Body Resonator Factoring

A valuable way of shortening the excitation table in commuted waveguide synthesis is to factor the body resonator into its most-damped and least-damped modes.

- The most-damped modes are then commuted and convolved with the external excitation.
- The least-damped modes can be left in parametric form (recursive digital filter sections)

Advantages:
- Excitation table is shortened
- Excitation-table signal-to-quantization-noise ratio is improved
- The most important resonances remain parametric, facilitating real-time control.
- Multiple body outputs become available (e.g., for more diverse spatialization)
- Resonators are often available in a separate effects unit, making them “free”
- Provides a fairly continuous memory vs. computation trade-off

Mode Extraction Techniques

The goal of resonator factoring is to identify and remove the least-damped resonant modes of the impulse response. This means finding and removing the narrowest “peaks” in the frequency response.

Two Basic Methods:

1. Complex spectral subtraction (equivalent to subtracting a second-order impulse-response)
   \[ H_i(z) = H(z) - \frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \]
   - Must accurately estimate phase and amplitude, as well as frequency and bandwidth
   - Requires resonators in parallel with residual
     \[ \Rightarrow \text{residual not readily commuted with string} \]

2. Inverse-filtering
   \[ H_i(z) = H(z) \left( 1 + a_1 z^{-1} + a_2 z^{-2} \right) \]
   - Factored resonator components are in cascade (series)
   - Residual (damped) modes more easily commuted with the string
   - Only need to measure frequency and bandwidth, not amplitude and phase

Mode Extraction by Inverse Filtering

Various methods are applicable for estimating spectral peak parameters:

1. Direct amplitude-response peak measurement on FFT magnitude data
   - Center frequency
   - Bandwidth

2. Weighted digital filter design

3. Linear prediction
   (special case of (2))

4. Sinusoidal modeling
   (like (1) but looking across multiple time frames)

5. Late impulse-response analysis
   (useable with all methods)

6. Work over Auditory frequency scale such as the Bark scale
   (useable with all methods)
Example of Body Resonator Factoring

Example of weighted digital filter design using `invfreqz` in Matlab:

```
Equation Error Fit, Noise = 1, In-Band Weighting = 100 (tsid.m)
```

```
"Measured" modal peak
Parametric fit to modal peak
```

```
Frequency (Hz)
```

```
Weighting function bandwidth
```

```
Magnitude (dB)
```


classical guitar body impulse response body before and after removing the first peak (main Helmholtz air resonance) using a second-order inverse filter:

```
Shortened Body Impulse Response
```

```
Shortened excitation can be truncated to \( \approx 100 \) ms
```

```
Shortened excitation can often be replaced by a filtered noise burst
```

Corresponding Amplitude Response

Normalized Bark-warped amplitude response, classical guitar body, before and after second-order FIR inverse filtering:

```
Localized Second-Order Mode Elimination Filter
```

```
\( H_r(z) = \frac{A(z)}{A(z/r)} = \frac{1 + a_1 z^{-1} + a_2 z^{-2}}{1 + a_1 r z^{-1} + a_2 r^2 z^{-2}} \)
```

```
A(z) = inverse filter determined by
- peak frequency
- peak bandwidth
```

```
A(z/r) = same polynomial with roots contracted by \( r \)
```

```
For \( r \approx 1 \) (but \( < 1 \) for stability), poles and zeros substantially cancel far away from the removed mode \( \Rightarrow \) mode removal is localized.
```

```
Similar in spirit to dc blocker
```

```
r \ can be interpreted as the new pole radius for the "canceled" pole.
```

```
Ideally, \( r \) should equal the radius of the neighboring poles so that all will decay at the same rate (recall late reverb synthesis story)
Matlab for Localized Peak Removal

freq = 104.98; % estimated peak frequency in Hz
bw = 10; % peak bandwidth estimate in Hz
R = exp( - pi * bw / fs); % pole radius
z = R * exp(j * 2 * pi * freq / fs); % pole itself
R = (1, -z + conj(z), z + conj(z)) % inverse filter numerator
r = 0.9; % zero/pole factor (notch isolation)
A = B .* (r .^ [0 : length(B)-1]); % inverse filter denominator
residual = filter(B,A,bodyIR); % apply inverse filter

Localized Peak Removal Example

In this example, r = 0.9 (arbitrary choice - not critical)

Close-Up on Localized Peak Removal Example

First eighth of previous figure.