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On the Twelve Basic Intervals in South Indian Classical Music

Arvindh Krishnaswamy[1]

[1] *Center for Computer Research in Music and Acoustics, Dept of Electrical Engineering, Stanford University*

Correspondence should be addressed to Arvindh Krishnaswamy (`arvindh@ccrma.stanford.edu`)

ABSTRACT

We discuss various tuning possibilities for the twelve basic musical intervals used in South Indian classical (Carnatic) music. Theoretical values proposed in certain well-known tuning systems are examined. Issues related to the intonation or tuning of the notes in Carnatic music are raised and discussed as well.

1. INTRODUCTION

Though it amazing that such a fundamental issue like the number of musical intervals used is a topic still open to debate in Indian music today, there are a number of reasons why this situation is understandable. There are those who have, in our opinion, correctly maintained that Indian music employs only 12 intervals whose intonation is somewhat flexible. But while (i) unfortunate misinterpretations of highly revered historical texts, (ii) the abundance of authors who have written authoritatively and convincingly on the subject, (iii) the existence of a popular easy-to-grasp and highly marketable 22-sruthi theory, (iv) a dearth of experimental data to test theories and claims, (v) a sense of pride in believing that Indian music and musicians are special, and (vi) a lack of understanding of the nature and capabilities of the human auditory system all contribute to a certain extent, we feel that the main force sustaining the popular belief that more than 12 musical intervals are used by the two main Indian classical music systems (Carnatic and Hindustani) is the fact that many practitioners and fans of these arts (including the present author) *can actually hear* more than 12 musical "atoms" or "entities" in an octave.

Recently we argued that there are only 12 *distinct constant-pitch* intervals used in present-day Carnatic music [1, 2], and similar to Western music, the 12 main tones seem to be roughly semitonal in nature. However, we also noted that pitch inflexions, which are an integral part of this music system, may lead to the perception of additional "microtonal" intervals. But even in the case of inflected notes, the 12 basic intervals serve as anchor points, which makes them all the more important.

Natural questions to ask are: (i) what are the tunings of the 12 basic intervals? (ii) how does actuality compare to theoretical tunings which have been proposed in the literature? and (iii) how good or accurate does the intonation of a top musician need to be?

Measurements of performances of North Indian music [3, 4, 5] and South Indian music [6, 1, 2] have shown that intonation in practice can be highly variable, similar to results of experiments done in the Western world [7].

However, an important distinction has to be made between *performances* and *preferences*: while a musician may be inconsistent in intonation when performing due to several factors, he or she may still have an underlying tuning preference for the various musical intervals, which only perceptual experiments may reveal. For example, Vos's data [8] suggest that Western musicians may find the equal tempered fifth ($2^{7/12}$ or 700 cents) more "acceptable" than the just or natural fifth (3/2 or 702 cents). Since Western music has *chosen* to use equal tempered tuning and Western musicians are constantly exposed to it, this result makes sense, but extracting such a minute difference of roughly two cents from analyses of music performances, which in general have shown no strong preference for any tuning system [7], may be next to an impossible task.

But analyses of performances are not useless by any means either. Ultimately, performances are what people hear and enjoy, and they reveal a lot about what is really important in the corresponding music systems. When Levy [5] finds that intonation in North Indian music performances is highly variable and does not agree with any standard tuning system, it does give us clues about the number and nature of the musical intervals used as well as how accurately they should be intonated. For example, expecting musicians to produce and audiences to distinguish intervals that are only around 22 cents apart would seem unreasonable given the variability in intonation. Also, though we can measure or hear different "shades" of the constant-pitch intervals, the variability in intonation tells us that these are not distinct interval categories. On the other hand, Levy's work can't rule out the possibility that there may exist an underlying preference for particular tunings of certain intervals among North Indian musicians. And while it is very likely that these preferences may also vary with melodic context, a useful first step is to figure out the preferred intonation of long and sustained notes.

There is a significant issue to keep in mind while investigating intervals and intonation: the musical abilities of different people with respect to the performance or perception of music may vary significantly, even among accomplished musicians. For example, when Jairazbhoy [3] reported that he found a huge variation in the intonation of the major third, in the recordings he analyzed, all the way from 375 cents to 439 cents, it is very unlikely that every single listener in the Hindustani music world will find any value in this whole range equally acceptable. In fact, Jairazbhoy himself claims that he was initially motivated to pursue his investigation because he had heard an unusually high major third (439 cents). Of course the musicians he chose to analyze were quite accomplished and Jairazbhoy also comments on the human (his) ear's remarkable ability to adapt to this sharp tuning upon repeated listening. Nevertheless, an enhancement to simply measuring and reporting the range of intonation of notes in performances is to have various human subjects rate their acceptability at the same time. For example, we may find that 439 cents is a little too high for the majority of listeners as far as the major third is concerned. We may also find a variation between various groups of human subjects like trained musicians, average listeners and musical novices. Such studies have been done many times in the past, and the only reason we mention them here is to highlight the fact that the next steps have to be taken in the investigation of Indian music. Just to be sure, we are not pointing out any flaw in any of these past analyses of Indian music. In fact, they have saved us the trouble of looking for "22 sruthis" today and let us focus on the 12 basic intervals in South Indian classical music.

