CCRMA MIR Workshop 2014
Wavelets and multiresolution representations

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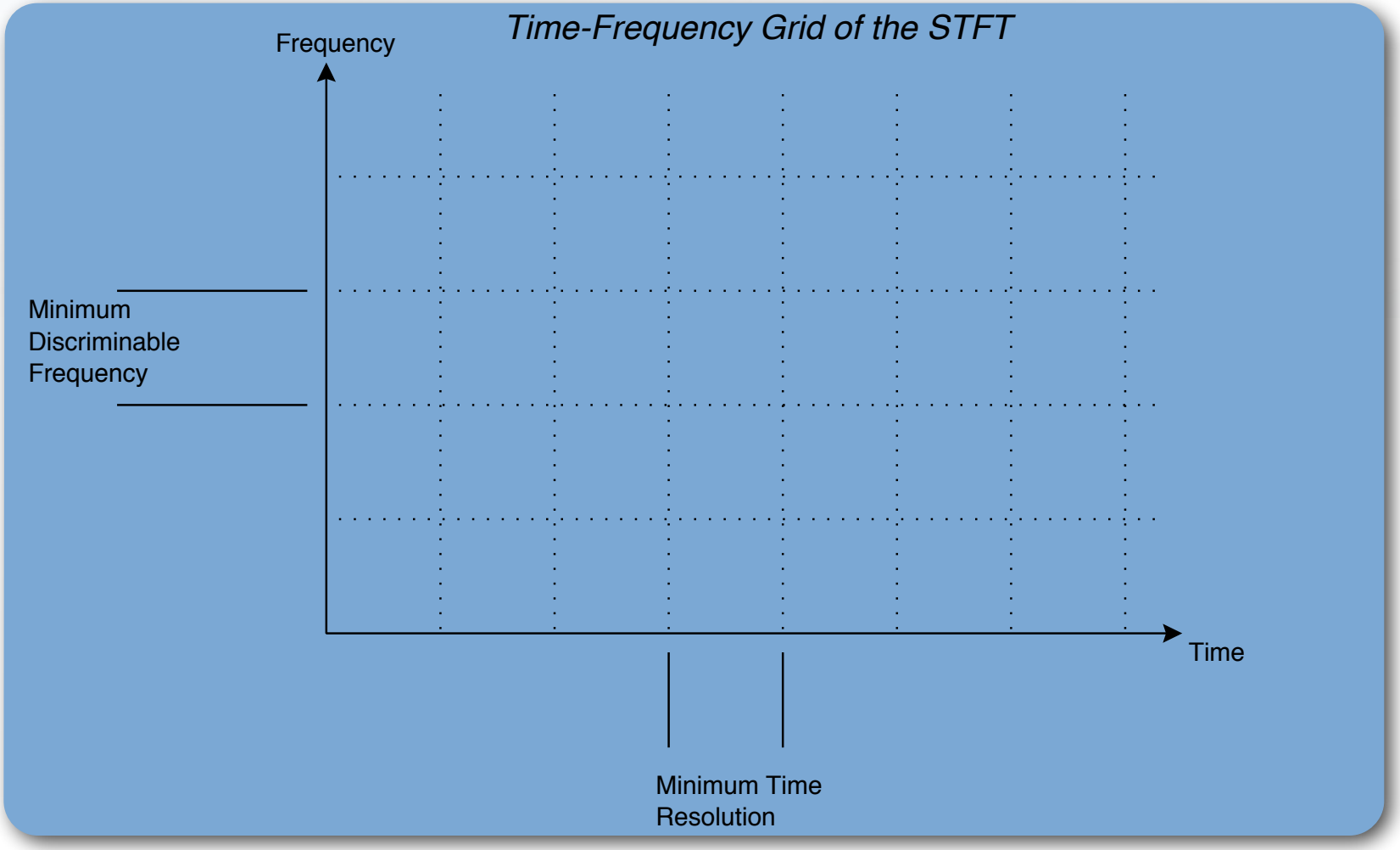
Basic system overview

