F9488-105

Equipment with which RSF
made experiments
on wood samples.

See charts
M. Grain

OPERATING INSTRUCTIONS

FOR

MAGNATEST ELASTOMAT TYPE FR-500

The Magnatest FR-500 Elastomat instrument provides a convenient means for rapid, accurate measurements of the dynamic moduli of elasticity and damping factor of materials. The modulus determinations are made by measuring the resonant frequency of a sample of simple geometric shape. The modulus is then calculated from this resonant frequency, the dimensions, and the density of the sample. The sample may be excited to vibrate in the torsional, longitudinal or flexural mode to obtain the desired modulus.

The resonant frequencies of samples of complex shaped can be measured, providing they are within the frequency range of the instrument of 600 to 25,000 cycles. The damping factor may also be measured on complex shapes.

The temperature dependence of the modulus of elasticity and damping may be investigated (up to 1000°C) by operating the sample in the specially constructed oven.

The dynamic method of measuring the elastic modulus is known as the "free-free" or floating beam method. A uniform test rod is suspended at its nodal points, by adjustable cross wires. Mechanical vibration is transmitted to the sample from a piezoelectric transducer by means of a fine coupling wire. In turn, a similar system receives the mechanical vibration from the specimen. Two systems are available for the excitation of the test rod at its resonant frequency. It may be vibrated by means of a variable frequency oscillator or by means of a positive feed-back system. In that case, the sample becomes the frequency determining element of the system. The resonant frequency is measured by an electronic counter to an accuracy of .01 to .001%.

Damping is determined by observing the decay in amplitude of vibration during free oscillation after the energy fed to the test specimen is interrupted. The number of cycles which occur when the amplitude decreases from a value of A to A/e is counted. The reciprocal of this number is the damping constant or logarithmic decrement.

The following subjects are covered by these instructions:

I. Operation of the Elastomat

    A. Location of the Elastomat
    B. General information concerning the instrument
    C. Checking of the Counter Unit
    D. Checking the triggering points for damping measurements.

II. The Sample and the Excitation System

    A. The sample
    B. The transducer systems
        a. Piezoelectric transducers
        b. Magnetic transducers
    C. Suspension systems.

I. OPERATION OF THE ELASTOMAT

A. Location of the Elastomat

The Elastomat should be located in a vibration-free, quiet room. It is suggested that the excitation system be placed on a separate table so that the power line hum and the operating distrubances from the oscillator component are not mechanically transferred to the excitation system. A foam rubber cushion beneath the excitation system also aids in damping toublesome mechanically vibrations.

The two electronic components - the oscillator unit and the counter unit - can be placed either next to each other, or one on top of the other. In the latter case, a piece of sheet metal must be placed between the two instruments so that the warm air emitted from the bottom instrument does not enter the top instrument. The distance of the sheet from the bottom instrument must be at least 1 inch.

Both the oscillator and counter components operate on 220 V. The voltage is supplied by the separate 110/220 power transformer unit. The Elastomat must be grounded. A suitable ground wire should be applied to the grounding sleeve loacted on the oscillator. The instruments are fussed by 2 Amp. 3 AG fuses.

B. General Information Concerning the Instrument

The 8-pin plug from the oscillator component is connected with the corresponding receptacle of the counter unit. Cables (32 & 42)* of the receiver and transmitter transducer (3 and 2) are connected to the similarly marked receptacles on the back panel of the oscillator unit. The units may be turned on by pressing any one of the keys from the row of keys (16 or 29), however the key marked "control" on the counter unit and the "damping Control" on the oscillator unit are preferred. These positions will allow a check of the proper operation of the instruments. When the units are turned on, the power line signal lampes (12 and 31) should glow. Furthermore, the thermostat lamp (30) will indicate that the temperature control of the crystal oven is operating. In this oven is located the highly constant, temperature compensated 10 KC quartz crystal which gives the time base for the frequency measurements.

Exactly timed pulses at 1 or 10 second intervals for the gate control of frequency counting are obtained by means of a frequency divider. These pulses are also available for external control at the "1 and 10 second" receptacle on the back panel of the counter. They have a time period of 27 μ seconds and a peak of 13.6 volts. The output load must be higher than 50K ohms. The 10KC frequency of the control quartz crystal is available from a high impedance output, with a voltage of about 30 volts (RMS). The counter may be used to measure external frequencies up to 30KC. This is done by applying the signal voltage of 3 to 6 volts (RMS) to the "counter" jack.

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* Refer to photographs

C. Checking of the Counter Unit.

A warm-up period of about 10 minutes is required before the instrument can be checked. The counter "control" key is turned on. In this position, the 10KC frequency of the quartz crystal is connected to the input of the counter. After a complete measurement period of 10 seconds, the counter should indicate exactly the value 00000.

This value will remain for about 8 seconds, is then erased and the counting process will again be repeated.

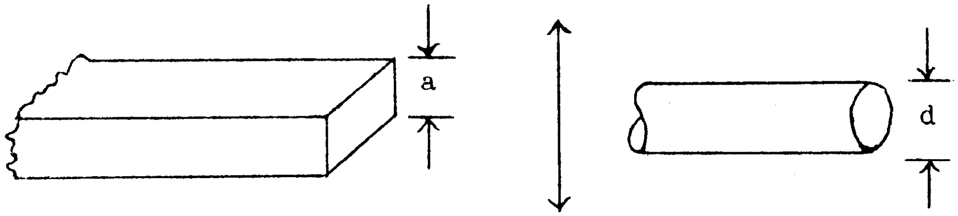
D. Checking the Triggering Points for Damping Measurements.

Turn on the "damping control" (16) on the oscillator component and the "damping" (29) on the counter; pull out the "generator amplitude" control and adjust it until the pointer of the meter points to 100 scale divisions. If one slowly decreases the amplitude, then the upper counter should start counting when the meter pointer reaches the red mark A2. At the lower mark A1, the counting is stopped. In case it is necessary to adjust the triggering points, the controls marked "A1" and "A2" (26 and 27) can be adjusted with a screw driver. Care must be taken that the adjustment is made in the A2 (turn-on point, amplitude = 1).

II. THE SAMPLE AND THE EXCITATION SYSTEM.

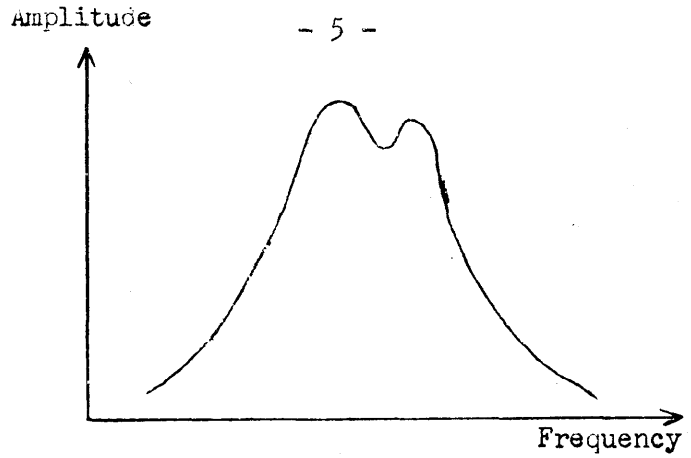
A. The Sample

In order that the natural frequency of the sample (1) be in the measurement range of the Elastomat, it is necessary to make the sample rods approximately 100 to 200 mm long and 8 to 20 mm thick. The nominal size is 160 mm long and 10 mm in diameter. In case of transverse vibrations, the frequency is proportional to the edge length "a" of the test rod and to the diameter of round test rods (see sketch).