In this paper, we do not present any empirical data. Rather, we discuss several theoretical issues that may affect the intonation of notes in Carnatic music.

Results in psychoacoustics suggest that pitch perception is categorical in nature and also that intonation or tunings of intervals are *learned* rather than based on some psychophysical cues like beats [7]. The so-called "natural" intervals in the just intonation (JI) tuning system may not necessarily be preferred by someone who has been trained with some other system of tuning, and there seems to be no underlying reason, based on the human auditory system, as to why the natural intervals should be universally more pleasant or acceptable to all humans, though they may have had a role to play originally in the evolution of musical intervals and scales [7]. Furthermore, simple integer ratios and the phenomenon of beating may have nothing to do with the sensations of consonance or dissonance either, as many people seem to believe; there are other, better accepted theories today [9, 10].

Nevertheless, we still discuss historical tuning systems and integer ratios in this paper just for argument's sake and mainly because a lot of Indian authors have written about them. We will present some very complicated interval ratios, but that does not mean that we endorse them or suggest that they are all relevant to Carnatic music.

We start by introducing the names of the notes used in Carnatic music and then proceed to review popular tunings of 12-interval systems. We then discuss various factors that need to be considered in the tuning or intonation of Carnatic music notes. As before [1, 2], our work is focused on present-day Carnatic music, though some of our statements may generalize to North Indian music also. We do not however offer any insight into music systems of the past.

2. THE TWELVE SWARASTHANAMS

In most Carnatic music sessions today, there is a drone instrument, the tambura, constantly sounded in the background, which provides the pitch reference (the tonic) to the performing musicians. Carnatic music is also melodic in nature, and harmonies and chords, if they occur at all, are usually unintentional.

Thus, whenever we refer to "musical intervals" in Carnatic music, we are talking about the relative pitch of each note with respect to the tonic or note positions (swarasthanams) in an octave, rather than the distance between adjacent notes which may not be very important today.

The names and symbols for the notes in Carnatic music are given in Table 1. For convenience, we will only be using the note names and symbols given in the left columns of this table. Our goal in this paper is to discuss possible numerical interval values for these twelve notes.

3. TUNING SYSTEMS

In this section, we consider some popular tuning systems, list some interval values and comment on them.

3.1. Pythagorean Tuning

In Pythagorean tuning, any interval can be expressed as a rational number using only powers of 2 and 3: $r = 2^m \cdot 3^n$. Such ratios can be generated simply by engaging in cycles of fourths and fifths as shown in Table 2. Usually the maximum value of n, the exponent of the factor 3 in a Pythagorean ratio is 6, but some people have included higher powers of 3 as well in their list of ratios [7]. It is well known that such cycles of fourths and fifths never return to their starting point (since no power of 2 equals a power of 3 for non-zero exponents), and this "discrepancy" shows up usually as two possible values for the augmented fourth [11]. Table 2 contains some "intimidating" numbers indeed, and one has to wonder about their usefulness or relevance to Carnatic music.

3.2. Just Intonation and 5-Limit

In [1], we used a loose definition of "Just Intonation"(JI) where we took this term to mean any ra-

Name	**Symbol**	**Western**	**Symbol**	**Name**
Shadjam	Sa	P1		
Suddha Rishabham	Ri_1	m2		
Chathusruthi Rishabham	Ri_2	M2	Ga_1	Suddha Gaanthaaram
Saadharana Gaanthaaram	Ga_2	m3	Ri_3	Shadsruthi Rishabham
Anthara Gaanthaaram	Ga_3	M3		
Suddha Madhyamam	Ma_1	P4		
Prathi Madhyamam	Ma_2	+4		
Panchamam	Pa	P5		
Suddha Dhaivatham	Da_1	m6		
Chathusruthi Dhaivatham	Da_2	M6	Ni_1	Suddha Nishadham
Kaisiki Nishadham	Ni_2	m7	Da_3	Shadsruthi Dhaivatham
Kaakali Nishadham	Ni_3	M7		

Table 1: *The Twelve Swarasthanams of Carnatic Music. The four notes shown on the right are enharmonic to the corresponding notes on the left.*

