Sitar spectrum properties. A. H. Benade and W. G. Messenger (Department of Physics, Case Western Reserve University, Cleveland, OH 44106)

The main strings of a sitar are tuned to pitches corresponding closely to F₃#, C₂#, G₂#, G₃#, G₄#, and C₅# (i.e., in 4ths, 5ths, and octaves). Playing is predominantly on the F₃# string, the tonic (sa) being at the 7th fret (C₄#). There are also 11 sympathetic strings tuned to the notes of the raga. The measured and calculated inharmonicity is far less than that of Western stringed instruments. Sitar strings are thin, giving small stiffness inharmonicity and low wave impedance that reduces random inharmonicities from bridge/belly resonances. String length variations from first mode rolling and/or sliding on the curved bridge profile produces FM and AM sidebands of ±f₁, from the upper partials. These components join with both plucked and sympathetic string partials to give harmonically related narrow-band clumps. The resulting tone has complex time behavior but well-marked pitch. Tuning to just frequency ratios is required since the beats arising from excitation of a mistuned string are not smeared by roughness due to inharmonicity. Raman's observation that plucking at L/4 need not remove the 4th partial from the tone is confirmed. Bridge-generated FM/AM provides part of the explanation, shock excitation of sympathetic strings via the bridge provides the rest.

JASA 71:583 (1982)
STRING INHARMONICITY

(A) STRING STIFFNESS \[ f_n = n f_0 \left(1 + \frac{J}{J_0} n^2\right) \]
\[ f_0 = \frac{1}{L} \sqrt{\frac{T}{\pi \rho}} \]
\[ J = \left(\frac{\pi}{2}\right)^3 \frac{V}{L^2} r^4 \]
\[ T = 4\pi r^2 f_0 \rho \]

SITAR (Sa)  
\[ f_0 = 277 \text{Hz} \]
\[ 2r = 0.28 \text{mm} \]
\[ L = 613 \text{mm} \]
\[ J = 1.45 \times 10^{-5} \]
\[ \frac{J_p}{J_o} = 11.2 \]

PIANO C_4  
\[ f_0 = 262 \text{Hz} \]
\[ 2r = 1 \text{mm} \]
\[ L = 625 \text{mm} \]
\[ J = 1.62 \times 10^{-4} \]

(B) PLATE RESONANCE EFFECTS
DATA SHOWS THESE ARE SMALL
SITAR "SA" MEZZOFORTE SPECTRUM
ROOM AVERAGES: 5 SEQUENCES OF 32 SAMPLES
RMS AVG OF EACH SEQUENCE

PLAYER SHIFTED, SEVERAL MINUTES BETWEEN SEQUENCES
MIKE CARRIED AROUND ROOM ON VARIOUS PATHS DURING EACH SEQUENCE.

HIGHLY STABLE

BLUE: TRACE OF ONE ROOM AVERAGE SEQUENCE WITH DOTS TO SHOW PEAK VALUES OF OTHER SEQUENCES.
RED: CLOSE-UP SPECTRUM (30 CM FROM BOWL)
NOTE (A) GREAT AMOUNT OF SIGNAL FROM OTHER STRINGS (ETC.) BETWEEN PEAKS FOR "MAIN" PARTIALS.
(B) ROOM AVERAGE LEVELS NOT MUCH LOWER THAN CLOSE-UP.
**Sound Spectrum**

Red: Room average pp level
Blue: Room average mf level

Mean level up ~ 6 db.
Spectral pattern more irregular.

Red: Tone has little "zing"
Blue: Tone has strongly marked "zing" at beginning.

Red: Bridge driving force spectrum ~ \( \frac{1}{f} \)

\( \frac{\text{implied driving point}}{\text{impedance} \sim \frac{1}{(\text{mass})}} \)

If top plate is small

IE bridge + top plate looks elastic.

(Result curious, oversimplified calculation)

But not crazy.

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**Experimental Setup**

- LP Filter
- Magnet
- Midpoint of string loop gain
- Only for...

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**Feedback Loop**

- PWR AMP
- LP Filter
- Attenu
- Meter + Preamp
- Vib pickup

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**Curved Bridge Left End String**

- Screw
- Curved Bridge (adjustable)
- Fiberglass/Epoxi

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**EXP. BRIDGE**

- Brass
CURVED BRIDGE OSCILLATOR
Max Excitation (Large Loop Gain at \( f_1 \))

INTERMEDIATE EXCITATION

THRESHOLD OF OSCILLATION

VIBRATION PICKUP SIGNAL
RESPONSE TO TAPPING

VIBRATION PICKUP SIGNAL LEVEL (dB)

FREQUENCY (Hz)

KNIFE-EDGE BRIDGE OSCILLATOR
(Maximum Loop Gain)

(Even harmonics present because vibration pickup is sensitive to tension variations)

VIBRATION PICKUP SIGNAL LEVEL (dB)

FREQUENCY (Hz)
We can easily get $\frac{1}{q} = 0.02$

**Review The Kinematics**

Via this Coupling
Feet All Higher Modes
Dissipation Since IT Must
Much Greater Than 20X
Feed Back Run With
Oscillation Under Mode 1

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**ITEM 3**

Mode 1
Amplitude Even Larger Than
The Tenth (10x) Largest At Larger
Only Can Higher Modes (Above)
Oscillator Feed Back To Mode 2

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**ITEM 2**

Reduced Coupling To 0
10-fold Reduction of 1st Mode
On Overall Amplitude of Oscillation
Coupling Is Strongly Dependent
Iner Mode Coupling
The Curved Bridge Gives Enormous

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**ITEM 1**

Frequency

Vibration Pickup Signal Level (dB)

Both Curves Have Same Reference Level

Red: Curved Bridge Oscillator
Blue: Curved Bridge Strongly Plucked String Decays Rapidly

 Recall: Energy Input to This Spectrum

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$\frac{1}{q} = 0.02$
CURVED BRIDGE GIVES VARIABLE STRING LENGTH DURING OSCILLATION

GROSSLY OVERSIMPLIFIED MODEL
For string plucked at x₀ modal amplitude

\[ A_n = \text{Const} \times \sin \frac{n \pi x_0}{L} \times \left( \frac{1}{n^2} \right) \]

So \( A_i \) is by far the biggest.

Use it to calculate frequency modulation side bands of higher modes.

The sideband of n-th mode frequency

\[ f_{n\nu} = f_n \pm \nu f_1 \]

Amplitude of this side band is

\[ A_{n\nu} = A_n M^{\nu} = \frac{1}{n^2} M^\nu \]

where \( M \) is the modulation index \( \frac{\nu}{\nu_0} \)

Rapid convergence of \( A_n \approx \sqrt{n} \), and small \( M \) implies effect is important only at low \( n \), and \( \nu = 1 \).

\[ f_1 \quad f_2 \quad f_3 \quad f_4 \quad f_5 \quad f_6 \]

\[ (A_1 + M A_2) \quad A_4 + M(A_2 + A_2) \]

Numbers far too small, no effect at higher mode numbers.

Look a little closer.

A) Upper limit of swing
B) Lower limit of swing if bridge is knife edged
C) Lower limit of swing if bridge is curved
D) Time-varying (non-sinusoidal) force required to push string up into shape of curved bridge

Wave shape (bandwidth) depends directly on existing spectrum.

\[ \text{Wave shape (Bandwidth)} \text{ depends directly on existing spectrum.} \]

[Diagram of wave shapes and sidebands]

\[ \text{Time} \rightarrow \]

\[ \text{Period} \rightarrow \]