CENTER FOR COMPUTER RESEARCH IN MUSIC AND ACOUSTICS
MARCH 1991

Department of Music
Report No. STAN-M-70

PSYCHOACOUSTIC FACTORS IN MUSICAL INTONATION: BEATS, INTERVAL TUNING, AND INHARMONICITY

Douglas Fleming Keislar

CCRMA
DEPARTMENT OF MUSIC
Stanford University
Stanford, California 94305

PSYCHOACOUSTIC FACTORS IN MUSICAL INTONATION: BEATS, INTERVAL TUNING, AND INHARMONICITY

A DISSERTATION

SUBMITTED TO THE DEPARTMENT OF MUSIC

AND THE COMMITTEE ON GRADUATE STUDIES

OF STANFORD UNIVERSITY

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY

By

Douglas Fleming Keislar

March 1991

Abstract

An important acoustical phenomenon for the theory and practice of musical intonation is the beating of partials of nearly coinciding frequency in simultaneous notes. It is well-known that beating partials are used when tuning certain instruments. It is also often assumed that intervals sound more in tune when the beating is slower, and there is some experimental evidence to this effect. However, in acoustic (non-electronic) musical instruments, the rate of beating is inextricably coupled to the tuning of the interval, making it difficult to ascertain whether the "out-of-tune" percept is caused by the tuning of the interval itself (as measured by the logarithm of the frequency ratio) or by the concomitant beat rate. With synthesized sound, on the other hand, one can manipulate the beat rate independently of the tuning by frequency-shifting the appropriate spectral components.

The question motivating this research was to what extent beat rate itself is responsible for the sensation of "out-of-tuneness" that is normally only found in tandem with a mistuned fundamental frequency ratio. Three psychoacoustic experiments were conducted using synthesized stimuli and musically experienced subjects. In the first two experiments, the interval was the perfect fifth F4-C5; in the third experiment it was the major third F4-A4. The beat rate was controlled by two different methods. The first was simply to change the size of the interval, as in traditional instruments, and the second was to frequency-shift one partial of each pair of beating partials, without changing the overall interval tuning. The second method introduces inharmonicity. In addition, two levels of beat amplitude were introduced by using either a complete spectrum of 16 equal-amplitude partials for both notes, or by deleting one partial from each pair of beating partials.

The results of all the experiments indicate that, for these stimuli, beating does not contribute significantly to the percept of "out-of-tuneness," because it made no difference statistically whether the beat amplitude was maximal or minimal. By contrast, changing the interval size had a great effect. For the fifths, frequency-shifting the appropriate partials had about as much effect on the perceived intonation as changing the interval size. For thirds, this effect was weaker, presumably since there were fewer inharmonic partials, and they were higher in the harmonic series. Subjects were less consistent in their judgments of thirds than of fifths. The major third of just intonation was not preferred overall to the equal-tempered third.

Since it is unlikely that beats would be more audible in real musical situations than under these laboratory conditions, these results suggest that the perception of intonation in music is dependent on the actual interval tuning rather than the concomitant beat rate. In addition, they indicate that inharmonicity can affect perceived intonation, and that the tuning of fifths is less ambiguous than that of thirds. If beating partials are unimportant vis-à-vis interval tuning, this strengthens the argument for a cultural basis for musical intonation, as opposed to the acoustical basis set forth by Helmholtz and others.

For my parents

Marvin and Mary Keislar

Acknowledgments

First, I should like to thank my advisor, John Chowning, for having accepted me into the Ph.D. program at Stanford, and for encouraging my research from its inception. John's keen insight and rare humanity have sown the seeds for much of the creative and scholarly work accomplished under his benevolent directorship of CCRMA. It was he who first suggested I base my graduate research on my interest in tuning and microtonality.

I cannot begin to express enough gratitude to Earl Schubert; the design and execution of this interdisciplinary research would simply have been impossible without his active and continuing interest. His patient and generous spirit, coupled with his vast knowledge of the psychoacoustic literature and experimental methodology, were indispensable in fueling my research efforts. Any merit in this work stems from his expert guidance.

My appreciation also extends to John Pierce and Max Mathews. Both of them have been interested in the relation of acoustics to intonation for many years, and they have been able to offer penetrating insights into this research problem. Discussions with them, individually and in committee, helped clarify the course of this thesis.

An important phase in the development of my research was a three-month stay at IRCAM in Paris, where Stephen McAdams and David Wessel were extremely helpful. Carol Krumhansl and Eric Clarke also participated in enlightening discussions about my pilot studies at IRCAM. I was pleased to be able to meet with Rudolf Rasch and Joos Vos in Holland during this time, since their writings on intonation are among the most important in the field.

A number of friends and colleagues have provided thoughtful conversations about this work and encouragement for its completion, including (but not limited to) Amnon Wolman, Janet Yale, Xavier Serra, Richard Karpen, John Strawn, and Patte Wood. Members of my family have been similarly helpful.

Recognition must be given those who helped supply the software and hardware environment that made this research possible; among them are Bill Schottstaedt, Tovar, Jay Kadis, Chris Chafe, and Glen Diener. Finally, I am grateful to all the volunteers who agreed to take time from their studies to serve as subjects in the experiments.

Table of Contents

List of Tables

List of Illustrations

Chapter

1 Introduction and Review of Related Literature

This dissertation investigates the role of beating partials in the perception of musical intonation. When the two simultaneous notes of a consonant harmonic interval are mistuned slightly from their "pure" tuning—in other words, their tuning in the system of just intonation—there are usually several pairs of beating harmonics. In each of these pairs, a harmonic of one tone almost coincides in frequency with another harmonic of the other tone, creating an interference pattern within the ear that is audible as beating. An influential theory holds that this phenomenon not only explains consonance and dissonance, but also makes the intervals of just intonation sound the most in tune. In this study we examine whether the beating itself actually has any effect on the intonation as judged by musically trained listeners, or whether it is just the size of the interval that is relevant. This chapter reviews the related literature, and Chapters 2 - 4 present the results of three psychoacoustic experiments designed to shed light on this question.

1.1 Introduction to the Problem

One of the earliest discoveries of a numerical principle embodied in physical phenomena was the finding, attributed by the Greeks to Pythagoras and by the Chinese to Ling Lun, that the fundamental musical intervals correspond to ratios of small integers. Pairs of otherwise identical strings or pipes produce consonant musical intervals when the lengths of their vibrating portions are related by small-integer ratios, such as 2:1, 3:2, and 4:3 (which correspond to the musical intervals of the octave, perfect fifth, and perfect fourth). Small-integer ratios have been essential to Western music theory, and have also appeared prominently in the musical traditions of non-European cultures, including China, India, and the Arab world. At the beginning of the Scientific Revolution these ratios of string lengths (or pipe lengths) were found to be identical to the ratios of the vibration frequencies of the sounding bodies.

Furthermore, the degree of simplicity of the ratios has some correlation with the degree of consonance as traditionally defined by music theory. In addition to the "perfect consonances" just mentioned, which were accepted by the ancient Greeks and in turn by Medieval European music theorists, Renaissance theorists recognized new "imperfect" consonances of 5:3, 5:4, 6:5, and 8:5 (corresponding to the musical intervals of the major sixth, major third, minor third, and minor sixth). The dissonant musical intervals corresponded to yet more complex ratios (such as 9:8 or 10:9 for the major second, 15:8 for the major seventh, 16:9 for the minor seventh, 16:15 for the minor second, and 45:32 for the tritone). Similar schemes have been present in the music theory of other cultures. This apparent correspondence between consonance and simplicity of ratio did not escape the attention of theorists; on the contrary, it was not only a fundament of Western music theory but also the object of scrutiny by a number of eminent scientists and philosophers from the late 16th century on. The phenomenon of consonance formed a bridge between order in the natural world and order in the human mind, and thus was of interest for mathematics, physics, and philosophy.

It was Hermann Helmholtz in the 19th century who first offered a thorough, acoustically based explanation of consonance that is still scientifically satisfactory. The phenomenon of beating had long been known, as had the harmonic series, but Helmholtz put the two together to propose that beats of upper partials were responsible for dissonance. For example, in a perfect fifth, the third partial of the lower note coincides in frequency with the second partial of the upper note, and when the interval is mistuned these two partials beat with each other. The more the fifth is mistuned, the faster the beating, until at some point maximal roughness is reached. Intervals with more complex ratios have fewer coinciding harmonics and tend to have more beating pairs. The smaller the integers in the interval's frequency ratio, the more of these pairs of coinciding harmonics there are.

Much earlier, theorists had been on a similar track, in seeking to explain dissonance by interfering vibration patterns, but their explanations of interference dealt with the vibrations of the fundamental frequencies rather than of upper partials, and they were at a loss to explain exactly how the interference caused dissonance. Helmholtz' familiarity with Ohm's law and the harmonic series enabled him to think about musical tones in terms of their constituent frequencies, and furthermore his understanding of physiology permitted him to see how neighboring frequencies would interfere within the ear, causing roughness. These capabilities resulted in the first well-founded physiological explanation of the phenomenon of consonance.

Not only did Helmholtz use his theory to explain the historical genesis of musical scales, but he also felt that the intervals of just intonation (those based on simple integer ratios) sounded more in tune than tempered intervals (those that deviate from such ratios). The implication is that beating partials not only explained the origin of the culture's musical materials, but also contributed to the perception of intonation: beating made intervals sound out of tune.

It is this second question which is addressed in the present study—not the remote origin of the consonances, but whether beating is a significant component of intonation for the contemporary listener. If listeners hear beat rate as contributing to an interval's goodness of intonation, this would indeed support the theory that beats were responsible for the adoption of small-integer ratios in music. If beating is unimportant for contemporary listeners, this weakens the theory. However, in either case one can argue that listeners judge intonation according to learned references, whether the reference is a familiar beat rate, in the first instance, or a familiar interval size, in the second.[1] Thus judgments of intonation by modern listeners can offer only evidence, not proof, of the origin of musical intervals. In any case, the contemporary musician is more concerned with the practical question of modern listeners' perception. If beats are important, they represent a possible constraint on a composer's choice of tuning system, and systems that minimize beating, such as just intonation, would have a greater theoretical justification. But if beats are not significant, the composer interested in nonstandard tunings might equally well choose some arbitrary system with no small-integer frequency ratios.

1. In the former case the argument is somewhat tenuous, since the beat rate of any interval depends on its register, and thus listeners would need to learn many more reference points than in the latter case, where one need only learn the size of 12 intervals.

In the course of answering the question of whether beat rate affects perceived intonation, this dissertation will shed further light on a related question that has already been the object of a good number of studies in the twentieth century—whether listeners do in fact judge the intervals of just intonation to be more in tune. The present study attempts to disentangle these two questions—whether beating is important, and whether just intervals are preferred—by using a technique in which the rate of beating is manipulated independently of the fundamental frequencies of the two notes in the given interval. I believe this approach to be original to this dissertation. To my knowledge, no one has previously studied the perception of ostensibly just intervals that contain rapid beating, for example. The role of beating is examined in this research by separating beat rate from interval tuning. One of the techniques used—frequency-shifting certain partials to control the beat rate without changing the interval size—has the side effect of introducing inharmonicity, as discussed further on page 50. Thus we are interested in previous literature on the topics of beating, interval perception, and inharmonicity, as these relate to the perception of intonation.[2]

The subsequent sections of this chapter summarize some of the relevant literature, both musicological and psychoacoustic. We begin by briefly reviewing theories of consonance, since—as we have already seen—these are closely related to matters of intonation and beats.

2. We shall not digress into a detailed treatment of the mathematical relations between beat rates and tuning systems. The interested reader is referred to Rasch (1984) for a thorough description of the mathematics of beats and intervals, and to Rasch (1983) and Blackwood (1985) for the mathematics of tuning systems.

1.2 Consonance Theories

> Εστι δε συμφωνια μεν κρασις δυο φθογγων, οξντερον και βαρυτερου. Διαφωνια δε τουναντιον δυο φθογγων αμιξια, μη οιων τε κραθηναι, αλλα τραχυνθηναι την ακοην.
>
> [Consonance is the blending of a higher with a lower tone. Dissonance is incapacity to mix, when two tones cannot blend, but appear rough to the ear.] [3]
>
> —Euclid

It is useful to examine consonance theories as a backdrop for reviewing the studies on beats and intonation, since many of these studies were designed to shed light on the question of consonance. First a semantic issue must be addressed: most of the psychoacoustic studies on consonance have assumed that it is a property observable in isolated harmonic intervals. In this limited sense, consonance is a property of certain frequency ratios that is independent of musical context, but which makes these intervals suitable for certain uses in musical contexts. More specifically, it is often assumed that some acoustic feature of small-integer ratios is responsible for their importance in traditional music theory, which describes or prescribes the harmonic use of such intervals at points of lessened tension in musical passages. But while it is true that music theory has made much of the small-integer ratios as seemingly absolute entities, it is also true that musical practice and theory have often defined consonance and dissonance relative to a musical context, taking into account contrapuntal motion, tonal centers, modal or scalar structure, style, and the like. As an example, the perfect fifth F# - C# would have a dissonant effect if embedded in a traditional cadential passage in the key of C. Modern psychoacousticians with some musical awareness have generally understood this discrepancy, using words like "tonal consonance" or "sensory consonance" to refer to psychoacoustic aspects that are still present in isolated intervals, and terms like "musical

3. *Euclides*, ed. Meibomius, p. 8. Cited by Helmholtz (1877/1954), p. 226, trans. Ellis.

consonance" to refer to aspects dependent on context and style.[4] The studies of consonance reviewed here do not address the contextual treatment of intervals. They simply attempt to explain the importance of small-integer ratios in music and the fact that music theory's traditional ordering of intervals by consonance roughly follows the order of complexity of the respective ratios. The validity of such studies holds, in spite of some authors' apparent ignorance of the more flexible connotation of "consonance" among musicians.

