Acoustic properties of miter bends

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Miter bends in tubing introduce perturbations in the characteristic impedance of the tube. Techniques and dimensions are provided to compensate the effect so that at frequencies of interest the perturbations are eliminated.

Introduction

The air columns of many wind instruments are often folded to keep them from being inconveniently long. This calls for portions of the column to be bent, usually in the form of a portion of a toroid. The characteristic impedance of bends will differ from that of a straight tube of the same diameter Such a change amounts to a perturbation in the bore diameter, which may give rise to undesired perturbations in intonation. This article provides information on how this perturbation may be compensated for, particularly when the bend is in the form of a mitered joint

Toroidal bends

Nederveen[1] has reviewed the literature on this subject, and presented various treatments of the effect.. Simply stated, the compliance (compressibility) of such a section depends only on its volume, whereas the inertance is altered because the vibrating motions tend to concentrate toward the inner wall of the curve, in effect taking a shortcut in the path around the section. The effect is a function of the ratio of r, the radius of the tube, to r_0 , the radius of the centerline of the toroidal bend. The proper characteristic impedance can be restored by decreasing slightly the diameter of the tubing in the curved portion of the bend. For a 180-degree toroidal bend, a reasonably good approximation of the factor (slightly less than 1) by which the tube diameter should be multiplied is:

$$G = 1 - 0.063\,(r/r_0)^2 \qquad (1)$$

The acoustic length of the bend will be shorter than the length measured along the toroid centerline. The new acoustic length can be calculated by multiplying the physical length by a factor equal to G^2.

Mitered bend

For the amateur instrument maker, a toroidal bend is very difficult to produce. A much simpler method is to cut the tubing at 45 degrees and solder or cement the tubes to produce a 90-degree mitered bend. Two such bends, not far apart, can be used to fold a tube by 180 degrees. Measured along the centerline, the volume and therefore the compressibility of the section bounded by the planes at the beginning and end of the cuts remains the same. The inertance however, is decreased as a result of the vibrations taking a “shortcut” around the bend, so that the characteristic impedance is altered and the effective acoustic length is decreased. Dequand et al[2] calculated and measured the reflection coefficients due to a variety of 90 degree sharp bends, among them the mitered bend described above. From their values of the reflection coefficient it is possible to derive the shortening of the section due to inertance change. This turns out to be

$$F = 0.61 \qquad (2)$$

where F is the factor by which the distance around the bend (which is equal to ID, the internal diameter of the tubing) is to be multiplied to get the effective length of the section. This is the shortening for the case where the bend is located at a pressure minimum, i.e. a vibration maximum. The authors do not describe any means by which the characteristic impedance of the section may be restored to that of the main tubing. It is the intention of this article to describe several ways of accomplishing this.

Compensation of miter bend

Compensation can be achieved either by introducing additional inertance to restore the inertance to that of a straight section, or by reducing the volume of the section so as to restore the characteristic impedance to that of a straight tube. In either case, operating on one usually involves a small change in the other. Values of dimensions for compensation were determined experimentally.

Increasing inertance

A tube of ID = 17.86 mm was cut near one end at 45 degrees. It could be reassembled to form a straight tube 380 mm in length, or to form a tube with a 90-degree miter bend having a short section 40.7 mm long, measured along the center line. A dynamic driver at the far end excited the tube while a smal probe microphone close to the driver measured the response. When driven at its lowest resonance frequency the assembly acted as a quarter-wave resonator, with high particle velocity and low pressure at the cut where the bend was made. Comparisons of the resonance frequencies for the straight and bent configurations gave a measure of the shortening effect of the bend. Correction was made for the fact that the bend was located a short distance from that of the maximum acoustic particle velocity. The effective shortening due to the bend was measured as 0.36 ID, close to the value of 0.39 ID predicted by Equation (1). The same setup was used to find by experiment the proper dimensions for the compensation methods described below.

In the first method, shown in Figure 1a, after the tubes are joined, a slot is cut to the depth shown, and a relatively thin plate or card with a straight edge is inserted. After sealing with an appropriate cement (or solder for metal construction) the outside of the plate can be cut away to configure to the outside wall of the tube. The plastic card I used for a 17.86 mm ID PVC tube had a thickness of 0.73 mm, close to the thickness of the hacksaw blade used to make the cut. It is desirable to round the edge of the card to reduce turbulence. At an insertion length of 0.35 ID the resonance frequency was restored to that of the straight tube.