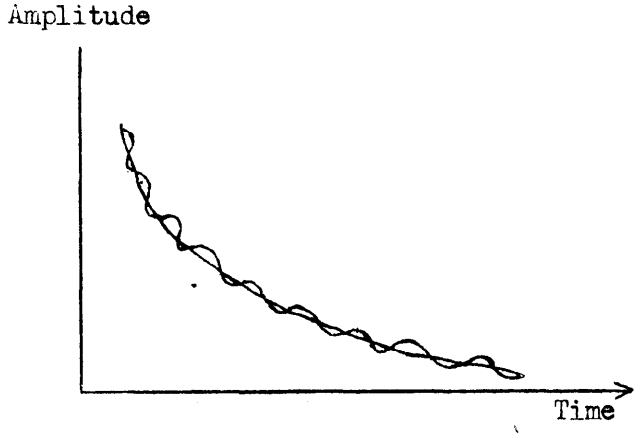


Direction of Vibration

Thus a rod with a rectangular cross-section will generally give two different frequencies for the fundamental of the transverse vibration. Even for rods with a square cross-section, the edge lengths differ slightly from each other or if the diameter of a round rod deviates in any direction, two resonance frequencies will appear which are close together, giving a resonance curve as shown in the sketch.



This complicates the measurement and decreases the measurement accuracy, since now the natural frequency is no longer exactly defined. Furthermore, if the two frequencies lie close together, beats can appear, which will be troublesome especially during the damping measurement. During free vibration of the rod, the vibration energy will transfer back and forth between the two vibration directions and will cause a variation of the amplitude decay curve (see sketch). Therefore it is advisable not to use test rods with square cross sections. It is best to work with round samples. To facilitate mounting the sample and to determine the direction of vibration, a small flat surface may be ground along the rod.



The distance between the two suspension wires (37) on the excitation system is adjusted according to the test rod length. This distance may be directly obtained by measuring the test rod length using the ruler (34) and then adjusting the movable supporting plate (40) to corresponding length on the scale (38). The suspension wire will have the correct spacing for the nodal points of the fundamental in transverse vibration. (Nodal points are 0.2242 times the rod length away from the ends.) The test rod must be supported symmetrically on the two wires. The laterial supporting points are not critical, however.

B. The Transducer Systems

a. Piezoelectric Transducers

The piezoelectric transducers should be moved on the magnet holders such that the wires for the mechanical energy transmission lightly touch their ends to the faces of the test rod, approximately 1-2 mm away from the edge of the faces. Favorable conditions are obtained when the wires are slightly bowed and flexable when pressing against the test rod. The most suitable mechanical coupling can be selected with the aid of the two fine adjustment screws (33) after the resonant frequency or its vicinity has been attained. In order to prevent slipping of the wires from the test rod, small indentations can be made in the test rod with a punch or the wire tips can be mounted with wax. It is also possible to weld the wires to metallic test rods.

The two transducers are completely similar and can therefore be interchanged. The energy transmitting wires are mounted in shellac. When exchanging or replacing the wires, the protecting caps (44) from the transducers should be removed and the small tubes carefully warmed (soldering iron, small flame). The wire can then be easily pulled out. The small tubes can be cleaned with a small brush and alcohol. The new wires should be remounted in shellac.

b. Electromagnetic Transducers

The electromagnetic transducers are connected in the same manner as are the piezoelectric systems. The electromagnetic transducers are similar and interchangeable. However, only ferromagnetic specimens can be excited and only into longitudinal or transverse vibration.

Longitudinal vibration is brought about by placing the transducers on the holders about 1 mm from the end of the test rod. The measuring procedure is the same as used for the piezoelectric system.

Transverse vibration is obtained by attaching the transducers to the mounting blocks (45). Then, in turn, they are positioned over the ends of the test rod at a distance of about 1 mm.

In order that direct electromagnetic coupling between the transducers may be avoided, the laminations of each transducer must be positioned so they are mutually perpendicular.

An advantage of the magnetic system is that only the desired mode of vibration and its harmonics are excited. Furthermore, since the coupling is countact-free, no additional damping is encountered. A disadvantage, however, is that torsional vibration cannot be excited since the system does not have a tangential force component.

C. Suspension Systems

Proper tension on the syspension wires which support the sample is obtained by means of tightening screws (35) and clamping screws (36). When replacing wires, the new wire is first attached to the back clamping screw, then placed in the two notches of the bridge and pulled across the lug, them wound once around the front clamping screw and finally wound around the tightening screw. After the wire is sufficiently tightened by means of the tightening screw, the front clamping screw can then be tightened.

III. MEASUREMENT OF THE RESONANT FREQUENCY.

The free vibration method of determing the modulii requires that the resonance frequencies be known. Two methods of measuring these frequencies may be employed by the Elastomat. These are:

1. Separate excitation by means of the oscillator.

2. Self excitation of the rod by means of positive feed-back system.

A. Separate Excitation.

Separate excitation of the test rod is normally used and will generally produce satisfactory results. All three modes of vibration, transverse torsional and longitudinal, can be excited by this method, if the specified rod dimensions are used. This is possible because the thin coupling wires will always have a component in the desired mode of oscillation. As previously mentioned, the optimum excitation conditions are obtained by adjusting the transducers or by suitable positioning of the coupling wires. Torsional vibrations may be more easily obtained with a coupling which is perpendicular to the rod. This is done by placing the magnetically held transducer units (2 and 3) on the blocks (45), which then are placed adjacent to the test rod. It must be pointed out that the finding and adjusting of the proper excitation conditions require a degree of experimental experience, because the changing of electrical energy to mechanical energy and back to electrical is a critical matter.

The most suitable adjustment is achieved when the control for the "receiver amplitude" (19) is turned up as far as possible (positions 8 to 9), while the "generator amplitude" (14) remains small (at positions 2 or 3). This arrangement will avoid distortion of the coupling wires. The built-in push-pull switch of the amplitude control serves for a greater range of adjustment of the amplitude in a ratio of 1:5.

In the event that external interferring vibrations (acoustic interference, for example) are a problem, the arrangement is made less sensitive to interference by reducing the receiver amplitude and increasing the generator amplitude. Thus an optimum adjustment between the receiver and generator amplitude is obtained depending on experimental conditions.

The fundamental resonant frequency of transverse vibration is always the lowest frequency. For metals, it will usually be in the first frequency range of .6 to 1.3 KC. Assuming of course, that the test rod is of nominal dimensions. Its identity can be checked by its overtones whose multiples of the fundamental frequency are given in Table 2. The approximate resonant frequency of longitudinal vibration can be calculated from the transverse frequency according to Equation 6 and the torsional frequency is approximately 30% below the longitudinal frequency. Harmonics of these modes, which are integer multiples of the fundamental, can also be used to help identify their fundamental frequencies.

The expected resonant frequencies of many materials can be approximately determined with the aid of Table 1, thus saving time in scanning. When test rods are of unknown characteristics, it is necessary to search through the entire frequency range, from 16 to 25 KC. The entire spectrum of resonant frequencies must be recorded. Thereupon, a comparison of frequency ratios can determine the fundamental and harmonics of the various vibrational modes.

When the resonant frequencies are to be determined by means of separate excitation, the expected frequency range is selected by the upper left knob (22), and the "Separate" control is selected. This frequency range can then be scanned by switching on the motor drive (17) of the oscillator. At the same time, the built-in oscilloscope is observed. The approach of a resonant point is heralded by the appearance of an elliptical pattern on the oscilloscope screen, whereupon the motor is switched off and manual adjustment (20) is made to a maximum meter deflection. An approximate value of frequency is given by the oscillator scale (23). The exact frequency however, is obtained from the counter. When a measuring period of one second is used, the measuring accuracy of the frequency is 1 cycle, however with a 10 second measurement period, the accuracy is .1 cycle.