1 / 1	3^0 / 2^0	Sa	2^{20} / 3^{12}	1048576 / 531441
3 / 2	3^1 / 2^1	Pa	2^{18} / 3^{11}	262144 / 177147
9 / 8	3^2 / 2^3	Ri_2	2^{16} / 3^{10}	65536 / 59049
27 / 16	3^3 / 2^4	Da_2	2^{15} / 3^9	32768 / 19683
81 / 64	3^4 / 2^6	Ga_3	2^{13} / 3^8	8192 / 6561
243 / 128	3^5 / 2^7	Ni_3	2^{12} / 3^7	4096 / 2187
729 / 512	3^6 / 2^9	Ma_2	2^{10} / 3^6	1024 / 729
2187 / 2048	3^7 / 2^{11}	Ri_1	2^8 / 3^5	256 / 243
6561 / 4096	3^8 / 2^{12}	Da_1	2^7 / 3^4	128 / 81
19683 / 16384	3^9 / 2^{14}	Ga_2	2^5 / 3^3	32 / 27
59049 / 32768	3^{10} / 2^{15}	Ni_2	2^4 / 3^2	16 / 9
177147 / 131072	3^{11} / 2^{17}	Ma_1	2^2 / 3^1	4 / 3
531441 / 524288	3^{12} / 2^{19}	Sa	2^0 / 3^0	1 / 1

Table 2: *Derivation of the Pythagorean intervals through cycles of fourths (going from the bottom to the top) and fifths (going from the top to the bottom). Powers of 3 greater than 6 are uncommon but are still listed by some authors [7].*

(Ni_3)	(Ga_2)	(Pa)
256/135*	32/27*	40/27*
Ma_2	Ni_2	Ri_2
64/45	16/9	10/9
Ri_1	$\mathbf{Ma_1}$	Da_2
16/15	**4/3**	5/3
$\mathbf{Da_1}$	**Sa**	$\mathbf{Ga_3}$
8/5	**1/1**	**5/4**
Ga_2	**Pa**	Ni_3
6/5	**3/2**	15/8
Ni_2	Ri_2	Ma_2
9/5	9/8	45/32
(Ma_1)	(Da_2)	(Ri_1)
27/20*	27/16*	135/128*

Table 3: *Intervals in Just Intonation (JI). Going down the columns involves cycles of fifths, and going up the columns involves cycles of fourths. The ratios marked with a "*" are not usually included in lists of JI intervals (like in [7]). Note that there are two possible values for Ri_2, Ni_2 and Ma_2 even among the more widely accepted set of ratios.*

tional ratio. However, some people use a stricter definition and give a list of values drawn from the numbers presented in Table 3. In this framework, the ratios contain only powers of 2,3 and 5, and furthermore, the exponent of the prime factor 5 is either -1,0 or 1.

There are also people who propose "5-limit" systems where each interval uses ratios composed of prime factors up to the number 5. Such ratios theoretically can have higher powers of 5. Similarly, a "7-Limit" system can use any combination and powers of the prime factors 2,3,5 and 7.

Many people refer to the ratios in JI as "ideal" or "natural" intervals [11]. It should be interesting to note that usually the major second, minor seventh and the augmented fourth are assigned two different interval values. Various ratios in JI are related by cycles of fourths and fifths as shown in Table 3, but it is not possible to generate all the ratios starting from the tonic and using only fourths or fifths. Somehow the factors of 5 and 1/5 need to be introduced to produce some of the intervals.

3.3. Harmonic series ratios

Similar to Levy [5], we define the "harmonic series" ratios as those ratios arising from or suggested by the partials or harmonics present in a simple and ideal, freely vibrating string. In Table 4 we list the ratios arising out of the first 32 partials of strings tuned to Sa, Pa and Ma_1. It has often been observed that some notes like Ga_3, due to the 5th harmonic, and Pa, due to the 3rd harmonic of Sa can be heard on a plucked tambura string. Also, in many tamburas, when playing the lower octave Sa, the harmonic minor seventh, 7/4, can be distinctly heard as well. But not all of the partials given in this table can be easily heard or guaranteed to be present with a strong amplitude, but we have listed quite a few of them just for illustrative purposes. Most likely, only the first few ratios may be significant in a practical setting.

