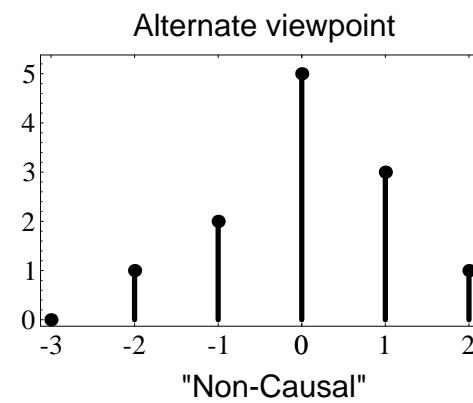
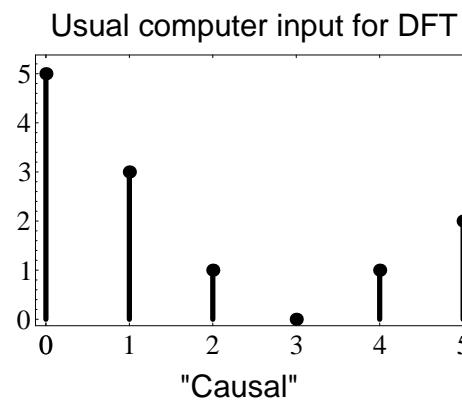


- Modulo Sequences
- Flipping
- Repeating
- Shifting
- Stretching
- Decimating
- Aliasing
- Convolving
- Correlation
- Zero Padding

• Modulo Sequences

Definition: The DFT converts a periodic signal into a periodic spectrum.

A DFT takes one period of that signal, interpreting the first element as the time=0 element. Earlier elements are found to left, but wrap around to the far right of the sequence.



- Flipping (Reversing)

Let $y(n) = \{0,1,2,3,4\}$, then the $\text{FLIP}(y) = \{0,4,3,2,1\}$.

$$\text{flip}_n(y) = y(N - n)$$

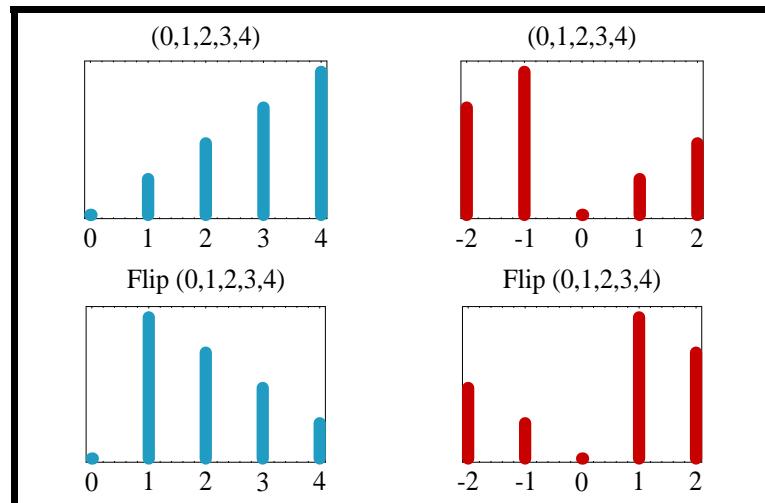
If you consider $y(n)$ as an N -modulo infinite sequence, then

$$\text{flip}_n(y) = y(-n)$$

The plots to the right show the flip of $y(n)$ from two perspectives:

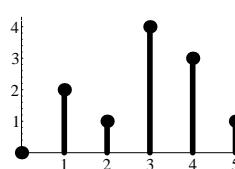
Left column is from a causal viewpoint.

Right column is from a non-causal viewpoint.

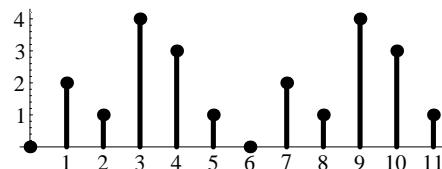


- Repeating

Let $y(n) = \{0,2,1,4,3,1\}$, then the $\text{Repeat}_2(y) = \{0,2,1,4,3,1,0,2,1,4,3,1\}$.



Repeat 2:

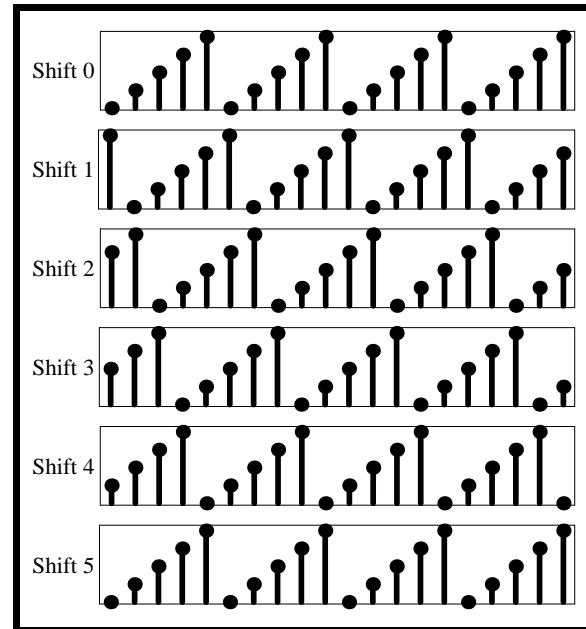


- **Shifting**

$\text{SHIFT}_l(y) \triangleq l\text{-sample circular right-shift of } y$

example:

$$\begin{aligned} y &= [0, 1, 2, 3, 4] \\ \text{SHIFT}_1(y) &= [4, 0, 1, 2, 3] \\ \text{SHIFT}_{-1}(y) &= [1, 2, 3, 4, 0] \end{aligned}$$

