CHARACTERIZATION OF APERIODICITY IN NEARLY PERIODIC SIGNALS

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Abstract

Nearly periodic waveforms produced by musical instruments fluctuate from period to period both in the length of the period and in the shape of the waveform of each period.We describe a measure of those fluctuations that is sensitive to correlations in waveshapes for medium time scales - up to 16 in the present implementation. The method also produces two dimensional color or gray-scale plots that allow visualization of the complexity of waveforms produced by acoustic or electronic instruments over hundreds of periods of the waveform.

I. Introduction

Tones from musical instruments are not exactly periodic. Both the period (the time between successive crossings nearly a period apart of a waveform through a given signal level), and the shape of the waveform vary from period to period. The variations are not necessarily random; they often reflect fundamental physical processes in the instrument. For example, sub-harmonics in bowed string notes have been documented and explained[1], and similar phenomena with totally different origins can be expected at least for some notes in woodwinds that change registers to reach the higher pitches of their ranges. Because of the relationship to this medium time scale organization of the fluctuations of waveform shape and period to the underlying physics of the oscillations, the method described in this paper was developed to allow the detection of subharmonic behavior for whole periods and portions of the period. The method allows a visualization of the complexity of a waveform in a two dimensional color format from which one can make qualitative judgments about the differences in complexity of tones produced by different instruments. It also establishes both quantitative and qualitative criteria for the success of computer simulations of acoustic instruments.

II. Technical Description

The fundamental idea is to compare the shape of a period with the shape of another period, or to make the comparison of a fraction of a period with the same fraction of all other periods from a continuous sound file ("note"). The waveforms have been digitized at the standard sampling rate of 44.1 kHz . Each period, measured in units of the sampling period, has a different (non-integral) number of samples. Typical fluctuations of the periods are no larger than 3-4% for a note with vibrato, and often are only a few cents. To make a quantitative comparison of different periods it is necessary to resample each period at a fluctuating rate so that there are equal numbers of samples in each period (period synchronous resampling). The new number of samples per period, an integer, is chosen so that there are slightly more samples than in the longest period of the note. Resampling is done in software.

Each period i, $1<i<N$, in an N-period note, is then a d-dimensional period vector

(1) $$X^i = (S_1^i,\ldots,S_d^i).$$

With these N vectors, a variety of calculations can be made. Each vector can be compared with an arbitrarily chosen period vector j component by component, forming the N d-dimensional vectors

(2) $$C^{ij} = (|(S_1^i-S_1^j)|,\ldots,|S_d^i-S_d^j)$$

If for some fixed j these N vectors are rescaled to the range of a color pallette, the resultant d by N color display shows which portions of the waveform fluctuate most strongly as the note progresses. This plot provides useful clues for picking out a contiguous subset of the period ("feature vector") for subsequent display and analysis.

One can also calculate a difference norm: the norm of the difference between the vectors of the ith and jth feature or period vector:

(3) $$\mathbf{NORM}^{ij} = \|\mathbf{X}^i-\mathbf{X}^j\|$$

For quantitative analysis the **NORM** is most useful if one choses **X** to be a feature vector. We have found that by selecting features guided by the color plot of **C**, an array of length N is produced for each comparison feature vector, $\mathbf{X}^j$, which can then be Fourier transformed (DFTed) to yield spectral information about the long time organization of the difference between a feature in the i[th] and j[th] waveform, $1<=i<=N$. Figure 1 shows such spectral information for the case of a note, concert F5, of a clarinet. The result of the DFT is plotted so that the axis represents the subharmonics of the original frequency, with the scale extending from subharmonic two (the Nyquist frequency for this transformation) to subharmonic 16. One sees a strong peak at subharmonic 3, an understandable result for a note played in the clarion register of a clarinet. Figure 2 shows the DFT spectrum of **NORM**^{ij} for arbitrary j for the note F5 of a flute. See figure caption for more details.

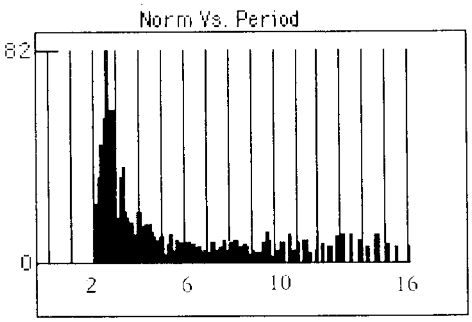

Figure 1. Discrete Fourier Transform of **NORM**^{ij}, j fixed, for a feature from the clarinet note concert F5. The abscissa is plotted in an inverse scale so that it is linear in harmonic number, from 2 to 16. There is a weak, inaudible third subharmonic in the waveform, clearly visible in the display.