One may sum up theories of consonance with the categories "numerical," "physiological," and "environmental."[5] By "numerical," I mean theories, such as virtually all explanations up to the 16th century, that rely on the actual or supposed properties of the ratios themselves, without considering the physical means by which such numbers might be apprehended by the human ear and brain. With the advent of the Scientific Revolution, scholars began to attempt to explain the physiological process, whether they felt that the brain could apprehend the ratios as such, or (as was more common later) that some intermediary mechanism was affected by the vibrations in such a way as to give the simple-integer frequency ratios a special characteristic. By "environmental," I refer to theories that assume that people would not hear such sounds as consonant if they did not learn to do so through frequent exposure to them.

4. See, for example, Terhardt (1977) or Rasch and Plomp (1982).
5. Other writers have categorized consonance theories. Linda Roberts (1983) refers to "psychoacoustic" and "relativistic" theories. (She also mentions some early 20th-century explanations, now abandoned, that are based on heredity.) Her "psychoacoustic" category consists of temporal mechanisms, whether concerning "fusion" or some sort of ratio detection, and spectral mechanisms such as Helmholtz'. Under "relativistic" she includes "learning," "contextual," and "cultural" explanations. Burns and Ward (1982), taking a less historical view, refer to three competing theories: ratio detection, beating (à la Helmholtz), and learning based on the harmonic series (à la Terhardt [1977]). H. F. Cohen (1984), considering the points of view during the early stages of the Scientific Revolution, discusses "numerological," "mathematical," "experimental," and "mechanistic" approaches, and subsequently mentions the later theories based on the harmonic series and on beats.

The oldest approaches are numerical, but they persist today. For the Pythagoreans, the world was simply an embodiment of mathematics, as proven by the presence of the small integers in the consonant musical intervals. The numbers two and three had special significance, as evidenced by the Pythagorean tuning, which uses only these two prime numbers. (For example, the Pythagorean major third is 81:64 rather than 5:4.) Numerological treatment of intervals spanned cultures and centuries. During the Middle Ages in Europe, music was considered one of the four mathematical disciplines of the Quadrivium, along with arithmetic, geometry, and astronomy. The leading music theorist of the Renaissance, Gioseffe Zarlino, formulated the *senario*, a largely numerological explanation of why the consonant musical intervals used only the first six whole numbers in their string-length ratios. (The one exception, the 8:5 ratio of the major sixth, occasioned additional forensic contortions.) Some later philosophers like Kepler cast their numerical treatments in a slightly more rational mold, while still being preoccupied by the mathematics themselves, which in Kepler's case took a geometric turn. A tradition of numerical, as well as numerological, treatment of musical intervals has continued unbroken to our time. While modern musical thinkers are not unaware of scientific approaches, there is considerable discussion today of the properties of various prime numbers for just intonation. In some cases, the participants appeal to physiological theories, such as that of Boomsliter and Creel (1961). The idea that different prime numbers correspond to different affects seems to be particularly appealing.[6] This idea has not been experimentally tested. However, Maher (1980) found "only partial support for the popular notion that each musical interval has a unique psychological effect." In the results of his psychological experimentation, many intervals were not discriminated from each other on any affective

6. For examples, see Makeig (1979 - 1980, 1982), and the interview with Ben Johnston in Keislar (1991).

scale, including all the intervals from the minor third through the major sixth. Certainly discrimination on the basis of prime factors would be even less likely.

A necessary addendum to any numerical theory of consonance is the concept of tolerance. Any number describable as a ratio of small integers has, of course, an infinite number of extremely nearby neighbors that cannot be so expressed (whether they be large-integer ratios or irrational numbers), yet the ear could not possibly discriminate these from the "pure" interval even if a means of producing sound so accurately could be found. Theories of consonance based on physiology have less difficulty explaining tolerance, since physical phenomena (such as the vibration of the basilar membrane in the ear) have limits of resolution.

Physiological explanations of consonance came to prominence during the Scientific Revolution. H. F. Cohen (1984) traces in great detail the development of theories of consonance in the 16th and early 17th centuries, which saw the emergence of a scientific approach involving both human physiology and the physics of vibrating objects. The major figures cited by Cohen are: Giovanni Battista Benedetti, Vincenzo and Galileo Galilei, and Marin Mersenne, as representatives of "the experimental approach"; Johannes Kepler and Simon Stevin ("the mathematical approach"); and Isaac Beeckman and René Descartes ("the mechanistic approach"). Some of these thinkers will be mentioned again below in the discussion of beats. The prevailing theory of consonance during this epoch was what Cohen calls "the coincidence theory," first formulated by Benedetti in 1563.[7] The connection between vibration and pitch had been discovered; scholars were now aware that the ratios of musical intervals, which had traditionally described the relative lengths of vibrating strings, also applied to the vibration frequencies. Thus it was natural that they should attempt to explain consonance by the synchrony of some aspect of vibration. They supposed, for example, that every third pulse of a lower note would coincide with every second pulse of a note a perfect fifth above it. Some writers such as Beeckman were more specific and postulated sound particles or globules that struck the eardrum periodically to produce a certain pitch. It was assumed that the periodic coincidence of two such pulses or impacts upon the ear yielded greater pleasure. Apparently no one until Isaac Newton recognized the fatal weakness of this coincidence theory: it was helpless to explain consonance when the two vibrations are out of phase.

The major new theory of consonance in the 18th century was Rameau's assertion that the harmonic series was responsible for the importance of small-integer ratios (Rameau [1737]). In his earlier *Traité de l'harmonie* (1722/1971), Rameau followed the tradition of explaining consonance in terms of numbers and string lengths. After reading the acoustical writings of Joseph Sauveur, however, he reformulated his theory of consonance on the basis of overtones.[8] The major triad's appearance in the first few partials of the harmonic series provided a "natural" basis for musical art, the emphasis on nature reflecting a major theme of the 18th-century *zeitgeist*. Not being a scientist, Rameau was less concerned with physiological investigations; it was sufficient to discover in nature the most important chord of music theory. Rameau's might thus be considered an "environmental" approach; one creates music organized by principles audible in natural sounds. He also introduced the principle of the *basse fondamentale*, or the root of a chord, which he used to establish a new

7. Cohen, a science historian, says that around this time Benedetti wrote a letter to the composer Cipriano de Rore which "ends with a brief, 40-line theory on the generation of the consonances through the 'cotermination of percussions'...Benedetti now takes as his starting point the regular strokes that are produced by a vibrating string. He realizes that at different intervals the number of these strokes per unit time is inversely proportional to the lengths of the strings that are sounded. Apparently he takes this property to be self-evident, as there is no trace in his account of even an attempt to prove it..." (Cohen [1984], pp. 75, 77).
8. This debt to Sauveur is emphasized by Philip Gossett in the introduction to his translation of Rameau's earlier treatise (Rameau [1722/1971]), p. xxi.

approach to harmonic analysis in which chords retained their essential identity through different inversions. Some modern writers[9] have cited Rameau as a forerunner of temporal or pattern-matching theories of consonance, because his notion of the *basse fondamentale* bears some similarity to later ideas such as Schouten's (1938) "residue pitch," the "long pattern" of Boomsliter and Creel (1961), or the "virtual pitch" of Terhardt (1974a).

In the 19th century Hermann Helmholtz combined the concept of interference, inherited from the 17th-century coincidence theory, with an understanding of the harmonic series, found in the 18th-century writings of Sauveur and Rameau. The resultant synthesis was a theory of consonance based on interfering harmonics that solved problems of both the earlier consonance theories.[10] Helmholtz had read d'Alembert's presentation of Rameau's theories, of which he says:

> In his book there is no mention of beats, and hence of the real source of distinction between consonance and dissonance. Of the laws of beats very little indeed was known at that time...[11]

Further, Helmholtz' understanding of both auditory anatomy and Ohm's law (based on Fourier's theorem) enabled him to propose a physiological model of hearing that explained consonance with unprecedented thoroughness. He was convinced that beats of upper partials, interfering within the ear, explained dissonance:

> ...it is apparent to the simplest natural observation that the essence of dissonance consists merely in very rapid beats...Consonance is a continuous, dissonance an intermittent sensation of tone...
>
> Hence I do not hesitate to assert that the preceding investigations, found upon a more exact analysis of the sensations of tone, and upon purely scientific, as distinct from esthetic principles, exhibit the true and sufficient cause of consonance and dissonance in music.[12]

---

9. for example, Terhardt (1974a), Mathews and Pierce (1980), Roberts (1983).
10. H. F. Cohen (1984) presents this insight into the synthetic nature of Helmholtz' achievement.
11. Helmholtz (1877/1954), p. 233.
12. Helmholtz (1877/1954), pp. 226 and 227.

In a sense, Helmholtz moved the old coincidence theory from the time domain to the spectral domain: the emphasis was now on coinciding frequencies, rather than coinciding pulses. Helmholtz understood from current anatomical research that the basilar membrane is in effect a series of resonators, so that each location along the membrane vibrates in response to a different portion of the audible frequency spectrum. Since the membrane has finite elasticity, two frequencies whose locations are near to each other, but not quite coinciding, will interfere, causing beating. When the sound source consists of two complex tones (i.e., tones containing harmonics), certain pairs of their component frequencies coincide when the two fundamental frequencies are related by small-integer ratios. If such consonances are mistuned, beating occurs. The dissonant intervals can be viewed as extremely mistuned consonances.

More will be said below, under "Beats," about Helmholtz' influential theory, as well as 20th-century extensions to it. (See pages 20 - 38.)

Stumpf (1898) proposed an alternative to Helmholtz' theory, saying that consonance is created by the sensation of fusion, i.e., how well the tones blend into a unitary sound. This theory offers little in the way of a physiological explanation, but was influential for some time. Brues (1927) asked subjects to rate the "fusion" or "unitariness" of all the intervals in the quarter-tone scale. The perceived fusion of an interval was not dependent on the simplicity of the ratio, providing evidence against Stumpf's theory. DeWitt and Crowder (1987) found limited support for Stumpf; subjects were slower to identify octaves and fifths as having more than one tone.

Helmholtz' model relies on the "place" mechanism, i.e., the spatially distributed frequency sensitivity of the basilar membrane. The other important sort of physiological explanation of consonance relies on temporal hearing mechanisms (although the anatomical component tends to be less explicit in such explanations). Irvine (1946) noted that the waveforms of harmonic intervals with small-integer frequency ratios had shorter periods than those with more complex ratios, and hypothesized that the length of this cumulative period was responsible for the degree of consonance. Resnick (1981) offered a refinement of this idea, connecting it with the minimum time required to perceive the pitch of a tone. According to this model, if the period of the tone is shorter than the minimum time for pitch perception, the tone is consonant. Boomsliter and Creel (1961) offered a theory, based on synchrony of neural firings, that can be viewed as an updated physiological enhancement of the old coincidence theory. They conducted a number of musical studies, making rather extensive claims about musicians' preference for various integer ratios.[13] Roberts (1983) cites Houtsma and Goldstein (1971) as proposing that a central processor of pitch (as opposed to the auditory periphery) is responsible for consonance and dissonance. Terhardt (1974a) proposed a pattern-matching scheme in which a "virtual pitch" is calculated from the frequencies of the harmonics in a stimulus. This pattern-matching mechanism would also be sensitive to simple ratios, a fact that Terhardt thought explained "musical consonance," as opposed to the "psychoacoustic consonance"—based on roughness—that Helmholtz treated. Although this pattern-matching theory bears some similarity to temporal theories, it is based on detection of frequency patterns across the spectrum.