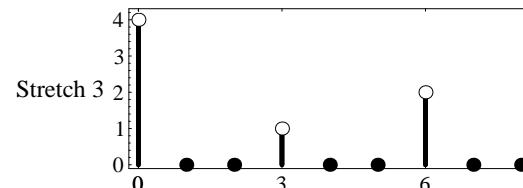
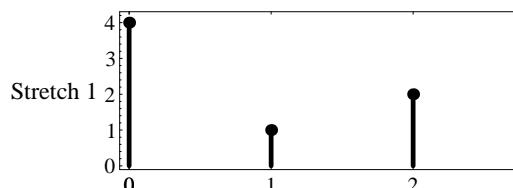


- **Stretching**

– Changes the length (dimension) of the signal by inserting $L-1$ zeros between samples.

$$\text{STRETCH}_{L,n}(x) \triangleq \begin{cases} x\left(\frac{n}{L}\right), & L|n \\ 0, & \text{elsewhere} \end{cases} \quad L \geq 1, \quad L \in \mathbf{Z}^+$$

$$\begin{aligned} x &= [4, 1, 2] \\ \text{STRETCH}_3(x) &= [4, 0, 0, 1, 0, 0, 2, 0, 0] \end{aligned}$$

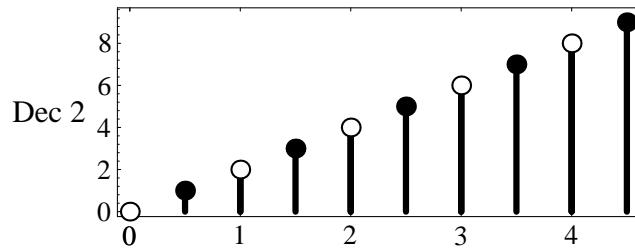


- **Decimating**

$$\text{DEC}_{L,m}(x) \triangleq x(mL), \quad m = 0, 1, \dots, \frac{N}{L}-1, \quad L|N$$

removes L-1 samples between samples.

$$x = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9] \\ \text{DEC}_2(x) = [0, 2, 4, 6, 8]$$



- **Aliasing**

$$\text{ALIAS}_{L,m}(x) \triangleq \sum_{l=0}^{L-1} x\left(m+l\frac{N}{L}\right)$$

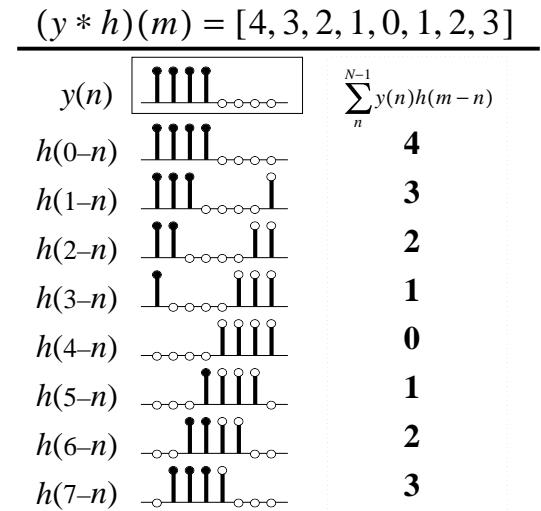
- "Add L equal partitions"
- Aliasing is not invertible: once you have it you can't go back.

- Convolving

$$(x * y)(m) \triangleq \sum_{n=0}^{N-1} x(n)y(m-n)$$

example:

$$\begin{aligned} y &= [1, 1, 1, 1, 0, 0, 0, 0] \\ h &= [1, 0, 0, 0, 0, 1, 1, 1] \\ \text{FLIP}(h) &= [1, 1, 1, 1, 0, 0, 0, 0] \\ y * h &= \sum_{n=0}^{N-1} y(n)\text{FLIP}(h) \\ &= [4, 3, 2, 1, 0, 1, 2, 3] \end{aligned}$$



– this is circular convolution.

To simulate linear convolution, add dimensions of vectors and subtract one

- Correlation

$$(x \star y)(m) \triangleq \sum_{n=0}^{N-1} \overline{x(n)}y(n+m)$$

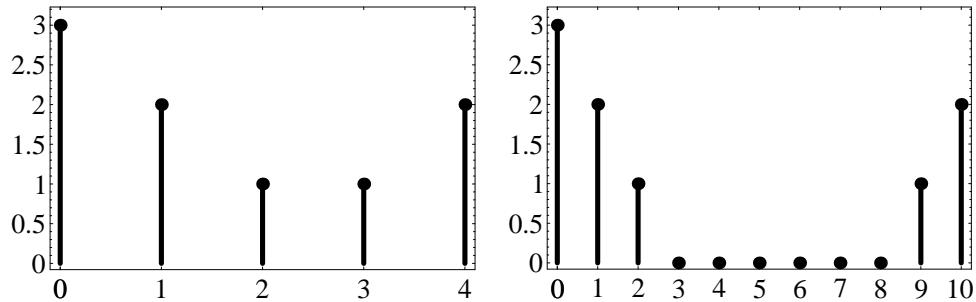
– Similar to Convolution, but second signal is not flipped.

– Used for Pattern matching.

- Zero Padding

– used to approximate the DTFT using the DFT

standard computer viewpoint:



More obvious from the non-causal viewpoint:

