DESIGNING THE SCALE OF THE BOEHM FLUTE

by John W. Coltman

The last several years have seen a renewed interest in devising proper scales for the Boehm flute, a result of the recognition that most flutes were not really well suited to the A 440 standard adopted internationally in 1953. There has also been during this period considerable progress in our understanding of the acoustics of the flute, and of those factors that influence the pitch that the musician obtains from the instrument. While there is much in the scientific literature that describes the phenomena involved, most of it is not easily accessible to those without training in physics, and so it has been of little help to the craftsmen and artists who are interested in improving the instrument. In this paper we attempt to remedy that situation by describing the requirements for producing a good scale, showing in quantitative terms how the various factors influence the pitch, and providing tables of important constants. We show how this information may be used in the design or modification of a Boehm flute so as to improve its intonation, without calling on elaborate mathematical procedures. A particular design and its resultant intonation are discussed.

What is a desirable scale?

The standard of pitch in universal use is the equal tempered scale based on a frequency of 440 Hz (hertz, or cycles per second) for the note A. This is a mathematically exact series of notes in which the octave bears a frequency ratio of 2 and is divided into 12 equal semitones. For purposes of measuring intonation, the semitone is further divided into 100 cents.

The ear is very discriminating, and some people can distinguish successive steady tones that differ by as little as one cent. In musical performance deviations of 5 cents are usually unnoticeable. We will use in this article the number of cents departure from an A 440 equal tempered scale as a measure of the intonation properties of a flute.

While an equal tempered scale provides, because of its uniformity, a very convenient standard for comparison purposes, it must not be supposed that this is what musicians play. There is, of course, the complex (and controversial) matter of various temperaments for keyboard instruments, and players may at times prefer intervals closer to a just scale than to an equal tempered one. But an overriding characteristic is that, in fact, musicians prefer (and usually play) a scale that is stretched. This means that the frequency ratio of the octave is greater than the sacred value of 2 assigned to it by mathematicians since the time of Pythagoras.

Dr. John W. Coltman is a retired research executive of the Westinghouse Electric Corporation and has devoted much of his spare time to the study of the flute in its historical, musical, and acoustical aspects. His collection of some 190 flutes and flute-like instruments illustrates both the variety of types to be found all over the world and the historical development of the orchestral flute. He has played flute and piccolo since school days and is an active participant in amateur chamber music.

It is well established[1] that given the opportunity to adjust manually the setting of an octave between two successively presented tones, musicians and non-musicians alike will set the ratio at more than 2, by amounts that are small in the mid-range but increase to 20 cents or more in the upper ranges of the flute.

The pitches employed by a musician may differ when playing in an ensemble, where concordance with the upper harmonics of lower-voiced instruments is desirable, than when playing solo, where more freedom is permissible. Pianos are typically tuned[2] with a stretch in the upper and lower ranges, so the soloist working with piano accompaniment will want to use a stretched scale also. The ideal flute scale then appears to be one in which the first register follows nearly equal temperament, but with a sharpening trend as the scale ascends, amounting to about 15 cents at the high end. Most important, the scale should be smooth, so that the flutist does not have to compensate for the errors of individual notes. Most flutes today exhibit the rising trend (some to an excessive degree), but they also show, to varying degrees, undesirable deviations in individual notes.

Factors determining flute intonation

From an elementary standpoint, the flute may be looked at as a simple tube, open at both ends, in which the air column vibrates at a frequency determined by its length, the velocity of sound, and the number of segments in which it is vibrating. In principle, the column terminates at the first open hole, and this length determines the frequency of the fundamental and of the octave, twelfth, etc. produced by overblowing. While these considerations suffice for a first approximation, a large number of perturbing factors must be taken into account.

The velocity of sound is not, in fact, constant along the tube; it depends on the temperature and composition (water vapor and carbon dioxide) of the air, which are modified by the performer's breath. The internal diameter of the tube is not uniform — in the body there are cavities under the keys, and the headjoint is tapered. An open finger hole does not completely terminate the column; some of the vibration is transmitted to the next open hole, so that the real termination is a succession of apertures. At the other end of the tube the restricted mouth hole, which is covered to a greater or lesser degree by the flutist as he plays, constitutes a major variable in the problem of scale design. There is a cork cavity above the mouth hole so that the column is not terminated by the mouth hole alone. The embouchure hole is also acoustically coupled to the flutist's mouth cavity, which has a resonance in the middle range that can affect the resonance frequency of the system[3]. Finally, the strength of blowing can force the frequency above or below the natural resonance frequency of the system.

The presence of these perturbing factors makes the design of a properly tuned instrument a complex process that is hardly ever attempted from first principles. Instead, reliance is placed on copying existing instruments, and making changes to correct deficiencies. A knowledge of the nature and degree of influence of these perturbing factors is important to this process, and in this article we will define quantitatively these factors and show how they may be used in the design process.

Measuring Flute Intonation

The frequencies produced by a given instrument are greatly affected by the manner in which it is blown. Each individual flutist has his own blowing habits; moreover his ability to produce the same frequency for a given note on separate trials is limited.

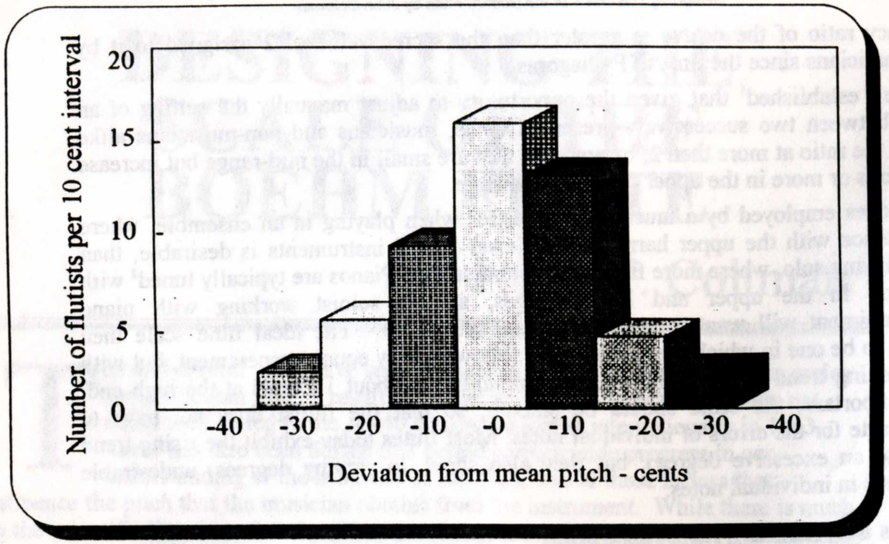

Figure 1 - Distribution of flutists according to the mean pitch each produced for the tuning A.

Measurement of the intonation characteristics of an instrument is therefore not a simple task.

A great deal was learned about the pitches produced by players on the flute in an experiment carried out at a National Flute Association meeting in 1975[4]. In this experiment fifty flutists played specified notes on a single flute, while the frequencies they produced were measured with an electronic counter. Thirty professionals and 20 non-professionals took part. We recap here some of the results.

Stability

A flutist will not produce exactly the same pitch each time a note is played. In the above test it was found that, on average, a flutist can repeat the pitch only to within six cents. This means that one can expect the average flutist will play a particular note more than six cents sharp or more than six cents flat (relative to his own mean value) one third of the time.

Stability varies from one flutist to another. The professionals had scores ranging from 2.8 to 9.8. The amateur scores ranged from 3.3 to 8.2, differing insignificantly from the professionals. My personal score (this relates to Figure 5 in this article) was 5.6 cents, close to the average.

Variation among flutists.

While the variations produced by a particular flutist are substantial, they are much less than those associated with different flutists. Figure 1 shows how the flutists are distributed according to the mean pitch produced by each for the note A in the first octave. Of the fifty flutists, 16 were within plus or minus 5 cents of the grand average, while 6 were more than 15 cents flat, and 2 sounded more than 25 cents sharp! Three out of the six flattest were professionals.

Intonation testing by playing.

From the above results, it is apparent that testing a flute by playing it requires considerable care in arranging and performing the tests, together with a great many readings, in order to get a meaningful average.

It is difficult to make recommendations about selecting a player. A highly trained and skilled musician is probably to be avoided - with a fine ear and lots of experience, even good intentions on the flutist's part may not avoid unconscious compensation for instrument errors. It is probably better to select a few good music students, have them make some preliminary tests on a few notes, and choose a player (or better, more than one) whose stability is high and who produce results consistent with the average. In my own case I play the flute myself, reassured, somewhat, by the fact that my own readings in the NFA test were close to average. This has the advantage that different flutes are compared using the same player, which, of course, is desirable if one is selecting or designing a flute for one's own use.

It is preferable to test all the notes on a flute, but this involves a large number of readings, and one may settle for a selection throughout the registers. The notes should include the middle and upper C#'s, two notes that are troublesome. A simple procedure is to write out a piece of "music" that presents the notes in some random order. One should avoid familiar musical intervals. Each note to be tested should appear just three times somewhere in the piece. After tuning the flute to A440, the subject should play the notes without vibrato at moderate loudness, sustaining each for a time long enough for an electronic tuner to settle and an observer to record the pitch. It is well to instruct the player that the test is meant to measure what the flute produces, not to find out how well the player can play in tune. This may allay attempts to bend notes to suit what the player thinks is a proper intonation. The player should, of course, not see the tuner during the test, nor the recorded results until all the sessions are complete. This test should be repeated at least four times, with intervals during which the player is engaged in some

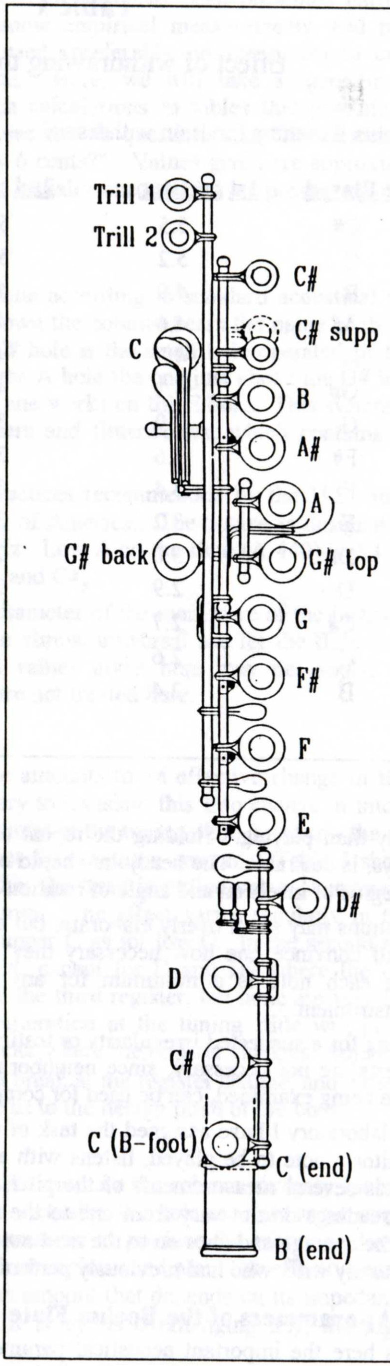

Plate I.

Table I

Effect of withdrawing the headjoint

Cents flattening for 1mm withdrawal —(100 cents = 1 semitone)

Note Played	1st register	2nd register	3rd register
C#	5.5	5.4	4.4
C	5.2	5.2	4.2
B	4.9	4.9	4.9
A#	4.6	4.6	4.6
A	4.3	4.3	4.0
G#	4.1	4.1	4.2
G	3.9	3.9	3.9
F#	3.6	3.7	3.8
F	3.4	3.5	4.1
E	3.2	3.3	3.2
D#	3.0	2.7	3.6
D	2.9	3.1	4.1
C#	2.7		
C	2.6		
B	2.4		

other activity than playing. Holding the repeat tests in separate sessions, or even on different days, is desirable. The headjoint should be marked (Scotch tape is handy here) so that its degree of insertion and angle of rotation can be accurately duplicated.

Such precautions may seem overly elaborate, but a look at the variation in the readings obtained will convince one how necessary they are. A total of twelve independent readings on each note is a minimum for any serious attempt at characterizing a particular instrument.

When looking for a suspected irregularity or testing the effect of making a change, such elaborate tests are not necessary, since neighboring notes, played in close conjunction with the one being examined, can be used for comparison.

In my own laboratory I have assigned the task of observation to a computer. It presents on the monitor a note to be played, listens with a microphone to the player's sustained sound, makes several measurements of the pitch every second, and when it finds that successive readings do not vary from one to the next by more than two cents, records and stores the average and goes on to the next note. The process is fast, objective and a great boon to my wife, who had previously performed the duty of observer.

Acoustical parameters of the Boehm Flute

We discuss here the important acoustical parameters that affect the intonation of the flute and provide quantitative information that can be applied directly in flute design.

The manner in which these parameters can be used has been presented earlier in a paper[5] that gives the theoretical basis, some empirical measurements, and provides formulas and procedures that may be used (preferably on computer) to calculate resonance frequencies of the entire flute. Here, we will take a more simplified approach, and present the results of such calculations in tables that give numerical values relative to changes. For example, we answer questions like "How much should this hole be moved to sharpen the note by 6 cents?". Values given are approximations that are valid over the range of variation usually encountered in present day Boehm flutes.

Conventions

In this paper we name the holes in the flute according to standard acoustical practice (see Plate I). The first open hole going down the column bears the name of the note it produces in the first register. Thus the C# hole is the small hole operated by the first finger, the C hole is the thumb hole, and the A hole the one just above the G# lever. If one wishes to change the intonation of E, one works on the E hole. This system differs from the one often used by music teachers and flutemakers, which contains several confusing anomalies.

The naming of the octave follows the practices recommended by the U.S. Standards Association and by the Acoustical Society of America. The octave is designated by a subscript number, starting at zero for 16 Hz. Low A on the flute (A 440) is A4 and the first register runs from B$_3$ or C$_4$ through C$_5$ and C#$_5$.

For all of the cases discussed, the inside diameter of the main bore of the instrument is assumed to be 19.0 mm, a value that is in almost universal use for the flute in C. For other diameters, some of the numerical values given here may not apply. Scaling methods are available for such cases, but are not treated here.

Changing the column length

In several cases, an acoustic change made amounts to an effective change in the total length of the air column, and it is necessary to translate this into change in intonation. A simple example is withdrawing the headjoint at the tuning slide or tenon - the pitch of each note on the flute is changed, but not by the same amount. Table I shows the flattening (in cents) for each millimeter that the head is pulled out or, conversely, the sharpening produced by pushing it in 1 mm. The effect varies regularly in the first register, being just twice as great for the upper C as for low C. In the second register, the values closely match those in the first, except for D and D# where the C# vent comes into play. Values vary irregularly in the third register, but none are very far from 4 cents. It is apparent that substantial alteration at the tuning slide will produce a sloping intonation characteristic, with a break where the register changes. Adjusting the headjoint to give a smooth scale, without a break at the register change, and measuring the resulting A will give an important clue as to the design pitch of the body.

The holes and their keys

Ideally, an open tone hole in the flute would terminate the column abruptly, isolating all the tube below it from acoustic vibration. Actually, the hole exhibits some impedance to the vibration; that is, it takes acoustic pressure to produce the vibratory flow in the hole, and this pressure acts also on the column below the hole. The effective end of the air column then lies beyond the hole, by an amount that depends on its impedance. The smaller the hole, or the more restricted it is by its overhanging key, the higher its impedance.

Table II

Value of h(mm) for various hole diameters, key styles and key rise. The normal rise for most keys on the flute is 2.0 mm at the center.

	Hole Diameter, mm.	Rise at Center, mm. 1.5	2.0	2.5	3.0
Solid key, 18.9 mm outside diameter:	12.5	31.7	28.7	27.1	26.4
	13.0	31.1	27.9	26.2	25.2
	13.5	30.2	27.0	25.2	24.0
	14.0	29.0	26.0	24.1	22.8
	14.5	27.6	24.7	22.9	21.6
	15.0	26.0	23.3	21.6	20.4
Solid key, 19.9 mm outside diameter:	12.5	32.1	29.2	27.5	26.7
	13.0	31.6	28.4	26.6	25.6
	13.5	30.7	27.5	25.6	24.4
	14.0	29.5	26.5	24.5	23.1
	14.5	28.1	25.2	23.2	21.8
	15.0	26.4	23.8	21.9	20.4
	15.5	24.4	22.3	20.5	19.0
	16.0	22.2	20.5	19.0	17.5
Perforated key, 18.9 mm outside diameter:	12.5	26.6	25.2	24.6	24.0
	13.0	25.6	24.2	23.3	22.6
	13.5	24.5	23.2	22.1	21.4
	14.0	23.4	22.1	21.0	20.2
	14.5	22.2	21.0	19.9	19.1
	15.0	21.0	19.8	18.9	18.1
Perforated key, 19.9 mm outside diameter:	12.5	26.9	25.6	24.9	24.2
	13.0	25.9	24.6	23.7	22.8
	13.5	24.9	23.6	22.5	21.6
	14.0	23.8	22.5	21.4	20.5
	14.5	22.7	21.4	20.3	19.4
	15.0	21.5	20.3	19.2	18.5

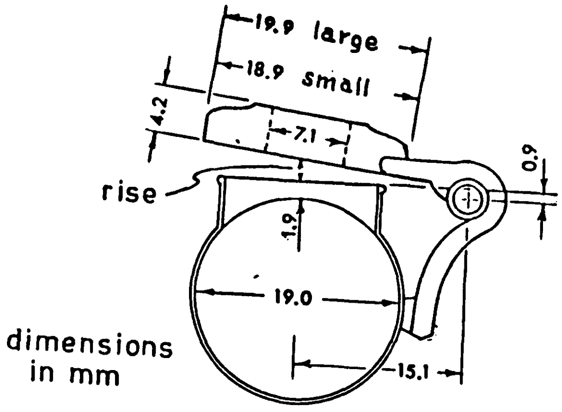

Figure 2 - Geometry and dimensions of the large holes and keys measured.