Terhardt (1974a) also proposed that one develops a template for simple ratios by hearing the harmonic series in speech sounds starting from infancy. This theory would be classified as an "environmental" one, and is somewhat like Rameau's ideas. Also in this

13. See page 51 for more on this study.

"environmental" category would fall all explanations based on cultural factors. Cazden (1962), for example, offers an eloquent rebuttal of reductionist approaches to art, specifically Helmholtz' notion that music can be reduced to terms of physiological sensation. Theories of consonance that appeal to culture, learning, or context imply a higher level of processing than that which psychoacoustics delineates. Certainly what has been referred to as "musical consonance" (in opposition to "sensory consonance") must fall primarily into the cognitive, rather than psychophysical, realm. There is ample evidence that the perception of the simplest musical elements, such as the pitch of notes in a musical scale, is dependent on a cognitive framework. Shepard and Jordan (1984), for instance, found that subjects made significant errors in judging the pitch relation of notes in a stretched major scale. The study of Ayres, Aeschbach, and Walker (1980), discussed on page 24, presents the idea that unfamiliarity with musical materials (in their case, quarter-tones) affects judgments of consonance even in an experimental setting that follows the psychoacoustic paradigm. From the cognitive perspective, the assumption that a certain arrangement of frequencies must be perceived as having this or that degree of consonance is analogous to the notion that a certain succession of phonemes must mean the same thing in every language. Since the focus of this dissertation is not on consonance, but on beats and intonation, we shall not belabor this point with respect to consonance, nor review all the related literature on music cognition. However, the same caution holds with respect to intonation; and when we examine the results of the psychoacoustic experiments presented in this dissertation, we shall again encounter the fact that subjects judge intonation with reference to a learned framework.

We close this review of consonance theories with a final caveat concerning the historical origins of consonant intervals. It has usually been assumed, but it can probably never be proven, that the use of musical intervals corresponding to small-integer ratios preceded awareness of the numerical relationship they embodied. Although the association of consonances with small integers was taken to be evidence of order in nature, it is conceivable that the causality is reversed—namely, that ancient musicians treated these intervals as special because of the theoretical principles they embodied, rather than because of any innate properties requiring the intervals to be heard as consonant. If so, then the hierarchy of consonances would be a cultural phenomenon based on non-universal materials. Such a view would explain the use, in some cultures, of scales that lack these intervals, such as the slendro tunings of Indonesian gamelan.[14]

The truth may lie somewhere in between these two viewpoints—perhaps acoustical phenomena determined the choice of, say, the octave, fifth, and fourth; and once the ratios of these intervals were ascertained, other small-integer ratios were consciously added to the system. The octave and fifth, at least, could scarcely have gone unnoticed (for example, in wind instruments). In combination, these intervals yield the fourth, and a cumulative series of fifths (or fourths) yields the entire set of Pythagorean intervals. The "Ptolemaic" major third (5/4) may simply have arisen as a numerical approximation to the Pythagorean third (81/64), although it is not unreasonable to imagine that it was originated by ear instead. It is worth noting another point, seldom addressed by modern advocates of just intonation: ancient theorists had little choice other than to express all intervals in terms of integer ratios, because decimal fractions—not to mention logarithms—were still unknown.

14. The inharmonicity of the gamelan has been mentioned as a counter-argument for this specific example, but see the comment about Pressing (1980) on page 37.

1.3 Beats

1.3.1 Early Writings

It is difficult to imagine that the ancients could have been unaware of the phenomenon of beating; certainly they could not have failed to notice the beating of mistuned unisons. Musicians probably listened to the rate of beating to tune pairs of strings (if not pipes) to the beat-free just intervals described by ancient theorists.[15] However, I am unaware of any description from ancient times that would verify this supposition. Beating was not part of Pythagoras' treatment of musical intervals, as it has been handed down to us in the descriptions of later writers.

Since the sustained tones of the organ make beats easy to hear, it is likely that Medieval organ tuners listened to beat rates to tune the pure fifths of Pythagorean intonation, which was the tuning advocated by theorists of the time. Beats were used in the Renaissance to tune organs to tempered intervals (such as those of meantone temperament), and the same was probably true for harpsichords, if not other stringed instruments. H. F. Cohen cites the organ composer Arnolt Schlick, in his *Spiegel der Orgelmacher und Organisten* (1511) as indicating, "though in a purely qualitative fashion, the rate of beating necessary for tempering the fifths and thirds to the extent he thinks desirable."[16]

Marin Mersenne, who carried on a prolific correspondence with contemporary scholars and musicians, was also aware of this practical use of beat rate in tuning organs, and he describes beating in his *Harmonie universelle* of 1636/37. According to Cohen,

15. Scholars believe, for example, that the strings of the *kithara* and *lyra* were tuned in anhemitonic pentatonic scales, yielding many fourths and fifths (Marcuse [1975], p. 322); and the Chinese legend of Ling Lun, discoverer of music, has him tuning bamboo pipes by the circle of fifths.
16. H. F. Cohen (1984), p. 248.

Mersenne didn't attempt to explain the phenomenon, other than to note that beating shouldn't be confused with the putative coincidence of pulses that Mersenne held to be responsible for consonance. Mersenne was also an early advocate of equal temperament (although not exclusively; he also described a system of just intonation with 18 notes per octave as being "the most perfect" system). On page 20 of the volume of *Harmonie universelle* entitled *Nouvelles observations physiques & mathematiques*, he describes the use of beats to tune equal temperament:

> Quelque-uns croyent qu'ils peuvent trouver l'accord precedent des demitons égaux...par le nombre de tremblemens, ou batemens que font la Quinte & les autres consonances temperées: par exemple, la Quinte bat une fois dans chaque seconde minute, lors que la Quinte est temperée comme il faut, tant sur l'Orgue que sur l'Epinette, au lieu que quand elle est iuste, elle ne bat plus.
>
> [Some people believe they can find the aforementioned tuning of the equal semitones...by the number of tremblings or beats that the fifth and other tempered consonances make: for example, the fifth beats once each second, when it is tempered as necessary (on the organ and the spinet alike), whereas when it is just, it does not beat anymore.]

Mersenne doesn't mention the register here, but this would be the middle register. According to a rule of thumb cited by Alexander Ellis,[17] equal-tempered fifths in the octave above middle C should beat once a second, and fourths in this register beat three times in two seconds.

It was Isaac Beeckman (1588 - 1637), according to Cohen, who first offered an explanation of the phenomenon of beating, and more importantly for our discussion, who first associated beating with dissonance. Again, Beeckman learned of the practical use of beats for tuning from an organist. The organist would first tune fifths exactly pure (so they were beat-free), and then

> ...he taps the pipes in such a way that the sounds run counter to each other as if they said wow, wow, wow, the one wow differing from the other in time as much as one pulse in the radial artery from

17. in his 1885 translation of Helmholtz (1877/1954), App. XX., Sect. G, p. 489.

the other, and then all is well. But if he makes them even more unequal, the wows come still 5 or 6 times closer together; and if he makes them worse again, the sound passes into something like rattling.[18]

Note the use of one's pulse as a reference for judging beat rate. (Musical references have also been used; Barthold Fritz (1780)[19] gave tuning instructions that compared the beat rate to eighth-notes in common time.[20]) Beeckman implies that a beat rate of approximately 1 Hz is fine, a rate of several beats per second sounds worse, and higher rates have a different quality—"rattling"—which may approach what in this century has usually been called "roughness."

Beeckman continues by describing that if two organ pipes are tuned to a fifth, there is no beating. If the upper pipe is slightly mistuned,

it overtakes [the other] every 30th time or so, which begins to take away the agreeability of the fifth. But if it is still worse, it rattles and is really vicious, since now the ratio is no longer as 2 to 3, but as 17 to 18 or 20 to 21 or 10 to 11, etc., which are all dissonances, for instead of the one string [sic] moving three times against the other one twice, the strokes now come together only once every eleven or 12 times, or so.[21]

Beeckman's numbers are specious: the ratios he mentions (17:18 and so on) are closer to a minor second than to a fifth. But the passage can be corrected without changing the substance of its argument, by substituting ratios such as 30:19. As such the passage could almost be taken as describing "waveform beats," which Plomp (1967) called the "beats of mistuned consonances"—a periodic change in the cumulative waveform of two simultaneous sinusoidal tones. But this phenomenon is very weak; the beats clearly heard in mistuned organ pipes are the beats of upper partials. At this time people did not realize

18. Cohen (1984), p. 144, from Beeckman (1604 - 1634/1939 - 1953), vol. 3, p. 51.
19. Cited by Barbour (1953), p. 48.
20. In earlier centuries the beat, or *tactus*, was a more absolute unit of time than when tempo designations came into use.
21. Cohen (1984), p. 145, from Beeckman (1604 - 1634/1939 - 1953), vol. 3, p. 51.

that it was upper partials, not the fundamentals, that beat in a mistuned perfect fifth, so Beeckman attempted to explain beating in terms of the relation of the fundamental frequencies. He did, however, understand that beating was a process of one vibration pattern lagging behind the other, and that the period of the beat is equal to the time it takes the two patterns to coincide again.

Later writers touched upon the relation between the consonance of small-integer ratios and their lack of beating. Christiaan Huygens considered ratios based on the number seven to be suitable for consonances, a possibility that Mersenne had already entertained but, for the most part, rejected. Huygens noted in 1676 that the close approximation in meantone temperament to the ratio 7/4 "appears to be consonant. For it emits a sound that is agreeable to the ear, and it does not beat."[22]

At the beginning of the eighteenth century, Joseph Sauveur published various writings, such as his *Principes d'acoustique et de musique* (1700/1701), in which he set forth a number of new acoustical findings and procedures. Sauveur authoritatively described the overtone series and included the idea that dissonance might be caused by beats.[23] He also originated the use of logarithmic units of frequency to measure intervals, and set forth a method of establishing a standard reference frequency, using beats of organ pipes.

Other 18th- and 19th-century figures wrote about beats and tuning. Georg Andreas Sorge has been cited as setting forth ideas relating beats to dissonance in the mid-eighteenth century, more than a century before Helmholtz.[24] Robert Smith analyzed the mathematics

22. Cohen (1984), p. 226, from Huygens 20:60.
23. Cohen (1984), p. 235.
24. Sorge (1745 - 47), cited in Plomp and Levelt (1965).

of beats and frequency ratios, publishing in his *Harmonics* of 1749 tables of the beat frequencies in different tunings, as did later writers such as Alexander Ellis.[25] Kirnberger (1779) discussed tuning by beats of fifths.[26] Scheibler (1834) developed a set of tuning forks to use as a tuning reference; a given note of an instrument's would be tuned to beat four times per second with the corresponding fork.

1.3.2 Helmholtz

As mentioned above (page 3), Hermann Helmholtz is generally credited with being the first to understand the beating of harmonics and to base a theory of consonance primarily on this phenomenon. Even if there were forerunners of this idea in the writings of Beeckman, Sauveur and Sorge, it was certainly Helmholtz who gave it a physiological basis and who crafted it into a monumental treatise purporting to explain musical harmony. Helmholtz mathematically analyzed the strengths of beating partials[27] and used this to draw a graph of roughness versus interval size over a two-octave range.[28] The graph shows peaks whose relative heights roughly correspond to the traditional ordering of the consonances and dissonances.

Helmholtz had some understanding that, apart from beat rate, there was an effect related to interval size:

> ...the 66 beats of the interval b" c"' are much more distinct and penetrating than the same number in the whole Tone b$^{\flat}$' c", and the 88 of the interval e"' f"' are still quite evident, while the 88 of the minor Third a' c" are practically inaudible... Hence it is not, or at least not solely, the large number of beats which renders them inaudible [when the beat rate is increased beyond some threshold].

25. in his 1885 translation of Helmholtz (1877/1954), App. XX., Sect. G, p. 489.
26. Kirnberger (1779), 2nd part, 3rd Division, pp. 179 ff. Cited by Barbour (1953), p. 64.
27. Helmholtz (1877/1954), App. XV, pp. 415 - 418: "Calculation of the Intensity of the Beats of Different Intervals."
28. Helmholtz (1877/1954), Fig. 60 A and B, Chapter X, p. 193; also Fig. 61, p. 333.

