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He joined the Asahi Chemical Industry Corporation, Tokyo, Japan in 1972, where he has been engaged in the research and design of A/D LSIs using CMOS technology. His current research interest is in digital filters and high-resolution A-to-D and D-to-A converters.

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# LETTERS TO THE EDITOR

## COMMENTS ON "THE IMPLEMENTATION OF RECURSIVE DIGITAL FILTERS OF HIGH-FIDELITY AUDIO"\*

I feel comment is required on the above paper by Jon Dattorro. In particular I would like to clarify the relationship between Mr. Dattorro's work as presented and the digital filter technology actually implemented in Lexicon's OPUS digital audio production system. The biographic summary states that "Mr. Dattorro was with Lexicon Inc. until 1987, where he was the principal engineer for the Model 2400, their premier time-compression device. His responsibilities also included the digital filter design for OPUS, their recently announced digital audio workstation." This is somewhat misleading. Although the bulk of the work presented in this paper was compiled by the author as part of the early stages of OPUS system development at Lexicon (and published without Lexicon's review or approval), this work is in no way related to the actual digital equalization implementation as introduced for OPUS at the conventions of the Society for Motion Picture and Television Engineers and the Audio Engineering Society this past fall.

Insofar as this is a potential source of confusion to present and prospective OPUS users we request that this clarification be published at your earliest possible convenience.

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### Author's Reply<sup>2</sup>

The above paper does not purport to be a design review of the Lexicon OPUS digital parametric equalization filters. Although the equalization subsystem for OPUS was my responsibility until 1987, I never indicated that the paper represented that which was offered to the customers. The system, OPUS, is only

mentioned in the biography because my design experience is relevant to the paper's topic. I also designed the original OPUS mainframe backplane and I shared the console filter-strip facade and electronics design. I imagine that much has changed during the interim, between my completion of the task in 1987 and the introduction of the equalization subsystem this past November at the 85th Convention of the Audio Engineering Society. Indeed, Mr. Eagle was not a Lexicon employee or consultant prior to my departure. Mr. Eagle has declined to share any technical information since I joined ENSONIQ, so I have no knowledge of the extent to which my work was used.

I would like to note several errors which appeared in the above paper.

- 1) On page 858, in the last paragraph in Sec. 1.2.2, line 11 should have read: "gain or loss. The . . . "
  - 2) Table 1, page 863, should have read as follows:

Table 1. Error feedback zeros.

| $K_1$ | <u>K<sub>2</sub></u> | Region θ      |
|-------|----------------------|---------------|
| +2    | -1                   | 0 Twice       |
| -2    | -1                   | π Twice       |
| 0     | +1                   | 0 and $\pi$   |
| +1    | -1                   | $\pi/3$ Once  |
| -1    | -1                   | 2π/3 Once     |
| +1    | 0                    | 0 Once        |
| -1    | 0                    | π Once        |
| 0     | -1                   | π/2 Once      |
|       |                      | 101 100000000 |

- 3) In Sec. 2.5.1, page 864, line 26 should have started with  $(N^2\sigma^2)$ .
- 4) The first equation at the top left of page 870 should have read
  - $c_i$  + binary code  $(ec_i)/(2^{qc}2^{qe}) \simeq c_{Fi}$ .
- 5) An asterisk should have appeared in the caption for Fig. 14 preceding 'See TMS . . . '
- 6) In Sec. 4.1.2, on page 872, lines 21 through 24 in the left-hand column, "worsens when . . . limitcycle problem," should have been deleted.
- 7) On page 874 the approximation to the h(n) equation should take the absolute value on the left-hand side.

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<sup>\*</sup> Manuscript received 1989 March 6.

J. Dattorro, J. Audio Eng. Soc., vol. 36, pp. 851-878 (1988 Nov.).

Manuscript received 1989 April 26.

surements and could lead to erroneous results. Experience with practical horns has shown that these reflections are not only common in most designs, but can be very strong. A second point, which may be of importance, is the effect of the cutoff phenomenon associated with many horn shapes. At frequencies near cutoff, gross phase dispersion effects occur, leading to group delays which, in many designs, can be more than a few milliseconds. This dispersion may significantly affect measurements taken using time windows of very short duration, as is necessary for the elimination of mouth reflections.

Some examples of our measurements along with theoretical predictions using a one-parameter, finite exponential element model based on Webster's horn equation are shown in Figs. 1 and 2. These results form part of a technical report which is near completion, an abridged form of which will be submitted shortly for possible publication.

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# COMMENTS ON "THE IMPLEMENTATION OF RECURSIVE DIGITAL FILTERS FOR HIGH-FIDELITY AUDIO" 11

In the above paper<sup>12</sup> Dattorro discussed in Secs. 2.5 and 2.6 a digital filtering scheme he calls "truncation error cancellation." I think the scheme has been explained and interpreted incorrectly, and I will offer my

explanation and interpretation and show where the paper is not complete.

