PID Control of a Plucked String

REALSIMPLE Project*
Edgar J. Berdahl and Julius O. Smith III
Center for Computer Research in Music and Acoustics (CCRMA), and the
Department of Electrical Engineering
Stanford University
Stanford, CA

Abstract

This laboratory assignment should teach the student about physical modeling and feedback control. These disciplines are applied to a digital waveguide model of a vibrating string. Students should complete the monochord laboratory assignment,[1] the weighted monochord laboratory assignment,[2] the harmonics laboratory assignment,[3] the traveling waves laboratory assignment,[4] and the digital waveguide model laboratory assignment[5] first.

Contents

*Work supported by the Wallenberg Global Learning Network
[1]http://ccrma.stanford.edu/realsimple/lab_inst/
[2]http://ccrma.stanford.edu/realsimple/weighted_mono/
[3]http://ccrma.stanford.edu/realsimple/harmonics/
[4]http://ccrma.stanford.edu/realsimple/travelingwaves/
[5]http://ccrma.stanford.edu/realsimple/waveguideintro/

1 Introduction

This laboratory assignment will help the student apply feedback control to a model of a musical instrument in a simulated laboratory environment. First we introduce feedback control, which helps further motivate the need for physical models. Next, we remind the student about the analogy between a mass-spring-damper system and a plucked vibrating string. Feedback control of the mass-spring-damper system is derived theoretically, and then feedback control of the more detailed plucked vibrating string model is tested in `pd`. In particular, the effects of Proportional-Integral-Derivative (PID) control on the harmonic content of the vibrating string are demonstrated.

2 Summary of Objectives

- Explain some of the basic ideas behind *feedback control* and how they relate to *physical models*.
- Describe how these disciplines may be applied to a *vibrating string*.
- Explain what *instability* is and how it may arise.
- Describe how modifying the parameters of a simple digital waveguide vibrating string model affect the *modeled harmonic content*.

3 Feedback control

Feedback control is the discipline in which system dynamics are studied and altered by creating feedback loops. It is applied in many different fields such as electrical engineering (circuit design), chemical engineering (process control), and mechanical and astro/aeronautical engineering (e.g. consider flight and propulsion systems, cruise control).

The block diagram for applying feedback control to a system is shown in Figure 1. This could be the block diagram for a cruise controller, where the car's velocity x would be driven to a target velocity x_{target}. u would be the rate at which gasoline were combusted, and r would be how far the gas pedal were pushed down by the driver. When no control were applied (i.e. $u = r$), certain system dynamics would describe the car's velocity x as a function of the gas r. A physical model describing this relationship could be developed and inserted into the block labeled "System" (see Figure 1). A simple physical model would probably assume that the road were flat, while a more accurate physical model would also take the steepness of the road into account. Either way, control theory could be used to derive appropriate contents for the "Controller" block in Figure 1 so that the composite, controlled system would behave in an altered, preferred manner.

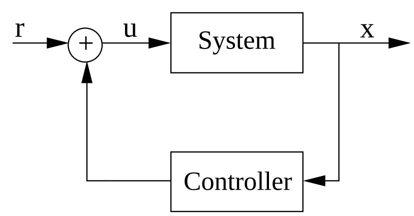

Figure 1: Standard block diagram for a feedback control system.

Figure 1 could just as well describe feedback control of a vibrating string. Here the "System" block would describe the vibrating string dynamics, and "Controller" would describe the electrical circuit connected between a string vibration sensor and string actuator.

4 PID Control

We will use the simplified physical model of a plucked vibrating string as explained in the monochord laboratory assignment[6]. Recall that as depicted in Figure 2, m is the mass in kg, K is the spring constant in N/m, and R corresponds to friction and is measured in N/(m/s). This kind of friction may for instance be implemented using a viscous resistance. Since we are studying musical instruments, the decay time of the displaced mass-spring-damper system will be long enough that the vibration will be almost periodic. The system may be considered "lightly damped", which is the same as having R be small. The m, K, and R parameters could be fit so that the mass-spring-damper system behaves like the lowest harmonic of a vibrating string.

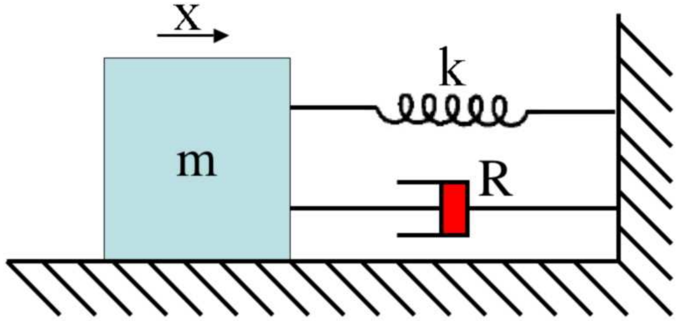

Figure 2: Mass-spring-damper system implementing a lightly-damped oscillator.

Apply Newton's second law $F = m\ddot{x}$, Hooke's law $F = Kx$, and the idealized friction law $F = R\dot{x}$, we obtain the dynamics of the forced oscillator as follows:

$$m\ddot{x} + R\dot{x} + Kx = F \tag{1}$$

In other words, the applied force F is balanced at all times by the sum of the resisting inertial, friction, and spring forces. In the case when $F = 0$, there are no external forces acting on the system. We have $f_0 \approx \sqrt{\frac{K}{m}}$ and the decay time constant $\tau = \frac{2m}{R}$.

However, if we implement the feedback law $F = P_D\dot{x} + P_P x$, then we arrive at the following differential equation [2]:

$$m\ddot{x} + (R - P_D)\dot{x} + (K - P_P)x = 0 \tag{2}$$

This feedback can be realized physically by using a sensor to measure the displacement x, scaling x by P_P, and adding the result to $\dot{x}$ scaled by P_D. The signal processing can be implemented using simple op-amp circuits, and the output signal can be fed to a motor that exerts the force F. This *controlled* system is equivalent to a system with friction coefficient $\hat{R} = R - P_D$ and spring constant $\hat{K} = K - P_P$. As a result, we have $\hat{f}_0 \approx \sqrt{\frac{\hat{K}}{m}} = \sqrt{\frac{K-P_P}{m}}$ and the decay time constant $\hat{\tau} = \frac{2m}{R-P_D}$.

[6] http://ccrma.stanford.edu/realsimple/lab_inst/

If instead we use $F = P_D \int x\,dt$, then we have

$$\hat{\tau} \approx \frac{2m}{R + \frac{P_I}{4\pi^2 f_0^2}}$$

although the analysis is more complicated [1]. Implementing $F = P_P x + P_I \int x\,dt + P_D \dot{x}$ is known as proportional-integral-derivative (PID) control. These controllers are often used to modify the system dynamics since the controller is simple and the user may freely choose the parameters P_P, P_I, and P_D.

In summary, for a single lightly damped oscillator, P_P may be used to alter the frequency of vibration f_0, and P_I and P_D alter the damping. Note that integral control P_I will do better at damping resonances with lower frequencies because integration provides a frequency weighting by $1/\omega$ (since $\int \cos(\omega t) = (1/\omega)\sin(\omega t)$, where $\omega = 2\pi f$ is radian frequency).

5 Instability

Consider the case where $P_D > R$. This implies that the net damping $\hat{R} = R - P_D < 0$, which corresponds to a scenario in which the magnitude of the vibration increases exponentially over time. At first this might seem to violate energy conservation since the kinetic and potential energy of the system increase over time. However, the power supply for the amplifier connected to the motor serves as the source of additional power.

Occasionally, certain kinds of instability may be desirable, but usually the goal is to quickly drive a quantity to zero. As a result, much of feedback control involves proving that controlled systems are stable. More sophisticated proofs revolve around proving how fast system quantities decay to zero.

6 Procedure

You will now carry out some simple experiments in `pd` . This will eliminate the need for specialized laboratory equipment and make it possible to ask some *"What if?"* questions. The simple mass-spring-damper model used in the previous sections was sufficient for understanding the roles of the PID controller coefficients P_P, P_I, and P_D. It turns out that the PID controller also works in a similar way when applied to a real vibrating string. In the following sections, we use a much more detailed digital waveguide vibrating string model, which is from the field of digital waveguide synthesis [3]. The model more accurately captures the physical behavior of a vibrating string, and so it also produces much more realistic simulated string sounds.