Short Term Fourier Transform

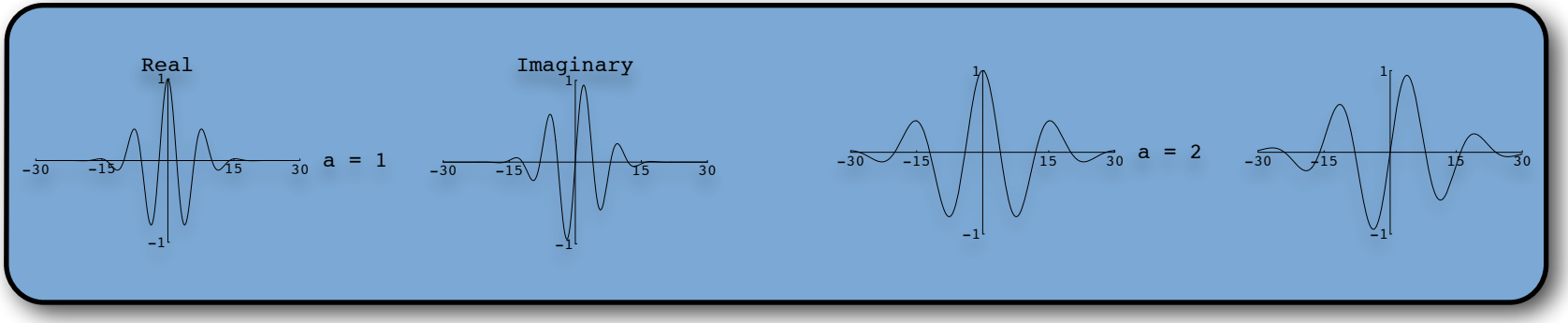


Wavelet time-frequency analysis

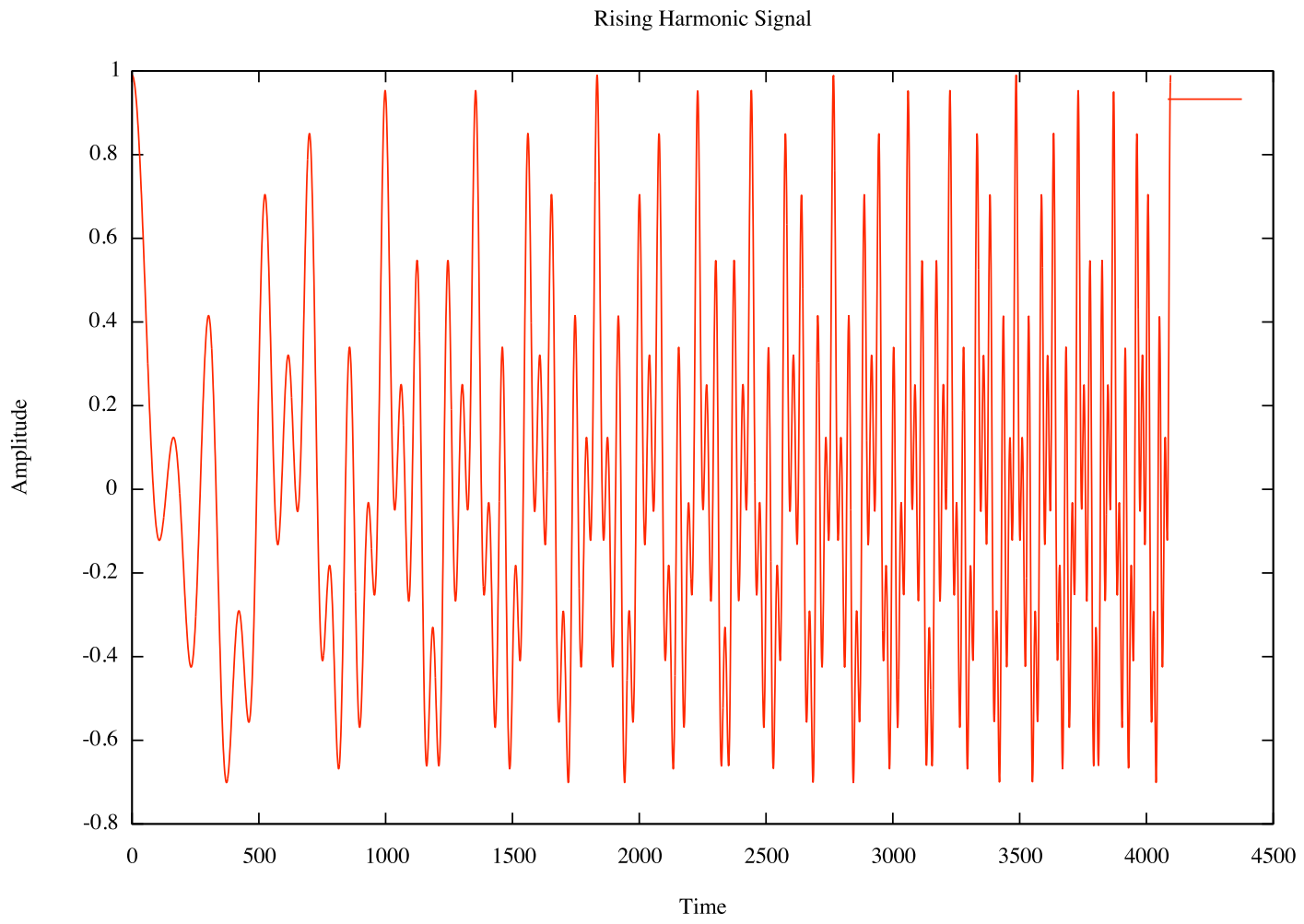
Continuous wavelet transform (CWT) decomposes (**invertibly**) a signal onto scaled and translated instances of a finite time "mother function" or "basis".