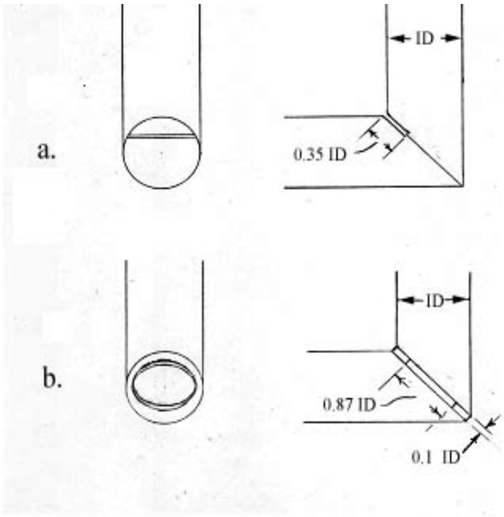

Fig 1. Two ways to compensate miter perturbations. Inside dimensions are shown.

In the second method a somewhat thicker plate having a round hole 0.87 times the ID of the tube is cemented between the two tubes as they are joined, as shown in Fig. 1b.. The compensation will depend somewhat on the thickness of the plate used, but some variation from that specified can be tolerated, say .05 to 0.15 rather than 0.1 store the inertance in a miter bend. Inside ID. Again, rounding of the hole edge is desirable. When calculating distances for finger hole placement, the effective length of the tube is that measured along the center lines of the tubes, plus the thickness of the plate.

Decreasing compliance

A third method for compensation is to reduce the compliance (volume) at the bend to

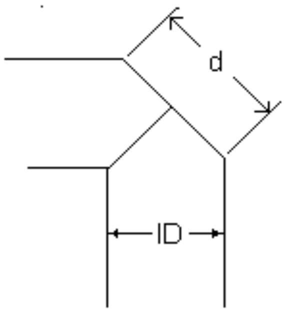

Figure 2. Inside dimensions of beveled miter bend

match the inertance, giving a characteristic impedance for the section equal to that of the main tube. A simple means of doing this is to bevel the sharp corner by cutting it off at 45 degrees and cementing a flat plate over the cut. Figure 2 shows a cross-section of the inside dimensions of this arrangement.

Measurements were made in the laboratory to determine the proper amount to cut off. A somewhat different technique was employed for these measurements. The tubes were arranged with an electromagnetic driver closing one end, while the other end was closed with a plastic disc. A small probe microphone close to the driver measured the acoustic pressure signal. Resonances were measured by taking pressure measurements vs. frequency on either side of the maximum and a measurement close to the maximum. Solving the resonance equation for these three points yielded the frequency of the pressure maximum and the quality factor Q of the resonance to a high degree of precision.

The first resonance mode of the assembly is a half-wave resonance with a pressure max at the two ends and a pressure minimum at the middle, where the bend is located. Here the acoustic velocity is at a maximum. The second mode is a full wavelength, with a pressure maximum at the ends and also at the bend, with a velocity minimum at the bend. The volume of the bend is unchanged from that of a straight tube of the same length measured along the center line. Therefore, the bend does not affect the resonance frequency of the second mode, since only compression is involved and there is no change in the volume being compressed. However, in the first mode, the flow lines of the acoustic velocity take a "short cut" around the bend, shortening the total effective length of the assembly, and raising the frequency of the first mode. The ratio of these two mode frequencies is then less than 2; for the case tested it was 1.941. The assembly will act like a straight tube if and only if the frequencies of the two modes retain their normal relationship, close to a factor of two. This factor is not exactly 2 because dissipation at the walls of the tube results in a slight frequency dependence of the velocity of sound in the tube. A measurement on a straight length of tubing yielded a value of 2.004 for this ratio in the case considered here. Thus we wish to design compensation arrangements that will make the measured ratio of the frequencies of the first two modes equal to 2.004.