In the operating position of separate excitation, the AC voltage from the receiver is applied to the counter, therefore the frequency appearing is the actual rod frequency. If, for example, the rod is excited by 1000 cycles and a figure eight on its side appears on the oscilloscope screen, the counter will indicate 2000 cycles. The rod, therefore is being excited by the second harmonic of the generator.

The meter (21) deflection must be larger than 35 divisions before the counter will respond and measure the frequency. This is achieved in most cases with the available amplitude. If the indicated rod amplitude on the meter does not reach 35 divisions, due to large damping, excitation of harmonics, etc., the frequency may still be measured. It is done by pressing the key marked "frequency measurement" (16). This adjustment applied the RC generator voltage directly to the counter. Since at resonance, the test rod and the generator frequencies correspond, the counter will indicate the rod frequency.

The phase of the two deflecting frequencies for the oscilloscope can be varied independently of the excitation, in the case of "separate excitation" or "frequency measurement". After the excitation frequency is adjusted to resonance, the "phase shift" control (17) can be adjusted so that a slanted line appears on the scope. This gives a sharp criterion for frequency changes and adjustments. It is of particular use for measurements over a long period of time. For example, changes of the modulus due to temperature, will change the resonant frequency and will require the adjustment of the oscillator to a new frequency.

B. Self Excitation

In many cases, it will be possible to determine the various vibrational modes by means of natural or self excitation. The arrangement forms a feed-back system in which the test rod is the frequency determining element. The method has an admantage in long term experiments, since the frequency does not require readjusting. Self excitation is obtained by pressing the corresponding key (16) and adjusting the phase angle control (18) to the most suitable feed-back conditions. The phase control is equipped with a push-pull switch, which will shift the phase by an additional 180°.

Excitation is obtained by carefully increasing the generator and receiver amplitudes from zero. At the same time, the phase angle must be correspondingly varied, until excitation occurs.

The amplitude of the feed-back system is stabilized by a built-in control that is independent of the adjusted amplification, that is the position of the generator or receiver control. The amplitude will stabilize at a little over 100 divisions on the indicating meter.

If excitation occurs suddenly without stabilizing itself, then the coupling of the wires is not suitable or stable. It is also possible that several modes are simultaneously excited, eventually one will dominate. In any case, measurements are possible only when the meter indication is proper and an ellipse appears on the oscilloscope screen.

A disadvantage of this method is that the oscillation will be excited for which the feed-back conditions are most suitable. It is, however, worth while to experiment with the coupling or the phase adjustment, to get proper excitation. Once this correct adjustment is found, very exact and convenient measurements are possible, which do not require frequency or amplitude readjustments, even in cases where considerable changes in the modulii or damping occurs.

C. Determination of the Approximate Resonance Frequency

It is frequently practical to determine the approximate expected frequencies of a specimen, therefore saving time consuming scanning work. Table 1 gives the resonant frequencies for samples of several materials, having the dimensions of 10 cm in length and 1 cm in diameter. From these values and the proportionalities 1a, 4a, and 7a the approximate frequencies may be computed.

If the material being investigated does not appear in Table 1, the following simplified expressions will also give the approximate resonant frequencies.

a. Longitudinal Vibration

The frequency of this mode of vibration is given by:

$$f_p^{\ell} = \frac{p\, v_\ell}{2} \cdot \frac{1}{\ell} \qquad (1)$$

where:p is an integer multiple of the fundamental (p 1,2,3, ...).

is the length of the test rod

v_ℓ is the velocity of sound and is given by:

$$v_\ell = \sqrt{E/\rho} \qquad (2)$$

where: E = modulus of elasticity

ρ = density

therefore:

$$f_1^{\ell} \; a \; {}^{1}/_{\ell} \qquad (1a)$$

b. Transverse Vibration

For specimens of circular cross-section the transverse resonant frequency is given by:

$$_{(o)}f_p^t = m_p^2 \frac{v_\ell}{8\pi}\frac{d}{\ell^2} \qquad (3)$$

where : d = diameter

m_p^2 = a constant depending on the overtone. Several values are compiled in Table 2.

The ratio m_p^2/m_1^2 given in Table 2 is the factor with which the fundamental frequency is multiplied to obtain the (p-1)th overtone.

For specimens of rectangular cross-section

$$_{(\square)}f_p^t = m_p^2 \frac{v}{4\pi\sqrt{3}} \cdot \frac{a}{\ell^2} \qquad (4)$$

where: a = length of edge which is parallel to the direction of vibration.

For rectangular rods the symbol (□) is used as an index, and for round rods the symbol (o) is used.

Therefore:

$$_{(o)}f_1^t \text{ a } \frac{d}{\ell^2} \qquad (3a)$$

$$_{(\square)}f_1^t \text{ a } \frac{a}{\ell^2} \qquad (4a)$$

when a = d the transverse frequencies of a circular rod and rectangular rod are related by:

$$_{(o)}f_p^t = 1.153 \; _{(\square)}f_p^t \qquad (5)$$

The longitudinal frequency can be obtained from the transverse frequency by the following expressions:

$$f_1^{\ell} = .561 \; \frac{\ell}{d} \; {}_{(o)}f_1^{t}$$

$$f_1^{\ell} = .486 \; \frac{\ell}{a} \; {}_{(\square)}f_1^{t} \qquad (6)$$

c. Torsional Vibration

The torsional resonant frequency of a circular rod is obtained from

$$ {}_{(o)}f_p^{tor} = \frac{p \cdot v_{tor}}{2} \cdot \frac{1}{\ell} \qquad (7)$$

where: $v_{tor} = \sqrt{G/\rho}$ (8)

G = shear modulus

therefore:

$$f^{tor} \; a \; \frac{1}{\ell} \qquad (7a)$$

For rods of rectangular cross-section, the right side of Eq.(7) must be multiplied by a factor K, which is a function of the rod's cross-sectional dimensions ratio a/b. These factors are given in Table 3. The torsion frequency of a rectangular rod therefore is

$$ {}_{(\square)}f_p^{tor} = K_{(a/b)} \cdot \frac{p \, v_{tor}}{2} \quad \frac{1}{\ell} \qquad (9)$$

IV. DETERMINATION OF THE DYNAMIC ELASTIC MODULUS FROM THE RESONANT FREQUENCIES.

A previous section was concerned with determining approximate frequency values. Now, since the measuring accuracy of the Elastomat is very great it will be necessary to take into account factors which were omitted in the simplified expressions.

A. E-Modulus from Longitudinal Vibration

The solution of Eqs.(1) and (2) gives the E-modulus from the longitudinal frequency

$$E = 4.0775 \cdot 10^{-8} \rho \ell^2 (f_1^{\ell})^2 \qquad (10)$$

where: E* is in Kg/mm, ℓ in cm, and ρ in g/cm3.