$$W_s(b,a) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(\tau) \cdot \bar{g}\left(\frac{\tau - b}{a}\right) d\tau, \; a > 0$$

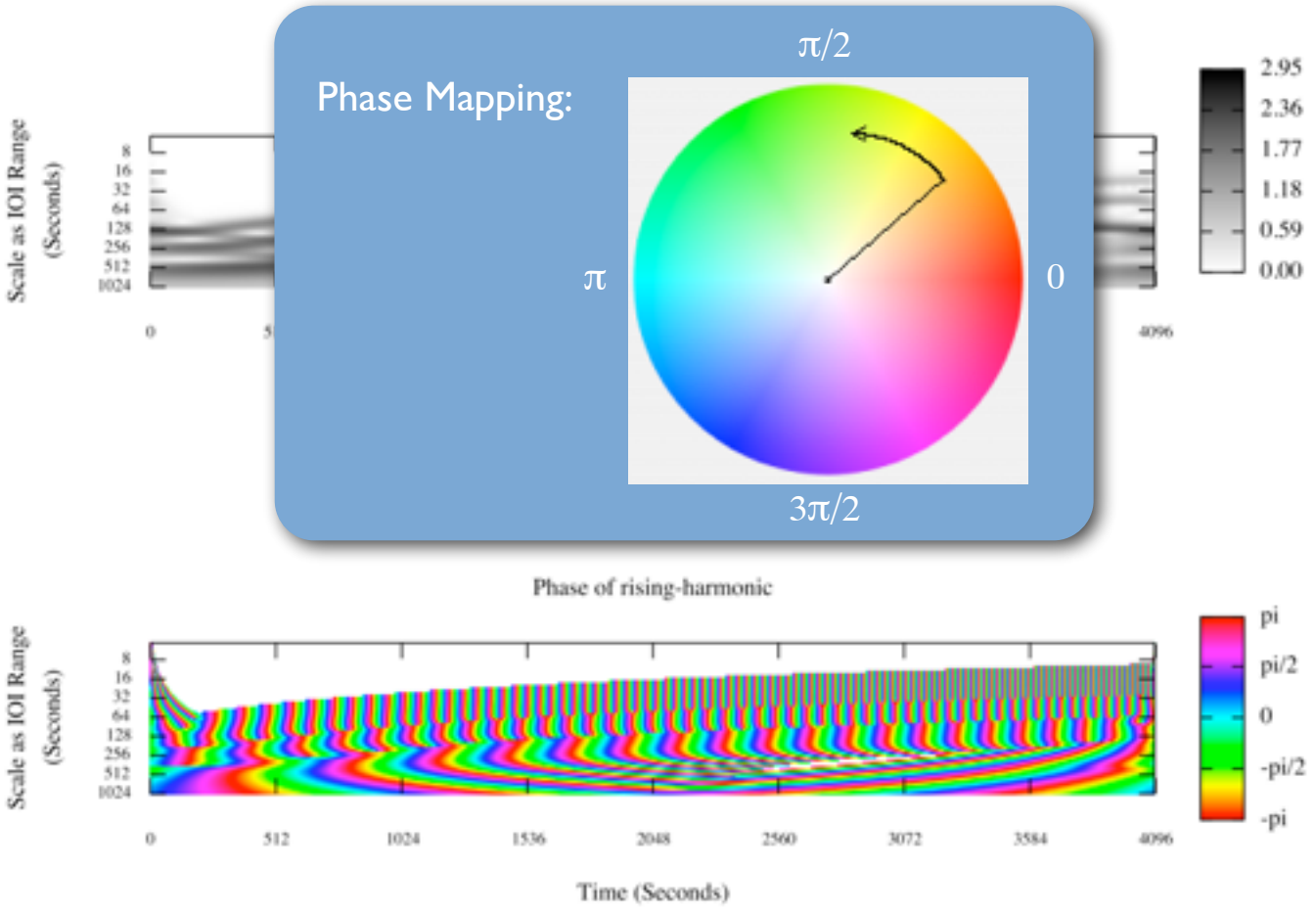
$$g(t) = e^{-t^2/2} \cdot e^{i\omega_0 t}$$