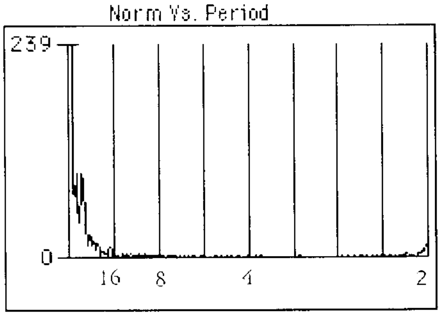

Figure 2. Similar to figure 1, for the note F5 of a flute, played with vibrato. Here the abscissa and ordinate are linear; the numbers are subharmonics, as in Figure 1. The weak peak at the right extremity shows there is a second subharmonic in the spectrum. This subharmonic is audible during part of the vibrato period of the note. The vibrato period shows up as an approximately 50th subharmonic. The abscissa is labeled by subharmonic number.

There are some pitfalls to be avoided. The use of period synchronous sampling largely removes effects produced by beating between the sampling frequency and the frequency of the note. However, because of the finite precision of a 16 bit ADC some remnant of this beating can be observed with computer generated periodic waveforms. The time of crossing of the signal over the level chosen to signal the start of a period has been determined by linear interpolation in the present work. Quantization errors have not been important for the "noisy" signals used here, but if they were a problem, they could be reduced by quadratic or quartic fits to more samples near the level crossing.

The method is extremely sensitive to aliasing. If the bandwidth of the original signal is not severely limited to less than half the sampling frequency, the high frequency components on the signal "fold back" to frequencies lower than the Nyquist frequency. The signature is seen readily on the N by N color plot of **NORM** as a beating effect between the frequency of the source and the sampling rate which goes away when the aliased signals are digitally filtered out.

The ordinary DFT of the signal waveform might be expected to reveal subharmonic behavior as well. We have conducted some experiments with synthetic notes as well as with examples from acoustic instruments. A noisy waveform was generated with 5 components to give a roughly saw-tooth shape, and an eighth subharmonic at -46 db with respect to the first harmonic was added with about 18° random phase noise. The DFT and the present method were equally effective in detecting the subharmonic. However, the DFT of the clarinet waveform (figure 1) showed no subharmonic at a frequency one third of the fundamental, but the power spectum showed well-defined peaks at harmonics 5, 8, 10, and 11 of the third subharmonic. This and other examples illustrate the difficulties that often arise in interpreting DFT spectra when the positions of peaks are not known in advance, and their signal to noise is poor.

The real advantage of the present method is the presentation of a time domain picture of the evolution of a waveform. The use of the "averaging" technique of the DFT on the difference norm plot should not obscure the fact that this method is rooted in the examination of time domain behavior.

III. Validation

The claim we make that this method of signal analysis detects subharmonic behavior, and by extension, the correlation of patterns of periodicity over time scales of up to at least sixteen periods, is validated by its application to examples produced from acoustic instruments under circumstances designed to produce audible (except for the clarinet example of figure 1) subharmonics. In addition we present in this section some examples that suggest the usefulness of the method for fuller understanding of sounds produced by brass instruments, and by singers.

The exception of the case of the clarinet tone is that, while expected, the subharmonic is not audible. However, the explanation of the existence of the third subharmonic is convincing. Any note in the clarion register is produced with the aid of a register key that kills the Q and shifts the frequency of the lowest frequency mode of the cylindrical bore. The oscillation then becomes based on the impedance peak at frequency three times that lowest mode[2]. The existence of a third subharmonic simply shows that enough impedance is left after the register hole is opened to allow a measurable standing wave of about one third the F5 frequency to exist, albeit at a very low level. The fact that the third peak in the subharmonic spectrum in figure 1 is not quite at the third subharmonic may be an indication that the opening of the register hole has shifted down in frequency the lowest frequency impedance peak while simultaneously lowering its Q. The existence of two incommensurate oscillations is consistent with current understanding of multiphonics in woodwinds, although the third subharmonic may be more reasonably explained as the response of the tube to noisy random excitations.

The flute example (taken, along with the clarinet example, from the McGill collection of acoustic instrument sounds [3]) shows on the norm difference plot (not shown) that the second subharmonic appears during only certain parts of the vibrato cycle. This effect is easily audible to the listener. Again, one is referred to [2] for the explanation of octave production in flutes.