If the weight P in grams is substituted one obtains for a circular rod

$$E = 5.1916 \times 10^{-8} P \frac{\ell}{d^2} (f_1^{\ell})^2 \qquad (10a)$$

for the rectangular rod:

$$E = 4.0775 \cdot 10^{-8} P \frac{\ell}{a\, b} (f_1^{\ell})^2 \qquad (10b)$$

Greater accuracy in the calculation of E is obtained from the longitudinal frequency when the effects of shape, size, and Poisson's ratio are taken into account. This is done by multiplying the right side of equations 10, 10a, or 10b by the correction factor K. This factor is given for circular cross-section as:

$${}_{(o)}K_1^{\ell} = 1 + \frac{d^2}{2} \cdot 1.2337 \mu^2 \qquad (11a)$$

for rectangular cross-section as:

$${}_{(r)}K_1^{\ell} = 1 + \frac{a^2 + b^2}{\ell^2} \cdot .82247 \mu^2 \qquad (11b)$$

B. E-Modulus from Transverse Vibration

The E-modulus is obtained from the transverse fundamental frequency by the solution of Eq. (3) or (4).

$$E = 1.2864 \cdot 10^{-8} \cdot \frac{\rho \ell^4}{d^2} \cdot \left({}_{(o)}f_1^{t} \right)^2 \qquad (12a)$$

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* The conversion factor to change Kg/mm^2 to psi (lbs/in2) is 1422.28 psi/Kg/mm^2. For example E-modulus of aluminum is approximately 7040 Kg/mm^2. In psi, therefore, E = 7040 Kg/mm^2 x 1422.28 psi/Kg/mm^2 = 10 x 10^6 psi.

or

$$E = .96478 \cdot 10^{-8} \frac{\rho \ell^4}{a^2} \cdot \left({}_{(\square)}f_1^{t}\right)^2 \qquad (12b)$$

If the weight P (in grams) is substituted for the density then

$$E = 1.6379 \cdot 10^{-8} \frac{P}{\ell} \left(\frac{\ell}{d}\right)^4 \cdot \left({}_{(o)}f_1^{t}\right)^2 \qquad (12c)$$

$$E = .96478 \cdot 10^{-8} \frac{P}{b} \cdot \left(\frac{\ell}{a}\right)^3 \cdot \left({}_{(\square)}f_1^{t}\right)^2 \qquad (12d)$$

Again the effect of size, shape, and Poisson's ratio is taken into account by the correction factor K which for a circular rod is

$${}_{(o)}K_1^{t} = 1 + \frac{d^2}{\ell^2} \cdot \left(3.092 + .854 \frac{E}{G}\right) - \frac{d^4}{\ell^4} \cdot 2.172 \frac{E}{G} \qquad (13a)$$

and for a rectangular is

$${}_{(\square)}K_1^{t} = 1 + \frac{a^2}{\ell^2} \cdot \left(4.123 + 1.230 \cdot \frac{E}{G}\right) - \left(\frac{a}{\ell}\right)^4 \cdot 4.17 \frac{E}{G} \qquad (13b)$$

where $E/G = 2\,(\mu + 1)$

C. Shear Modulus from Torsional Vibration

The shear modulus is determined from the torsional resonant frequency and by solving Eqs. (7), (8), and (9). For round rods

$$G = 4.0775 \cdot 10^{-8} \rho \ell^2 \left({}_{(o)}f_1^{tor}\right)^2 \qquad (14a)$$

for rectangular rods

$$G = 1.0194 \cdot 10^{-8} F_{(a/b)} \cdot \rho \ell^2 \left({}_{(\square)}f_1^{tor}\right)^2 \qquad (14b)$$

If, again, the weight P is substituted for the density ρ, (14a) and (14b) become

$$G = 5.1916 \cdot 10^{-8} \; P \; \frac{\ell}{d^2} \cdot \left({}_{(o)}f_1^{tor} \right)^2 \qquad (14c)$$

$$G = 1.0194 \cdot 10^{-8} \; F_{(a/b)} \; \cdot \; P \; \frac{\ell}{a \cdot b} \cdot \left({}_{(\square)} f_1^{tor} \right)^2 \qquad (14d)$$

G from the above equations will be in Kg/mm.

D. Other Correction Factors

a. Effect of Thermal Expansion

The influence of thermal expansion of the test rod at high temperatures must be considered when exact measurements are required. This is accomplished by multiplying Eqs. (10), (13), or (15) by the factor $F_{(t)}$ where

$$F_{(t)} = \frac{1}{1 + a\,t} \qquad (15)$$

where: a = coefficient of thermal expansion

t = temperature

b. Effective Diameter of a Tubular Specimen

In the event that the specimen is a hollow tube with an inside diameter of d_1 and an outside diameter of d_2, the expression 16 must be substituted for d in the appropriate equation

$$d = \sqrt{d_1^2 + d_2^2} \qquad (16)$$

V. DETERMINATION OF POISSON'S RATIO AND THE VELOCITY OF SOUND

A. Poisson's Ratio

Poisson's ratio can be obtained from E and G by Eq. (17) or directly from the frequencies by Eqs. (17a) to (17b)

$$\mu = 1/2 \quad E/G - 1 \qquad (17)$$

circular rods:

$$\mu = 1/2 \left(\frac{f_1^{\ell}}{{}_{(o)}f_1^{tor}} \right)^2 - 1 \qquad (17a)$$

$$\mu = .1577 \left(\frac{\ell}{d} \right)^2 \cdot \left(\frac{{}_{(o)}f_1^{t}}{{}_{(o)}f_1^{tor}} \right)^2 - 1 \qquad (17b)$$

rectangular rods

$$\mu = \frac{2.0}{F_{(a/b)}} \cdot \left(\frac{{}_1f}{{}_{(\square)}f_1^{tor}} \right)^2 - 1 \qquad (17c)$$

$$\mu = \frac{.473}{F_{(a/b)}} \cdot \left(\frac{1}{a} \right)^2 \cdot \left(\frac{{}_{(\square)}f_1^{t}}{{}_{(\square)}f_1^{tor}} \right)^2 - 1 \qquad (17d)$$

B. Velocity of Sound from the Resonant Frequencies

The velocity of sound may be computed from transverse or longitudinal resonant frequency by the following relationships:

a. Longitudinal frequency

$$v_{\ell} = 2\,\ell f_1^{\ell} \qquad (18)$$

b. Transverse frequency

(circular rod)

$$v_t = 1.0812 \; \frac{\ell^2}{d} \; {}_{(o)}f_1^{t} \qquad (19a)$$

(rectangular rod)

$$v_t = .9465 \; \frac{\ell^2}{a} \; {}_{(\square)}f_1^{t} \qquad (19b)$$

VI. EVALUATION OF FREQUENCY MEASUREMENTS AND RESULTS

A. Damped and Undamped Natural Frequency.

The above equations which contain the natural frequencies of a test rod are theoretically correct only for materials without internal damping. The natural frequency without damping f_σ and with damping f_σ are related in the manner of Eq. (20).

$$\frac{f_\sigma}{f_\sigma} = \frac{1}{\sqrt{1 + \left(\frac{\sigma}{2\pi}\right)^2}} \qquad (20)$$

Therefore Eq. (10), (12) and (14) may be corrected by multiplying with the factor

$$1 + \left(\frac{\sigma}{2\pi}\right)^2 \qquad (21)$$

where: σ = damping constant or logarthmic decrement.

However, the effects of (21) on the measured frequencies are very small and can be neglected in practically all cases. Even when damping is .05, the deviation in frequency is only .06%.

B. Adiabatic and Isothermal Modulii

The dynamic measurement of this method does not allow thermal diffusion within the specimen during the time of a quarter cycle. Therefore, the E-modulus determined from the above equations is the adiabatic E-modulus, E_a. This modulus is slightly larger than the isothermal E-modulus, E_i, obtained by static measurements. The relationship between the adiabatic and the isothermal modulii is given by

$$\frac{E_a}{E_i} = 1 + a^2 T \frac{E_a}{\varrho \cdot C_p} \qquad (22)$$

where: a = coefficient of thermal expansion

T = absolute temperature

C_p = specific heat at constant pressure

ϱ = density.

