# PROBLEMS OF EXPERIMENTAL DETERMINATION OF AIRWAY AREA FUNCTIONS BY ACOUSTICAL METHODS

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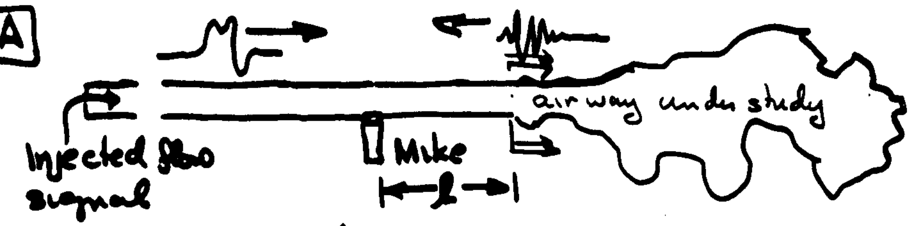
THERE ARE Two approaches to the measurement of airway AREA FUNCTION

(via acoustical measurements from one end)

A




Compare the incident signal with the return signal, and deduce the area function from the alteration

The distance ℓ is only to provide time delay so that one can record the two signals.

In the time domain this problem is essentially insoluble ... except via

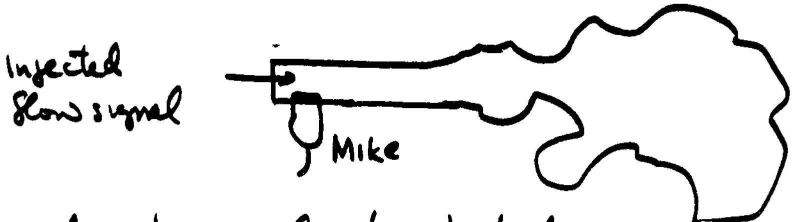
Computationally one analyzes into Fourier collection

$$p_{inc}(t) = \sum a_n \cos(\omega_n t + \phi_n) = \int a(\omega) e^{j(\omega t)} d\omega$$

$$p_{refl}(t) = \sum b_n \cos(\omega_n t + \psi_n) = \int b(\omega) e^{j(\omega t)} d\omega .$$

1 GET SHAPE FROM $[b(\omega)/a(\omega)]$

$\boxed{B}$




Record mike signal at instant of original injection, <u>and all subsequent reflections</u>

Let $\mathcal{R}$ be the operator that changes any arbitrary incident signal into its reflected version.

$$\text{Obs signal} = p_i(t) + 2\sum_{1}^{\infty} \mathcal{R}^n p_i(t)$$

$$= \left[2\sum_{0}^{\infty} \mathcal{R}^n - 1\right] p_i(t)$$

$$= \left[\frac{2}{1-\mathcal{R}} - 1\right] p_i(t)$$

This is formally ok but not ~~easily~~ interpreted physically ..... However if we make the Fourier analysis

$$p_i(t) = \sum c_n \cos(\omega_n t + \beta_n) = \int C(\omega) e^{j\omega t}$$

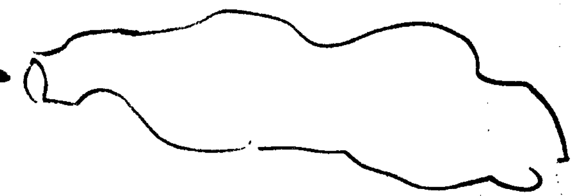
Then $\mathcal{R}(\omega)$ is <u>refl coeff for each component</u> is easy.

② It is equal to the $[b(\omega)/a(\omega)]$ of method $\boxed{A}$

What then <u>IS</u> $\mathcal{R}(\omega)$ ?

(the pressure refl. coeff for a sinusoid having frequency $\omega$)

Every air column has an input impedance

$Z_{in} \rightarrow$ 

$$Z_{in} = \frac{p(\omega)}{u(\omega)}.$$

Fundamental measurement procedure:
<u>Have known injected flow</u>, <u>measure pressure response</u>

STEADY STATE, SINGLE FREQUENCY

$Z_{in}$ depends on structure and materials of the air column

Method $\boxed{A}$ measures $\mathcal{R}(\omega) = \dfrac{Z_{in} - R_o}{Z_{in} + R_o}$

where $R_o = Z_{in}$ for an infinitely long cylindrical lead-in pipe.

$$\boxed{R_o = \sqrt{B\rho}/\pi a^2}$$

③ So Method $\boxed{A}$ gives $Z_{in}^{(\omega)} = R_o\left[\dfrac{1+\mathcal{R}(\omega)}{1-\mathcal{R}(\omega)}\right]$

METHOD $\boxed{B}$ gives $Z_{in}$ directly

because $P_i(\omega) = R_0 \cdot U(\omega)_{injected}$

and $\left[\frac{2}{1-R} - 1\right] = Z_{in}/R_0$ !!