3.4. Simple Ratios

There are many so-called "simple" ratios that are not present in the harmonics of Sa and Pa. For example, the ratios 4/3, 5/3 and 8/5 are not present in the harmonic series of Sa or Pa, but they are included in JI. Such simple ratios may be important because they usually can be "locked onto" through the use of the acoustic cue of beating. For example, if two harmonic sounds with a relative pitch of 4/3 are played together, the interaction of the fourth harmonic of one sound with the third harmonic of

Partial Number	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Sa String	1/1	1/1	3/2	1/1	5/4	3/2	7/4	1/1
Pa String	3/2	3/2	9/8	3/2	15/8	9/8	21/16	3/2
Ma String	4/3	4/3	1/1	4/3	5/3	1/1	7/6	4/3
Partial Number	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
Sa String	9/8	5/4	11/8	3/2	13/8	7/4	15/8	1/1
Pa String	27/16	15/8	33/32	9/8	39/32	21/16	45/32	3/2
Ma String	3/2	5/3	11/6	1/1	13/12	7/6	5/4	4/3
Partial Number	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Sa String	17/16	9/8	19/16	5/4	21/16	11/8	23/16	3/2
Pa String	51/32	27/16	57/32	15/8	63/32	33/32	69/64	9/8
Ma String	17/12	3/2	19/12	5/3	7/4	11/6	23/12	1/1
Partial Number	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**
Sa String	25/16	13/8	27/16	7/4	29/16	15/8	31/16	1/1
Pa String	75/64	39/32	81/64	21/16	87/64	45/32	93/64	3/2
Ma String	25/24	13/12	9/8	7/6	29/24	5/4	31/24	4/3

Table 4: *Ratios arising from the harmonic series of ideal vibrating strings tuned to Sa, Pa and* Ma_1.

the other can be heard in terms of beats. (In fact, there will be many pairs of beating partials.)

It is reasonable to assume that the simpler the relative ratio, the easier it is to detect the beats. To quantify the "simplicity" or complexity of a ratio, p/q, we have arbitrarily used a measure $\mathcal{C}(p,q) = (p+q) + \frac{p \times q}{1000}$ to list an ordered set of simple ratios as given in Table 5. In Table 6, we have sorted these ratios according to their cent value. Compared to JI for example, this table may offer simpler or alternate ratios for certain intervals. However, notice that most of the simplest ratios appear in the JI scheme anyway.

In his book, Ayyar [12] has dismissed some of the more complicated rationals like 256/243 but has made some bold claims about the usage of simpler ratios in Carnatic music. He suggests that ratios like "7/6" *are* used and that musicians vary the particular *ratio* they use for each note depending on its melodic context. While such assertions may have been valid for *his own* violin playing, they seem, in general, premature, and may not hold for most other Carnatic musicians.

3.5. Equal Temperament

In the Western Equal Temperament (ET) tuning system, the intervals between adjacent semitones are all equal. Since there are 12 notes in an octave, a semitone is $2^{1/12}$ or 100 cents. Though people claim that certain intervals are "mis-tuned" and sound a little worse than their JI counterparts, ET has served Western music very well over the years and does not suffer from interval size problems whenever there is a modal shift of tonic.

3.6. Miscellaneous

There have been other tuning systems tried in the West, but which did not really gain mass popularity and survive. Some of these divided the octave into more than 12 notes, and others tuned each interval in a 12-note scheme slightly differently. Since we believe that these tuning systems do not have much relevance to Carnatic music, we will not discuss them here.

3.7. Theoretical Indian Tunings

Even today, many Indians seem to believe that a set of 22 intervals are used in Carnatic music as listed in [13] and reproduced in Table 7. People have derived the values in this table in many fanciful ways, but one should note that this list is little more than a merging of the well-known JI and Pythagorean intervals, which really provide alternate, slightly different tunings for a basic set of 12 intervals! In [1, 2], we dismissed the veracity of this list for various reasons. However, that doesn't mean that every single entry in this list is completely invalid as we will discuss later.

