- **Definition of the DFT/IDFT**
- **DFT in terms of Geometric Signal Theory**
- **Frequency Bins**
- **Non-Harmonic Frequencies**

- **Definition of the Discrete Fourier Transform (DFT)**

DFT: converts a signal to its spectrum

$$\mathrm{DFT}_k[x(n)] = X(k) \triangleq \sum_{n=0}^{N-1} e^{-j\omega_k nT} = \sum_{n=0}^{N-1} e^{-j2\pi nk/N}$$

Inverse DFT: converts a spectrum to its signal

$$\mathrm{IDFT}_n[X(k)] = x(n) \triangleq \frac{1}{N} \sum_{k=0}^{N-1} e^{+j\omega_k nT} = \sum_{k=0}^{N-1} e^{+j2\pi nk/N}$$

– Note: several ways to define DFT/IDFT: (1) sign change, and (2) reallocation of normalization factor 1/N.

Transform notation to show the relation between signal and spectrum:

$$x \leftrightarrow X$$

• DFT in terms of Geometric Signal Theory

$$\mathrm{DFT}(x) = \sum_{n=0}^{N-1} x \, e^{-j\omega_k nT} = \sum_{n=0}^{N-1} \langle x, \hat{s}_k \rangle$$

••The DFT is a change of coordinate systems from the "samples" which are coefficients relative to the natural basis in C^N to "spectral samples" relative to the sinusoidal basis.

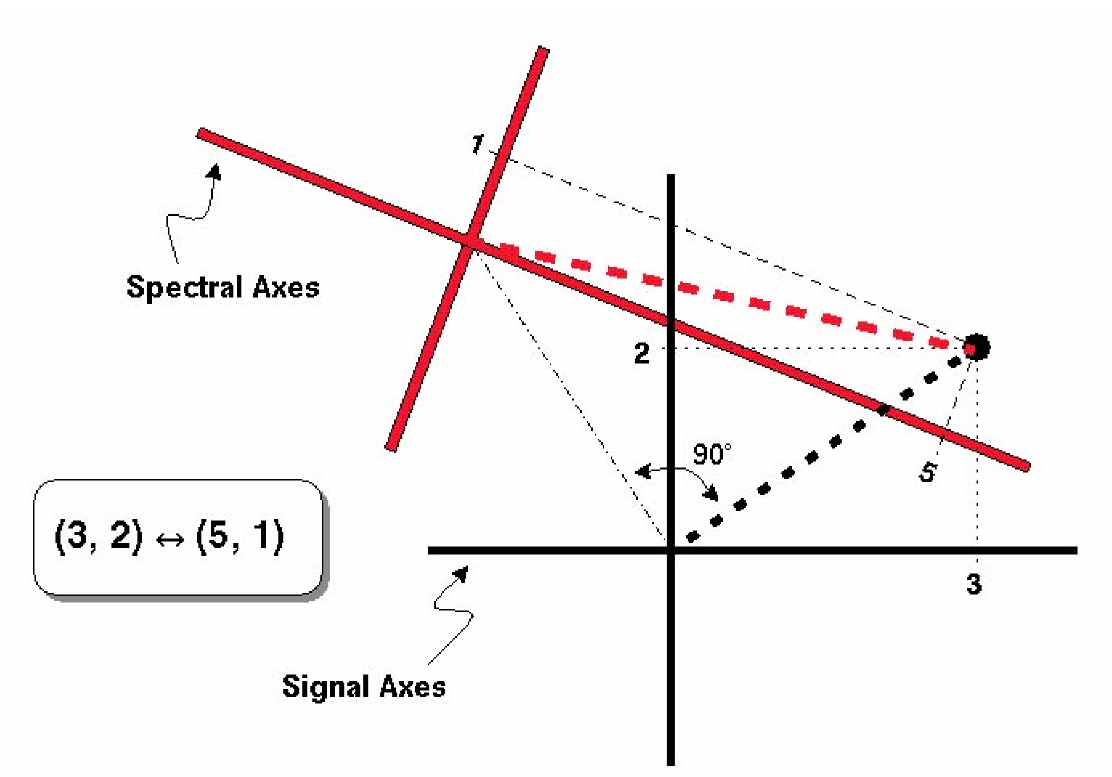
Example for N=2:

$$DFT([a,b]) = [a+b, a-b]$$

$$\text{Let } x = [3,2]$$

$$x = [3,2] \leftrightarrow X = [5,1]$$

Note that the signal and the spectrum represent the *same thing* **– just within different coordinate systems.**



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