The dimension d shown in Figure 2 is the long axis of the near-elliptical opening created by beveling the mitered bend. A series of these was measured, and interpolated to find the value that gave a mode-frequency ratio of 2.004. The value found was:

$$d = 1.26\ \mathrm{ID} \qquad (3)$$

The acoustical distance around the bend is shortened by an amount:

$$s = 0.32\ \mathrm{ID} \qquad (4)$$

Double mitered bend

A pair of ninety degree miter bends positioned close together may not behave simply as two bends spaced far apart. There are also additional options for compensating the disturbances in acoustic impedance. Accordingly, experiments were carried out on the arrangement shown in Figure 3. The tubing used was PVC pipe ¾ inch nominal size, Type S21. This type has relatively thin walls, allowing the bend to be quite compact. Dimensions were 23.8 mm ID, wall thickness 1.46 mm and 26.7 mm OD. The two lengths of tubing were in contact, so that the inside walls were spaced by 2.9 mm .

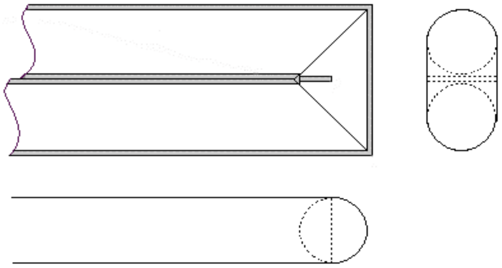

Figure 3, Cross-section and inside dimensions of double miter bend and segmented disk

Measurements were made of the resonance frequencies of a straight tube, a single 90 degree miter bend and of a double mitered bend as in Figure 3. The single bend exhibited an effective first-mode acoustic shortening of 9.2 mm. The double bend showed a shortening of 18.8 mm. This is only slightly more than twice that of the single bend, so the very close spacing of the two 90 degree bends had little effect. We can conclude that the wall thickness of the tubes, which determines the minimum spacing of the bends in a double bend, can be ignored as a variable in making compensation.

An effective way to produce the desired compensation is to introduce a segment of a thin disk at the midpoint of the bend. The disk diameter is the same as the ID of the tube, and it is cut off on a chord such that its height from the perpendicular of that line to the circumference has a value h. The segment is placed at the narrowest part of the tube bridging the bend. The width here is twice the tube wall thickness; in our case 2.9 mm. The disk thickness used was 2.3 mm. In mounting this segment in the short portion of the bend tube, I found it useful to turn on the lathe a wooden cylinder that fit into the tube, facing it off to provide a perpendicular plane against which the disk can be held while being cemented into place. The bridging tube can then be fitted into the mitered ends of the main tubes. With the disk segment in place, the acoustic flow path is lengthened, lowering the frequency of the first mode, but leaving that of the second mode unchanged except for a small amount due to the volume taken up by the disk segment.

Experiments with three disk segments of different heights gave a value for h of 10.34 mm to provide complete compensation. The open area above the segmented disk is thus somewhat larger than that of the disk segment. An experiment with a thinner disk gave a rule for including disk thickness in determining the segment height. The rule for providing compensation then becomes:

$$\text{Segment height } h = 0.49\,\text{ID} - 0.5\,t \qquad (5)$$

where ID is the inside diameter of the tubing and t is the thickness of the disk, all in the same units. The acoustic length of this compensated bend will be shortened by about 0.4 t where t is thickness in millimeters.

Beveled miter, double bend

Experiments were made with the double bend shown in Figure 3, omitting the disk and beveling the two corners as in Figure 2. The results specified a distance d for the long axes of the openings:

$$d = 1.33\,\text{ID} \qquad (6)$$

This is somewhat more than the value for a single 90-degree bend given by equation (3), implying an decrease in inertance of the bend section, consistent with the finding of greater shortening for the close-spaced double bend than for two 90-degree bends.

The acoustic length of this section is shortened compared to the distance along the center line by an amount:

$$s = 0.69\,\text{ID} \qquad (7)$$

Conclusion

Mitered bends are simple to make and can be readily compensated to act acoustically, at most frequencies of interest, as if there were no perturbations in a straight tube. Expressions for the acoustic length of such modified sections are easily applied.

References

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2. Dequand, S., Hulshoff, S. J., Ayregan, A., Huijnene, J., ter Riet, R., van Lier, L. J., Hirschberg, A., Acoustics of 90 degree sharp bends. Part 1: Low-frequency response, Acta Acustica Vol. 89 (2003), pp. 1025-1037