> The magnitude of the interval is a factor in the result, and consequently we are able with high tones to produce more rapid audible beats than with low tones.
>
> ...equally large intervals by no means give equally distinct beats in all parts of the scale. The increasing number of beats in a second renders the beats in the upper part of the scale less distinct...The beats of a whole tone, which in deep positions are very distinct and powerful, are scarcely audible at the upper limit of the thrice-accented octave [say at 2000 vib.—*Ellis*]. The major and minor Third, on the other hand, which in the middle of the scale [264 to 528 vib.—*Ellis*] may be regarded as consonances, and when justly intoned scarcely shew any roughness, are decidedly rough in the lower octaves and produce distinct beats.
>
> On the other hand we have seen that distinctness of beating and the roughness of the combined sounds do not depend solely on the number of beats...The roughness arising from sounding two tones together depends, then, in a compound manner on the magnitude of the interval and the number of beats produced in a second.[29]

He goes on to discuss the anatomical reason for the latter observation, namely that if the interval is wide enough, the vibrating parts of "Corti's organs" will be too far apart to permit interference. However, he does not seem to propose that the beat rate yielding maximal roughness might change as a function of frequency; this would remain to be contributed by the critical band theory in the 20th century.[30]

As H. F. Cohen points out,[31] Helmholtz' theory helps explain why more consonant intervals are more susceptible to mistuning (namely, because they contain more coinciding or nearly coinciding partials that can beat). However, some writers have argued that the fifths can withstand more mistuning than the major thirds, and indeed meantone temperament is based on this principle. (For more on the relative sensitivity of thirds versus perfect intervals, see the comments about Hall and Hess [1984] and Vos [1986] on page 27.) Helmholtz also notes that although the less consonant intervals will have their beat rate

29. Helmholtz (1877/1954), pp. 171 - 172.
30. Where Helmholtz thought the two important factors were beat rate and *interval size*, discussions of the critical band theory often present the two factors of beat rate (or frequency separation) and *frequency*. Any two of these factors are sufficient to determine the third. To use Helmholtz' example, given a beat rate of 66 Hz and an interval of a minor second, the frequency of the lower tone must be b", or about 988 Hz.
31. H. F. Cohen (1984), p. 240.

increase more quickly as the interval is mistuned, this will cause the beats to disappear sooner into inaudibility:

> ...if the amount by which one of the tones is put out of tune remains constant [in cents], the number of the beats increases according as the interval is expressed in larger numbers. Hence for Sixths and Thirds the pitch numbers of the tones must be much more nearly in the normal ratio, if we wish to avoid slow beats, than for Octaves and Unisons. On the other hand a slight imperfection in the tuning of Thirds brings us much sooner to the limit where the beats become too rapid to be distinctly separable.[32]

Helmholtz devotes considerable discussion to the beats of combination tones with each other and with upper partials, but says:

> But since all tones which are useful for musical purposes are, with rare, exceptions, richly endowed with powerful upper partial tones, the beats due to these upper partials are relatively of much greater practical importance than those due to the weak combinational tones.[33]

Helmholtz also noticed that the pitch of a mistuned unison could be heard to fluctuate subtly during the minimum-amplitude portion of the beat cycle. He included in his book an appendix with a trigonometric derivation of this phenomenon, given to him by G. Guèroult (who translated Helmholtz' book into French).[34] The phenomenon has been further studied in our century (Feth [1974]; Feth, O'Malley, and Ramsey [1982]).

1.3.3 Extensions and Tests of Helmholtz' Theory

Following Helmholtz, a number of other papers treated consonance along similar lines (Wundt [1880], Ogden [1909], Montani [1947], Husman [1953]).[35] The next major addition to Helmholtz' theory, however, was based on the concept of critical band (Zwicker, Flottorp, and Stevens 1957), which defined, as a function of frequency, the maximum

32. Helmholtz (1877/1954), p. 185.
33. Helmholtz (1877/1954), p. 180.
34. Helmholtz (1877/1954), Appendix XIV, pp. 414 - 415.
35. These papers are all cited by Plomp and Levelt (1965).

frequency ratio within which two sinusoids would still interfere with each other on the basilar membrane. Plomp and Levelt (1965) used this idea to relate perceived consonance of sinusoids to Helmholtz' model. They presented listeners with simultaneous sinusoidal tones in various frequency ranges and beat rates, and asked for judgments of "consonance," which was defined as "beautiful" or "euphonious" for subjects unfamiliar with the term. (The subjects' musical experience was not reported.) Their listeners judged tones to be most dissonant when they were separated by a quarter of a critical bandwidth, and intervals larger than this were judged as increasingly more consonant. Thus the maximally dissonant beat rate was found to vary with register, whereas Helmholtz had suggested a beat rate of 30 - 40 Hz, independent of register.

Helmholtz had extrapolated from his computations of the beat intensity of sinusoids, to graph a curve of dissonance for complex tones. Similarly, Plomp and Levelt extrapolated from their psychoacoustic data on sinusoids to the condition of simultaneous complex tones, assuming that the consonance of such tones could be computed as a linear sum of the judged consonances of each pair of sinusoidal components in the tones. For six partials per tone, the resultant curve has peaks at the consonances of just intonation, with relative heights that roughly follow the traditional ordering of consonances. This pattern is nearly independent of frequency over a large range, but below a critical frequency that depends upon the interval, the interval becomes more dissonant, as reflected in musical practice.

This prediction was tested by Kameoka and Kuriyagawa (1969a, 1969b) with psychoacoustic experiments using both sinusoidal and complex tones. For sinusoidal tones, their results were somewhat different from Plomp and Levelt's: maximal dissonance was not simply proportional to the critical bandwidth, but increased with both frequency and sound pressure level. Complex tones again showed peaks at the simplest frequency ratios.

Kameoka and Kuriyagawa developed a somewhat different model from Plomp and Levelt's linear model for predicting the consonance of arbitrary intervals with arbitrary spectra. (See Vos [1986] for a critique of Kameoka and Kuriyagawa's model.)

Hutchinson and Knopoff (1978, 1979) provided another formula for calculating the consonance of chords from their component frequencies, in an attempt to reach a result useful for music theory. Danner (1985) used this model to characterize pitch-class sets.

Ayres, Aeschbach, and Walker (1980) studied the perceived consonance of dyads consisting of complex tones with eight partials each. The intervals included not only the standard equal-tempered intervals, but also intervals from the quarter-tone scale (i.e., the division of the octave into 24 equal parts). They found a number of cases where a quarter-tone interval was judged more dissonant than either of its chromatic neighbors. Ayres, Aeschbach, and Walker held that Plomp and Levelt's model could not predict this, and concluded that their subjects judged the quarter-tone intervals as more dissonant because they were more unfamiliar. While the latter hypothesis seems reasonable, it is difficult for the reader to evaluate the authors' statement that Plomp and Levelt's model was inadequate, for the predictions they show are based on Plomp and Levelt's curves for tones with six (presumably equal-amplitude) partials, but the stimuli that Ayres, Aeschbach, and Walker used in their experiment had eight partials with unequal amplitudes. Still, the study serves as a reminder that cognitive factors might override psychoacoustic ones in such experiments. (Another illustration of this point is the experiment on auditory illusions by Shepard and Jordan [1984].)

Nordmark and Fahlén (1988) also conducted some experiments with complex tones to test Plomp and Levelt's model. The subjects had some musical experience. In the first experiment, the stimuli were harmonic intervals with six equal-amplitude partials per tone and a duration of four seconds. The task was to rate the stimuli's dissonance. The results of this experiment matched Plomp and Levelt's model fairly well, although removing some of the partials in a dissonant interval (the minor ninth) did not increase its consonance as predicted. A second experiment was run with tetrads (four-note chords) in different inversions and tunings (equal temperament, just intonation, Pythagorean intonation, and an "irregular" tuning). Neither Plomp and Levelt's nor Kameoka and Kuriyagawa's model matched the data for these chords.[36] For example, a tempered chord could be more consonant than the just version, although the models predict the reverse. Nordmark and Fahlén attempt to explain the results with recourse to periodicity pitch, but it seems that learning would provide a better explanation. With these more explicitly musical stimuli, it is likely that the subjects' exposure to the Western musical tradition played a definitive role.

Terhardt (1974b) studied roughness in beating pairs of sinusoids as well as in amplitude-modulated tones, frequency-modulated tones, and pulse trains. Some of his findings are: (1) Roughness is determined mainly by relative amplitude envelope fluctuations. It is proportional to the square of the ratio of amplitude fluctuations to steady-state amplitude value. (2) Roughness is also dependent on frequency of the stimulus, as well as on the rate of amplitude fluctuations. It is maximal at about 50% of the critical bandwidth. (Plomp and Level had found maximum roughness at about 25% of the critical bandwidth.) (3) The roughness of a pair of beating sinusoids is about half that of a 100%-amplitude-modulated tone having the same frequency and a modulation rate equal to the pair's beat rate.

36. Nordmark and Fahlén rejected Hutchinson and Knopoff's formula, since it allows a given interval to vary by factor of 10 over a three-octave range.

Roberts and Mathews (1984) found that some subjects preferred chords with beating, while others preferred chords tuned in just intonation. The stimuli in their study had a "typical bland electronic organ timbre": 10 harmonics, with a 9 dB/octave rolloff. The chords were the major and minor triads of conventional just intonation, plus two nontraditional chords (with frequency ratios 3:5:7 and 5:7:9, respectively). The middle note of each chord was tempered by –30, –15, 0, +15, or +30 cents. There were 13 subjects, ranging from professional musicians to musically untrained people. All subjects were always able to discriminate between the temperings. When asked to judge which of a pair of chords was more in tune, they fell into two groups: the "rich" listeners who preferred beating and the "pure" listeners who preferred the just tuning. (This pattern did not hold for the minor triad.) When asked which of the pair was more smooth or more pleasant, the "rich" listeners responded the same as when asked which was more in tune, but the "pure" listeners showed distinctions between these tasks.

Roberts and Mathews offered two possible explanations for the preference of the "rich" listeners: either they simply preferred some beating (which may have helped enliven the bland electronic timbre[37]), or they preferred equal temperament. The latter explanation does not account for all the data, but the former does not explain why in some conditions (minor chords, and in I - IV - V progressions) the "rich" listeners preferred tempering in the direction towards equal temperament but not in the opposite direction. This problem suggests an experiment separating beating from tuning, such as the experiments presented in this dissertation.

37. Synthesizer manufacturers, for example, often add beating to their "patches" in order to make them richer and more evocative of natural sounds.

Hall and Hess (1984) had musically trained subjects evaluate the acceptability of the tuning of a large number of intervals (including the entire range from the unison to the octave, quantized no more coarsely than in 4-cent increments). The stimuli in this study were isolated harmonic intervals of three seconds' duration, using an "oboelike" timbre based on a pulse waveform (whose spectrum has equal-amplitude harmonics), filtered with a low-pass filter whose cutoff frequency was 2.5 kHz and whose rolloff was 24dB/octave. For the most part, however, the authors only show representative per-subject graphs and data analysis, making it difficult for the reader to determine average results. Hall and Hess state that subjects gave the highest ratings to the intervals of just intonation; none of their seven subjects was a "rich" listener. Consonances were easier to judge than dissonances. The intonation of perfect intervals (e.g., fourths and fifths) was more ambiguous than that of thirds and sixths when the physical mistuning was expressed in terms of cents, but less when expressed in terms of beat rate. Hall and Hess interpreted their results to mean that both beating and interval tuning contribute to intonation judgments. We shall return to this question in the discussion of Joos Vos's work.

1.3.4 Vos's Research on Pure and Tempered Intervals

Certainly the most extensive and thorough research on the importance of beats for intonation has been that conducted by Joos Vos, most of which was included in his doctoral thesis (Vos [1987]). A series of psychoacoustic experiments (Vos [1982, 1984]; Vos and Vianen [1985a, 1985b]) found that beating partials were the main factor in discriminability between pure and tempered harmonic intervals; later studies (Vos [1986, 1987]) found beats to be important for the perceived intonation of both isolated intervals and musical passages. This research merits the more detailed treatment in the following paragraphs.

Vos (1982) examined thresholds for discrimination between pure and mistuned fifths and major thirds. The stimuli were simultaneous complex tones of various durations and beat frequencies. The "discrimination threshold" was measured as a difference in level between the two tones of the interval; thus one tone was typically at a much lower sound pressure level than the other. (In other words, the threshold was the maximum difference in level at which the pure interval could still be discriminated from the mistuned one). One of the experiments also measured "identification thresholds," in which the subject had to identify the direction of mistuning (whether the interval was larger or smaller than the "pure," i.e., just, interval). The conclusions of this study were:

(1) Mistuned fifths are more easily discriminated from the pure interval than are thirds. Unlike the results of Hall and Hess (1984), this finding is true both as a function of beat rate and as a function of mistuning in cents.
(2) Slow beating makes discrimination more difficult. (The beat rates ranged from .5 to 32 Hz.)
(3) Longer durations make discrimination easier, especially at low beat rates. (Durations ranged from 1/4 to 1 second.)
(4) Both fifths and thirds must be mistuned by 20 to 30 cents to identify the direction of mistuning reliably.
(6) For both fifth and third, the just interval was the point of "mean subjective purity," i.e., the tuning where an equal number of responses say the interval is compressed as stretched.
(7) Sensitivity to beats is about the same for thirds and fifths, and is usually independent of the beat rate.
(8) Discrimination is determined mainly by beats, but also by perception of the interval size, especially for fifths when the lower tone is quieter than the higher.

Vos's next study (Vos [1984]) determined that the earlier results, which concerned thresholds where one tone was much quieter, were a good predictor for supraliminal conditions. (There are some exceptions with the major third, however.) The thrust of this study was to determine the relation between the previously found discrimination thresholds and the pairs of beating harmonics in the stimuli. Vos found that the thresholds for discrimination between pure and mistuned intervals are determined mainly by the lowest pair of nearly coinciding harmonics (for example, in a fifth, the second partial of the upper tone and the third partial of the lower). This result is true for fifths but less so for major thirds, where harmonics other than the lowest pair enter into play. (In the major third, the lowest pair are the fifth harmonic of the lower tone and the fourth of the upper.)