$\mathbf{r = p/q}$	cents	$\mathbf{p+q}$	$\mathbf{p \times q}$	$\mathbf{r = p/q}$	cents	$\mathbf{p+q}$	$\mathbf{p \times q}$
1 / 1	0.0000	2	1	13 / 11	289.21	24	143
3 / 2	701.96	5	6	16 / 9	996.09	25	144
4 / 3	498.04	7	12	14 / 11	417.51	25	154
5 / 3	884.36	8	15	13 / 12	138.57	25	156
5 / 4	386.31	9	20	17 / 9	1101.1	26	153
7 / 4	968.83	11	28	15 / 11	536.95	26	165
6 / 5	315.64	11	30	17 / 10	918.64	27	170
7 / 5	582.51	12	35	16 / 11	648.68	27	176
8 / 5	813.69	13	40	14 / 13	128.30	27	182
7 / 6	266.87	13	42	17 / 11	753.64	28	187
9 / 5	1017.6	14	45	15 / 13	247.74	28	195
8 / 7	231.17	15	56	19 / 10	1111.2	29	190
9 / 7	435.08	16	63	18 / 11	852.59	29	198
11 / 6	1049.4	17	66	17 / 12	603.00	29	204
10 / 7	617.49	17	70	16 / 13	359.47	29	208
9 / 8	203.91	17	72	15 / 14	119.44	29	210
11 / 7	782.49	18	77	19 / 11	946.20	30	209
12 / 7	933.13	19	84	17 / 13	464.43	30	221
11 / 8	551.32	19	88	20 / 11	1035.0	31	220
10 / 9	182.40	19	90	19 / 12	795.56	31	228
13 / 7	1071.7	20	91	18 / 13	563.38	31	234
11 / 9	347.41	20	99	17 / 14	336.13	31	238
13 / 8	840.53	21	104	16 / 15	111.73	31	240
11 / 10	165.00	21	110	21 / 11	1119.5	32	231
13 / 9	636.62	22	117	19 / 13	656.99	32	247
15 / 8	1088.3	23	120	17 / 15	216.69	32	255
14 / 9	764.92	23	126	20 / 13	745.79	33	260
13 / 10	454.21	23	130	19 / 14	528.69	33	266
12 / 11	150.64	23	132	17 / 16	104.96	33	272

Table 5: *All possible rationals p/q such that $1 \leq p/q < 2$ and $(p+q) \leq 33$, sorted by $\mathcal{C}(p,q) = (p+q) + \frac{p \times q}{1000}$.*

r = p/q	cents	p + q	p × q	r = p/q	cents	p + q	p × q
1 / 1	0.0000	2	1	7 / 5	582.51	12	35
17 / 16	104.96	33	272	17 / 12	603.00	29	204
16 / 15	111.73	31	240	10 / 7	617.49	17	70
15 / 14	119.44	29	210	13 / 9	636.62	22	117
14 / 13	128.30	27	182	16 / 11	648.68	27	176
13 / 12	138.57	25	156	19 / 13	656.99	32	247
12 / 11	150.64	23	132	3 / 2	701.96	5	6
11 / 10	165.00	21	110	20 / 13	745.79	33	260
10 / 9	182.40	19	90	17 / 11	753.64	28	187
9 / 8	203.91	17	72	14 / 9	764.92	23	126
17 / 15	216.69	32	255	11 / 7	782.49	18	77
8 / 7	231.17	15	56	19 / 12	795.56	31	228
15 / 13	247.74	28	195	8 / 5	813.69	13	40
7 / 6	266.87	13	42	13 / 8	840.53	21	104
13 / 11	289.21	24	143	18 / 11	852.59	29	198
6 / 5	315.64	11	30	5 / 3	884.36	8	15
17 / 14	336.13	31	238	17 / 10	918.64	27	170
11 / 9	347.41	20	99	12 / 7	933.13	19	84
16 / 13	359.47	29	208	19 / 11	946.20	30	209
5 / 4	386.31	9	20	7 / 4	968.83	11	28
14 / 11	417.51	25	154	16 / 9	996.09	25	144
9 / 7	435.08	16	63	9 / 5	1017.6	14	45
13 / 10	454.21	23	130	20 / 11	1035.0	31	220
17 / 13	464.43	30	221	11 / 6	1049.4	17	66
4 / 3	498.04	7	12	13 / 7	1071.7	20	91
19 / 14	528.69	33	266	15 / 8	1088.3	23	120
15 / 11	536.95	26	165	17 / 9	1101.1	26	153
11 / 8	551.32	19	88	19 / 10	1111.2	29	190
18 / 13	563.38	31	234	21 / 11	1119.5	32	231

Table 6: *Rationals in Table 5 sorted by cent value.*

Cents	Ratio	Note	Note	Ratio	Cents
0.0000	1 / 1	Sa			
90.225	256 / 243	Ri_{1_1}	Ni_{3_2}	243 / 128	1109.8
111.73	16 / 15	Ri_{1_2}	Ni_{3_1}	15 / 8	1088.3
182.40	10 / 9	Ri_{2_1}	Ni_{2_2}	9 / 5	1017.6
203.91	9 / 8	Ri_{2_2}	Ni_{2_1}	16 / 9	996.09
294.13	32 / 27	Ga_{2_1}	Da_{2_2}	27 / 16	905.87
315.64	6 / 5	Ga_{2_2}	Da_{2_1}	5 / 3	884.36
386.31	5 / 4	Ga_{3_1}	Da_{1_2}	8 / 5	813.69
407.82	81 / 64	Ga_{3_2}	Da_{1_1}	128 / 81	792.18
498.04	4 / 3	Ma_{1_1}	Pa	3 / 2	701.96
519.55	27 / 20	Ma_{1_2}			
590.22	45 / 32	Ma_{2_1}	Ma_{2_2}	64 / 45	609.78