VII. DAMPING MEASUREMENT.

A. Damping from the "N" value.

The test rod is brought to vibration by either self or separate excitation and the amplitude is adjusted so that the deflection on the meter is larger than 90 divisions. However, the ellipse on the oscilloscope screen must still lie within the screen range, so that overloading of the tubes does not occur. The counter unit is switched to "Damping" (29) and the actual measurement is made by pushing the button "Damping Start" (25). By so doing, the energy fed to the rod is interrupted and the rod amplitued decreases exponentially. The oscillations which take place while the pointer of the meter is between the red marks A^2 and A^1, are now counted and can be read from the counter. This is the number of cycles counted as the amplitude decays from the value A to A/e or to 36.7% of amplitude A. The reciprocal value of this number "N" will then give the damping constant or the logarithmic decrement.

$$\sigma = \frac{1}{N} \qquad (23)$$

The button "Exciting" (24) will return the counter to zero and the rod will again begin to vibrate. Thus several measurements can be carried out in a short time, and will give a relatively accurate average value of damping.

B. Damping from the Width at Half-Maximum of the Resonance Curve.

If the meter deflection cannot be increased to above 90 divisions, the damping can still be determined from the width at half-maximum value of the resonance curve. The value is obtained simply by finding the resonant frequency at which the meter will indicate maximum amplitude and then locating the two frequencies, above and below the resonant frequency, at which the amplitude has decreased to half of the resonance amplitude. Damping is then obtained from the difference of the two frequencies and the resonant frequency by Eq. (24).

$$\sigma = 1.8136 \frac{\Delta f}{f_\sigma} \qquad (24)$$

A possible source of error, in this method of measuring damping, may be due to the mechanical damping, at the supports of the test rod. To minimize this, the test rod must be suspended as exactly as possible at its nodal points. The scale on the suspension system applies only to the fundamental of transverse vibration. Nodal points for the various overtones and harmonics are given in Tables 5 and 6. The nodal distances, in terms of rod lengths, are given from the end of the rod.

d. Preparing a Circular Test Rod for Damping Measurements

Whenever damping measurements are to be made on circular rods, it is advisable to grind a flat surface on the rod. The measurement of transverse frequency in this case must be made before the surface is ground, or if it is measured with the ground surface, the frequency may be corrected to that of an unground rod by the reduction coefficient R (h/d). Table 4 gives values for the coefficient R (h/d) for several ratios of h/d. It is also shown graphically in Fig. 3. Values are for ground surface in the direction of vibration R^{II} and perpendicular to that direction R^{I}. Therefore the corrected frequency is

$$(o)\, f_1^t = R\, f_1^t \quad \text{(ground rod)} \qquad (25)$$

and may be substituted in Eq. (13a) or (13c).

VIII. CIRCUIT DESCRIPTION

The circuit of the Elastomat will be explained through simple block diagrams.

Figure 5 illustrates the Elastomat set up for "separate excitation". The variable frequency oscillator supplies a driving voltage to the sending transducer. The transducer converts the electrical oscillations into mechanical vibrations and couples them to the sample. At the other end, the mechanical vibrations of the sample are reconverted into electrical signals by the receiving transducer.

These electrical signals are amplified and their amplitude is indicated on the meter and displayed on the oscilloscope screen.

The amplified signal is also applied to the counter unit, which counts the frequency. As explained in the text, if the signal from the amplifier is too weak to operate the counter unit, the "frequency measurement" button is pressed, which connects the counter unit directly to the variable frequency oscillator (dashed line) instead of to the amplifier. In this case the counter counts the number of cycles per second of the oscillator directly.

In Figure 6 the self excitation system is illustrated. The feedback amplifier, in connection with the sample and transducers, forms an oscillator circuit. The frequency of oscillations is determined by the resonant frequency of the sample as explained in the text. The receiving transducer is connected to a second amplifier which works into the metering circuit and the oscilloscope circuit. The frequency counter counts the number of cycles per second of the osicllation frequency.

In damping measurements, either the basic circuit of Figure 5 or Figure 6 may by used, as desired. As explained in the text, damping measurements are made by interrupting the excitation to the sample and counting the number of cycles that it takes for the amplitude of oscillations of the sample to decay from a reference amplitude to 1/e of this amplitude. Thus the block diagrams for damping measurements are identical with Figure 5 and Figure 6 with the addition of a switching circuit that starts and stops the counter at the proper signal amplitudes.

A brief description of the circuit of the main electronic component shown in Drawing No.______________ follows.

The power supply includes a voltage regulating power transformer and a separate high voltage transformer for the oscilloscope tube. Full wave bridge selenium rectifiers are used for the B+ voltage supplies. V7 and V8 form the variable frequency oscillator circuit. The signal from the oscillator is amplified by V5 and V6 before it is applied to the sending transducer. The signal from the receiving transducer is amplified by V1 and V2 before it is measured on the meter and by the additional stages V9 and V10 before it is displayed on the oscilloscope tube V11. V3 and V4 are additional amplifying stages used when the "self excitation" method is employed. A nonlinear resistance marked K1 100 in the cathode of V4 helps to stabilize the amplitude of the oscillations in the "self excitation" system.

The chart at the lower right of the drawing indicates which switches are closed by pushing the various push buttons on the front panel.

A brief description of the circuit of the counter unit shown in Drawing No.________ follows.

Two voltage regulated power transformers are used. Again full wave bridge selenium rectifiers are used for the B supplies.

V1 is the crystal controlled 10 KC oscillator that serves as the time standard for the frequency counter. The crystal is shown twice in the drawing, once at the upper left to show its function in the circuit and at the middle left to indicate its pin connections. V1 is the crystal oscillator tube. V2, V3, V4 and V5 perform amplifying and wave shaping functions for the first series of decade counters. V6 and V7 are the tubes in the first decade of this counter. The purpose of this series of counters is to count the frequency of the fibrations of the sample.

V16, V17 and V18 are amplifiers and wave shapers for the second series counter decades. The purpose of this series of counters is to generate timing signals that accurately control the period of counting for the first series of counters (one second or 10 seconds, as desired). These accurate timing signals are obtained by counting the cycles of the 10 KC crystal oscillator. The output from this counter is amplified and shaped by V29, V30 and V31 to control the first counter.

The first series of counter decades (V6 through V15) is the one that shows on the front panel. The second counter series (V19 through V28) is for timing purposes only and is not visible without removing the chassis from the cabinet.

The key at the lower right again shows which switches are closed when the front panel puttons are pushed.

IX. OPERATING INSTRUCTIONS FOR THE ELASTOMAT FURNACE.

The two-piece furnace serves for the investigation of specimens at temperatures up to 1000°C. The furnace is constructed so that the electric heating elements in the two portions completely surround the test compartment. In addition, in order that a constant temperature is assured over the entire sample lenght, a special specimen chamber is provided.

The test rod should be of a circular cross-section, having a maximum length of 170 mm and a maximum diameter of 15 mm. It is suspended by means of two wire loops from the upper portion of the test chamber. The loops are placed in the 15 mm wide grooves and always located so that the sample hangs from its transverse fundamental nodal points. The length of the loop is adjusted in order that the center of the test rod is 1 to 2 mm above the center of the test chamber. For high temperature work, the use of platinum wire .15 to .2 mm in diameter is recommended.