The measure of impedance that is used here is the effective height "h" of the hole. This is defined as the length of tubing of bore diameter the same as that of the main bore, that would have an equal impedance at low frequencies. Whenever we use the term "hole height" or "h" we are referring to this measure, not to an actual dimension. To a good approximation, the hole height "h" is all we need to know about an open hole in order to specify how it affects flute intonation.

Figure 2. shows the geometry of the holes and keys whose acoustic heights were measured, and defines their dimensions. The rise of the key was measured at the center, rather than at the edge, because this makes the results independent of reasonable changes in the length of lever arm. The chimney dimension is measured from the inside of the bore to the rim, on which the pad seats, at the point where this dimension is a minimum.

Table II gives values of hole height "h" for large holes and keys of several types as a function of hole diameter and key rise. If you have a hole and key of a particular geometry and type, you may find it in the table or, if it does not fit exactly, interpolate between neighboring values to estimate its "h" to a close approximation. If the chimney length you use is greater or smaller than 1.9 mm, add (or subtract if less) that difference multiplied by the square of the ratio of the bore diameter to the hole diameter. For example, an added .5 mm of chimney would add .5 x $(19/13)^2$ or 1.1 mm to "h" for a hole 13 mm in diameter.

Figure 3 gives curves to find the values of "h" for small holes such as the C# and trill keys. We will show in a later section how "h" is used to determine intonation properties.

Cavities

When a key is closed it leaves a cavity underneath, whose dimensions depend on the hole diameter, chimney length and any perforation in the key. The effect on flute intonation of a single cavity depends on the state of vibratory motion at the hole location. If the hole is at a place where the sound pressure is maximum, the extra

Table III

Cavity effect constants for several keys with dimensions as in Fig. 2.

Hole Diam., mm	Key	"a", mm	"b", mm
7.4	Solid	0.12	0.06
12.9	Solid	0.8	0.5
12.9	Perforated	1.3	0.7
13.9	Solid	1.2	0.6
13.9	Perforated	1.7	0.8
15.6	Solid	1.8	1.0
15.6	Perforated	2.2	1.2

Figure 3 - Hole height "h" of small holes with keys.

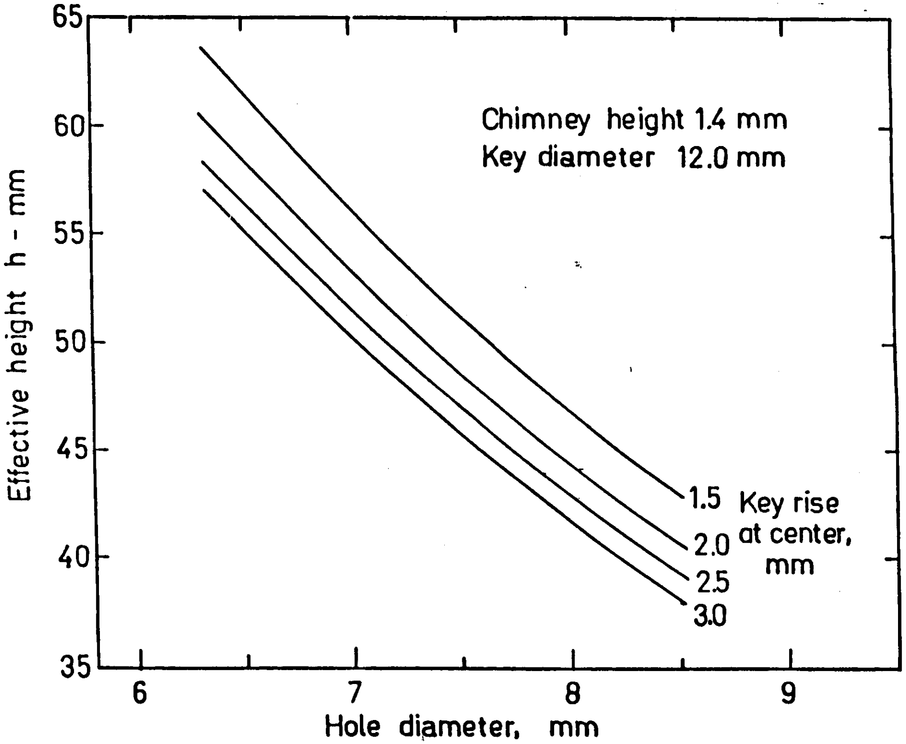

Table IV

Cumulative lengthening (mm) due to cavities for an open key flute, and octave shrink, cents.

Fingered note	1st register change, mm	2nd register change, mm	Octave shrink, cents
C (lowest)	7.8		
D	5.8	6.0	3.0
E	2.6	5.9	11.1
F	1.6	5.3	13.2
G	.1	3.5	13.6
A	-.3	1.4	7.6
B	0	0.6	3.0
C	0	0.5	2.7

volume of the cavity acts to effectively lengthen the air column. If it is at a place where the sound pressure is low but the vibratory motion high, it effectively shortens the column. The amount of this maximum effective lengthening "a" or shortening "b", in millimeters, is given for several types of keys in Table III. At locations between pressure maxima and minima the effects combine, so that at some positions there may be zero effect.

