

# ON THE CAPILLARY METHOD OF IMPEDANCE MEASUREMENT — A BRIEF NOTE

May 11, 1979

Mr. Xavier Lurton  
Laboratoire d'Acoustique  
Université Paris VI  
4, Place Jussieu  
Tour 66, 5<sup>e</sup> étage  
75230 Paris Cedex 05 FRANCE

Dear Mr. Lurton:

After much delay I sit down to give a brief response to your letter about the capillary method of impedance measurement.

To begin, you will find a more recent paper by John Backus dealing more explicitly with the behavior of his annular capillary. As a practical matter, the capillary that Jansson and I used gives quite smooth (measured) response, being down some 3 dB around 2000 Hz. Backus' original paper presents a completely inadequate view of the mechanism. His second paper, with a much more detailed analysis (in response to comments by me!) shows the problem is much less than he thought before. In practice the problem is less yet.

Here is an excellent way to test (or calibrate) any capillary you may wish to use: The input impedance of a cylindrical tube of length  $L$  and characteristic impedance  $R_0 = \rho c / \pi a^2$  varies with frequency according to

$$Z_{in} = R_0 \left[ \frac{1 + j Q_0 \tan kL}{Q_0 + j \tan kL} \right]$$

where  $Q_0$  is a measure of the down-and-back attenuation due to wall losses and the terminating reflection loss. If the returned wave has an amplitude  $F$  for unit wave sent into the system

$$Q_0 = \frac{1+F}{1-F} = \left\{ \frac{(R_0/R_t) + \tan k \alpha L}{1 + (R_0/R_t) \tan k \alpha L} \right\}^2$$

Notice that when  $kL$  is an even or odd multiple of  $\pi/2$  that  $Z_{in} = R_0 Q_0^{\pm 1}$ . This is the key to the procedure.

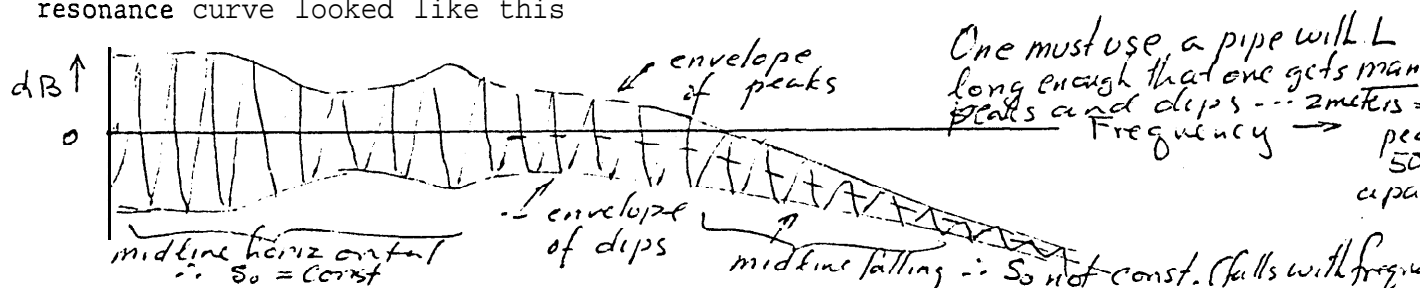
If you excite the pipe with a (capillary) source of strength  $S(\omega)$  (which you hope is a constant) and measure the pressure  $p_i$  at the input end,

$$P_i = S_0 Z_{in}$$

Mr. Xavier Lurton  
 Page 2  
 May 11, 1979

Plotting this on dB graph paper gives a curve whose peaks and dips are above and below the midline  $S_R$  by an amount  $\pm 20 \log Q$ . As long as  $Q$  is slowly varying, you can draw the envelope of the peaks, and of the dips. The midline drawn between these is now an estimate of  $S_R$ . If this is a constant, you have established that  $S$  is constant. Deviations from constancy give a direct measure of nonconstancy of  $S$  and thence of the properties of the capillary.

It is to be emphasized that as long as the entryway of the test pipe is cylindrical, any sort of (complex) termination is permissible, even a trumpet bell, or tuft of wool inserted a few cm from the open end. I have enjoyed using the technique even in extreme cases in which the resonance curve looked like this



Do not forget that even if the capillary is not frequency-independent, its behavior is smooth enough that a simple RC equalizing circuit can usually be designed for use in either the excitation feedback loop or in the pressure microphone channel to get an effectively constant system.

You will find it helpful to discuss these matters with René Causse at IRCAM in Paris. He has set up equipment of this sort, and its relatives. He and I have worked together both here and in Paris.

Yours sincerely,

A H Benade  
 A. H. Benade

1r