IN BOTH METHODS, WE GET $Z_{in}(\omega)$
and then try to figure out
what shape (etc) of air column gives

What is the choice of method?
Your choice, freely. How is
your computer programmed?
What sort of $U(t)_{injected} \rightarrow U_{inj}(\omega)$
can you generate?

AT CWRU WE HAVE OUR CHOICE FOR FINDING $Z_{in}(\omega)$

Not true (nearly) everywhere else.



AS A MATHEMATICS PROBLEM

(a) Calculation of $Z_{in}$ for a given air column
is fairly straightforward

(b) Calculation of air column for given $Z_{in}$
is not straightforward at all!

NOTE
(a) partial information on the air column gives decent $Z_{in}$
(b) partial information on $Z$ in may give disastrous air col shape.

THIS IS CRUCIAL

For example: If only the poles of $Z_{in}(\omega)$ (or zeros) are given
There are TWO possible air columns.

ambiguity is only resolved if we have both poles and zeros.

Also: need great precision in many cases

IF WE CAN'T CALCULATE $Z_{in}(\omega)$
WE DONT HAVE A CHANCE AT
CALCULATING THE AIR COLUMN.

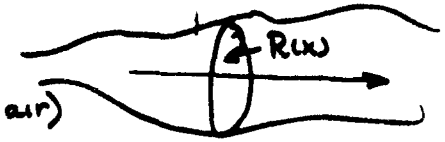
LET US LOOK NOW AT CALCULATING $Z_{in}$.

Suppose:

$R = R(x)$ (radius)

$\rho = \rho(x)$ (density of air)

$B = B(x)$ (bulk modulus of air)




At the frequency $\omega$ TO GOOD APPROX

$$\frac{\partial^2 p}{\partial x^2} + \left[\frac{2R'}{R} - \frac{\rho'}{\rho}\right]\frac{\partial p}{\partial x} + \left(\frac{\rho}{B}\right)\omega^2 p = 0$$

Geometry

density

both density and bulk modulus

THIS IS NOT BAD AT ALL

(1) IF $R''$ is not too bumpy ... (more later)

(2) IF NO NET FLOW OF AIR.

(3) IF WALLS ARE RIGID

(4) IF NO DISSIPATION

ALL THE AREA INFO IS HERE IN THE $\frac{\partial p}{\partial x}$ TERM

THIS WILL PROVE IMPORTANT



(1) If $R''$ is bumpy, we can put in a decent correction thus:

Replace $\rho(x)$ by $\rho(x)\left[1 + \frac{R|R''|}{(R')^2}\right]$

(This can be a nuisance, but no big deal in calculating $Z_{in}$ for specified air column)

(2) If net flow $U_0 \neq 0$ we can make corrections all of which depend on $[1 \pm M^2]$

where $M(x) = [U_0/\pi R^2(x)]/c$ 345 m/sec

[for example if $U_0 = 1$ liter/sec, $\pi R^2 = 1\ cm^2$] $M = .03$

(more anon)

(3) Non-rigid walls

Big Mess

Replace $R(x)$ by

$$R_0(x) + r_e(x) \cdot \left[\frac{\omega_0^2}{\omega_0^2 - \omega^2 + jg_0\omega}\right]$$

(4) If we take dissipation into account

Replace $\begin{cases} B(x) & \text{by } B(x) + j\beta(x) \\ \rho(x) & \text{by } \rho(x) + j\sigma(x) \end{cases}$

$\beta$ and $\sigma$ depend on R and $\sqrt{\omega}$

THERE IS ALSO SIGNIFICANT DISSIPATION ASSOCIATED WITH FLOW ($u_0/\pi R^2$) THAT GOES IN WITH $\rho(x)$ MOSTLY

(3a) THE PRESENCE OF FLOW ALSO ALTERS THE R" CORRECTION FOR $\rho(x)$! INCREASES

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THE PRESENCE OF FLOW CAN DO EVEN MORE .... AND IT CAN BE VERY SERIOUS

(a) Swirling etc can have a big effect on the $Z_{in}(\omega)$.

Area function + dissipation

IN A RIGID-WALL SYSTEM.