The transducer mounting devices used on the standard excitation system are used with the furnace. They are removed from the dovetail guides after the limiting screws have been unscrewed. They are then placed on the corresponding guides of the furnace. A special transducer arrangement is provided for furnace use. It has a longer coupling wire mounting tube which permits the wire to be guided, contact-free, through the opening of the test chamber. Excitation of the specimen is accomplished by attaching the coupling wire to the suspension loops by means of little hooks which may be bent at the end of the wires. The coupling wires are arranged in a slightly declined fashion without contacting the sample or test chamber.

At the beginning of temperature experiments, the transducer mounting device should be adjusted so that the thermal expansion of the rod can be compensated by means of the "fine" adjustment screw. The adjustment screws are first completely turned in, then as the temperature increases, the coupling wires are slightly tightened from time to time. In this manner an optimum condition for energy transfer is maintained. This operation is reversed when the temperature decreases to compensate for thermal contraction. This contraction may pull the wires from the suspension loops or displace the suspension points.

The furnace temperature should not exceed 1000°C. Furthermore, to protect the heating elements when operated above 850°C; the line voltage must be decreased to about 95 volts and the current to about 10 amperes. A nickel-nickel-chromium thermocouple is located in the lower test chamber for temperature measurements.

Technical Data:

Voltage - 110 V
Warm-up load - 1.5 Kw
Max. Furnace Temperature - 1000°C
Voltage at 1000°C - 95 V
Load at 1000°C - 1 Kw
Warm-up time to 1000°C - 2 hours
Max. test rod length - 170 mm
Max. test rod diameter - 15 mm

X. MISCELLANY - PHOTOGRAPHS, TABLES, CURVES

Photograph Legend:

1. Test specimen
2. Excitation transducer
3. Receiver transducer
4. Transducer holder

5. Supporting and coupling wire tube
6. Upper portion of test chamber
7. Furnace, upper part
8. Lower portion of test chamber
9. Thermocouple
10. Furnace, lower part
11. Focus control for oscilloscope screen
12. Power line signal lamp, oscillator unit
13. Oscilloscope screen
14. Control knob for the generator amplitude
15. Brightness control for cathode ray tube
16. Operating keys
17. Motor switch
18. Phase adjustment control knob
19. Amplitude control knob for receiver
20. Frequency adjustment "fine"
21. Amplitude indicating meter
22. Frequency adjustment "course"
23. RC generator scale
24. "Excitation"button for damping measurement
25. "Start" button for damping measurement
26. Control screw A_1. lower turn-off point adjustment
27. Control screw A_2. turn-on point adjustment
28. Counter decades
29. Operating keys of the Counter unit
30. Signal lamp for the thermostat
31. Power line signal lamp
32. Connecting cable
33. Fine adjustment screw
34. Scale for locating fundamental transverse nodal points
35. Suspension wire tightening screw
36. Suspension wire clamping screw
37. Suspension wire
38. Adjustment mark for nodal distance
39. Tightening screw for movable dovetail guid support
40. Dovetail guide support (movable)
41. Tightening screw for the transducer holder
42. Connecting cable
43. Limiting screw
44. Protector cap
45. Transducer mounting block.

TABLE I

Resonant Frequencies for Sample of Circular Cross Section

Length = 100 mm. Diameter = 10 mm

Material	f_1^{long} d = 10 mm	f_1^{trans}	f_1^{tors}
Aluminum	25,200	4,488	15,400
Aluminum-Oxide	48,000	8,546	
Antimony	17,000	3,027	
Lead	6,000	1,068	3,500
Cadmium	12,000	2,136	7,440
Ebonite	7,800	1,380	
Iron	25,850	4,604	16,150
Ivory	15,050	2,680	
Plaster	11,550	2,060	
Glass,-Flint	20,000	3,560	12,800
Glass,-Crown	26,500	4,720	17,100
Granite	19,750	3,520	
Cast Iron	18,000	3,210	11,340
Hard Rubber	7,850	1,400	
Resin	8,000	1,420	
Wood	15,000 to 20,000	2,670 to 3,560	
Iridium	24,500	4,360	
Cobalt	23,600	4,200	
Constantan	21,500	3,830	13,180
Copper	18,550	3,300	11,300
Magnesium	24,500	4,360	
Manganin	19,500	3,470	11,960
Nickel Silver	18,000	3,200	10,880
Nickel	23,930	4,260	14,800
Palladium	15,000	2,670	8,900
Paraffin	6,950	1,240	
Platinum	14,000	2,490	8,400
Porcelain	27,400	4,880	17,800
Quartz Glass	26,850	4,780	17,580
Sealing Wax	6,850	1,220	
Silver	13,200	2,350	7,950
Stearin	6,900	1,230	
Tellurium	17,500	3,120	
Clay burnt	18,250	3,250	
Bismuth	8,950	1,590	5,490
Tungsten	21,550	3,840	13,100

TABLE II

	p	m_p^2	m_p^2 / m^2	Position of Nodes
Fundamental	1	22.3729	1.000	0.224
1. Overtone	2	61.6696	2.756	0.132/0.500
2. Overtone	3	120.9120	5.404	0.094/0.356
3. Overtone	4	199.8548	8.933	0.073/0.277/0.500
4. Overtone	5	298.5638	13.345	0.060/0.366/0.409
5. Overtone	6	416.9764	18.638	
6. Overtone	7	555.1678	24.814	
7. Overtone	8	713.0502	31.871	

TABLE III

a/b	K(a/b)	F(a/b)
1.00	0.918	4.7424
1.20	0.905	4.8961
1.40	0.871	5.2790
1.60	0.829	5.8245
1.80	0.784	6.5001
2.00	0.741	7.2882
2.50	0.642	9.6912
3.00	0.562	12.659

TABLE IV

h/d	R^{I} (h/d)	R^{II} (h/d)
0.05	1.0258	0.99203
0.10	1.0693	0.98120
0.15	1.1239	0.97142
0.20	1.1886	0.96403
0.25	1.2643	0.95074
0.30	1.3523	0.95898
0.35	1.4550	0.96211
0.40	1.5757	0.96958
0.45	1.7190	0.98195
0.50	1.8915	1.00000

TABLE V

For Transverse Vibration

Vibration Form	Position of Nodes
p = 1/Fundamental	0.224 - 0.776
p = 2/1. Overtone	0.132 - 0.500 - 0.868
p = 3/2. Overtone	0.094 - 0.356 - 0.644 - 0.906
p = 4/3. Overtone	0.073 - 0.277 - 0.500 - 0.723 - 0.927
p = 5/4. Overtone	0.060 - 0.226 - 0.409 - 0.591 - 0.774 - 0.940

TABLE VI

For Longitudinal and Torsional Vibration

Vibration Form	Position of Nodes
p = 1 Fundamental	1/2
p = 2 2. Harmonic	2/4 - 3/4 -
p = 3 3. Harmonic	1/6 - 1/2 - 5/6
p = 4 4. Harmonic	1/8 - 3/8 - 5/8 - 7/8
p = 5 5. Harmonic	1/10 - 3/10 - 1/2 - 7/10 - 9/10