Obviously, cavity effects present a complicated situation. For each note there are several cavities acting, each in a different position relative to the sound pattern characteristic of that note. In Table IV the total lengthening due to the combined action of all cavities is given for each of several notes on the flute. For example, at low F cavities add 1.6 mm to the column, but at middle F they add 5.3 mm, so that from this cause alone, the octave would be shrunk by 13.6 cents. It is clear that cavity effects are not negligible. If one uses higher chimneys than those specified in Figure 2 (1.9 mm), the effects are exaggerated. Values of "a" in Table III will be increased by an amount equal to the added chimney length multiplied by the square of the ratio of hole diameter to bore diameter. Values of "b" will be changed somewhat less.

Intonation changes due to hole changes.

Table V shows how changes in hole height or position affect the intonation of the note for which that hole is a termination. Values are given for 1mm change. Linearity is a good assumption for the small size changes normally encountered, so one can multiply these values by the size change in millimeters to obtain the total effect.

The first column shows how much the pitch (in cents) of a specified note in the first register will be changed by moving its hole 1 mm; flattened if moved down or sharpened if moved up the tube. Values in the second register are the same except that for the upper C# (an unusually small hole) the value becomes 3.1 cents, and, because of the vent, the value for D in the second register is 2.3 cents. Values for the third register present complications, because the hole involved is not necessarily the one named by the note sounded; for example, high D is affected primarily by the G hole. The venting also comes into play, and there are enhanced effects from open holes farther down the

Table V

Effect of changing hole parameters, flattening in cents.

Hole or key	Move center 1 mm down	Increase key height "h" 1 mm	Trade-off ctrmove/hchange
C# (conv.)	3.8	1.1	-.28
C	4.3	1.8	-.39
B	4.1	1.7	-.40
A#	4.0	1.8	-.43
A	3.8	1.8	-.45
G# (back)	3.6	1.6	-.43
G	3.4	1.6	-.44
F#	3.3	1.7	-.48
F	3.1	1.6	-.49
E	3.0	1.6	-.50
D#	2.8	1.5	-.52
D	2.7	1.5	-.53
C#	2.6	1.5	-.54
C	2.5	1.4	-.55

flute. Generally, the effects in the third register will be less than those in the first or second. At any rate, hole position will usually be governed by requirements of the first two registers.

The second column of Table V shows the change in pitch due to changing the value of "h" of the hole by 1 mm. The third column gives the trade-off between these, that is, how much the center position must be moved to compensate for a change of "h".

As an example of the use of these tables, suppose one finds that the note G on a particular flute is too sharp by 8 cents. Column 1 of Table V shows that 1 mm of movement will flatten this note 3.4 cents — so, to get 8 cents flattening it should be moved 2.4 mm down the tube. Again, if the G# was found to be 3 cents flat, we might correct it by decreasing its value of "h" by 1.9 mm (3 divided by the value of 1.6 found in column 2 for G). An examination of Table II, giving values of "h" for a key having the dimensions at hand, tells us that this can be accomplished by changing the key rise from 2 to 2.5 mm. One can do this easily by sanding the cork under the G# lever.

As a more elaborate example, suppose we wished to substitute perforated keys for plateau keys. One may examine Table II to find that the "h" for a particular key in question (say the E key) is 27 mm for the hole with a plateau key, but 23.2 mm for a perforated key of the same dimensions — a change in "h" of -3.8 mm. If this substitution were made without changing the hole position, it would sharpen the note E by 1.6 cents for each mm (column 2 of Table V) or a total of 6 cents. However, from column 3 of that table we find that 1 mm change in "h" is compensated by -.55 mm change in position, so we need to move the hole -.55 x -3.8 = 2.1 mm. Positive values

represent increases in air column length, so the hole should be moved down the tube. A similar calculation is to be made for each of the keys being substituted.

These procedures go faster, with a bit more precision, by using a computer program that incorporates the above data and techniques. It also permits scaling for different main bore diameters, and is applicable to piccolos, alto flutes, and bass flutes. *

The headjoint.

There are three aspects of the headjoint that affect intonation: the embouchure hole (including the lip position and blowing habits of the flutist); the cork cavity; and the taper. All of these interact, so that the situation is too complex to be treated here in detail. Some representative results that may be used as a guide to experiment are presented. More elaborate treatments may be found in References 6. and 7.

The mouth hole terminates the upper end of the air column. Like the finger holes, it has a hole height "h" that specifies how much tubing is effectively added to the column. This hole height is large because the hole is small. More importantly, the hole height varies with the note played because the flutist varies the amount of the hole that is covered by the lip, for the following reasons.

An air particle in the jet of air issuing from the flutist's lips takes a certain time to travel across the space between the lip and the edge of the embouchure hole. A governing principle of the sounding mechanism is that this time must be a certain fraction of the time of one cycle of oscillation. Thus the time must be adjusted by the flutist in order to sound the note he desires. He can shorten the time in two ways: by blowing with more pressure, which speeds the particle up; or by moving his lips forward to shorten the distance to the edge. All flutists do both as they ascend the scale, but some use greater pressure and longer distance, while others use lower pressure and bring the lips closer to the mouth hole edge. In any case, the coverage, and therefore the effective hole height, is larger for higher notes. How much larger depends on the particular lip positions used by the flutist.

The primary reason for tapering the headjoint is to compensate for the change of hole area by the flutist. Since this varies in degree from one flutist to another, it is desirable to make available headjoints having different tapers.

Beyond the mouth hole is a short length of tubing terminated by the cork. The cavity thus formed supplies some compressibility that the mouth hole lacks, so together they behave acoustically, much more like a short additional length of air column than does the mouth hole alone. The effect of this cavity depends only on its volume, which may be adjusted by moving the cork. Changing the cork position chiefly affects the upper portion of the flute scale, as exhibited in Figure 4.