It is not difficult to generate faintly audible subharmonics on the violin[1]. Their sound is reminiscent of noise passed through a band pass filter. We have analyzed an example with the signal generated by an accelerometer placed on the bridge of a violin, bowed while varying bow to bridge distance from approximately 1/6 to 1/3 of the finger to bridge distance during a single bow stroke. The fingered note was A6 on the E-string. The subharmonic sequence forms a D-minor arpeggiated chord based on D4, as subharmonics 6, 5, 4, and 3 are generated. Although the perception of the main pitch A6, is of a somewhat wobbly and uncertain note, it was periodic enough at about 1760 Hz to allow us to detect successfully all the expected subharmonics from a sequence of 0.25 sec extracts from the parts of the waveform that have clearly audible subharmonic sounds. Figure 3 shows the subharmonic plot of the DFT of the portion of the note producing the audible sixth subharmonic.

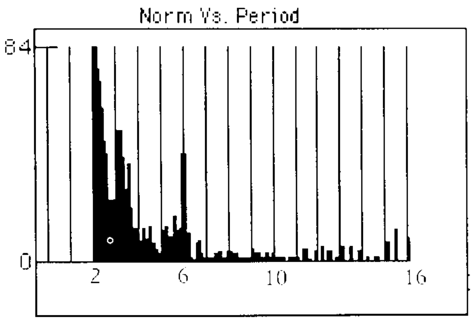

Figure 3. The DFT plot, subharmonic scale on abscissa, of the note A6 on a violin played so that the sixth subharmonic is audible. The abscissa is labeled by subharmonic number.

The feature comparison technique is capable of revealing unexpected results as well. Figure 4 shows a color (grayscale here) representation of the subharmonic plot of the DFT of the norm difference plot for five comparison periods of the note F4 played on a French horn. For each comparison period j each feature vector consists of one eighth of the full period. The DFTs of these eight features are displayed sequentially from top to bottom for each of the five comparison periods.

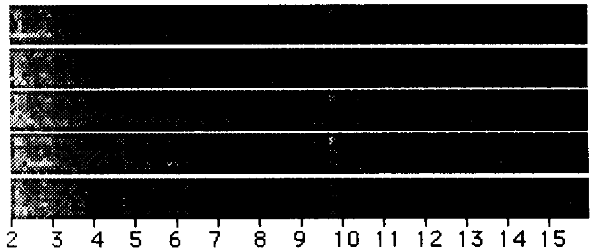

Figure 4. DFTs of norm difference plot for eight features, and five comparison periods, for a note of pitch F4 played on a French horn. Subharmonic numbers are labeled across the bottom. Note that subharmonic activity is strong for some portiions of the period, nearly absent for others.

There is a strong band just below the 10th subharmonic. Close inspection reveals that the subharmonic strength is stronger for some comparison features than others, and the strength varies particularly strongly with feature position within the period. There is in addition scattered activity at or near other subharmonic numbers, particularly five and six. The diffuse band near or just below subharmonic five may be a harmonic of subharmonic ten created, perhaps, by the nonlinear calculation that maps shape changes into the single number, the norm difference. The large amount of diffuse subharmonic activity near subharmonics two and three is characteristically present in most such pictures, and is in part caused by the "1/x" type mapping from the linear DFT plot along the abscissa, such as in figure 2, to the subharmonic scale, as in figures 1 and 3. Since white noise on the linear scale maps to a hyperbola on the subharmonic scale, the strong subharmonic near three in figure 1 is particularly impressive. Figure 3 is more characteristic, and makes detection of weak subharmonics of low number uncertain. For that reason, the linear abscissa scale was chosen for the flute example, figure 2. For comparison, a plot using a linear scale, and ten features, is shown in figure 5 for the French horn note F4 of Figure 4.

The note F4 is the eighth mode of the horn in F. A simple picture based on the notion that the player's lips emit "puffs of air" into the bore of the instrument suggests that since eight of them are emitted before the lips receive the message about when to open again from the returned and modified first one, one might expect an eighth subharmonic to be visible. The tenth subharmonic was seen, instead, not only on the note illustrated, but with less clear definition for F4 played in two separate CD recordings by professional players. The results suggest that lip motion may not be modelled with a simple single mass-spring, and that rather more complicated oscillations may be involved.