The study also found that the perceived beat rate is equal to the absolute value of $pf_2 - qf_1$, where f_1 and f_2 are the fundamental frequencies and their ratio closely approximates the small-integer ratio p:q.[38] This perceived rate usually corresponds to the beat rate of the first pair of beating harmonics, but it even holds for cases where the component at pf_2 is absent (in which case the first pair of beating harmonics, if any, would have a faster rate than the perceived rate). The perceived beat rate occasionally corresponded to that of the second lowest pair of beating harmonics, especially if the first pair was absent. It is surprising that even when the first pair was missing, it was more likely that $pf_2 - qf_1$ was heard as the beat rate. This rate was also heard faintly when there were no beating partials; Vos invokes waveform beats (Plomp's "beats of mistuned consonances") to explain this.

In order to generalize Vos's findings to other intervals, Vos and Vianen (1985a) studied discrimination thresholds for the intervals with the following frequency ratios: 1/1 (unison), 2/1 (octave), 3/1 (twelfth), 3/2 (fifth), 5/2 (major tenth), 4/3 (fourth), 5/3 (major sixth), 5/4 (major third), 7/4 ("subminor seventh"), 6/5 (minor third), 7/5 ("subminor fifth"), 8/5 (minor sixth), and 7/6 ("subminor third"). A main result was that, as in the previous study, discrimination thresholds were caused mainly by nearly coinciding

38. The convention here is that $f_1 < f_2$ and thus $p < q$, although the formula also works with $f_1 > f_2$ and $p > q$.

harmonics. Because of the relation between the harmonic series and the frequency ratios of these intervals, the thresholds were also strongly correlated with the complexity of the frequency ratio (as expressed by p+q). In other words, as the ratios get more complex, it becomes more difficult to discriminate between the pure and the tempered versions of the interval. Because the traditional order of the consonances corresponds roughly to the ordering by ratio complexity, Vos and Vianen concluded that discrimination thresholds seemed to be a measure of "tonal" or "sensory" consonance (as studied, for example, by Plomp and Levelt 1965). Although beating harmonics were again found to be important, the thresholds did not change when the spectrum changed from flat to a 6 dB/octave rolloff.

Vos and Vianen (1985b) generalized the early results on discrimination thresholds to other registers. They followed the same experimental procedure as Vos (1982), but extended the stimuli to the four-octave frequency range from C2 to C6. The results showed that thresholds are independent of register, when plotted versus beat rate. (Since a given interval beats faster in higher octaves, the effect of register was removed by plotting it this way.) This suggests that all the results of Vos's previous studies (for example, the effects of duration and spectrum) are valid over this entire frequency range. Also, the authors noted some implications for traditional tuning systems such as equal temperament and quarter-comma meantone temperament. Whereas tempered fifths in these systems are not very discriminable from pure fifths in the octaves below Middle C, major thirds are, at least in the range where they would occur in traditional music.

The study most relevant to the present research is Vos (1986), which examined subjective "purity" for isolated major thirds and perfect fifths. Vos found that beats affect purity judgments: the beat-free just intervals were rated the purest, and removing beating partials increased the purity of mistuned intervals. The study consisted of two experiments.

In the first, the fifths and thirds had complex spectra with a 6 dB/octave rolloff. The variables were interval type (fifth or major third), tempering from 0 to 50 cents, duration (1/4 or 1/2 second), and spectrum of the higher note (first 20 harmonics or first 10 odd harmonics). There were two different tasks: to rate the purity of a single interval on a scale from 1 to 10, and to decide which of a pair of intervals was purer. Graphs of the main results are presented in Figure 1. The most important result for our purposes is that deletion of beating partials resulted in higher purity ratings, especially for the major third. The curves of purity versus tempering in cents could be described by exponential functions. Fifths compressed by up to 15 - 25 cents or stretched by up to 10 - 20 cents were rated purer than the corresponding thirds, but the reverse was true for larger temperings. (This experiment is described in greater detail on page 152ff. of this dissertation.)

The second experiment of this study was similar, but used sinusoids rather than complex spectra, and also compared harmonic intervals with melodic. The variables were: interval (fifth or major third), tempering from 0 to 50 cents, simultaneous versus successive presentation, and loudness (0 dB attenuation versus 25 dB). The resultant curves were much flatter than for complex tones; i.e., the mistuned sinusoids sounded purer than the corresponding complex tones. 25dB attenuation of simultaneous sinusoids resulted in higher purity (except for compressed thirds); attenuated simultaneous sinusoids had about the same purity curves as successive sinusoids.

With the techniques used, Vos was unable to decorrelate interval tuning and beat rate.[39] The experiment with sinusoids was intended to shed light on the contribution of

39. Vos recognized that separating beat rate from interval tuning would be desirable. On. p. 243 of Vos (1986), for example, he suggests amplitude modulation as a means of inducing beats without changing the interval size.

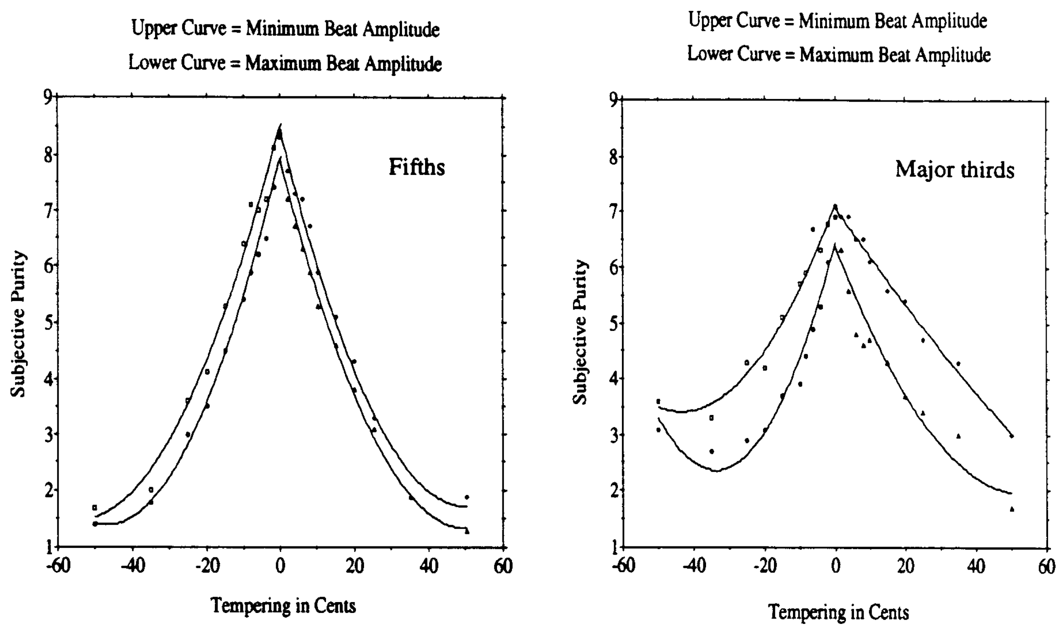

Fig. 1. After Vos (1984). Subjective purity of perfect fifths and major thirds, plotted vs. tempering in cents. The center point at 0 Hz is the just interval. Re-plotted here with second-order polynomial regression curves, and responses normalized to a range of 1 to 9, for comparison with results of the present research. (See "Comparison with Results of Vos (1986)" on page 152.)

interval tuning to purity judgments, since beats were eliminated in this experiment (at least at the quieter level). Because the sinusoids were rated as purer than the corresponding complex tones, Vos concluded that interval tuning was not as important as beating. This may not have been a correct assumption for two reasons. First, it is often more difficult to make musical judgments about sinusoids than about complex tones. The pitches of the sinusoidal tones may have been less clear, which would make judgments of interval tuning more difficult. (Indeed, Vos found in a later study [Vos and Vianen (1986), discussed below] that the use of sinusoids reduced discriminability of melodic intervals.) Secondly, there was a drastic change in timbre between the two sorts of stimuli; one must generally be hesitant about conclusions when there is a covarying variable. It is not clear that the timbral change would affect the perceived intonation (although there is some evidence for timbral effects on intonation), but, as discussed on page 158, it might affect judgments of "purity." In any case, Vos's interpretation that intonation judgments are based mostly on beats and roughness, rather than interval tuning, was apparently not derived from any rigorous quantitative analysis. As we shall see, the results of the present study lead to the opposite conclusion.

In his dissertation (Vos [1987]), Vos included a chapter titled "Subjective acceptability of various regular twelve-tone tuning systems in two-part musical fragments." In this research, partially presented at earlier conferences,[40] the findings of the previous study were extended to short musical passages in two voices extracted from Michael Praetorius' *Musae Sioniae*, Part VI (1609). The synthesized music was presented in various regular 12-tone tuning systems, as defined by Rasch (1983).[41] They included (in order of decreasing size of the perfect fifth) an extreme tuning with a fifth of 704 cents, Pythagorean tuning, equal temperament, Silbermann, quarter-comma meantone, Salinas, and an extreme tuning with a fifth of 692 cents. In these tuning systems the size of the major third is linearly related to the size of the fifth; since four fifths minus two octaves is a major third, a unit of decrease in the size of the fifth means four units' decrease in the size of the major third. Pythagorean tuning has a pure (3:2) fifth, quarter-comma meantone a pure (5:4) major third, and Salinas a pure (6:5) minor third. Vos found that subjects rated the tunings from

40. Fifth Workshop on Physical and Neuropsychological Foundations of Music, Ossiach, 1985; and 12th International Congress on Acoustics, Toronto, 1986.
41. Rasch uses "regular twelve-tone tunings" to refer to tunings of twelve notes per octave, forming a circle of fifths in which eleven of the fifths have the same size. This definition is similar to Blackwood's (1985) "recognizable diatonic tunings." (Blackwood's usage of the term "diatonic" is idiosyncratic.)

Pythagorean through meantone to be equally acceptable, and the others less acceptable. In a second experiment, the beating harmonics were deleted from the spectra, resulting in higher acceptability, with the same effect of tuning system as before. The most important finding of this study is that the acceptability of the musical passages could be predicted from a linear combination of the purity ratings of fifths and major thirds found in Vos (1986). This constituted a neat justification of the previous study, just as the 1985a and 1985b studies had generalized earlier studies to more intervals and registers, and just as the 1984 study had shown the first study (Vos 1982) to be applicable to supraliminal conditions.

Most of these studies had involved harmonic intervals. Vos and Vianen (1986) investigated discrimination thresholds for melodic intervals of the unison, major third, and fifth. Two spectral conditions were included—sinusoids, and complex tones of 20 harmonics with 6dB/octave rolloff. The average thresholds were 17 cents for the unison, 40 cents for the major third, and 39 cents for the fifth. Melodic intervals were more difficult to discriminate than harmonic intervals had been in the previous studies; the interval needed to be tempered about 10 cents further to discriminate it from the pure version. Especially for major thirds, intervals smaller than just were easier to discriminate from the just version than were intervals larger than just. Since just major thirds are already noticeably smaller than the equal-tempered major third, Vos and Vianen hypothesized that subjects were using equal-tempered major thirds as an internal reference.

Vos and Vianen found that for the melodic intervals, the sinusoidal stimuli were harder to discriminate, with thresholds about 7 cents higher. This suggests that the use of sinusoids in Vos (1986) had indeed reduced the sensitivity to interval size, instead of only eliminating beating as intended. As mentioned in the description of Vos (1986) above, Vos had concluded from that experiment with sinusoids that beating, not interval size, was the primary determinant of intonation judgments. That assumption appears to be weakened by the results of Vos and Vianen (1986). This point is noteworthy, since the present dissertation finds interval size to be much more important than beating.[42]

1.3.5 Beating and Consonance of Inharmonic Sounds

Several studies have used inharmonic stimuli to test whether coincidence of partials is more important than interval tuning for consonance. Pierce (1966) used spectra with a spacing that approximated the spacing of the harmonic series, but all partials were members of the diminished seventh chord built on the fundamental. In other words, each partial was placed at some quarter-octave point, with larger gaps towards the lower part of the spectrum. Two types of intervals were tested: those found in the diminished seventh chord, and those that differed from the first type by 150 cents (three quarter-tones). The former intervals, in which the partials of the two notes coincided or were at least a minor third apart, were judged to be more consonant. The latter intervals (members of the quarter-tone scale) had some partials separated by an eighth of an octave (150 cents) and were judged more dissonant. In light of the results of Ayres, Aeschbach, and Walker (1980), who found quarter-tone intervals to be judged more dissonant, it is worth asking what contribution the nonstandard intervals made to the perception of dissonance. One might also wonder whether musicians would not simply hear such stimuli as diminished seventh chords rather than single pitches, since inharmonic sounds are less likely to fuse. (The reported spectra indeed sound that way to my ear, at least with equal-amplitude partials.) If so, the intervals judged more consonant would be heard as a single diminished seventh chord, and the ones judged more dissonant would be heard as two diminished seventh chords having roots three

42. My intent is not, however, to de-emphasize the extensive contributions and superior caliber of Vos's research.

quarters of a tone apart—clearly more dissonant from a musical point of view, regardless of critical bands and beats.