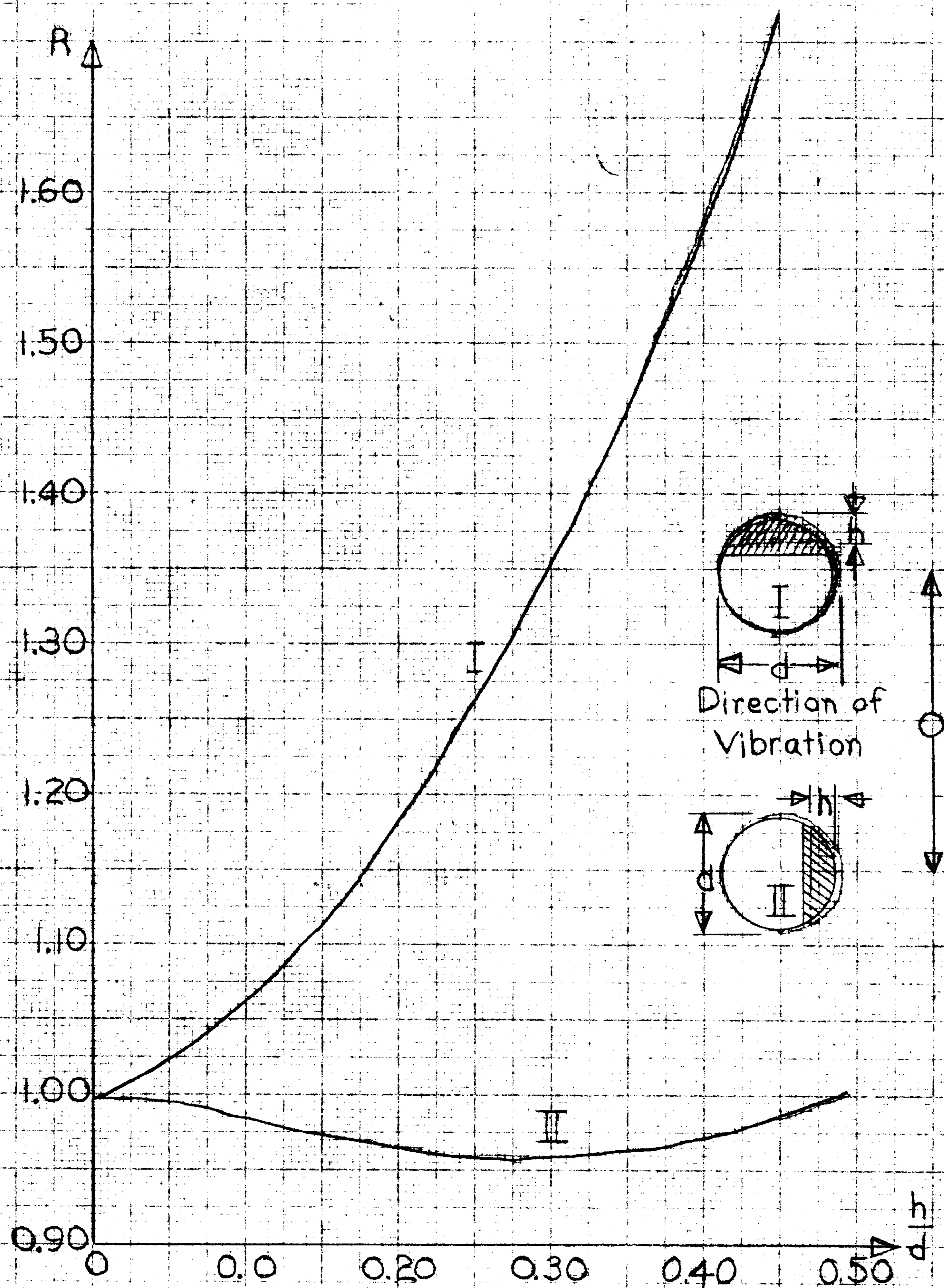

Figure 3. Reduction coefficient R - to adjust frequency of circular rod with flat to circular rod without flat.

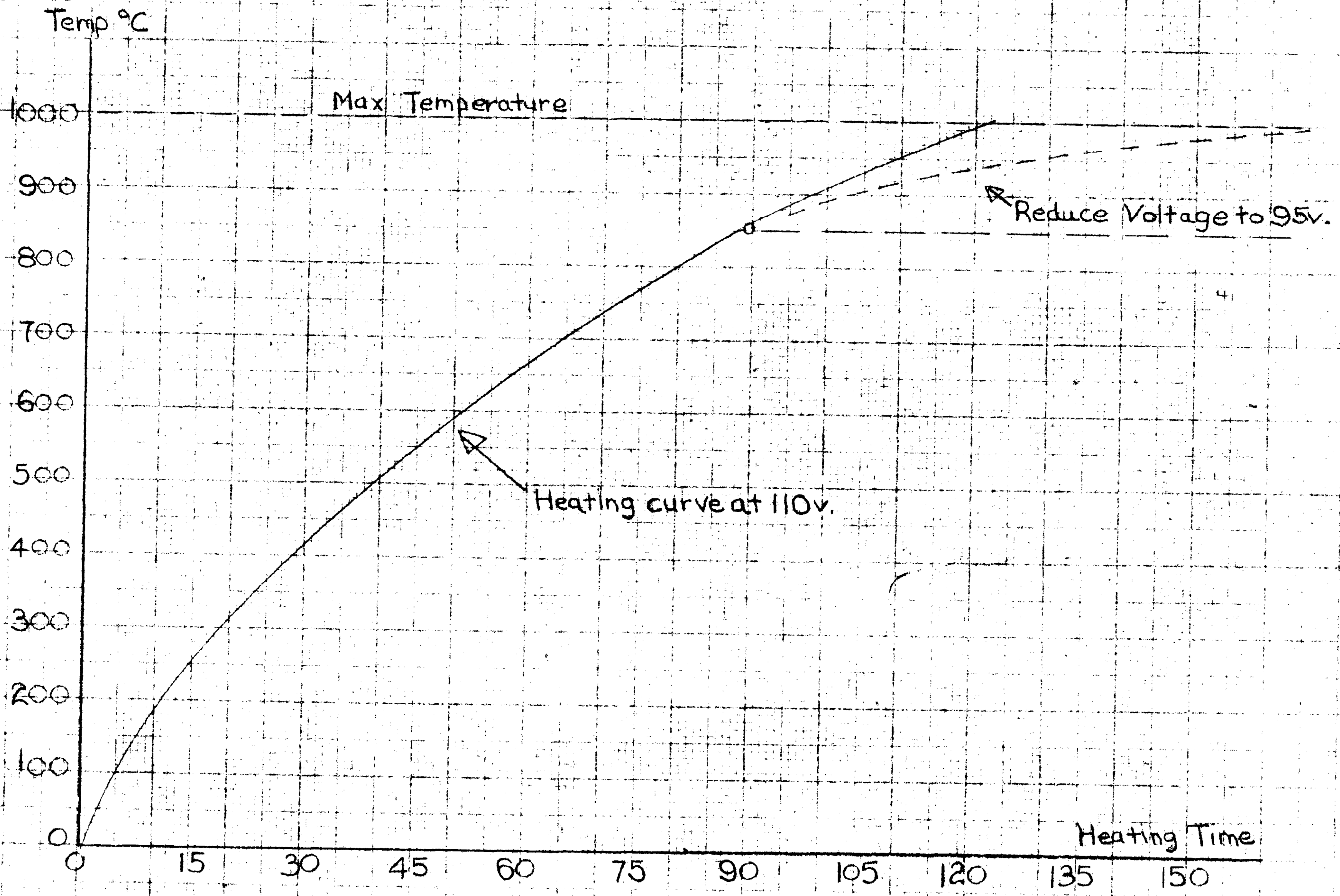

Figure 4. Heating Curve of the Elastomat Furnace.

