SIGGRAPH 2000
[NEW ORLEANS]

Strings and Bars

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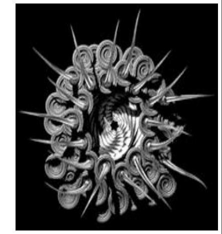
(also Music)

Views of Sound: Time Domain



Sound is produced/modeled by physics, described by quantities of

- Force — force = mass * acceleration
- Position — x(t) — actually < x(t), y(t), z(t) >
- Velocity — Rate of change of position — dx/dt
- Acceleration — Rate of change of velocity — dv/dt

Examples: ***Mass+Spring+Damper***

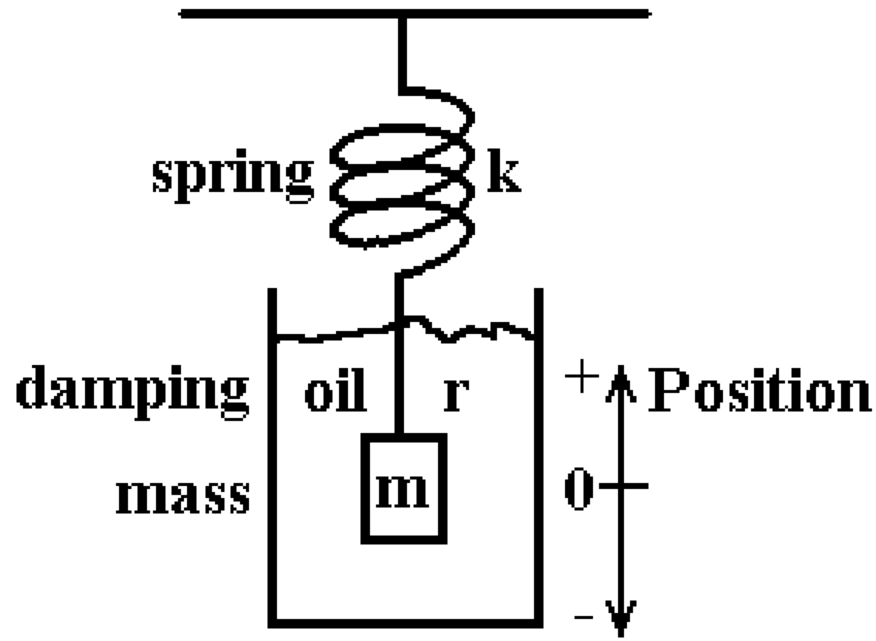
Wave Equation

Mass/Spring/Damper

F = ma = - ky - rv - mg

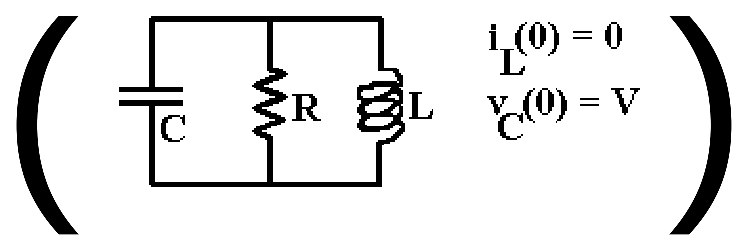
F = ma = - ky - rv

(if gravity negligible)

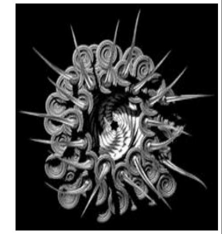


$$\frac{d^2 y}{dt^2} + \frac{r}{m}\frac{dy}{dt} + \frac{k}{m}y = 0$$

$$D^2 + Dr/m + k/m = 0$$



2nd Order Linear Diff Eq. Solution



1) Underdamped:

$$y(t) = Y_0 e^{-t/\tau} \cos(\omega t)$$

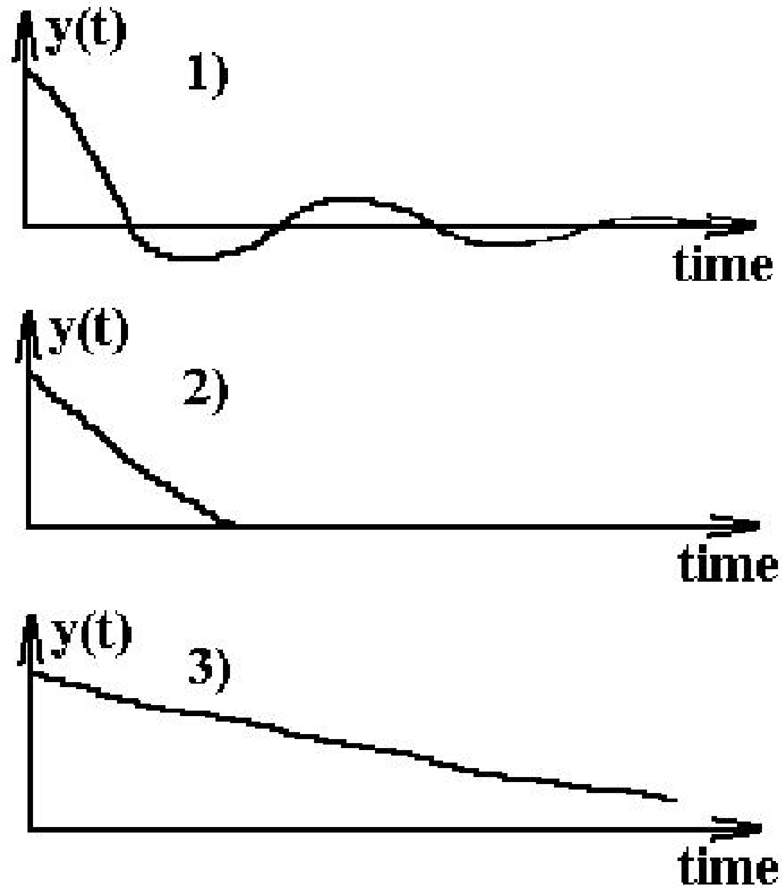
exp. * oscillation

2) Critically damped:

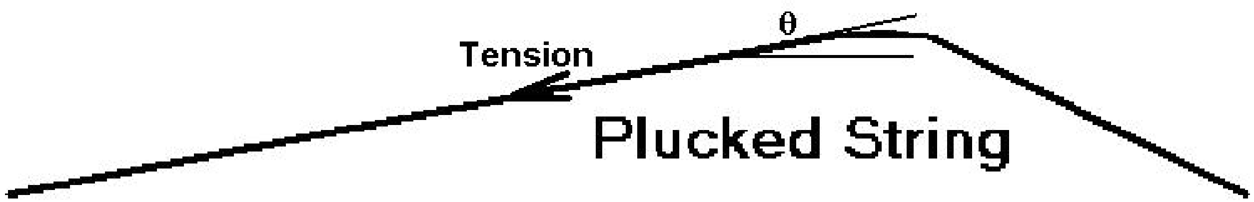
fast exponential decay

3) Overdamped:

slow exponential decay



Wave Equation



- **$df_y = (T \sin\theta)_{x+dx} - (T\sin\theta)_x$** ***(for each dx of string)***
- ***$f(x+dx) = f(x) + \delta f/\delta x \, dx + \ldots$*** ***(Taylor's series)***
- ***$\sin\theta = \theta$*** ***(for small θ)***
- ***$F = ma = \rho \, dx \, d^2y/dt^2$*** ***(ρ = mass/area)***