The author puts forth two conditions on the structure in Fig. 11 for implementing truncation error cancellation. The first is that the error feedback coefficients  $K_1$  and  $K_2$  are equal to  $b_1$  and  $b_2$ , respectively. The second condition is given in Sec. 2.6, where the author states that the error accumulation must be shifted right (in the case of Fig. 11, shifted right 16 bits) before adding to the signal accumulation. Under these two conditions, the filter can be redrawn as shown in Fig. 11'.

This filter has a 32-bit feedback delay path and a 48-bit accumulator for the feedback multiplication results. This gives the filter a wide dynamic range internally. The filter has two truncation points in it and thus has two error signals to consider. The first,  $e_1$ , is produced by truncating the 48-bit accumulation to 32 bits before the accumulation enters the feedback path. The second,  $e_2$ , is produced by truncating the filter output to 16 bits. Using the author's convention, the filter transfer function is

$$\hat{Y}(z) = X(z) \frac{\sum a_i z^{-i}}{1 - \sum b_i z^{-i}} - E_1(z) \frac{1}{1 - \sum b_i z^{-i}} - E_2(z) . \quad (1)$$

This equation is different from Eq. (17) in the paper because the truncation from 48 bits to 32 bits was ignored. It should have been taken into consideration in Sec. 2.6. Paragraph 2, sentence 2, states that the error feedback math of Fig. 11 "must be shifted to the right before combining it with the signal accumulation." Explicitly, the 32-bit accumulator results from the error feedback accumulations must be shifted right 16 bits. But this truncates the 16 least significant bits. This must be considered as another error source.

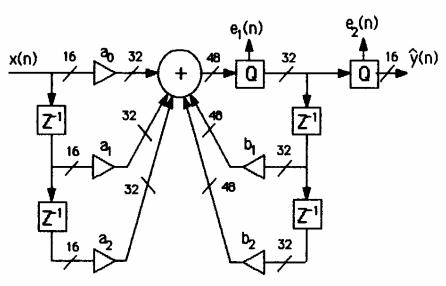


Fig. 11'.

Manuscript received 1989 February 6.

<sup>&</sup>lt;sup>12</sup> J. Dattorro, J. Audio Eng. Soc., vol. 36, pp. 851-878 (1988 Nov.).

This implementation being considered is most simply described as a filter that uses double-precision arithmetic in the feedback portion. The term "truncation error cancellation" is not proper. Truncation error is always present when using finite word length arithmetic in a recursive filter. The truncation error has simply been reduced by using a longer word length in the recursive section of the filter.

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### Author's Reply<sup>13</sup>

Mr. Neyrinck has pointed out an additional truncation error source in the truncation error cancellation circuit, which was first brought to my attention by Tom Hegg of Lexicon, and later by Richard Cabot of Audio Precision. I consider that error source to have impact only to a second degree on that circuit's performance, and so I chose to ignore any discussion of it, hoping to simplify the mathematics for the reader. Mr. Neyrinck's astute observation, however, has made me regret that omission. It is important to note that this second-degree source of truncation error is nonexistent in the truncation error feedback topologies discussed in Secs. 2.3 and 2.4. This is because the error feedback coefficients were trivial multipliers there.

As for my interpretation of the truncation error cancellation circuit, which I believe is correct, it is important to understand that truncation error cancellation is a special case of error spectrum shaping (ESS), which is truly a truncation error feedback technique and not a means of increasing purely numeric precision. The benefits of ESS are established in the frequency domain as a means of reducing truncation noise buildup. Truncation error cancellation may well be viewed as a doubleprecision implementation. In Sec. 2.5.0, a subtle distinction between truncation error cancellation and standard double precision was expressed in terms of the double-precision realization: 1) the signal feedforward paths remain in single precision, and 2) the multiplier inputs are never unsigned. I acknowledged the fact that other engineers had found the noise performance of the truncation error cancellation circuit to be equivalent to that of a standard double-precision implementation.

The two conditions under which the truncation error cancellation will be perfect were set forth in the last paragraph of Sec. 2.6: 1) the signal feedback coefficients  $b_1$  and  $b_2$  must be precisely equal to the error feedback coefficients  $K_1$  and  $K_2$ , respectively, and 2) no rounding (truncation) may be performed in the formulation of the error accumulation.

If the error accumulation must be shifted right before combining it with the signal accumulation (Sec. 2.6, paragraph 2), and if significant digits are lost as a result, then the second criterion is clearly violated.