Slaymaker (1970) also reported that coincidence of partials appeared to be critical for the amount of consonance in his inharmonic stimuli. Geary (1980) reported the most convincing evidence that coincidence of partials overrides interval tuning in determining consonance, at least in decidedly inharmonic stimuli. His stimuli had all components separated by an equal number of Hz, but the distance from the lowest component to zero Hz was different. (The spacing between components was $\sqrt{2}$ times the frequency of the lowest component. To my ear, such tones sound inharmonic, and have slightly ambiguous pitch, but not more so than carillon tones, for instance.) Geary found that traditionally consonant intervals are judged dissonant if the partials clash, and traditionally dissonant intervals are judged consonant if the partials coincide. Especially significant is the fact that this relationship was pronounced for musically trained subjects, whom one might expect to be influenced more by the interval tuning.

Mathews and Pierce (1980) used spectra with stretched harmonic series to study consonance in cadential passages. Most of their results argued against Helmholtz' theory that dissonance is caused by interfering partials, since the cadence's sense of finality was altered by stretching the harmonic series without altering the interference. However, by choosing an inharmonic spectrum and a cadential formula in which the penultimate interval contained beating but the final one did not, they were able to create a sense of finality that was not destroyed by stretching.

E. Cohen (1984) had subjects adjust the tuning of simultaneous notes having stretched or compressed harmonic series. The "pseudo-octave" ranged from a frequency ratio of 1.4, which is about a tritone (maximum compression), to 3.0, which is a twelfth (maximum stretch). In tuning the intervals (fifths or octaves), one subject used coincidence of partials exclusively. The others mostly used perception of interval size, but also used coincidence of partials for pseudo-octaves close to the normal octave (with frequency ratios from 1.9 to 2.1).

A sound example published by the Institute of Perception (Houtsma, Rossing, and Wagenaar [1987]) offers evidence that consonance might be related to the positions of partials. A Bach chorale is played with either normal tuning or a stretched tuning with a 2.1 "pseudo-octave," and with the partials either stretched or unstretched. The normal tuning with unstretched partials sounds acceptable, of course. Interestingly, the stretched tuning with stretched partials also seems to sound less rough than either the normal tuning with stretched partials or the stretched tuning with unstretched partials.

One of the arguments put forth against an acoustical basis for consonance and musical scales is the fact that a number of non-Western scales do not make use of small-integer ratios. The various gamelan scales of Indonesia are often cited. A counter-argument is that gamelans have inharmonic timbres and so should not be expected to use the same ratios; this sometimes leads to speculation that such scales might be derived from the inharmonic spectra of the corresponding instruments. However, Pressing (1980) informally reported investigating the spectra and tuning of a gamelan, using data gathered by John Grey, and he did not find this sort of acoustical basis for the tuning.

1.4 Perception of Interval Size

Several of the studies of beating mentioned above (Vos [1982], Hall and Hess [1984], Vos [1986], Vos and Vianen [1986]) made reference to a two-component model of interval perception, in which beat rate is considered to be one cue for judging intonation, and perception of the size of the interval the other. As mentioned, E. Cohen (1984) found that coincidence of partials and perception of interval size were both relevant in tuning an interval with inharmonic partials; however, she concluded that interval size was more important. All of these researchers had to use induction to evaluate the relative importance of beat rate and interval size, in contrast to the present research, in which the two are independent variables.

In this section, we review the literature dealing with the perception of interval size. (Note that in speaking of "interval size" we are concerned with the fine-tuning of interval perception. Clearly, competent musicians can accurately identify the intervals of the chromatic scale, but it is less obvious how well they perceive more minute gradations of frequency ratio.)

Although "perception of interval size" is a fundamental factor in the production and appreciation of music, it has not found an important role in the history of music theory, perhaps because it is viewed as a practical skill for performers rather than an explicit component of composition. One might, however, point to writers who emphasized the importance of determining interval tuning empirically rather than on a numerical basis. An often-cited father of musical empiricism is Aristoxenus, a practicing musician who decried the computations of the Pythagoreans, insisting that the ear must be the arbiter of musical scales and tuning. (Some writers have also attributed the invention of equal temperament to

Aristoxenus, since he asserted that six whole tones constitute an octave. A reading of Aristoxenus [translated by Macran 1902] reveals, however, that on the one hand he does make use of mathematical proofs—even though his calculations are fatally flawed, as in his "proof" that the fourth consists of two and a half tones—and that on the other hand he was probably too mathematically unsophisticated to appreciate the numerical difficulties of exact equal temperament.) A later empiricist was Vincenzo Galilei (father of Galileo), who disputed Zarlino's dogma that the intervals with small-integer ratios were the "natural" ones. H. F. Cohen (1984) translates this inflammatory excerpt from Galilei's writing:

> ...Now those musical intervals are as natural (as I have said) that are contained in the ratios of the Senario, as are the others that are outside those ratios, and the major third that is contained in 5:4 is as natural as the one that is contained in 81:64. Just so is it also as natural for the octave to be consonant in the ratio 2:1 as it is natural for the seventh to be dissonant in the ratio 9:5; and let Zarlino trouble his head about this as much as he wishes.[43]

If music theorists have relegated interval perception to an inconspicuous role, scholars in the sciences have certainly conducted many studies on the musical interval sense. We cannot review all the literature here. Pikler (1966) offers a historical survey, starting from Mersenne and concentrating on 19th- and early-20th century experiments. Burns and Ward (1982) provide a summary with an emphasis on more recent psychoacoustic research.

The psychoacoustic measure that most directly ascertains the perception of interval size is the JND (just noticeable difference) for frequency ratio. A canonical procedure would be to play two intervals (whether melodic or harmonic) and ask the subject to say which is larger (or smaller). However, relatively few experiments have studied ratio JND's. Using melodic intervals and sinusoidal waveforms, Houtsma (1968) found a JND of 16

43. Galilei (1589), pp. 92 - 93, quoted by Cohen, p. 80.

cents for the octave. JND's for all the intervals of the chromatic scale were studied for one subject; they ranged from 14 - 25 cents, with a tendency for the smaller intervals to have smaller JND's. Burns and Ward (1978) also studied successive sinusoids, for the intervals from the minor third to perfect fourth. Using an adaptive paradigm, their musically trained subjects had initial JND's of about 20 - 50 cents, which dropped to 15 - 30 cents after they reached asymptotic performance. Neither Houtsma nor Burns and Ward found any evidence that subjects were more sensitive at the small-integer ratios. Viemeister and Fantini(1987) studied simultaneous sinusoids with musically untrained subjects, with similar results. As mentioned above, Vos (1982) found thresholds of 20 - 30 cents for the perfect fifth and major third using simultaneous complex tones; and using melodic intervals Vos and Vianen (1986) found thresholds of 17 cents for the unison, 40 cents for the major third, and 39 cents for the fifth. I am unaware of any more extensive study of ratio JND's using simultaneous complex tones, which are of course more directly relevant to musical inquiry than are sinusoids.

Other experimental paradigms can shed light on the perception of interval size. In adjustment tasks, subjects control the tuning of a tone generator such as an oscillator, setting it to what they believe to be the correct tuning for the requested interval. The classic study of this type is Moran and Pratt (1926), who studied all the intervals of the chromatic scale and found average errors of 14 - 22 cents. A more recent experiment along similar lines is that of Rakowski and Miskiewicz (1985). They found average errors ranging from about –12 cents, for the minor second, to +22 cents for the major seventh. This study turned up neither an effect of timbre on the intonation of melodic intervals nor any evidence for either just or Pythagorean intonation. Their subjects tended to stretch the larger intervals and shrink the smaller, with respect to equal temperament.[44]

This tendency to stretch the larger intervals and contract the smaller ones is also found in some studies of intonation in performance, as summarized by Burns and Ward (1982), who also conclude there is no tendency towards either just or Pythagorean intonation. Where some earlier researchers such as Greene (1937) concluded that string players used Pythagorean intonation, Salzberg (1980) found that string players play sharper than Pythagorean, supporting the conclusion of Burns and Ward. Mason (1960) also found a tendency for woodwind players to play sharp; the more experienced players tended towards equal temperament. Hagerman and Sundberg (1980) found evidence contradicting the popular notion that barbershop quartets sing harmonies in just intonation. Besides the lack of support for just or Pythagorean intonation, the other important distillation from many studies of intonation in performance is the large variability. Ward (1970) reports ranges up to 78 cents and interquartile values up to 38 cents. With reference to singing, Sundberg (1982) stresses that some of the variations may be attributable to conscious inflections for expressive purposes, and that the use of vibrato may help disguise deviations from theoretically correct pitches.

Because musicians "overlearn" the intervals of the chromatic scale, it would not be surprising if they use these categories as reference points when judging the size of arbitrary intervals. A number of studies have found evidence for some degree of categorical perception of musical intervals, including Locke & Kellar (1973), Siegel and Siegel (1977a, 1977b), Burns and Ward (1978), Zatorre and Halpern (1979), Wapnick, Bourassa, and

44. Szende (1977) performed an extensive study on interval perception, which should mentioned simply because of its scope; almost 900 musical subjects were tested, and the results filled a book. Unfortunately, the methodology of this study is quite dubious. For example, the tunings used for some of the augmented intervals were all smaller than the equal-tempered version, which was not included. For theoretical reasons, the author prejudged the larger ones of these to be "sharp," and so a subject response that labeled such an interval as smaller than the (internalized) correct tuning was considered inaccurate, and "to be taken into consideration only occasionally"! (p. 50).

Sampson (1982), and Zatorre (1983). For example, subjects may discriminate best between two intervals when they lie on opposite sides of a category boundary. (The boundary is a quarter-tone, assuming the categories are those of equal temperament.) In evaluating such results, it should be noted that some of these experiments use procedures that elicit categorical perception. With an adaptive paradigm, Burns and Ward found that categorical effects tended to disappear when asymptotic performance was reached. Siegel and Siegel used a magnitude estimation task in one of their experiments, in which subjects were asked to judge the relative sizes of intervals without necessarily using the familiar interval categories. Another interesting task would be to have expert listeners evaluate interval sizes in cents, or to have them place the interval on a line chart having tick marks for the standard intervals. (I am unaware of any such experiments, aside from some pilot studies I conducted prior to my dissertation research.)

The overall conclusions we can reach from this summary of studies on interval size are:

(1) The just noticeable difference for interval size is from 15 - 40 cents, depending on the interval, waveform (sinusoidal versus complex), presentation (harmonic versus melodic), musical training, and experimental paradigm. It is not known to what extent musical training with microtones could reduce this threshold.
(2) In performance, a given interval can take on a fairly wide range of sizes and still be heard as in tune.
(3) Small intervals (e.g., the minor and major second) tend to be compressed while larger ones tended to be stretched, relative to equal temperament.
(4) Generally, people do not perform in just intonation, nor do they prefer just intervals. It is not known to what extent this is a result of exposure to equal temperament.
(5) Under conditions of stimulus uncertainty, listeners have a tendency to assimilate nonstandard intervals to the known intervals of the chromatic scale (displaying partially categorical perception).

It is not known to what extent these findings could be altered by training. Since musically trained individuals can discriminate and label intervals more easily than untrained subjects, is reasonable to hypothesize that microtonal ear-training might further reduce the JND for interval size and reduce categorical effects based on the chromatic scale.

1.5 Inharmonicity

Of the three psychoacoustic factors listed in the title of this dissertation, the final is inharmonicity. As previously mentioned, inharmonicity is a by-product of the technique used in this research to control beat rate independently of interval size; and as such it is not a primary focus of this dissertation, but needs to be addressed in order to explain the experimental results. The literature on inharmonicity is much scantier than that pertaining to the other two factors (beats and interval size). In this section we examine the literature on inharmonicity *per se*; see page 35 for experiments using inharmonic spectra to test consonance theories.

1.5.1 Inharmonicity in Instruments

Early writings dealing with inharmonicity concern instruments. The makers of percussion instruments have long been aware that the component pitches of their instruments were not all members of the series of ratios 1:2:3:4:5, etc. Bell founders, for example, were aware of these different pitches, and crafted bells so as to emphasize certain ones. Plomp (1987) cites the composer and carillonneur Jacob van Eyck as showing, in the year 1644 or so, how to tune the lowest five partials of a bell to the ratios 1:2:2.4:3:4, which became typical for carillons. These ratios correspond to the prime, octave, minor tenth, twelfth, and double octave. Note the inharmonic ratio 2.4 interposed in the harmonic series; this component gives carillons a characteristic minor-third sound. Since the founders' quest was to make their bells as musical as possible, they must also have been well aware of the greater inharmonicity in bells of poorer quality.