Example: Sinusoidal Signal

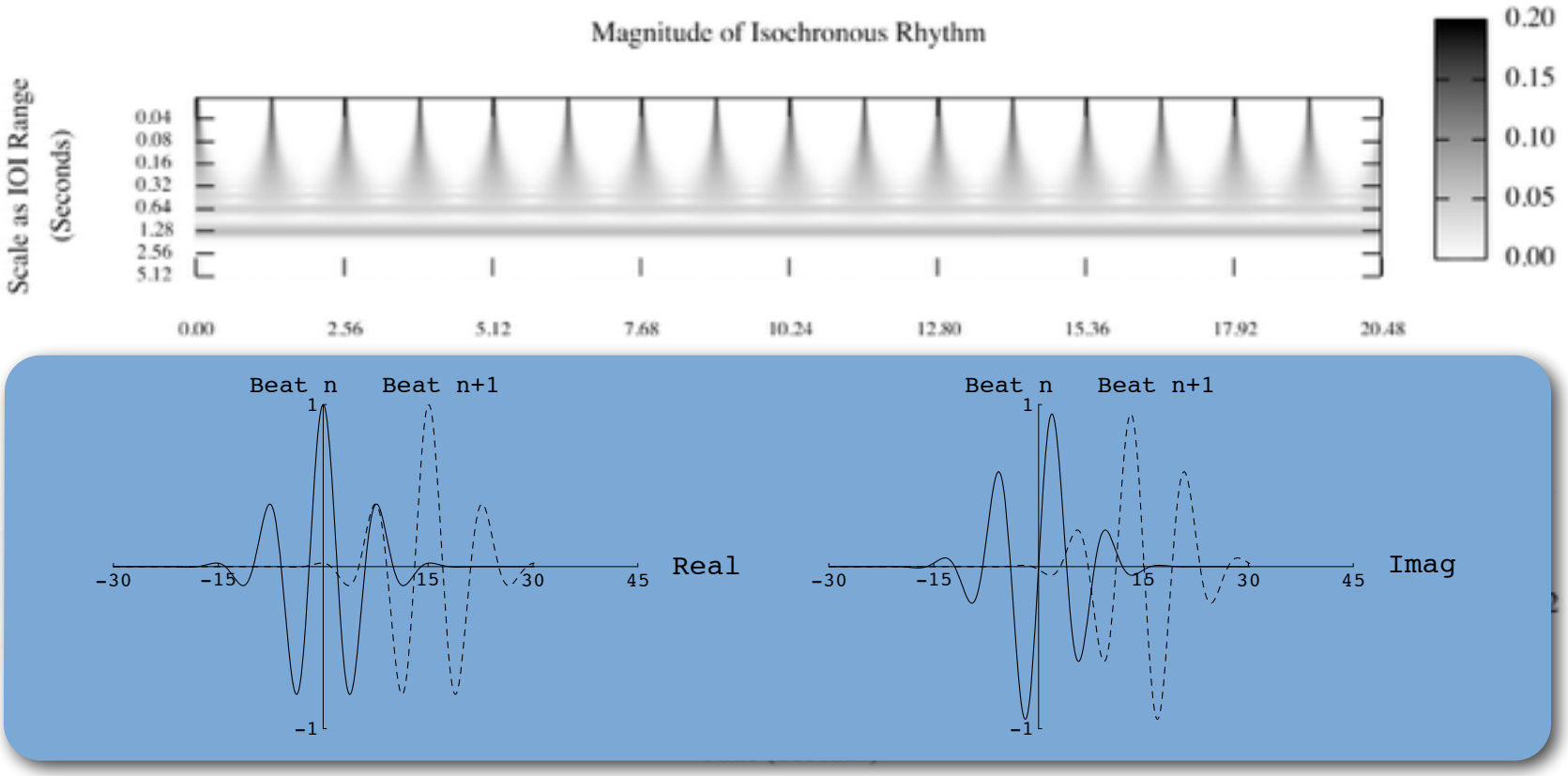


Example: Sinusoidal Signal



Example: Simple Rhythm

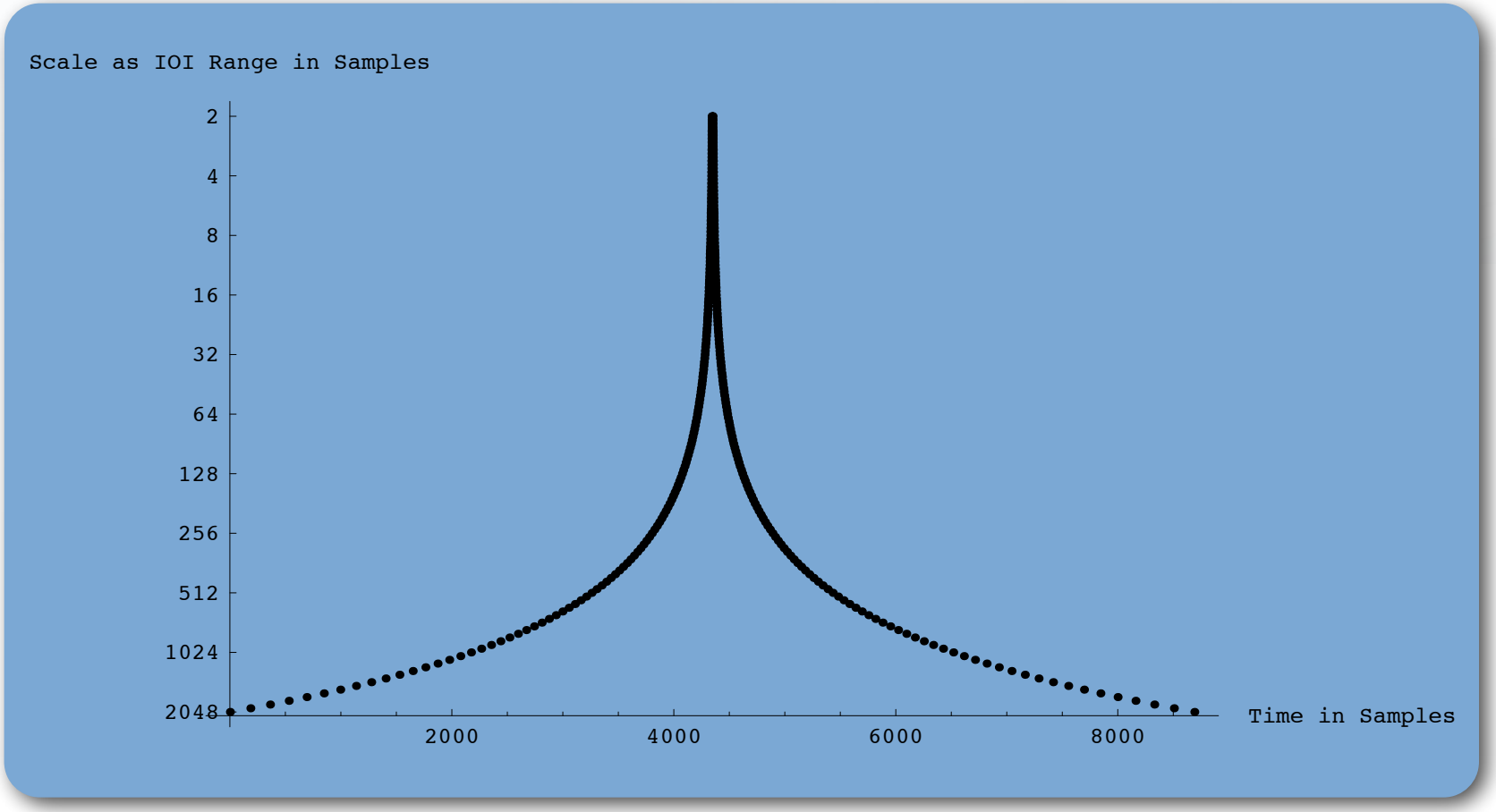
Scaleogram and Phaseogram of an isochronous pulse rhythmic signal:



Implementation

- Implemented as a set of complex value bandpass filters in Fourier domain.
- Scaling produces a “zooming” time window for each frequency “scale”.
- Creates simultaneous time and frequency localisation close to the Heisenberg inequality.