NOISE ALSO TO CONFUSE THE ISSUE FURTHER



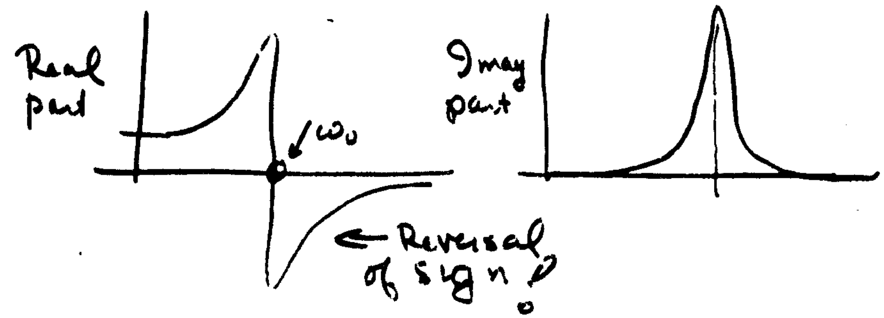
STEADY FLOW IN AN ELASTIC-WALLED AIRWAY GIVES RISE TO

**AN ADDITIONAL TERM IN THE WAVE EQUATION**

$$\frac{\partial^2 p}{\partial x^2} + [\quad]\frac{\partial p}{\partial x} + \left[\frac{u_0}{\pi R}\right]^2 \left[\frac{\rho(x)}{R(x)}\right]\left[\frac{\text{Const}\cdot\omega_0^2}{\omega_0^2 - \omega^2 + jg\omega}\right]\frac{\partial p}{\partial x} + \left(\frac{\rho}{B}\right)\omega^2 p = 0$$

Notice that $\frac{\omega_0^2}{\omega_0^2 - \omega^2 + jg_0\omega}$

Looks like this:




GENERAL REMARK

$u_0$ REALLY SCREWS UP ESTIMATES OF $R(x)$!!!

$$\left[\left(\frac{2R'}{R} - \frac{\rho'}{\rho}\right) + \left(\frac{u_0}{R^2}\right)^2 \frac{1}{R} \times \left(\frac{\omega_0^2 \times \text{Const}}{\omega_0^2 - \omega^2 + jg\omega}\right)\right]\frac{\partial p}{\partial x}$$

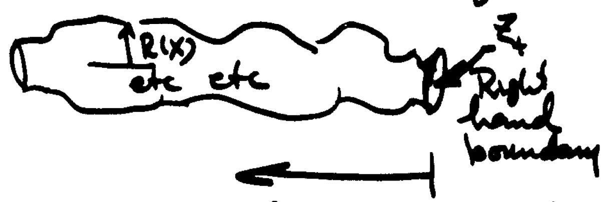
ORIGINAL AREA INFO

NEW FLOW DEP FREQ DEP AREA INFO!

# TAKE A BREAK FROM ALL THESE COMPLEXITIES

How does one get $Z_{in}$ from the wave equation?

$Z_{in} \rightarrow$




**EASY VERSION**

Integrate the wave equation from right to left starting from a specified ratio $Z_t$

$$Z_t = p \Big/ \left[ \frac{-\pi R^2}{j\omega\rho} \frac{\partial p}{\partial x} \right] \quad \text{evaluated at the last end}$$

this gives p and $\frac{\partial p}{\partial x}$ at the right hand end.

Then calculate $Z_{in}$ thus,

$$Z_{in} = p \Big/ \left[ \frac{-\pi R^2}{j\omega\rho} \frac{\partial p}{\partial x} \right] \quad \text{at the left end}$$

Note: p's Here are The true p(x) at the two ends.

Once this is done as a formal procedure, try to invent an inversion procedure

$$Z_{in}(\omega) \Longrightarrow R(x) \text{ etc etc}$$

OFTEN THIS REQUIRES KNOWLEDGE OF $Z_t$!

That is where we fail.



# FUNDAMENTAL PROBLEMS OF CALCULATING R(x) FROM $Z_{in}(\omega)$.

1. The wave equation involves R(x) in two ways, (one is frequency dependent) IF FLOW AND/OR ELASTICITY

2. Dissipative effects mix with R(x) in several ways, mostly frequency-dependent.

3. We usually do not (cannot) know $Z_t$ except in the vaguest sort of way.

THIS MAKES R(x) AMBIGUOUS EVEN IN THE SIMPLEST CASE

## CONCLUSION

It is not really clear how much area info is actually extractible. Consistent results tend to prove only consistent procedures.

MUCH MORE CHECKING MUST BE ROUTINE!

SOPHISTICATED, DETAILED