Table 7: *The popular list of 22 ratios supposedly used in Indian music. The "missing" entries are: 2/1 or 1200 cents (Sa an octave higher) and 40/27 or 680.45 cents (a flattened version of Pa).*

Incidentally, the number 22 seems to have arisen from an ancient text, the Natyasastra, attributed to Bharata Muni. However, there is evidence which suggests that Bharata was describing only 7 swarams or swarasthanams (not 22 swarams!) and was using the term/unit "sruthi" to roughly denote what the interval size of each swaram was [14]. However, the number 22 has stuck till today and its meaning has been warped somehow to accommodate 22 note positions in an octave. Different numerical values for the various sruthis have been drawn up over the years, and people seem to be under the impression that present-day Indian music is directly connected to ancient Indian music. Ramanathan makes a good point that the idea of sruthis as Bharata though of it may be relevant and applicable only to his music system at his time [14].

4. DRONES, BEATS AND RATIONALS

Do beats play a role in a Carnatic musician's intonation? How does the tambura affect a person's intonation, which has a lot of harmonics?

The pitch reference in a Carnatic music performance, the tambura drone, is indeed rich in partials which are harmonic, but it is a very difficult task to try to look for and minimize beats in the middle of a performance when (i) there are several instruments playing, (ii) the tambura is not tuned perfectly, (iii) when the tambura can't even be heard or (iv) fast phrases and short-duration notes are being played.

However, it is a different story when a musician is practicing at home and can take his time to get his intonation correct. In that case, the acoustical cues one gets from the tambura may be important. For instance, an artist may match his Ga_3 roughly with the third harmonic of the tambura.

Two examples of Ga_3 presented in [2] suggest that the ratio 5/4 may be significant, especially in slow passages when the musician has time to converge to a particular interval value. Though by no means conclusive proof of the usage of the ratio 5/4, these examples suggest that simple ratios or rationals should not be ruled out completely, even in the context of a recording or performance.

Nowadays, the electronic tambura is also very popular, and in some models which just output a constant tone or note, finding the beats to minimize is a much easier task when compared to doing the same with the acoustic tambura. Many people may have had their music careers influenced significantly by this instrument. But while most musicians are well aware of the phenomenon of beats, we can't expect everyone, however, to use these cues to intonate their notes. It will be interesting to find out what methods different musicians use to intonate each note in the presence of a clean, audible tambura or other drone sound.

Finally, one may interpret integer ratios as symbols rather than as exact numerical values: for example the "symbol" 5/4 may refer to the interaction of the fifth and fourth harmonics of two sounds rather than the number 1.25. Doing so accommodates the fact that no human or instrument in the real world is perfect.

Table 8 lists the more promising integer ratios as far as minimizing beats are concerned.

5. INSTRUMENT CONSTRAINTS

Certain musical instruments have implicit constraints that affect the intonation of the notes they produce. For example, instruments like the veena, mandolin and the guitar have frets that run perpendicular to all the strings. These frets impose a relationship between notes on different strings stopped by the same frets as shown in Table 9. If two strings of an instrument are tuned to *Sa* and *Pa* as shown, then the only solution that keeps the interval values of all the notes consistent is ET tuning. We get discrepancies if we try to impose Pythagorean or JI tuning on instruments with frets as illustrated by Tables 10 and 11. Notice that the cycles of fourths and fifths reappear in these "fret equations" since going from one string to another involves one cycle of a fourth or fifth.

In the Veena, where the string can be deflected to produce a higher pitched note, this may not be too much of a problem (if veena players are indeed even aware of this "problem" and really use this technique to "correct" it), but in instruments like the mandolin, JI tuning is not possible without some compromise somewhere, in the absence of "staggered frets."

In reality, for instance, Mandolin U. Shrinivas seems to be using ET tuning, and this tuning seems to work quite well for him. He is widely acknowledged as a top-notch musician by the veterans of Carnatic music, and many people enjoy his music completely. This implies that at least for his plucked instrument, the ET Ga_3 does fine, for example. Perhaps the discrepancy between JI and ET of the Ga_3 is more prominent and audible for sustained sounds like those from a violin. For example, there are violinists who play Da_2 on the *Pa* string at a position slightly lower than the corresponding Ri_2 on the *Sa* string to obtain the JI value for both.