The concept of inharmonicity depends by definition on the concept of the harmonic series. It is likely that early craftsmen simply viewed the components of bell spectra as pitches that did or did not coincide to those of the musical scale, rather than seeing in them departures from the harmonic series. Even musical thinkers who were aware of the latest acoustical investigations may not have had much knowledge of inharmonicity. For example, Helmholtz says of Rameau's notion that the major chord is the most "natural" of all:

> ...if Rameau had listened to the effects of striking rods, bells, and membranes, or blowing over hollow chambers, he might have heard many a perfectly dissonant chord. And yet such chords cannot but be considered equally natural. That all musical instruments exhibit harmonic upper partials depends upon the selection of qualities of tone which man has made to satisfy the requirements of his ear.[45]

Helmholtz' apparent exclusion here of the percussion family from "musical instruments" is at least partly a reflection of contemporary musical practice. Helmholtz was aware of Chladni's investigations into the inharmonic modes of vibration of plates. He devotes a chapter of his book to "Musical Tones with Inharmonic Upper Partials," discussing, and in some cases entabulating, the frequency components of tuning forks, straight elastic rods of glass, wood, or metal, elastic plates, bells, and stretched membranes such as tympani. With respect to the perceptual attributes of inharmonicity, Helmholtz says:

45. Helmholtz (1877/1954), p. 232.

> ...if the [inharmonic upper partials] are of nearly the same pitch as the prime tone [i.e., the fundamental], their quality of sound is in the highest degree unmusical, bad and tinkettly.[46] If the secondary tones are of very different pitch from the prime, and weak in force, the quality of sound is more musical, as for example in tuning-forks, harmonicons of rods, and bells; and such tones are applicable for marches and other boisterous music, principally intended to mark time. But for really artistic music, such instruments as these have always been rejected, as they ought to be, for the inharmonic secondary tones, although they rapidly die away, always disturb the harmony most unpleasantly...[47]

Although he devotes considerable attention to the harmonics of the piano, Helmholtz never mentions any inharmonicity, presumably being unaware of this property of piano strings. The phenomenon has been well-documented in the twentieth century, including its relation to the "stretched tuning" of the extreme registers of the piano (Shankland and Coltman [1939], Shuck and Young [1943] Young [1952], Rasch and Heetvelt [1985]). The acoustics of other instruments with inharmonic spectra, notably the percussion family, have also been better documented, but rather than digress further, we shall return to our primary focus on inharmonicity as a perceptual variable.

1.5.2 Pitch of Inharmonic Sounds

The effect of inharmonicity on pitch has only been studied in the last few decades, starting with de Boer (1956). Increasingly inharmonic sounds lose the unitary pitch perception characteristic of harmonic sounds (deBoer 1976). The pitch extraction models of Goldstein (1973) and Wightman (1973a, 1973b) assume a harmonic input; the pitch of an inharmonic stimulus would be determined by a best fit to a harmonic series. The model of Piszczalski and Galler (1979) removes inharmonic components from the decision-making process. Terhardt's (1974a) model of "virtual pitch" also assumes a harmonic

46. I presume this word (Ellis' coinage?) means "like a tin kettle."
47. Helmholtz (1877/1954), p. 73.

template-matching mechanism, to which inharmonic stimuli would be compared to find the most likely matching pitch or pitches.

Moore, Glasberg, and Peters (1985) used inharmonicity to investigate the relative dominance of individual partials in determining the pitch of complex tones. Their stimuli had 10 or 12 equal-amplitude harmonics, one of which was frequency-shifted up or down by an amount ranging up to 8% (133.2 cents). They found that only the first six harmonics have an effect on pitch. The maximum pitch shift (about 8.6 cents) occurred at a frequency shift of 4% (68 cents). For mistunings from 0 to 3% (51 cents), the shift in pitch is a linear function of the shift in the partial's frequency. (In this range, the pitch shift is about one sixth of the partial's shift.) However, it was impossible to determine which partials of the first six are dominant for pitch perception, as there were great individual differences. Thus the authors caution against fixed formulas, saying that the pitches of inharmonic stimuli may differ significantly across individuals. A conceivable weakness of this study is that the subjects were not asked to match the pitch they were judging, and thus they may have been listening to the shifted partial rather than the overall pitch. In my own listening to such stimuli, I find it possible to switch between analytic and synthetic modes; in the first case one can hear the shifted partial's pitch separately from the fundamental, and in the latter one hears a unitary percept whose pitch may change if the partial is frequency-shifted enough.

Moore, Peters, and Glasberg (1985) tested thresholds for the detection of inharmonicity, using stimuli similar to those of their previous study. They found that inharmonicity in the lower partials (up to about the fourth) was detected mostly by the partial's "standing out," while thresholds for higher harmonics appeared to be determined by detection of waveform fluctuations (the "beats of mistuned consonances" of

Plomp [1967]). The thresholds were approximately the same for different harmonics, when measured in Hz.

Similarly, Eggen and Houtsma (1986) frequency-shifted each partial of a bell sound in order to detect its contribution to the pitch. The pitches in the bell spectrum included the "hum" (an octave below the perceived pitch), prime, minor third, fifth, octave, twelfth, double octave, double minor tenth, and double eleventh. For this bell, the octave and twelfth were the most important in determining pitch, and the double octave, which was quieter, less so. It is difficult to make generalizations about complex tones from this stimulus, of course.

1.5.3 Inharmonic Partials and Multiple Sources

As a partial becomes increasingly inharmonic, it can be perceived as an individual tone separate from the rest of the complex tone of which it is ostensibly a part. Moore, Glasberg, and Peters (1986) determined the threshold at which one of the first six partials was heard as a separate tone. The threshold usually occurred at a shift in Hz that was between 1% and 3%of the harmonic's frequency. Since the partial can still affect the pitch when it has been shifted beyond this threshold, as reported by Moore, Glasberg, and Peters (1985), this again raises the possibility of a gray area where two modes of perception—analytic and synthetic—are possible, as suggested above.

1.6 Related issues

We have discussed the literature concerning beats, perception of interval size, and inharmonicity. There are some related issues which are important for music, and which will help put the findings of this dissertation into a broader perspective. These include the perception of the overall tuning system (instead of isolated intervals) and the effects of timbre and context on intonation.

1.6.1 Preference for Tuning System

The merits of different tuning systems, particularly just intonation versus equal temperament, have been hotly contested for centuries. We cannot trace the history of this issue here, nor mention all the tuning systems that have been used or proposed. (For a history of tuning in Western music, see Barbour [1953].) We shall mention Helmholtz' views, however, since these have been remarkably influential on twentieth-century acousticians, and even on composers such as Harry Partch (Partch [1974]). Moreover, Helmholtz' opinions are of particular interest for the present dissertation, which is essentially a test of the relevance of Helmholtz' theory to the perception of intonation.

Stated simply, Helmholtz felt that the coincidence of partials in the intervals of just intonation rendered it patently superior to equal temperament.

> As regards musical effect, the difference between the just and the equally-tempered, or the just and the Pythagorean intonations, is very remarkable. The justly-intoned chords, in favourable positions, notwithstanding the rather piercing quality of the tone of the vibrators [of Helmholtz' harmonium], possess a full and as it were saturated harmoniousness; they flow on, with a full stream, calm and smooth, without tremor or beat. Equally-tempered or Pythagorean chords sound beside them rough, dull, trembling, restless. The difference is so marked that every one, whether he is musically cultivated or not, observes it at once.

...In a consonant triad every tone is equally sensitive to false intonation, as theory and experience alike testify, and the bad effect of the tempered triads depends especially on the imperfect Thirds.[48]

After citing experiences in London with the singers of the Society of Tonic Sol-faists and with the Enharmonic Organ of General Perronet Thompson, as well as experiences accompanying singers with his own justly tuned harmonium, Helmholtz states:

I think that no doubt can remain, if ever any doubt existed, that the intervals which have been theoretically determined in the preceding pages, and there called natural, are really natural for uncorrupted ears; that moreover the deviations of tempered intonation are really perceptible and unpleasant to uncorrupted ears; and lastly that, notwithstanding the delicate distinctions in particular intervals, correct singing by natural intervals is much easier than singing in tempered intonation.[49]

Helmholtz' "experiments" in comparing just intonation to equal temperament were hardly rigorous by modern standards. In the twentieth century, a number of psychological experiments have focused on preference for tuning system. As noted above under "Perception of Interval Size" (page 38), studies of isolated intervals usually show no evidence of a preference for just intonation. There are important exceptions, notably Hall and Hess (1984) and Vos (1986) (discussed under "Beats" on page 16), which found just intervals to be rated as more in tune. A number of tests of tuning preference have been conducted using more extended musical materials, whether chords, melodies, chord progressions, or excerpted musical passages. Most but not all of these studies show that listeners prefer equal temperament to just intonation.

It is important to note, however, that unlike equal temperament, "just intonation" is not a single tuning, but rather a principle of tuning—the exclusive use of ratios of relatively small integers—which can find expression in a variety of scales. The form of just intonation that is often used in psychological experiments is limited to twelve notes per octave, and

48. Helmholtz (1877/1954), pp. 319 - 320.
49. Helmholtz (1877/1954), App. XVIII, p. 428

the exact tuning is seldom reported. (Part of the blame may be placed on acoustics textbooks, which typically give only one of these limited versions; keyboard instruments are largely responsible as well.) Such twelve-note tunings have a "wolf fifth," often between D and A, which generally is heard as very out of tune. By contrast, "extended" just systems offer microtonal variants of certain pitches to overcome such problems. Helmholtz used an extended just system on his harmonium, as did some of his contemporaries,[50] and most musicians working with just intonation today make use of such microtonal variants.

Vos (1987) cites Loman (1929) as running an experiment in which musicians listened to the same passage played successively on two pianos, one tuned in just intonation and the other in Pythagorean intonation. All listeners preferred the Pythagorean version of the passage, which was taken from the beginning of Wagner's prelude to "Die Meistersinger von Nürnberg." Van Esbroeck and Montfort (1946) used a special pipe organ called the "orthoclavier," which could play in equal temperament, just intonation, or Pythagorean tuning. Melodic or harmonic fragments were played for a large number of musicians and non-musicians. The subjects who could distinguish between the tunings preferred equal temperament. 55% of these subjects preferred equal temperament when compared to Pythagorean tuning, and 60% when compared to just intonation. Vos (1987) analyzed their data further and found that the differences were slight in the judgments of the harmonic fragments, but significant in the melodic case. Kok (1954, 1955) used an electronic organ capable of different tunings to test tuning preferences of musicians and non-musicians. The latter could not discriminate between equal temperament and meantone temperament; musicians could, however, and preferred meantone in certain types of chordal passages. According to Roberts (1983), Kok also found that equal temperament was preferred to just

50. For a review of microtonal keyboard instruments, see Keislar (1987).

intonation for both melodic and chordal passages. Ward & Martin (1961) compared just intonation and equal temperament on an electronic organ. The stimuli were ascending diatonic scales. Most subjects, including musicians, could not discriminate between the two tunings; a few musicians could, and seemed to prefer equal temperament. In a study of consonance using isolated triads, Roberts (1986) also found that nonmusicians could not distinguish between equal temperament, just intonation, and Pythagorean intonation. Musicians, however, rated equal-tempered triads as the most consonant, followed by just, then Pythagorean. As mentioned on page 33, Vos (1987) found that two-part musical passages were judged equally acceptable when the tuning was equal temperament, Pythagorean tuning, or meantone temperament. Just intonation was not included in that experiment.

A few experiments have shown overall preference for just intonation. Boomsliter and Creel (1963) used a special organ capable of playing many different intervals, including just, Pythagorean, and equal-tempered. They claimed that musicians who were asked to find melodies on this organ used successive small-integer ratios, rather than adhering to any of the canonical 12-note tunings (including the usual "textbook" just scale). However, the experiment was not at all rigorous; for example, the equal-tempered intervals were located on a separate keyboard, which could bias the performer. O'Keeffe (1975) played four-measure excerpts of electronic organ music, including harmony, from "Silent Night" and "America the Beautiful." The subjects were students aged 13 - 18. There were no significant differences between musicians and non-musicians, nor between the two pieces. However, 56% of the subjects preferred just intonation ($p < .01$), and boys showed a greater preference for just intonation than did girls ($p < .01$). O'Keeffe speculated that the harmony was more influential than the melody, since his results contradicted those of Ward and Martin (1961), who simply used scales.[51]

It was mentioned above that there is evidence that performers tend to expand the larger intervals and contract the smallest ones. Similarly, there is some evidence of a preference for a slightly stretched tuning in the case of musical passages (Kolinski [1959], Martin & Ward [1961], Terhardt and Zick [1975]). Terhardt and Zick (1975) found that a stretched intonation was preferred for cases with a high melody and a low accompaniment, i.e., a large separation; but that an unstretched tuning was preferred otherwise, and for chords with "high spectral complexity" (i.e., more frequency components), even a contracted intonation was acceptable.