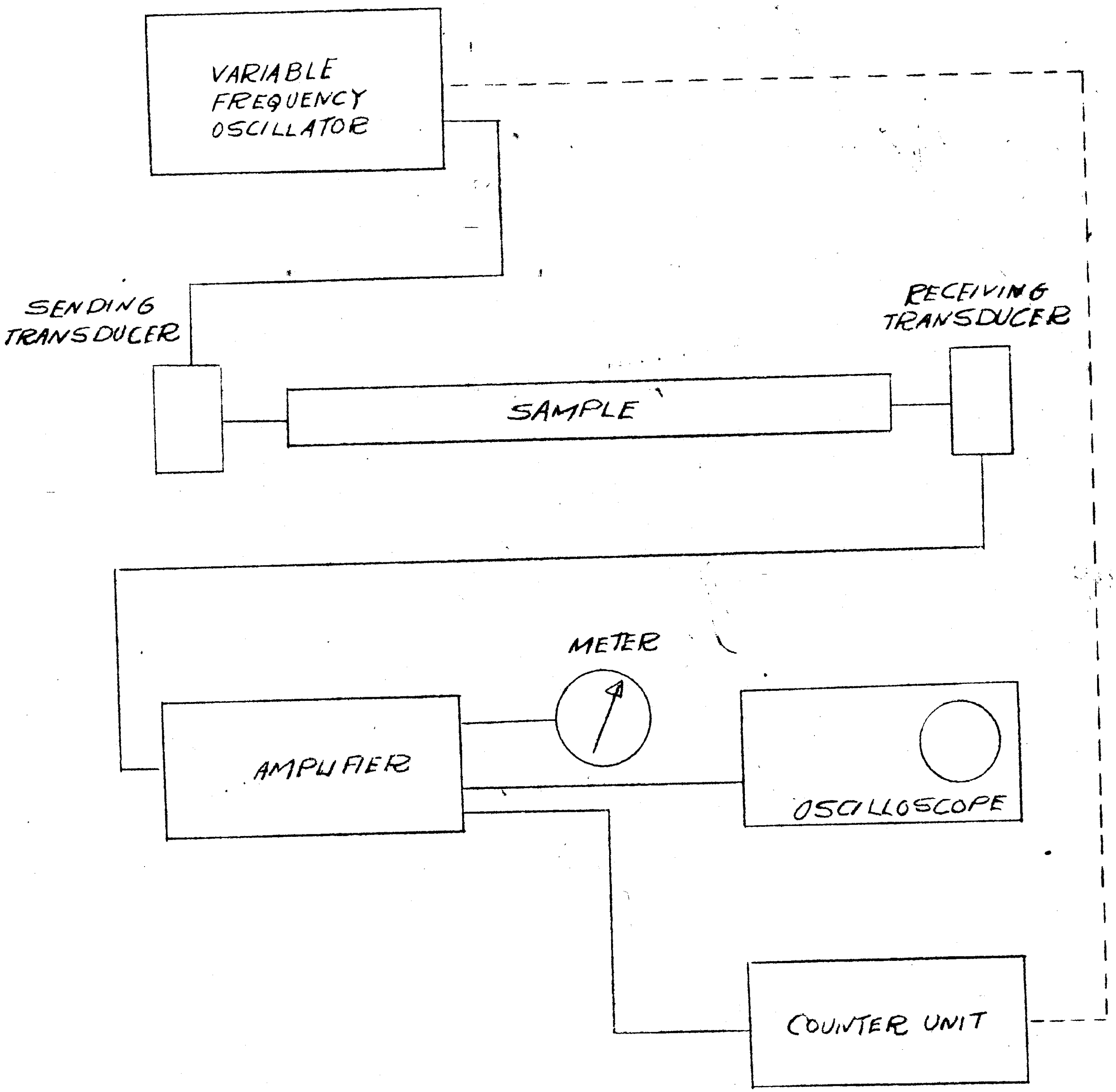

FIGURE 5
BLOCK DIAGRAM OF ELASTOMAT
SEPARATE EXCITATION SYSTEM

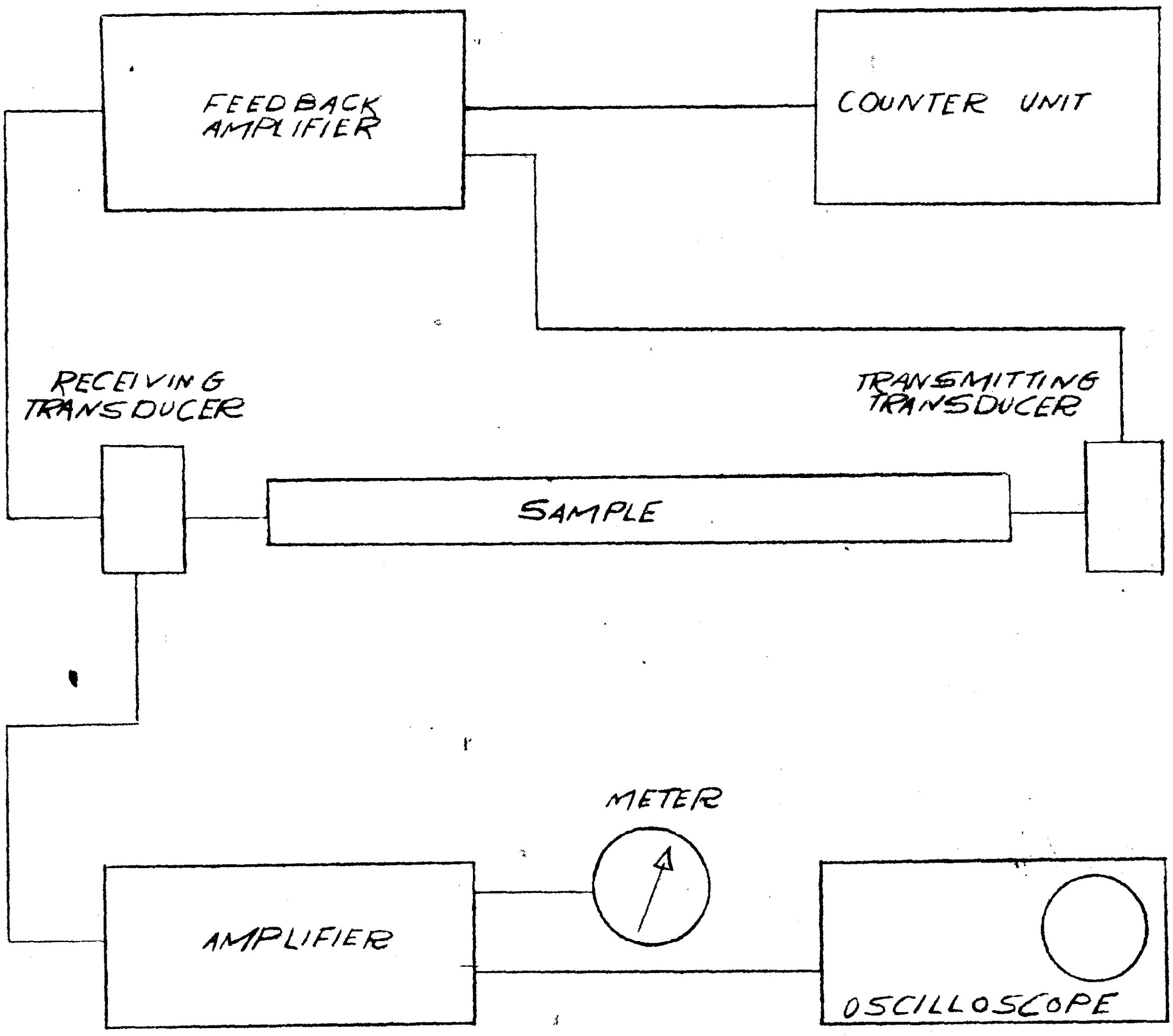

FIGURE 6
SIMPLIFIED BLOCK DIAGRAM OF
THE ELASTOMAT, SELF EXCITATION SYSTEM