The process of veena fretting may also offer some insight into the tuning of the intervals and the tolerance allowed in intonation. If the string harmonics are used to fix certain intervals, then this implies that these intervals may indeed be based on the harmonic series ratios. However, there is, at the same time, evidence that perhaps intonation is somewhat flexible when Vidya Shankar says that "there is no hard or fast rule" for veena fretting [15]. However the tuning of certain "simple " intervals in the veena may affect the tuning of the same intervals in the rest of Carnatic music.

There are other indigenous instruments as well as Western ones used in Carnatic music today like the flute and saxophone whose physical construction may also offer some insight into tuning or simply reveal that intonation and tunings are flexible.

6. GRAHABEDHAM

The practice of grahabedham or modal shift of tonic has been used by some to "derive" the various sruthis [16]. However, in today's music, which artist really derives the intonation of his swarams through such techniques? As we pointed out before, intonation is learned, and certainly no one uses grahabedham to teach intonation today. Furthermore, in [16], the author makes some bold assumptions about the interval values of the 12 basic intervals and then "derives" 11 more "sruthis" by using each of the initial intervals as a new tonic to get 22 "sruthis." (The original *Sa* is not included in his count.) Even after all these derivations, the author ends up adjusting the derived values of some intervals, after he had synthesized and listened to them, to suit current day music. In our opinion, the method of deriving the "sruthis" and values presented in this book is an exercise in futility.

Now what does happen when an artist performs grahabedham on stage? How are the tunings of the various notes affected? These questions have to be investigated carefully. Each performance or melodic context has to be studied separately, but it is possible that, in most cases, there may be no precise shifting or re-tuning of the intervals involved. The flexibility in intonation might be an important feature of Carnatic music that makes this whole process work quite well.

7. WESTERN INFLUENCES

Virtually every Indian musician today has been ex-

Note	**Ratios**
Sa	1/1
Ri_1	16/15, 17/16
Ri_2	9/8, 10/9
Ga_2	6/5, 7/6
Ga_3	5/4
Ma_1	4/3
Ma_2	17/12, 7/5, 10/7
Pa	3/2
Da_1	8/5, 11/7
Da_2	5/3
Ni_2	9/5, 16/9, 7/4
Ni_3	15/8, 17/9, 13/7

Table 8: *The most promising simple ratios selected from Table 6. The left column of this list is identical to the JI system except for* Ma_2.

Sa String				**Pa String**		
Western	**Note**	**Ratio (r)**	**Constraint**	**Ratio (r)**	**Note**	**Western**
P1	Sa	1/1	$1 \cdot g = g$	g	Pa	P5
m2	Ri_1	a	$a \cdot g = h$	h	Da_1	m6
M2	Ri_2	b	$b \cdot g = i$	i	Da_2	M6
m3	Ga_2	c	$c \cdot g = j$	j	Ni_2	m7
M3	Ga_3	d	$d \cdot g = k$	k	Ni_3	M7
P4	Ma_1	e	$e \cdot g = 2$	2/1	$\dot{S}a$	P8
+4	Ma_2	f	$f \cdot g = 2a$	$2a$	$\dot{R}i_1$	m9
P5	Pa	g	$g \cdot g = 2b$	$2b$	$\dot{R}i_2$	M9
m6	Da_1	h	$h \cdot g = 2c$	$2c$	$\dot{G}a_2$	m10
M6	Da_2	i	$i \cdot g = 2d$	$2d$	$\dot{G}a_3$	M10
m7	Ni_2	j	$j \cdot g = 2e$	$2e$	$\dot{M}a_1$	P11
M7	Ni_3	k	$k \cdot g = 2f$	$2f$	$\dot{M}a_2$	+11
P8	$\dot{S}a$	2/1	$2 \cdot g = 2g$	$2g$	$\dot{P}a$	P12

Table 9: *Constraints imposed by frets across two strings tuned to Sa and Pa. There are 11 unknown variables, a – k, and 11 non-trivial equations. Solving for g, or the ratio of Pa, one gets:* $g^{12} = 2^7$*, which has exactly one real and positive solution:* $g = 2^{7/12}$ *(ET tuning system).*

Sa String			**Pa String**		
Western	**Note**	**Ratio (r)**	**Ratio (r)**	**Note**	**Western**
P1	Sa	1/1	3/2	Pa	P5
m2	Ri_1	256/243	128/81	Da_1	m6
M2	Ri_2	9/8	27/16	Da_2	M6
m3	Ga_2	32/27	16/9	Ni_2	m7
M3	Ga_3	81/64	243/128	Ni_3	M7
P4	Ma_1	4/3	2/1	$\dot{S}a$	P8
+4	Ma_2	**1024/729**	$2 \cdot 256/243$	$\dot{R}i_1$	m9
P5	Pa	3/2	$2 \cdot 9/8$	$\dot{R}i_2$	M9
m6	Da_1	128/81	$2 \cdot 32/27$	$\dot{G}a_2$	m10
M6	Da_2	27/16	$2 \cdot 81/64$	$\dot{G}a_3$	M10
m7	Ni_2	16/9	$2 \cdot 4/3$	$\dot{M}a_1$	P11
M7	Ni_3	243/128	$2 \cdot$ **729/512**	$\dot{M}a_2$	+11
P8	$\dot{S}a$	2/1	$2 \cdot 3/2$	$\dot{P}a$	P12

