N)$ is first-order, $\mathrm{O}(N^2)$ is second-, and so on
1	real one
$\mathbf{1}$	vector of ones. $\mathbf{1}=\delta^2(\mathbf{1})$, $\delta(\mathbf{1})=I$
$\mathbf{1}_m$	$\mathbf{1}\in\mathbb{R}^m$
e_i	vector whose i^{th} entry is 1 (otherwise 0); i^{th} *member of the standard basis for* $\mathbb{R}^m$ (63)
I	Roman numeral one or capital i
I	Identity operator or matrix $I=\delta^2(I)$, $\delta(I)=\mathbf{1}$
I_m	$I\in\mathbb{S}^m$
$\mathcal{I}$	*index set,* a discrete set of indices

max — *maximum* [230, §0.1.1] or *largest element of a totally ordered set*

maximal — characterizes a maximum that is, somehow, *not necessarily* unique; *id est*, maximum $\not\Rightarrow$ unique maximum

$\underset{x}{\text{maximize}}$ — *find maximum of objective function w.r.t independent variables* x. Subscript $x \leftarrow x \in \mathcal{C}$ may hold implicit constraints if context clear; *e.g*, semidefiniteness

arg — *argument* of operator or function, or *variable of optimization*

$\sup \mathcal{X}$ — *supremum* of totally ordered set $\mathcal{X}$, *least upper bound,* may or may not belong to set [230, §0.1.1]; *e.g*, range $\mathcal{X}$ of real function

$\arg\sup f(x)$ — *argument* x *at supremum of function* f; not necessarily unique or a member of function domain

subject to — specifies constraints of an optimization problem; generally, inequalities and affine equalities. *Subject to* implies: anything not an independent variable is constant, an assignment, or substitution

min — *minimum* [230, §0.1.1] or *smallest element of a totally ordered set*

minimal — describes a minimum that is, in some sense, *not necessarily* unique; *id est*, minimum $\not\Rightarrow$ unique minimum

$\underset{x}{\text{minimize}}$ — *find objective function minimum w.r.t independent variables* x. Subscript $x \leftarrow x \in \mathcal{C}$ may hold implicit constraints if context clear; *e.g*, semidefiniteness

$\underset{x}{\text{find}}$ — *find any feasible solution, specified by the* (*"subject to"*) *constraints, w.r.t independent variables* x. Subscript $x \leftarrow x \in \mathcal{C}$ may hold implicit constraints if context clear; *e.g*, semidefiniteness. "find" denotes a *feasibility problem*; it is the third objective of Optimization

$\inf \mathcal{X}$ — *infimum* of totally ordered set $\mathcal{X}$, *greatest lower bound,* may or may not belong to set [230, §0.1.1]; *e.g*, range $\mathcal{X}$ of real function

$\arg\inf f(x)$ — *argument* x *at infimum of function* f; not necessarily unique or a member of function domain

iff — *if and only if*, *necessary and sufficient*; also the meaning indiscriminately attached to appearance of the word "if" in the statement of a mathematical definition, [148, p.106] [294, p.4] an esoteric practice worthy of abolition because of ambiguity thus conferred

rel — relative

intr — interior

lim — limit

sgn — signum function or *sign*; for $x \in \mathbb{R}^n$, $\operatorname{sgn}(x) = \left[\begin{cases} x_i/|x_i|\,, & x_i \neq 0 \\ 0\,, & x_i = 0 \end{cases}\right]$

round — round to nearest integer

mod — modulus function

tr — matrix trace

rank — as in $\operatorname{rank} A$, *rank of matrix* A; $\dim \mathcal{R}(A)$ (143)

dim — dimension, $\dim \mathbb{R}^n = n$, $\dim \mathbb{R}^{m\times n} = m\times n$
$\dim(x\in\mathbb{R}^n) = n$, $\dim \mathcal{R}(x\in\mathbb{R}^n) = 1$
$\dim(A\in\mathbb{R}^{m\times n}) = m\times n$, $\dim \mathcal{R}(A\in\mathbb{R}^{m\times n}) = \operatorname{rank} A$

aff — affine hull

dim aff — affine dimension r

card — cardinality, *number of nonzero entries* $\operatorname{card} x \triangleq \|x\|_0$
or N *is cardinality of list* $X\in\mathbb{R}^{n\times N}$ (p.261)

conv — convex hull (§2.3.2)

cone — conic hull (§2.3.3)

cenv — convex envelope (§7.2.2.1)

content — of high-dimensional bounded polyhedron, volume in $\mathbb{R}^{\mathbf{3}}$, area in $\mathbb{R}^{\mathbf{2}}$, and so on

cof — matrix of cofactors corresponding to matrix argument

dist — absolute distance between point or set arguments; *e.g*, $\operatorname{dist}(x\,,\mathcal{B})$

vec — columnar vectorization of $m\times n$ matrix,
Euclidean dimension mn (39)

svec — columnar vectorization of symmetric $n\times n$ matrix,
Euclidean dimension $n(n+1)/2$ (59)

dvec — columnar vectorization of symmetric hollow $n\times n$ matrix,
Euclidean dimension $n(n-1)/2$ (76)