7 Controlling The Plucked String Model

Now you will gain some intuition into adjusting the PID control parameters P_P, P_I, and P_D. Try to avoid adjusting the PID parameters in a manner that causes the system to become unstable. If instability occurs, the output from the `pd` patch will become louder and louder until the `pd` patch shuts itself off and sends the message *UNSTABLE* to the `pd` message window. If you ever see this message, close the patch and open it again. One way to avoid instability is to adjust the parameters slowly–backing off if ever the sound from the model starts becoming louder and louder. The blue button "Reset parameters for no control" will set $P_P = 0$, $P_I = 0$, and $P_D = 0$ and may also be triggered by pressing the space bar.

The sliders for adjusting P_I, P_D and P_P are set up to make as much of the useful parameter space as available as possible. For example, the default position for the P_I slider, which corresponds to $P_I = 0$, leaves little room for making P_I negative by moving the slider to the left. This is because even slightly negative values will cause the energy in the string to grow quite fast. Note that the actual values for each PID control parameter is shown in a box underneath the corresponding slider.

- Download the `pd` patch `4-1.pd`,[7] and open it in `pd` .
- Ensure that the patch is not in editing mode, and check the "compute audio" box in the main `pd` window.
- To virtually pluck the string, press any key or click the "PLUCK" button. You may optionally check the box "pluck repeatedly." Adjust the "Master volume" slider on the right until the volume level is comfortable.
- The four sliders on the upper left calibrate the characteristics of the vibrating string model. Consider reviewing the digital waveguide model laboratory assignment[8] to remind yourself of the meanings of these parameters.
- The fourth slider labeled "Sensor and actuator position" may be used to move the virtual sensor and actuator along the string. So that the string may be controlled successfully, the sensor and actuator must be located at the same position [1]. What sensor locations cause the sensor to measure no energy at the second harmonic? (Hint: If the sensor is placed near one of these locations, then the sensor will measure very little energy at the second harmonic. You may observe this effect by viewing the spectrum while adjusting the "Sensor and actuator position" slider.)
- How do you adjust P_D so that the overall decay time constant becomes shorter?
- Click the button "Reset parameters for no control" and adjust P_I for integral control damping. What differences do you observe in the spectrogram when integral control damping is applied?
- While damping behaves as predicted in the theory section, you will find that altering the pitch is more difficult. This might seem surprising given our derivation, but recall that we assumed that there was only one harmonic. Now P_P alters the pitch of all of the harmonics simultaneously; however, it also alters the damping some. The way in which it alters the damping is complicated, so for larger values of $|P_P|$, one or more of the harmonics becomes unstable. This means that the pitch may be altered significantly only in conjunction with additional damping required to preserve stability. Try to see how far you can shift the lowest harmonic while preserving stability. Write down $\hat{f}_0$, P_P, P_I, and P_D for making $\hat{f}_0$ both as large as possible and as small as possible.
- Compare your results with those shown in Figure 3 where f_0 is chosen so high that only the lowest harmonic appears on the spectra. *Flatted* notes have relatively lower pitch (see

[7]http://ccrma.stanford.edu/realsimple/pidcontrol/4-1.pd
[8]http://ccrma.stanford.edu/realsimple/waveguideintro/

Figure 3, top), while *sharpened* notes have relatively higher pitch (see Figure 3, bottom). You will probably find that you cannot shift the pitch quite as far because your PID controller parameters are limited to a smaller space.

- Do you notice any problems in the sound of the controlled tones when the frequency is altered? One thing you may notice is inharmonicity. Can you surmise why this might occur?
- *Challenge problem:* Modify the subpatch `pd controlled-string` to induce tremolo in the modeled vibrating string. For our purposes here, we will let tremolo be defined as periodic variations in the amplitude of a signal. See the hint in the subpatch to help you decide where to make the required changes. Save the result into the file `4-1.tremolo.pd`.

References

[1] E. Berdahl, J. O. Smith, and A. Freed, "Active damping of a vibrating string," in *International Symposium on Active Control of Sound and Vibration (ACTIVE 2006), Adelaide, Australia*, http://ccrma.stanford.edu/ eberdahl/Papers/Active2006BerdahlSmithFreed.pdf, 2006.

[2] C. Besnainou, "Transforming the voice of musical instruments by active control of the sound radiation," in *International Symposium on Active Control of Sound and Vibration (ACTIVE 99), Fort Lauderdale, FL*, 1999.