Wavelet Time-Frequency Resolution from Dilation (“Zooming")

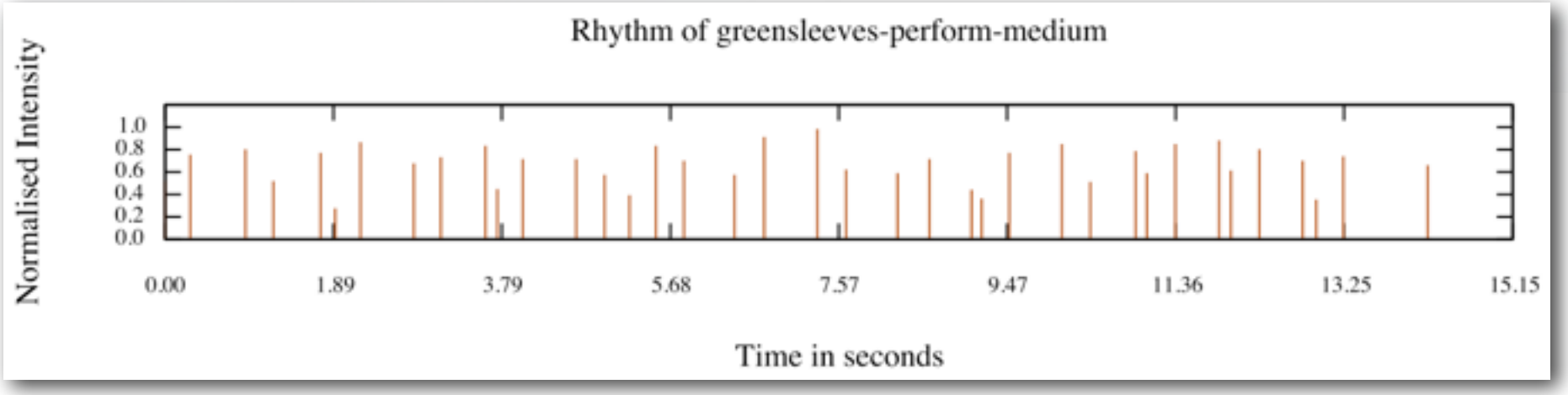


Wavelets for Rhythm (Smith & Honing 2008)

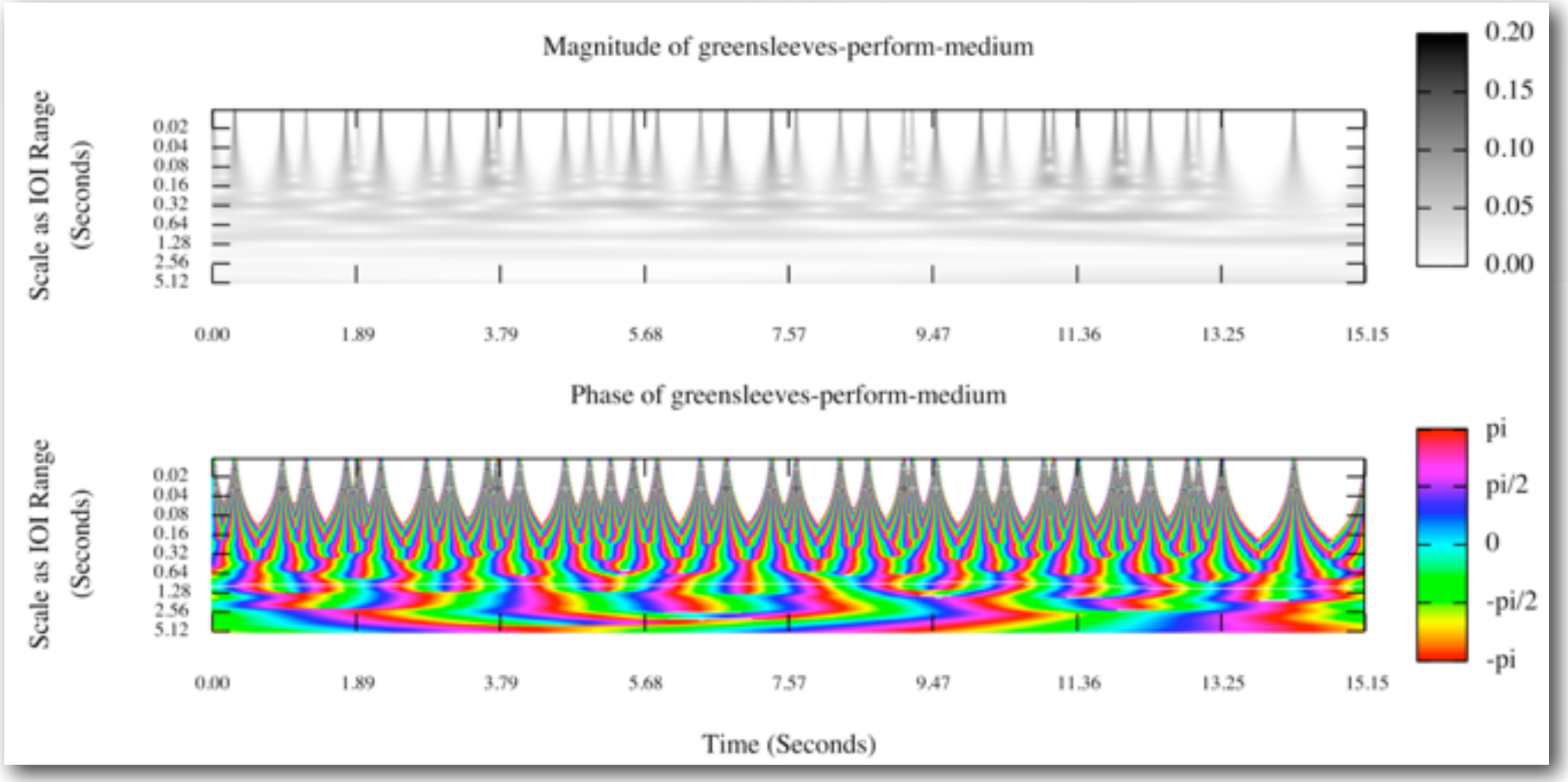
- The CWT enables representation of temporal structure in terms of time varying rhythmic frequencies.
- Produces magnitude and phase measures which reveal time-frequency ridges indicating the frequencies present in the input rhythm signal (collectively a skeleton, Tchamitchian & Torrésani '92).