The diameter of the bore at the mouth hole is usually about 17.3 mm. The diameter gradually enlarges over a distance of about 120 mm until it reaches the 19.0 mm diameter of the main column. The form, in most cases, is nearly conical, modified a bit to avoid sudden changes in slope where the cylindrical portions begin. The function of the taper is to shorten the effective acoustic length of the air column as the frequency increases, introducing a sharpening in pitch that compensates for the flattening produced by lip movement and other geometric effects as one ascends the scale.

The properties of the headjoint that affect intonation can be compactly expressed by

---

*** The program is available from the author, John Coltman, 3319 Scathelocke Road, Pittsburgh, PA 15235 at a cost of $35. IBM compatible only; specify disk size wanted.***

stating its equivalent acoustic length; that is, the length of tubing of constant 19.0 mm diameter that would give the same resonance frequency if substituted for the headjoint. This equivalent length varies with frequency for all of the reasons given above.

Figure 4 plots the equivalent length of a particular headjoint as a function of frequency over the entire range of the flute. It includes the variation of lip position with frequency typical of the average flutist. The curve varies smoothly over the range, starting with a length of 211 mm and decreasing to about 200 mm at the top end. The general effect is to sharpen the pitch as the scale is ascended — most rapidly in the middle register, less rapidly at the beginning and end. This sharpening characteristic compensates for some other flattening phenomena, such as, cavities and lack of complete termination by the finger holes. What remains gives rise to the stretched characteristic of the flute scale, which is a desirable feature. The effect on intonation of any particular note, due to changes in effective length of the head joint, may be obtained by the use of Table I.

The upper dotted curve shows how the trend is modified by moving the cork. We see the effects are largely confined to the third register, where 1 mm change in the cork distance can change the effective length of the column by 1.5 mm, flattening (for outward movement) these notes by roughly 6 cents. This is a valuable adjustment to have available, as flutists with different techniques produce different values of octave stretch. Any serious flutist in possession of a new flute should measure the intonation he or she produces and adjust the cork appropriately, rather than relying on the sometimes heard advice to leave the cork-setting where the manufacturer placed it.

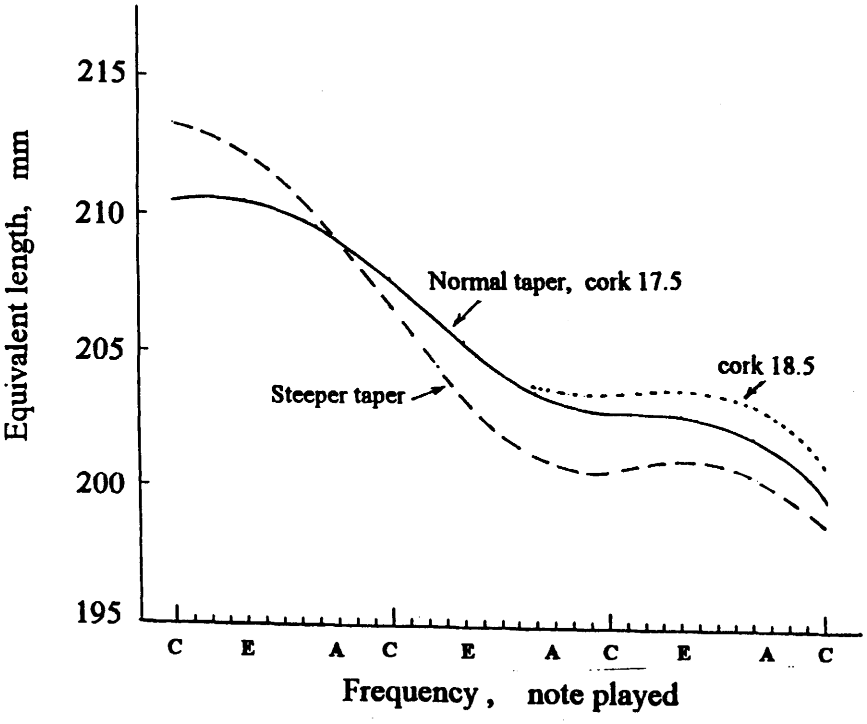

Figure 4 - Equivalent length of headjoints as a function of frequency. Normal taper 17.3 mm bore at mouth hole, steeper taper 16.8 mm.

While tapers can take different forms, the primary variable (assuming reasonably regular taper form) is the diameter of the bore at the mouth hole, to which the steepness of the taper must conform. To give some idea of what happens here, the dashed line curve is a characteristic for a similar taper fitting a mouth hole bore diameter of 16.8 rather than 17.3 mm. We see a substantial increase in slope, giving a much more pronounced sharpening (decrease of effective length) with frequency. The two curves have been brought to coincide at low A, as one would do with the tuning slide, but an octave higher the more tapered head has shortened its length 2.7 mm more, introducing an additional octave stretch of 12 cents. Thus, a quite small change in bore diameter at the mouth hole can substantially influence the scale. A flutist who uses small pressure change, but large lip movement, in effecting the octave jump might find the steeper taper gives him better intonation.