The material in Sec. 2.6 stands by itself as it is. The analysis that follows would, hypothetically, be added as Sec. 2.6.1, where we would go to this second level of depth into the truncation error assessment. Fig. 11"

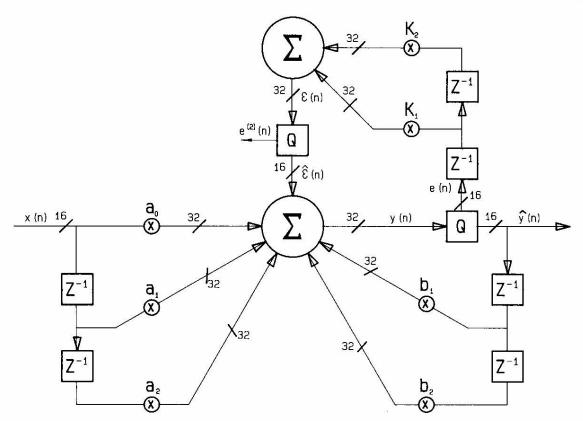


Fig. 11". Second-order truncation error cancellation showing all truncation errors.

<sup>&</sup>lt;sup>13</sup> Manuscript received 1989 September 21.

here shows the truncation error situation in every detail for a 16/32-bit processor having a 16-by-16-bit multiplier with 32-bit products and accumulations. Referring to the figure,

$$y(n) = \hat{y}(n) + e(n) \tag{2}$$

as before. We define a new variable,  $\epsilon$ , which is simply the accumulation of the truncation errors,

$$\epsilon(n) = \sum K_i e(n-i) . \tag{3}$$

In the text, Eq. (14) uses the accumulation  $\epsilon$  directly. A new truncation error source  $e^{(2)}(n)$  results when the truncation error accumulation  $\epsilon$  is shifted right (truncated) before it is combined with the signal accumu-

$$\epsilon(n) = \hat{\epsilon}(n) + e^{(2)}(n) \tag{4}$$

where  $|e^{(2)}(n)| << |e(n)|$ , and where the 2 raised in parentheses connotes the second-degree error. This second-degree truncation actually occurs in the TMS32010 program in Appendix 1 of the paper on the lines that read

The equation which in reality becomes implemented in the program is then

$$y(n) = \sum a_i x(n-i) + \sum b_i \hat{y}(n-i) + \hat{\epsilon}(n) .$$
 (5)

Contrast this with Eq. (14) in the text. Only the last term is different because we are adding a truncated version of the truncation error accumulation. It is important to realize that both the first and second terms in Eq. (5) here and in Eq. (14) in the text retain full numerical precision because none of the operands need to be truncated prior to multiplication. (Indeed,  $\hat{y}(n-i)$ is already truncated and accounted for.) What is the effect of the truncation of  $\epsilon$ , the truncated truncation error accumulation, the double truncation?

In the frequency domain,

$$\hat{Y}(z) = X(z) \frac{\sum a_i z^{-i}}{1 - \sum b_i z^{-i}}$$

$$- E(z) - \frac{E^{(2)}(z)}{1 - \sum b_i z^{-i}}$$
(6)

which agrees with Mr. Neyrinck's Eq. (1), except for the nomenclature. Contrast Eq. (6) here with Eq. (17)

in the text; Eq. (6) has an additional (third) term. Eq. (6) says that the truncated output spectrum is comprised of the ideal filter transfer minus the nonamplified output truncation minus the truncated truncation error accumulation  $E^{(2)}(z)$ .  $E^{(2)}(z)$  is much smaller in magnitude than E(z) but is amplified by the poles of the system.

Assuming a 16-bit signal and a -90-dB signal noise floor, then in a 16-bit system  $E^{(2)}(z)$  is ideally 16 bits lower in magnitude than E(z). (This would be possible using a processor such as the TMS320C25, where the products can be shifted left out of the multiplier prior to accumulation.) This means that, depending on the implementation, the noise power associated with  $E^{(2)}(z)$ can be 186 dB below unity. In a 24-bit system the noise power associated with  $E^{(2)}(z)$  is at least 24 bits lower in magnitude than that of E(z), and so 234 dB below unity. The bottom line is that this second-degree source of truncation error will only be problematic if the amplification of it by the system poles raises it above the signal noise floor. For this to happen, the amplification would need to be excessive; certainly, the use of a 24bit processor makes this much less of a likelihood.

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### COMMENTS ON "THE IMPLEMENTATION OF RECURSIVE DIGITAL FILTERS FOR **HIGH-FIDELITY AUDIO"14**

In the above paper<sup>12</sup> Appendix 1 needs to be amended as follows:

#### Addendum to Appendix 1

The first nine instructions in the TMS320 code are for I/O and must be changed or deleted to suit your purposes.

Note that the operands associated with the LTD instructions must be allocated storage in ascending order so that the program functions properly. For example,

XNL: EOU 7 XNM1L: EQU 8 XNM2L: EQU 9 etc.

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<sup>&</sup>lt;sup>12</sup> J. Dattorro, J. Audio Eng. Soc., vol. 36, pp. 851-878 (1988 Nov.).

14 Manuscript received 1989 October 16.