Table 10: *A possible "solution" obtained when solving the equations in Table 9, but after setting/forcing $g = 3/2$. We expect one discrepancy due to this additional constraint, as can be seen in the two values for Ma_2. (The point of discrepancy can be moved.) Notice that we have derived the Pythagorean tuning system above.*

Sa String				**Pa String**			
Western	**Note**	**Ratio (r)**	**r · g**	**r/g**	**Ratio (r)**	**Note**	**Western**
P1	Sa	1/1	3/2	1/1	3/2	Pa	P5
m2	Ri_1	16/15	8/5	16/15	8/5	Da_1	m6
M2	Ri_2	9/8	**27/16**	**10/9**	5/3	Da_2	M6
m3	Ga_2	6/5	9/5	6/5	9/5	Ni_2	m7
M3	Ga_3	5/4	15/8	5/4	15/8	Ni_3	M7
P4	Ma_1	4/3	2/1	4/3	2/1	$\dot{S}a$	P8
+4	Ma_2	f	$2 \cdot f$	**64/45**	$2 \cdot 16/15$	$\dot{R}i_1$	m9
P5	Pa	3/2	$2 \cdot 9/8$	3/2	$2 \cdot 9/8$	$\dot{R}i_2$	M9
m6	Da_1	8/5	$2 \cdot 6/5$	8/5	$2 \cdot 6/5$	$\dot{G}a_2$	m10
M6	Da_2	5/3	$2 \cdot 5/4$	5/3	$2 \cdot 5/4$	$\dot{G}a_3$	M10
m7	Ni_2	9/5	$2 \cdot$ **27/20**	**16/9**	$2 \cdot 4/3$	$\dot{M}a_1$	P11
M7	Ni_3	15/8	$2 \cdot$ **45/32**	$4/3 \cdot f$	$2 \cdot f$	$\dot{M}a_2$	+11
P8	$\dot{S}a$	2/1	$2 \cdot 3/2$	2/1	$2 \cdot 3/2$	$\dot{P}a$	P12

Table 11: *Discrepancies which arise when JI tuning is imposed on an instrument with frets. As expected, there are 3 notes with two different interval values, due to the additional constraints: $g = 3/2$, $d = 5/4$ and $h = 8/5$.*

posed to Western music and ET tuning. Therefore, the effects of ET tuning can't be ignored. It is possible that the intonation of at least some Carnatic musicians has been modified by constant exposure to Western music. It is possible that the more "difficult" intervals like Ri_1 and Ma_2 have been more vulnerable to Western influence than intervals like Ga_3 for which the tambura gives a stronger cue.

8. A HYBRID SCHEME

We believe that it is reasonable to expect some of the more consonant intervals to be governed by simple integer ratios while some of the more complex intervals like Ma_2 may be a lot more flexible or variable in intonation between artists. Perhaps a combination of ratios and cent values for the 12 intervals in Carnatic music will be most appropriate. One should perhaps be willing to accept a hybrid tuning scheme rather than try to fit Carnatic music to a preconceived scheme like JI. That said, Table 8 lists the most promising ratios for each note, selected for simplicity from Table 6. In addition to these ratios, the Western ET values should also be considered, especially for the more complex intervals. In the end, it is even possible that certain notes do not conform to any known tuning scheme or even allow themselves to be represented by a single numerical value. We would be surprised if more than 3 or 4 of the ratios listed in Table 8 are relevant, but perceptual experiments are the key to answer such questions.

9. CONCLUSIONS

We presented a few tuning systems and suggested tuning possibilities for certain notes. Without concrete empirical evidence, all of these values are, of course, mere speculation. There are reasons to believe that simple ratios may be useful for some of the more prominent intervals like the major third. But for others, Western ET or some other numerical range may be appropriate. Also, we saw that certain musical instruments can't be tuned "perfectly" but that does not seem to be a problem at all. Perhaps we will be able to come up with an underlying tuning preference for intervals in Carnatic music, but it won't be surprising at all if there is no agreement between preferred intervals of different artists trained by different gurus and allowed to evolve and mature on their own.

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