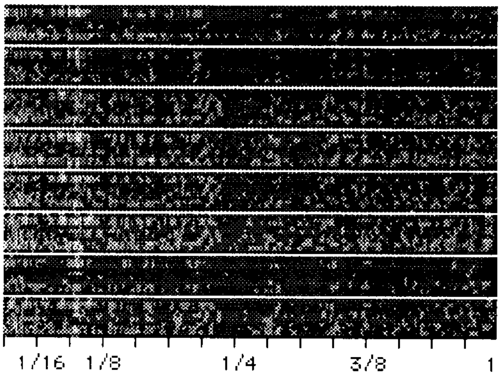

Figure 5. The same as Figure 4 except the horizontal scale is linear. The lowest frequencies (to the left) below the 32nd subharmonic have been suppressed. Light areas are largest DFT amplitudes. Comparison with Fig. 4 helps translation between the conventional linear display of the abscissa of the DFT and the subharmonic scale of Fig. 4. The linear scale is the same as in Figure 2, except for the truncation to the left of the first vertical line of Figure 2. Eight comparison periods are shown, and each period was divided into ten features.

That conclusion is similarly suggested by the examination of sung notes. Figure 6 shows the subharmonic display of the DFT of the norm difference plot for a note, F5, sung by Jussi Bjoerling[4] in the *recitative* before the aria *"Celeste Aida"*. Note the prominent subharmonic activity between numbers seven and eleven. Since there is not a high Q linear system following the glottis, as there is in the French horn following the lips, one is lead to speculate that the medium term correlations are produced by glottal activity.

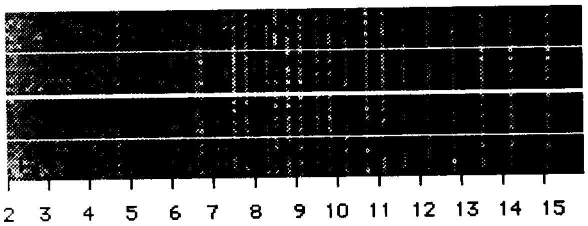

Figure 6. Subharmonic plot of F5 as sung by Bjoerling (see text). The light shades indicate large spectral components. The features were formed from the decimated period, and four comparison periods equally spaced over the 0.77 second sound sample are exhibited. Only about 250 periods are included in this note, so the DFT as represented on the subharmonic abscissa scale is more sparse that in figure 4.

IV. Discussion and Summary

The square colored pictures generated by **NORM**^{ij} provide information complementary to the DFT of **NORM**^{ij} with j fixed. The two dimension plots reveal that the results of such a DFT ("subharmonic hunting") depend on the period j from which the comparison feature is taken. To find subharmonic behavior it is often necessary to search for a comparison period that does not disguise medium term correlations and for a feature vector that exhibits them. The structure of the two dimensional picture often reveals what comparison periods will be most fruitful.

In addition the feature chosen may reveal or conceal interesting behavior. There is physical reason to believe, based on the discussion in reference [1], that for violin subharmonics parts of the waveform that originate near the slipping-sticking transitions will be more rich in subharmonic behavior. The allowance of choice of the same feature of each period for comparison was deliberately built in from the beginning of development in order to investigate such physical origins of transient subharmonic behavior.

The two dimensional colored plot on **NORM**^{ij} does not always easily reveal subharmonic behavior by visual inspection. One exception is the image associated with the flute example, because the existence of the lower octave is so strong. However, even casual inspection of the colored plots shows clear distinctions between notes with and those without subharmonic behavior. The regular organization of the former, even in the face of weak, short lasting subharmonics, is visually quite clear compared to the more random features of those notes without subharmonic behavior. The spectral analysis of the norm difference plot with selected comparison features and with selected comparison periods reveals that subharmonic behavior is detectable only for some comparison periods and for only some portions of a waveform. These remarks are illustrated in Figure 4, which also indicates that new physics, probably associated with the motion of the lips, is needed to understand fully the nature of the medium time scale organization of aperiodicities in the French horn. A similar survey of other woodwinds and the singing voice suggests that this technique can make new revelations about these sound sources as well.

By concentrating on shape changes of a waveform during the evolution of a note we have developed a tool, feature comparison, that will be useful for better understanding of the physics of sound production in acoustic instruments and the singing voice. It should also aid the developer of physical modeling algorithms for musical synthesis purposes in the production of more natural sounding electronic music.

References

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4. Jussi Bjoerling, "Operatic Arias", RCA CD 5934-2-RC (1987).