$\measuredangle(x\,,y)$ — *complex sinusoid phase* or *angle* between vectors x and y, or *dihedral angle* between affine subsets

$\succeq$ — generalized inequality, membership to pointed closed convex cone; *e.g*, $A\succeq 0$ means:

- *vector or matrix* A *must be expressible in a biorthogonal expansion having nonnegative coordinates with respect to extreme directions of some implicit pointed closed convex cone* $\mathcal{K}$ (§2.13.2.0.1, §2.13.8.1.1),
- **or** *comparison to the origin with respect to some implicit pointed closed convex cone* (2.7.2.2),
- **or** (when $\mathcal{K}=\mathbb{S}^n_+$) *matrix* A *belongs to the positive semidefinite cone of symmetric matrices* (nonnegative eigenvalues, §2.9.0.1),
- **or** (when $\mathcal{K}=\mathbb{R}^n_+$) *vector* A *belongs to the nonnegative orthant* (each vector entry is nonnegative, §2.3.1.1)

$\underset{\mathcal{K}}{\succeq}$ — as in $x \underset{\mathcal{K}}{\succeq} z$ means $x-z\in\mathcal{K}$ (189)

$\succ$ — strict generalized inequality, membership to cone interior; $A\succ 0$ means:

- *vector or matrix* A *must be expressible in a biorthogonal expansion having positive coordinates with respect to extreme directions of some implicit pointed closed convex cone* $\mathcal{K}$ (§2.13.2.0.1, §2.13.8.1.1),
- **or** *comparison to the origin with respect to the interior of some implicit pointed closed convex cone* (2.7.2.2),
- **or** (when $\mathcal{K}=\mathbb{S}^n_+$) *matrix* A *belongs to the interior of the positive semidefinite cone of symmetric matrices* (positive eigenvalues, §2.9.0.1),
- **or** (when $\mathcal{K}=\mathbb{R}^n_+$) *vector* A *belongs to the interior of the nonnegative orthant* (each vector entry is positive, §2.3.1.1)

$\nsucc$ not positive definite

$\geq$ scalar inequality, total order, *greater than or equal to*; comparison of scalars, or entrywise comparison of vectors or matrices with respect to $\mathbb{R}_+$

nonnegative for $a\in\mathbb{R}^n$, $a\succeq 0$; *id est*, nonnegative entries when w.r.t nonnegative orthant; coefficients of vector on boundary of or interior to pointed closed convex cone $\mathcal{K}$

$>$ *greater than*

positive for $a\in\mathbb{R}^n$, $a\succ 0$; *id est*, positive (nonzero) entries when w.r.t nonnegative orthant; coefficients to no vector on boundary of pointed closed convex cone $\mathcal{K}$

$\lfloor\ \rfloor$ floor function, $\lfloor x\rfloor$ is greatest integer not exceeding x

$\lfloor\downdownarrows\rfloor$ download button

$|\ |$ entrywise absolute value of scalars, vectors, and matrices

log natural (or Napierian) logarithm

det matrix determinant

$\|x\| = \sqrt{\sum_{j=1}^n |x_j|^2}$ *Euclidean norm* or vector 2-norm $\|x\|_2$ (§3.2)

$\|x\|_2^2 = x^{\mathrm T}x = \langle x\,,\,x\rangle$ *Euclidean norm square* (§3.1.1.1)

$\|x\|_\ell = \sqrt[\ell]{\sum_{j=1}^n |x_j|^\ell}$ vector ℓ-*norm*, convex for $\ell\geq 1$. nonconvex (not a norm) for $0<\ell<1$ (§3.2)

$\|x\|_0 = \operatorname{card} x$ "0-norm" or *cardinality of vector* x (§4.6.1)

$\|x\|_1 = \mathbf{1}^{\mathrm T}|x|$ 1-norm, *dual infinity-norm* (§3.2)

$\|x\|_\infty = \max\{|x_j|\ \forall j\}$ *infinity-norm* (§3.2)

$\|x\|_{\substack{n\\k}} = \sum_{i=1}^k \pi(|x|)_i$ k-*largest norm* (§3.2.2.1)

$\|X\|_2 = \sup_{\|a\|=1}\|Xa\|_2 = \sigma_1 = \sqrt{\lambda(X^{\mathrm T}X)_1}$ matrix 2-norm or *spectral norm*, (605) *largest singular value* [185, p.56]. For x a vector: $\|\delta(x)\|_2 = \|x\|_\infty$.

$$\|Xa\|_2 \leq \|X\|_2\|a\|_2 \tag{2316}$$

$\lVert X\rVert_2^*$	$= \mathbf{1}^{\mathrm T}\sigma(X)$	*nuclear norm, dual spectral norm*	(§C.2)
$\lVert X\rVert$	$= \sqrt{\sum_{i,j} X_{ij}^2}$	*Frobenius' matrix norm* $\lVert X\rVert_{\mathrm F}$	(§2.2.1)