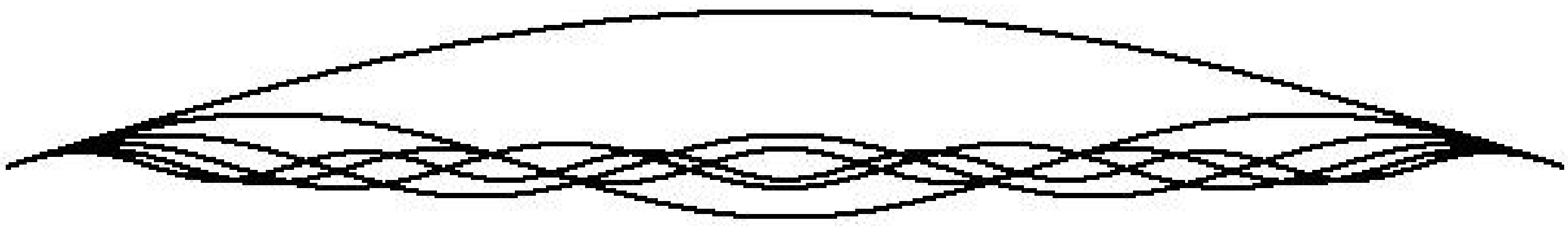
The wave equation:

$$\frac{d^2y}{dx^2} = \frac{1}{c^2}\frac{d^2y}{dt^2}$$

($c^2 = T / \rho$)

Modal Synthesis: Strings

Strings are pinned at both ends

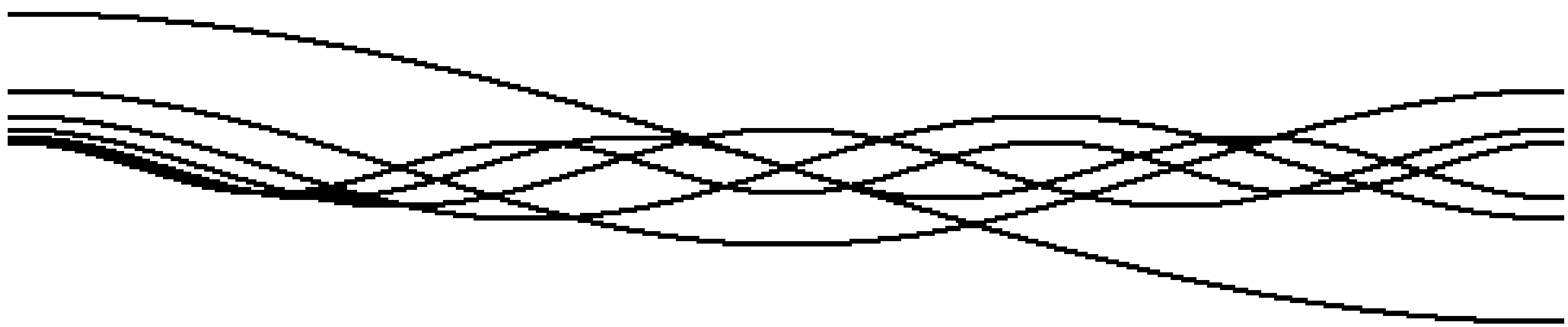


Generally harmonic relationship

Stiffness can cause minor stretching of harmonic frequencies

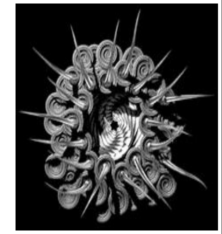
Modal Synthesis: Bars

Modes of Bars: Free at each end



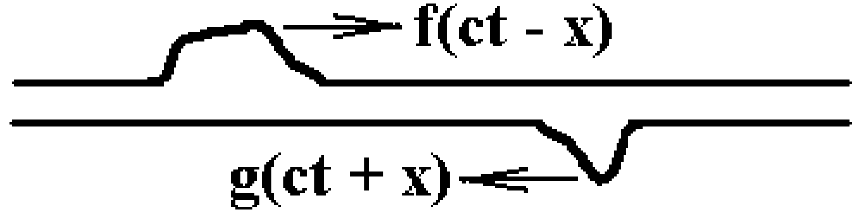
These would be harmonic, but stiffness of rigid bars stretches frequencies.