[3] J. O. Smith, *Physical Audio Signal Processing*, `http://ccrma.stanford.edu/~jos/pasp/`, Aug. 2006, online book.

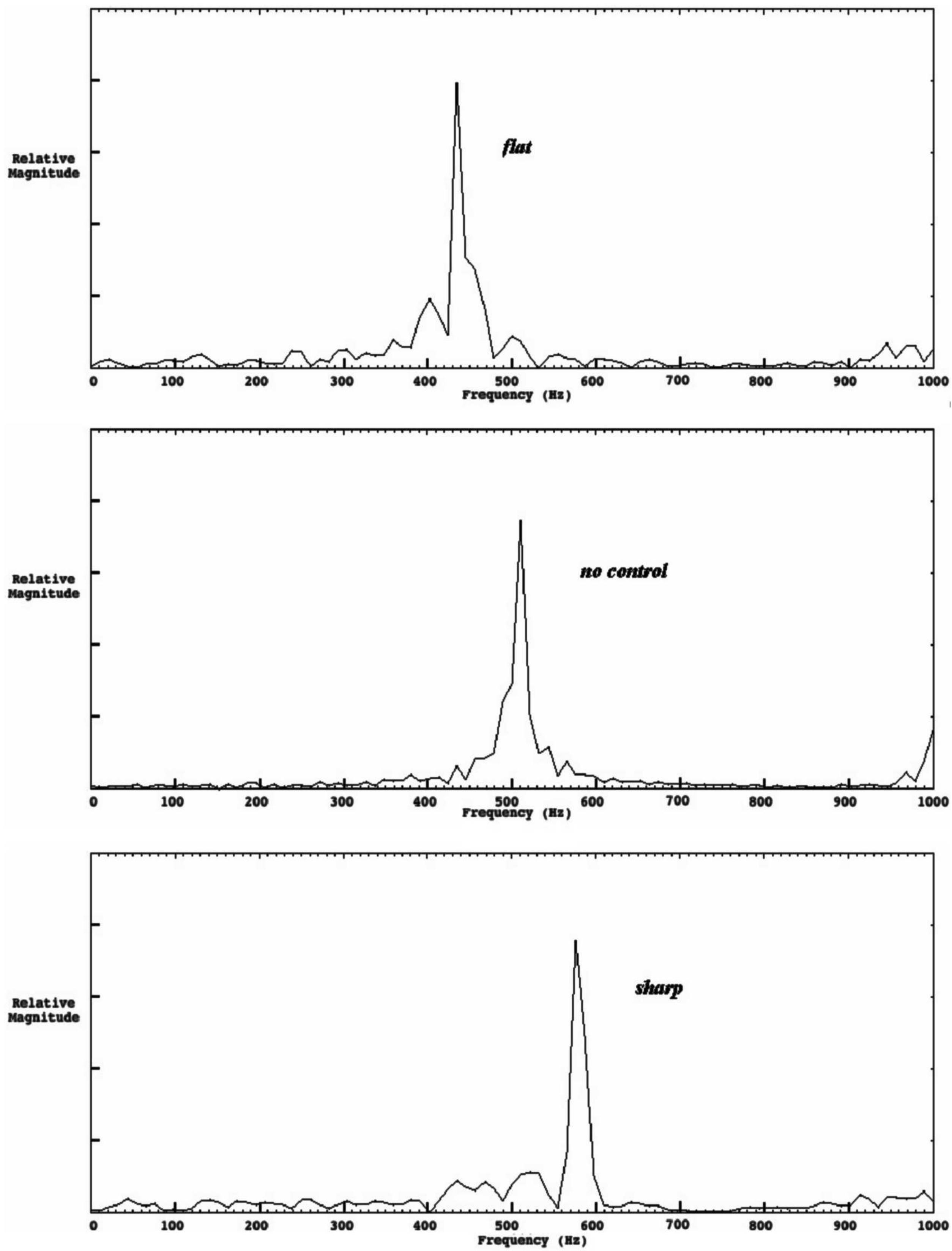

Figure 3: Example spectra for frequency shifting via PID control (top: flat (decreased pitch), middle: no control, and bottom: sharp (increased pitch))