Musical Example

- The rhythm of “Greensleeves”...

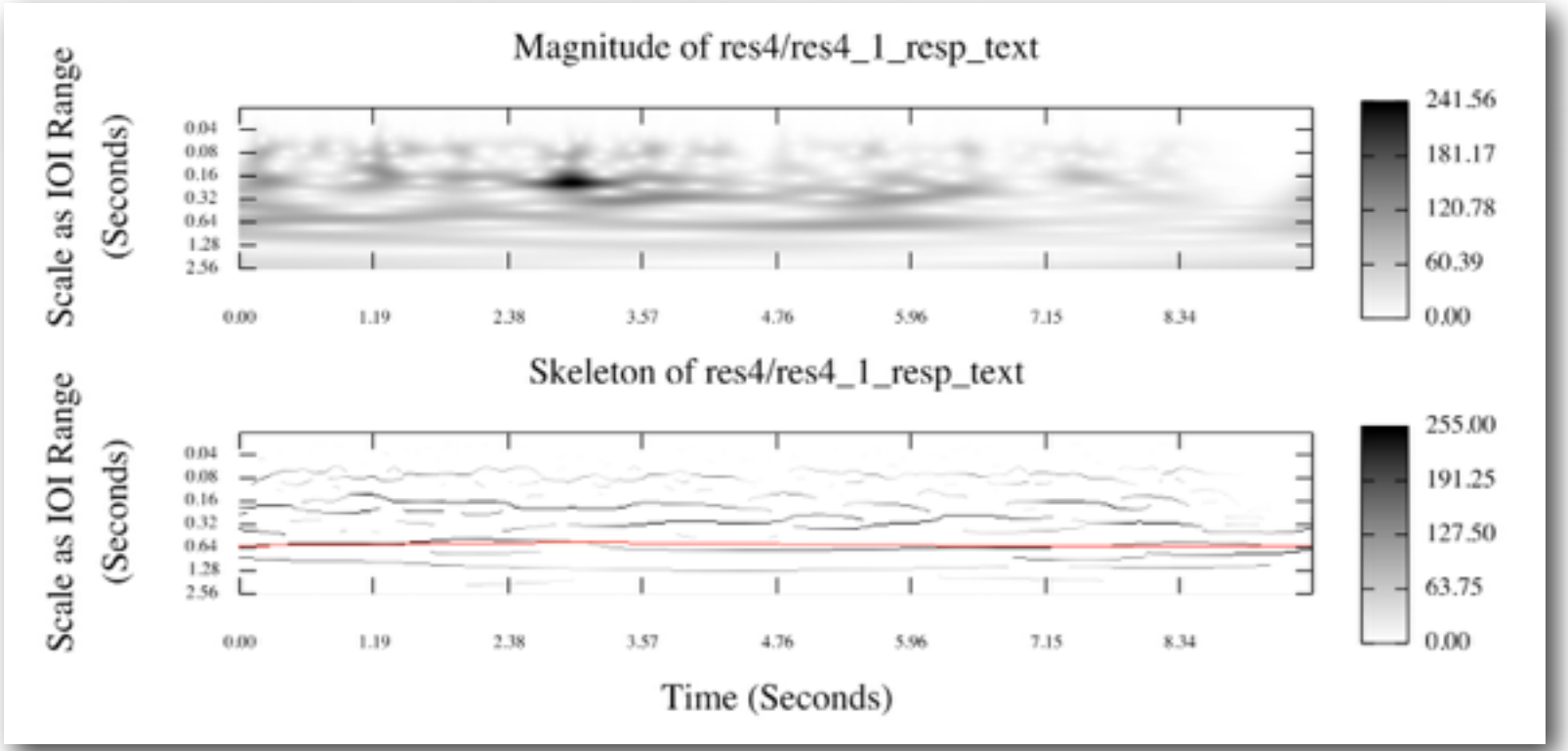


Greensleeves



Memory Based Tactus

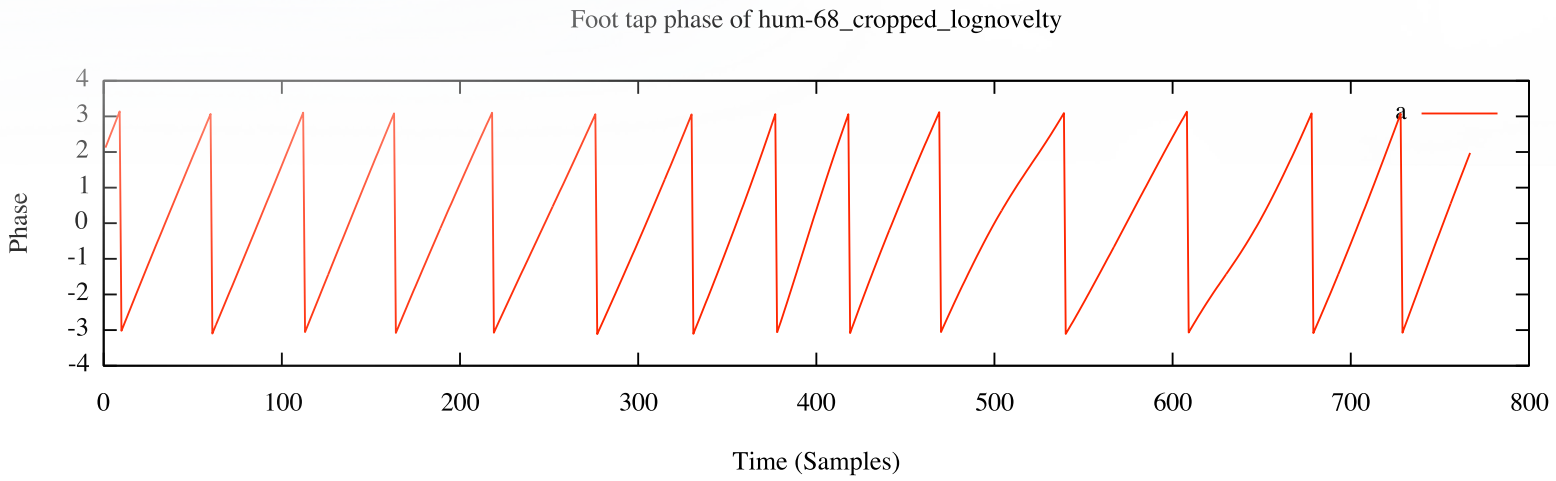
- Wavelet rhythm analysis is also applicable to continuous onset salience traces from auditory models (Coath, et. al 2009).



Memory Based Tactus

- Uses lossy windowed integrator to amass tactus likelihood.
- Suppress all but the magnitude coefficients of the extracted tactus ridge.
- Invert the extracted tactus ridge and original phase plane back to the time domain. Creates a single beat oscillation.
- Nominating a starting beat and noting its phase, all other foot-taps are generated for the same phase value.

Reconstructed Phase



Example: Foot-tapping to singing

- Singing examples of Dutch folk songs from the "Onder de Groene Linde" collection (Meertens Institute).
- Uses continuous wavelet transform of rhythmic signals (Smith 1996, Smith & Honing 2008) to derive tactus:
- Example 1: Original... + Accompaniment.
- Example 2: ...Original + Accompaniment.