Modes: 1.0, 2.765, 5.404, 8.933



Physical Modeling: Strings

Plucked string model



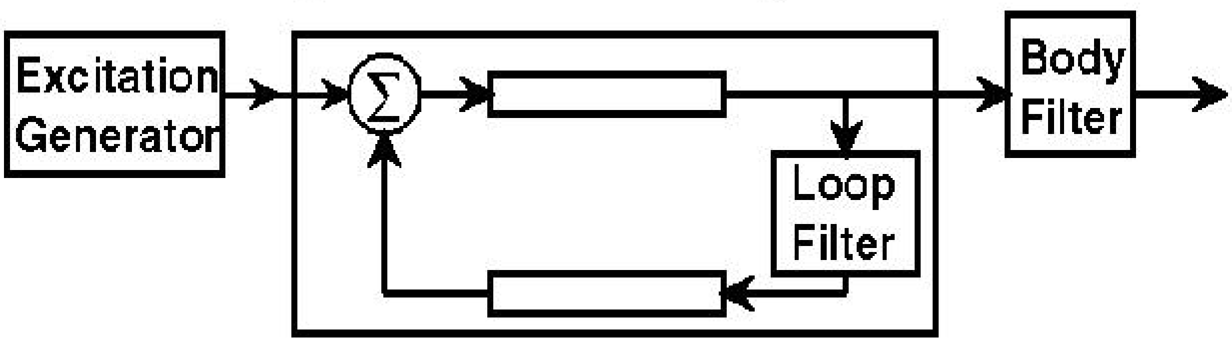
Use D'Alembert Solution

of 2nd order wave equation

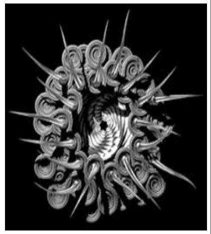
(left and right going components)

- Digital Waveguide Filter Solution

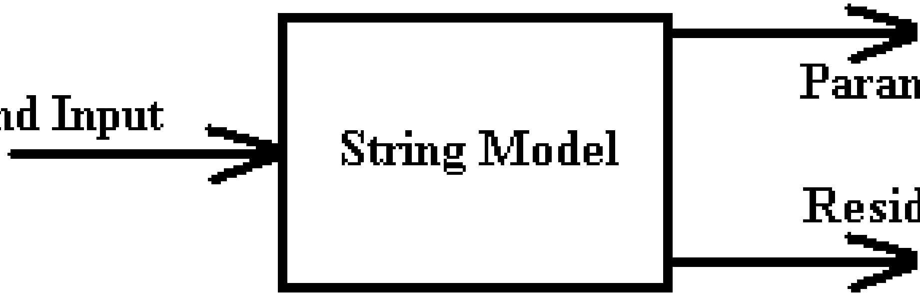
Simple Plucked String Physical Model Block Diagram



Physical Models: Strings



Can extract model parameters and residual excitation for plucked strings



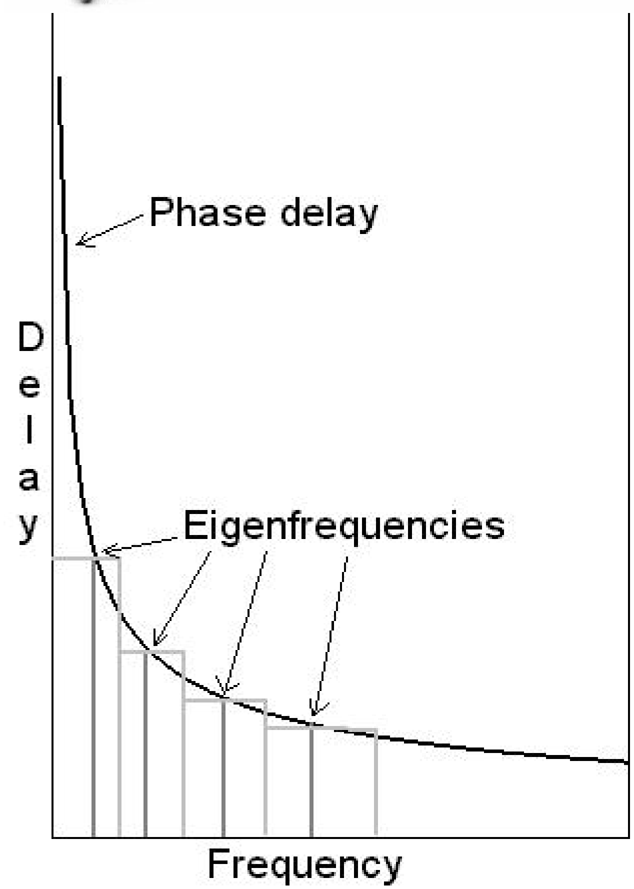
Bowed Strings: Model stick-slip friction curve of bow-string interaction