A Wimberly flute with the "Coltman Scale"

Readers who are interested in descriptions and analyses of the scales of some Boehm flutes of the past may wish to refer to articles 8., 9. and 10. in the list of references. Here we will describe in detail one specific example. The flute in question is one made for me, to my design, by David Wimberly, now in Halifax, Nova Scotia. The design was carried out with the help of a computer program that calculated the resonance frequencies of the column, including blowing corrections derived from my experience

Table VI

Mechanical parameters of a Coltman Scale
All dimensions in millimeters. Bore Diameter 19.0 mm

Hole or key	Center position	Hole Diam.	Key cup Diam.	Chim-ney	Center rise	Height "h"	Cavity a	b
Trill 1	-132.2	7.3	13.0	2.0	2.0	54.2	.3	.2
Trill 2	-112.3	7.6	13.0	2.0	2.0	51.2	.3	.2
C# conv	-94.3	7.6	14.5	2.8	2.0	57.5	.5	.3
{C# vent}	-92.5	5.6	13.0	2.2	2.2	77.2	.2	.1
{C# supp}	-71.9	9.5	14.5	1.3	2.2	38.4	.4	.2
C thumb	-62.3	13.5	18.5	1.3	3.0	22.5	.7	.4
B	-43.3	13.5	18.5	1.3	2.3	24.4	.7	.4
A# perf	-21.7	13.5	18.5	1.3	2.3	21.2	1.3	.6
A perf	0.0	13.5	18.5	1.3	2.3	21.2	1.3	.6
G# (back)	22.9	14.3	18.5	1.3	2.3	22.5	1.0	.5
G# (top)	23.7	12.5	18.5	1.3	2.3	25.9	.7	.4
G	47.0	14.3	18.5	1.3	2.3	22.7	1.0	.5
F# perf	73.9	14.3	18.5	1.3	2.3	19.6	1.5	.7
F perf	102.5	14.3	18.5	1.3	2.3	19.6	1.5	.7
E perf	131.4	14.3	18.5	1.3	2.3	19.6	1.5	.7
D#	160.1	14.3	18.5	1.3	2.0	24.1	1.0	.5
D	194.0	15.7	20.1	1.3	2.9	18.0	1.4	.8
C#	226.8	15.7	20.1	1.3	2.9	18.0	1.4	.8
C (b-foot)	262.4	15.7	20.1	1.3	2.9	18.0	1.4	.8
C end	270.0	-	-	-	-	5.8	.0	.0
B end	308.0	-	-	-	-	5.8	.0	.0

with existing flutes. Most of the hole diameters, key cup sizes, etc. were chosen with available tooling in mind, and the necessary acoustical parameters were calculated for these. We discuss here some of the considerations that went into this design, and some reasons for the intonation variations present in the resulting scale.

Table VI gives the critical mechanical dimensions and some derived parameters for this Wimberly flute, serial #42. The flute has a "Coltman C#"[11] that substitutes two holes for the conventional small C# hole. One of these acts as a vent placed to provide the best intonation for notes such as D_5 and $G\#_6$, while the second larger hole is opened in addition when playing $C\#_5$ and $C\#_6$. The action is automatic, requiring no fingering change. Intonation of several notes is, thereby, improved. While the flute described has this feature, the table includes parameters for a conventional C#.

The first column of Table VI names the hole according to the standard acoustical notation, and notes any particularities of the hole or key. The second column gives the distance (in mm, as are all the other dimensions) of the center of the hole relative to the center of the A hole. Since A is used for tuning, it is assumed that this note will be placed in tune by adjusting the tuning slide, and, therefore, it provides a good reference point for the other positions. Positive distances are in the direction away from the mouth hole.

The next four columns describe the important dimensions of the hole and its associated key. These may be used to estimate (using the techniques previously described) the essential measure "h" of the acoustic impedance of the open hole, and the cavity effects "a" and "b" of the closed hole. Results of these estimates are given in the last three columns. Note the variations from hole to hole — these result from differing hole sizes and from the fact that some of the keys are perforated. Columns 2, 6, 7 and 8 together comprise a complete acoustical characterization of this flute, not including the headjoint. If one has hole-position and "h" values for some other flute, one may compare directly its expected performance with that of this instrument by applying the rules expressed in Tables V and I. This assumes that a headjoint with the same taper and mouth hole characteristics is used, and that the cavity values "a" and "b" are not too dissimilar from those shown here. The effects of cavities under the closed keys are cumulative and are different in different registers, so they are not easily handled except by computer.

In order to maintain or possibly enhance the sounding power, the hole sizes were chosen so as to be close to those usually encountered, but on the slightly large side. Key rise, likewise, was increased to 2.3 mm (center) rather than the more common 2.0 mm. Most chimneys were made as small as mechanical considerations permitted (1.3 mm) in order to reduce cavity effects. For all these reasons, values of "h" are generally somewhat smaller than are encountered on most flutes. For the small holes, higher chimneys permitted an increase in hole diameter and a consequent reduction of eddy current losses[12] without reducing the high values of "h" required for proper venting action.

The results of extensive playing tests on this instrument are given in Figure 5. The tests were carried out with the author as player, following the computerized procedure described earlier. They were made in several sessions over a period of a week.

Altogether 33 readings of pitch were obtained for each note. The average of these 33 readings is plotted as a short line on the cents scale, while the small rectangle that the line crosses represents the standard deviation of the mean. That standard deviation is a measure of the precision of the test. For example, the rectangle at low G# extends from 1 to 5 cents. What this means is that the average for this series of tests was 3 cents, and we may expect that if a similar set of 33 points were taken again, under the same conditions, the probability is about one in three that its average would lie lower than 1 or higher than 5. Hence, when looking at this plot one must always keep in mind that any particular point has some random content, and one should not try to interpret the results too closely as belonging strictly to the mechanical design. With this caution, we will comment on the deviations exhibited here.

Overall, the scale stays within plus or minus 10 cents of an A 440 equal tempered scale for the first two registers. There is a gradual rising trend, which is musically desirable, though it is not as pronounced as in some instruments I have tested. The tendency to go a little flat at the low end of the first register is mostly due to cavities. Note that the D in the second register is somewhat sharp; a result, partly, of the venting used with this note. If we should move the D hole up to correct the low D, the middle D will become even sharper. The low D# is also a bit flat, but again we are constrained by the excessive sharpness exhibited by the D# in the third register. This last note is particularly sharp on most flutes — I have palliated the departure by using a smaller hole for the D# key than those on the rest of the footjoint, which gives a slight octave shrink. In addition, the G# hole is larger than usual and lower down on the column, thus diminishing its sharpening effect when used as a vent for the high D#. E and F are made a bit flat in the first register to diminish their sharpness in the third. It is more important to avoid excessive sharpness in these prominent high notes than to avoid flatness in the low register. The footjoint notes, low B, C and C#, are back in line since these holes are not employed elsewhere.

