Analyzing Hemiolas with the Discrete Fourier Transform

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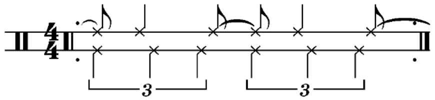
What is a hemiola?

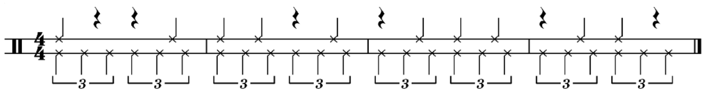
"A hemiola can be said to arise whenever **pulses in a 3:2 ratio are perceived to conflict**." (Cohn 2001)

However, other features might characterize hemiolas:

- **Phase difference** or **displacement** between conflicting pulses
- **Relative support for/strength** of conflicting pulses



Ex 1. Phase difference of ♪ between note onsets supporting ♩ pulse and triplet-♩ pulse.



Ex 2. Inconsistent support for ♩ pulse compared to triplet-♩ pulse.

Existing methods of hemiola representation

Method	Captures phase difference?	Captures relative strength?
Ski-hill graph (Cohn 2001)	❌	❌
Grouping dissonance (Krebs 1999)	❌ (✅ combined with **displacement dissonance,** Krebs 1999)	❌
Semimeters (Chung 2008)	✅	❌

References

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The Discrete Fourier Transform (DFT)

Recent music theoretic work using DFT mostly in the domain of harmony (Quinn 2006/2007, Yust 2020), but has been applied to meter and rhythm too (Chiu 2018, Yust 2021)

Input: sampled time-domain signal (e.g. a **rhythm**) → **Output**: complex-valued vector of **frequency components (pulse periods)** → Decompose complex output into **magnitude (strength)** and **phase (displacement)**

Formal definition: $X(k) := \sum_{n=0}^{N-1} x(n)e^{-j2\pi nk/N}$

- Variables: $X(k)$: kth component (period = N/k), k in [0, N-1], x: **sampled signal** length N, n: **sample index** in [0, N-1]
- Constants: $e = \lim_{n\to\infty}(1+1/n)^n$, $j = \sqrt{-1}$

For a complex number $z = a + bj$:

- magnitude $|z| = \sqrt{a^2 + b^2}$
- phase $\angle z = \tan^{-1}\frac{b}{a}$

Ex 3. Sibelius Violin Concerto, ii. Adagio di molto, bb. 32-36, solo violin.

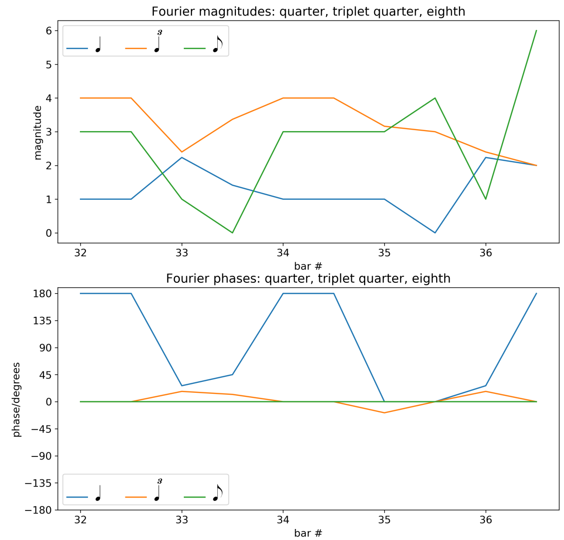

Fig 1. Fourier magnitudes and phases of Ex. 3.

Applying the DFT to a musical example

Rhythmic features of Ex. 3:

- **Constant triplet-♩s** from b. 32 to halfway through b. 36
- Untripleted line has **quite variable rhythms** (both **syncopated** and **unsyncopated**)
- Onsets line up **every half bar**
- Shortest common unit: **triplet 32nd**

Encoding the excerpt:

- For each of the upper and lower lines, sample for note onsets every **triplet 32nd** note (onset = 1, no onset = 0)
- Set window size and hop size = **half bar**
- Sum upper/lower line vectors making one vector of 0s, 1s & 2s

Interpreting the DFT output (Fig. 1)

Magnitude plot

- Triplet-♩ consistently stronger than ♩ pulse until halfway through b. 36; ♩ pulse marked more strongly by onsets in first half of b. 33
- ♪ pulse stronger than ♩ during syncopated passages; receives most support when onsets are isochronous (b. 35 second half)

Phase plot (total syncopation = 180° phase, unsyncopated = 0°)

- Relatively unsyncopated triplet-♩ and ♪ pulses
- ♩ pulse shifts rapidly between total syncopation and (near-)zero syncopation

Conclusions and future directions

- The DFT effectively captures **local variations in pulse strength and phase** since it is **sensitive to note onset information**
- Could help model **pulse salience** in the perception of complex hemiolas and other polyrhythms (polyrhythm perception: Vuust et al. 2006; Fujioka, Ross & Fidali 2014; rhythmic balance: Yust 2021), as well as a tool for larger corpus analyses
- DFT alone cannot provide a theory of meter (meter not exclusively determined by rhythm), but provides insights into **how rhythms reinforce metric interpretations**