Physical Models: Bars

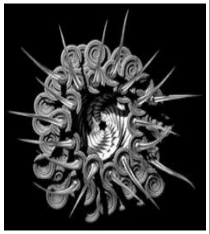
Stiffness in bars makes wave propagation frequency dependent

Idea:

- Model each mode with filter and delay
- Merge modal with waveguide
- Preserve spatial sampling
- Can strike, damp, bow



Banded Waveguides (Essl)



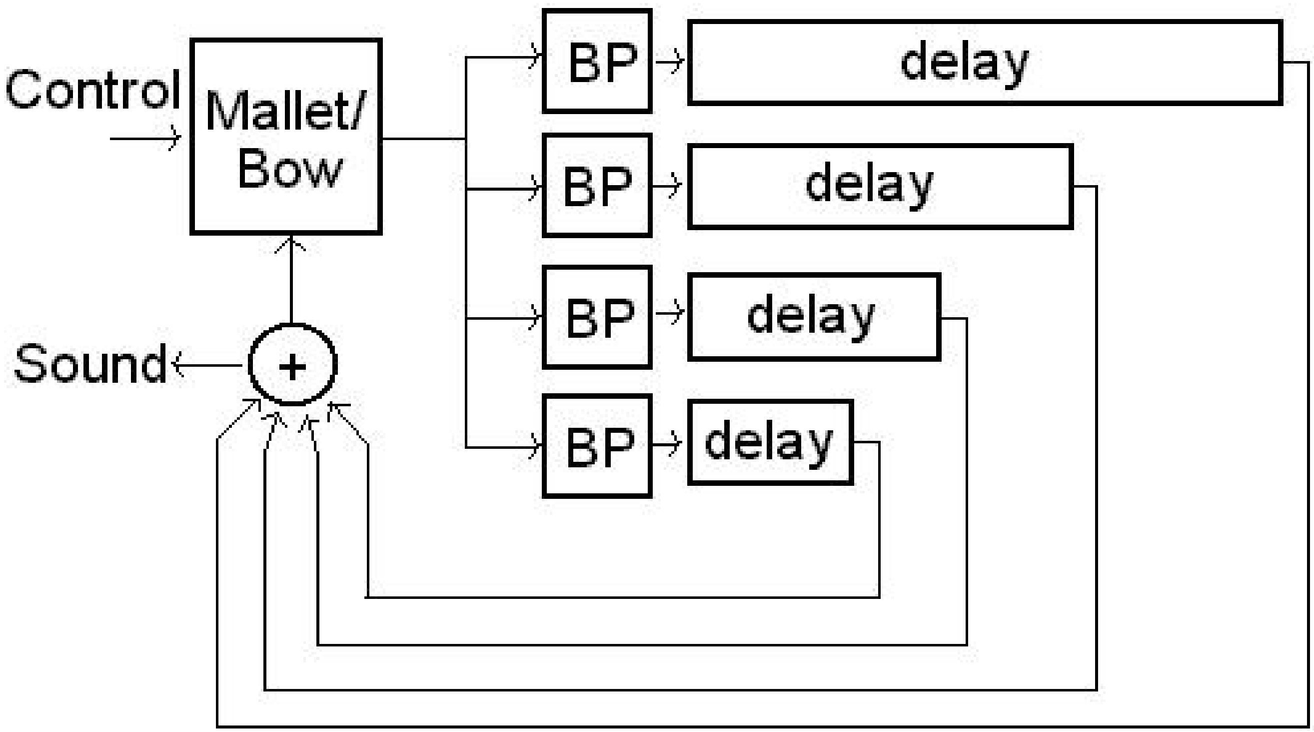
Acoustics:

- ***Approximate model of wave train closure and neighboring frequencies***

Filter:

- ***Comb filter with only one resonance***

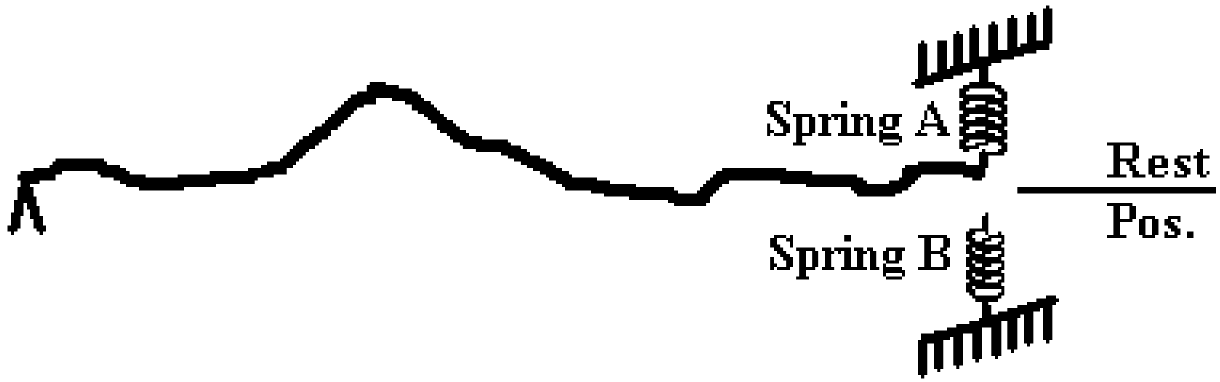
(high vs. low Q)



Physical Models: Non-linearity

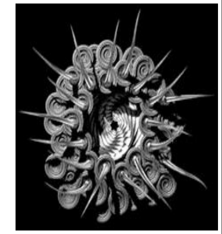
Add spring with position dependent constant

(one spring for positive displacement, another negative)



Acts to spread spectral components

Physical Models:
Bars , Plates, Non-Linearities



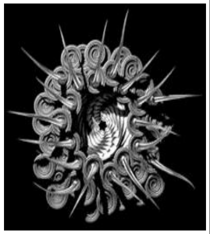
Strengths:

- Variety of sounds
- Computationally cheap
  (somewhat, under conditions)
- Physical (or pseudo) variables
- Naturalness of interaction

Weaknesses:

- No general system-ID techniques (2D + non-linear)
- Repeatability (not necessary a weakness)

References: Waveguide Modeling



For many references to specific physical models, please consult the reference sections of the papers attached as appendices to the course notes.   Other physical modeling papers:

Computer Music Journal, 1992-3, Two Special Issues on Physical Modeling, MIT Press, Vol. 16 No. 4 and Vol. 17 No. 1, Winter 1992 and Spring 1993.

Van Duyne, S. and J. Smith 1993. "Physical Modeling with the 2-D Digital Waveguide Mesh." In Proceedings of the ICMC, Tokyo, pp. 40-47.

J.O. Smith, “Acoustic Modeling Using Digital Waveguides,” in Roads et. al. eds., Musical Signal Processing, Netherlands, Swets and Zeitlinger, 1997.

Pierce, J. R. and Duyne, S. A. V. 1997, A passive non-linear digital filter design which facilitates physics-based sound synthesis of highly nonlinear musical instruments. *Journal of the Acoustical Society of America*, 101(2):1120-1126.

References: Friction

Siira J. and Pai D.K. 1996, “Haptic Textures, A Stochastic Approach,” IEEE International Conference on Robotics and Automation.

Fritz, J.P and Barner K. E. 1996, “Stochastic Models for Haptic Texture,” Proceedings SPIE Intl. Symposium on Intelligent Systems and Advanced Manufacturing.

Hayward, V., Armstrong, B. 1999. A new computational model of friction applied to haptic rendering. Preprints of ISER'99 (6th Int. Symp. on Experimental Robotics).