The G# plays about 3 cents sharp (relative to its neighbors) in both registers, and this was confirmed by another series of tests of the three notes G, G# and A. The computer simulation does not show this, and I believe that it is a result of the peculiar mounting of this key, whose axle is displaced farther around the tube than axles of other keys. As a result, when the key is opened it simultaneously retracts, giving a more "open" key

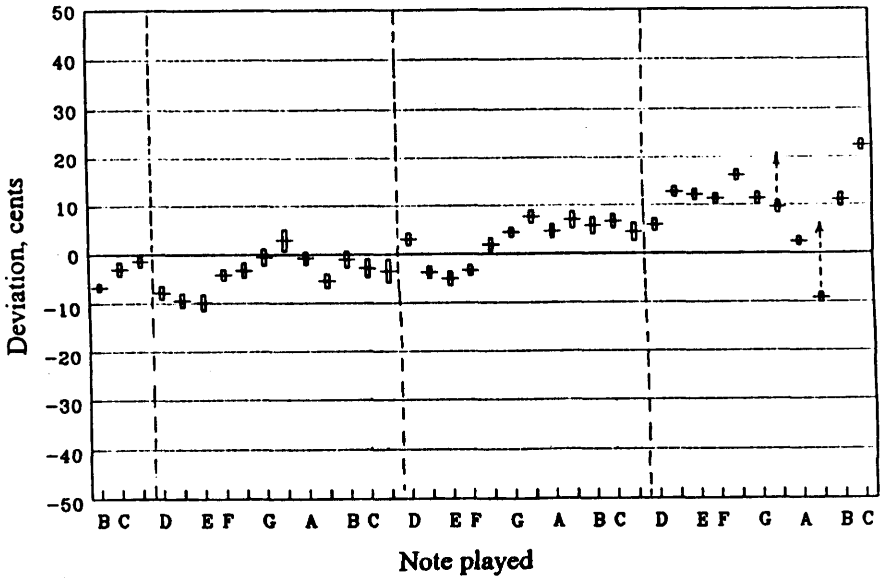

Fig. 5 Results of playing tests on Wimberly flute #42. Deviations from A440 equal tempered scale.

than was calculated using the parameters for other keys. I have since decreased the center rise of the back G# key to 1.9 mm so as to correct this.

Notes in the third register are notorious on all flutes for their variations. The palliative measures taken for high D#, E and F have already been mentioned. Due to the use of the "Coltman C#" arrangement, the high G# on this flute is quite in line with its neighbors. With a conventional C# key, this note would be 11 cents sharper than it is here, as shown by the arrow in Figure 5. This is a prominent fault on most flutes.

The high A is low compared to the preceding notes, and I have no suggestions as to how this might be changed. All flutes exhibit a somewhat flat (relatively) high A and a very flat high Bb. The latter can be played in tune, and very cleanly, by using a different fingering: Left hand first finger down; thumb on Bb lever; third finger down; right hand first finger on trill; third finger down as for F#; and no D# key. The greatly improved pitch for this fingering is indicated by the arrow in Figure 5.

The topmost C is very sharp, again, a standard feature of all flutes I have measured. Fortunately, this note is not often played, but when it is, it will have to be lipped down.

While this scale shows some faults, the deviations from a smooth scale are, over most of the range, substantially smaller than those encountered in normal playing variations. I have measured several modern flutes, some by playing tests and others by mechanical measurement and computer computation. None have exhibited a better scale than this one, though the departures in many modern "new scale" flutes are not excessive.

Where does one go from here? It is my opinion that merely making further changes in hole size and position will not provide substantial improvement. The needs lie largely in the third register, and significant change here calls for new approaches. These might include such things as extra holes brought into play, and local variations in bore diameter. The conservative tendencies of flutemakers — reinforced by the even more conservative attitudes of their customers — make it unlikely we will see such radical changes taken up in the near future. In the meantime, a properly made modern Boehm flute can provide a reasonably well-tuned instrument on which the player can achieve the pitches he desires without an inordinate amount of corrective effort.

References

1. J. G. Roederer, *Introduction to the Physics and Psychophysics of Music*, Springer Verlag, New York, 1973.
2. John Backus, *The Acoustical Foundations of Music*, W.W. Norton Co., New York, 1977.
3. John W. Coltman, "Mouth resonance effects in the flute", J. Acoust. Soc. Am. **54** p. 417 ff (1973).
4. John W. Coltman, "Fifty Flutists Play One Flute", Woodwind World, March, 1976.
5. John W. Coltman, "Acoustic analysis of the Boehm flute", J. Acoust Soc. Am. **65** (2) Feb. 1979 p. 499-506.
6. A. H. Benade and J. W. French, "Analysis of the flute headjoint", J. Acoust Soc. Am. **37**, 679-691 (1965).
7. J. W. Coltman, "The role of the head-joint in flute intonation", Flute Notes, (1985),15 Grandison Rd London SW11 6LS, England.

8. Robert W. Young, "Why an international standard tuning frequency", J. Acoust. Soc. Am. **27** No. 2,379-380, March, 1955.
9. John W. Coltman, "The intonation of antique and modern flutes", Part 4, The Instrumentalist, March 1975.
10. John W. Coltman, "Theobald Boehm and the scale of the modern flute", J. Amer. Musical Instrument Society, **IX**, 89-111, 1983.
11. John W. Coltman, "A new C# for the Boehm flute", The Instrumentalist, May 1977.
12. John W. Coltman, "Acoustical losses in the Boehm flute", The Flutist Quarterly, Fall 1993.

* * *