1.6.2 Timbre

The effect of timbre just mentioned contradicts the results of Rakowski and Miskiewicz (1985), mentioned on page 40, who found no effect of waveform. Vos and Vianen (1985a) likewise found no effect of spectral rolloff for discrimination between pure and tempered intervals (see page 29). Studies finding some effect of timbre on intonation include Greer (1970), Biock (1975), Platt and Racine (1985), and Geringer and Madsen (1981). These studies mostly concern musical instruments, rather than the electronic

51. These studies raise two issues worth noting about just intonation: the difference between "free" and fixed just intonation, and the possible importance of harmony. When constrained to a fixed set of twelve pitches, just intonation has some intervals that are not inexpressible as small-integer ratios (such as the "wolf fifth" D-A in one common version). Some experimental studies might not avoid these intervals, yielding a lessened overall acceptability of the tuning. With "free" just intonation, all intervals can be perfectly tuned, but passages with traditional cadences might wander in pitch. (See Blackwood [1985] for a discussion of this problem.)
If, as O'Keeffe suggests, harmony is an important factor in the preference for just intonation, perhaps the tuning is more critical for intervals in sustained and prominent chords. Experimental studies sometimes fail to make any distinction between such intervals and other intervals in the passage whose tuning might be expected to have more leeway.

stimuli used by Rakowski and Miskiewicz (1985) and Vos and Vianen (1985a). The effect of timbre on tuning is a multifaceted issue for which many questions remain unanswered.

1.6.3 Context

Another inherently complex aspect of intonation is the effect of context. The present dissertation, like many of the studies cited here, deals with isolated stimuli. Clearly many cognitive factors come into play when evaluating musical passages. We have alluded to the research of Shepard and Jordan (1984), in which subjective sizes of intervals depended on expectations based on the internalized traditional diatonic scale, rather than on absolute size. Intonation judgments can also be affected by the surrounding notes, rather than an internalized standard. Biock (1975) stated that context affected intonation judgments for his subjects. Wapnick, Bourassa, and Sampson (1982) found that musicians judged intonation more accurately when the intervals were presented in a musical context. Cuddy, Cohen, and Mewhort (1981) also found that the context influenced subjects' ability to perceive differences in tuning. Like timbre, the effect of context on perceived intonation is a fruitful area for future research.

1.7 Summary

A central feature of traditional music theory and acoustics is the rough correlation between the degree of consonance of an interval and the degree of simplicity of its frequency ratio. Helmholtz held that the beating of harmonics was responsible for this relation. A number of psychoacoustic studies have offered support for this view. The main competing theory of consonance explains it as a cultural phenomenon, mediated by cognitive processes. It may be that the acoustic factors described by Helmholtz were responsible for the historical origin of at least the "perfect" intervals (octave, fifth, fourth), but that cultural factors are largely responsible for listeners' perception of consonance and dissonance in music.

If beating causes roughness and dissonance, the intervals of just intonation might sound the most in tune. Helmholtz indeed stated that equal temperament sounded more out of tune than just intonation. Most studies of preference for tuning system find, to the contrary, that subjects prefer equal temperament to just intonation. Studies of intonation in performance again show little evidence for just intonation. Along with some psychoacoustic studies, they instead reveal a tendency to stretch larger intervals with respect to equal temperament and to contract the smallest intervals.

Several authors have presented a two-component model of intonation perception, in which the two components are beat rate and interval size. The relative contribution of these two components has not been thoroughly examined. No previous studies have controlled beat rate independently of interval size. However, Vos (1986) found that subjects rated fifths and major thirds as more pure when the beating partials were removed. The judged purity of these isolated intervals was also found to be a good predictor of the acceptability of the intonation of two-part musical passages (Vos 1987).

The purpose of the present dissertation research is to decouple beat rate and interval size, in order to help determine whether beating really is a significant component of intonation for musically trained listeners. Beat rate is objectively ascertainable, whereas knowing the "correct" interval size requires training, whether through education or simply through exposure to music. Thus the relative importance of beating versus interval size lends itself to interpretation as the relative importance of immediate versus learned properties, or psychoacoustic versus cognitive factors. The latter interpretation is an oversimplification, but a useful one.

Chapter

2

Experiment One: Pairs of Perfect Fifths

2.1 General Design of the Experiments

The purpose of the experiments was to determine the contribution of beating to judgments of intonation. Three independent variables were chosen:

(1) "Projected beat rate"
(2) Beat amplitude
(3) Method of controlling beat rate
 (a) changing interval tuning but keeping tones harmonic
 (b) keeping interval tuning constant but making beating partials inharmonic

The main thrust of this dissertation is to study beat rate independently of interval tuning. Because these two factors are normally coupled, however, they must be studied indirectly rather than being two of the independent variables themselves. The relation of beat rate and interval tuning to the three independent variables listed above requires some explanation. It may be helpful to refer to Figure 2 below, "Experimental design: independent variables."

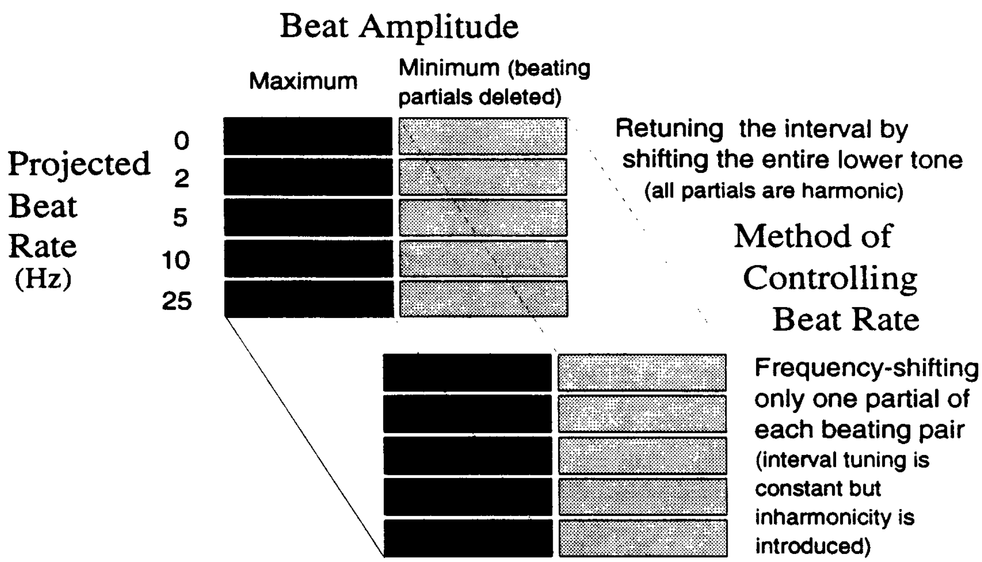

Fig. 2. Experimental design: independent variables.

In normal instruments, beat rate can only be changed by changing the interval tuning. With computer-generated sound, we can make interval tuning independent of beat rate by introducing another technique of controlling the beat rate—frequency-shifting the beating partials. With this technique, rather than moving all the partials of a tone in parallel (as would be the case in a normal instrument), we examine the partials of the two tones to see which pairs of partials are beating, and just move one partial of each pair in order to change the beat rate. In the stimuli used in the present study, several pairs of partials beat; the higher beating pairs always have frequencies that are integer multiples of those of the lowest beating pair. The beat rate of the lowest beating pair corresponds to the perceived beat rate (Vos 1984), and the rates at which the higher partials beat are always integer multiples of this rate. To change the beat rate, we find all the partials in the lower tone that beat with partials in the upper tone, and shift the former in concert, such that the beat rates themselves always stay as a harmonic series (which helps maintain a unified perceived beat rate.)[1]

Unfortunately, this technique introduces a covarying inharmonicity, which might be perceived and might affect the intonation judgments. As a check on this possible side effect, we do two things. First, we compare inharmonic sounds that have beating with the same inharmonic sounds with the beating removed. The beating is removed by simply deleting the appropriate partials of the upper tone. This yields the second independent variable listed above, beat amplitude. Secondly, we compare this frequency-shifting technique with the normal method of controlling beat rate (i.e., retuning the entire note). This constitutes the third independent variable listed above: method of controlling beat rate.

Although we are most interested in the effect of beat rate, and although we have two different methods for controlling it, each of these methods introduces another change (in either the tuning or the inharmonicity). To isolate the effect of beating from these two covarying variables, we need the third independent variable—we can remove the beating without affecting the harmonicity or the tuning (as the case may be).

Note that "projected beat rate" is defined even when the beat amplitude is zero. That is why I have used the qualifier "projected." "Beat rate" is not itself an independent variable, but is replaced by this qualified term, which refers to the amount of shift of the frequencies (whether all the frequencies of one note, in the case of the first method of controlling beat

1. Another approach would have been to make all the beat rates the same, instead of having them form a harmonic series. In this case each partial would have been shifted by a different amount in cents and the overall inharmonicity would have been less. However, it was decided to have as close a comparison as possible with naturally occurring stimuli, particularly with respect to beating. Having all the beat rates be the same would have unnaturally accentuated the primary beat rate in the "shifted partials" stimuli, in comparison with the "retuned interval" stimuli.

rate, or only those partials that beat, in the case of the second). When the beat amplitude is zero, "projected beat rate" does not describe a perceptible beating of partials, but it does still refer to the number of Hz that separates the partial of the lower note from that partial in the higher note with which it would beat had the latter's amplitude not been reduced to zero. In achieving this projected beat rate, the partial in the lower note may have been shifted to an inharmonic position, or it may have been moved along with all the other partials of the lower note, by retuning the interval.

To prevent the deletion of partials from affecting the degree of inharmonicity, we frequency-shift only partials of the lower note. We choose to frequency-shift the lower note's partials and delete the upper note's, rather than vice versa, because the beating partials of the lower tone are higher in the harmonic series, which means they will introduce less perceptual inharmonicity (Moore, Glasberg, and Peters 1985). The trade-off is that the upper note has more partials deleted—most of the even-numbered partials—which has a greater effect on the timbre. However, the results of the experiments vindicate this decision, as we shall see: inharmonicity appears to be significant for intonation, but removing partials from the spectrum does not.

The nature of the stimuli is described in the descriptions of the individual experiments. In addition, an exhaustive description of every stimulus is included in the Appendix (page 168).

2.2 Experiment One: Method

The first experiment studied intonation by means of relative judgments: subjects were presented with pairs of stimuli and asked how much more in tune one was than the other. Details on the stimuli and the experimental procedures are given below.

2.2.1 Stimuli

The stimuli were digitally synthesized sounds. The constant and variable features of the stimuli are listed below. Not all the component frequencies of each tone are listed here, since the algorithmic selection of partials to be shifted in frequency resulted in too lengthy a list. A more complete specification of the stimuli is given in the Appendix, along with the algorithm that selected the partials to be frequency-shifted or deleted.

Constants

Harmonic musical interval (two simultaneous notes)
Pitch of upper note: C5 (523.251 Hz, which is equal-tempered with respect to A440)
Pitch of lower note: approximately F4 (could be retuned; see variables below)
Time-invariant frequency spectrum
Flat frequency spectrum (0 dB/octave rolloff)
16 partials per note (1st 16 of harmonic series, though some may be deleted or made inharmonic in some stimuli)
Duration: 1.5 seconds
Trapezoidal amplitude envelope
Attack portion of amplitude envelope: .05 seconds
Decay portion of amplitude envelope: .5 seconds
Beginning phase of all partials is 0.0 (not randomized phase), sine phase

Variables

(1) Beat amplitude (2 levels):

(a) maximum (beating partials have equal amplitude)

(b) minimum (partial of upper note deleted, for each beating pair)

(2) Projected beat rate (5 levels): 0, 2, 5, 10, 25 Hz

Note that this variable is present even when beat amplitude is zero. This is because we want to test other effects of the methods used to control beat rate.

(3) Method of controlling beat rate (2 levels)

(a) Retuned interval

The entire lower tone (F4) is transposed (all partials). This method preserves harmonicity but changes the interval tuning. To achieve beat rates of 0, 2, 5, 10, and 25 Hz, the F4 is transposed to:
348.827, 349.493, 350.493, 352.160, 357.160 Hz
= 0.0, 3.3, 8.3, 16.5, 40.9 cents higher than the "just" F4
or –2.0, 1.3, 6.3, 14.5, 38.9 cents higher than the equal-tempered F4

(b) Frequency-shifted partials

The partials in the lower tone that beat with partials of the upper tone (or that would beat if the partials of upper tone were not deleted) are shifted in frequency. This method introduces inharmonicity but preserves the fundamental frequency ratio. F4 is always equal-tempered with respect to A440 (as is the upper note, C5), and thus has a constant frequency of 349.228 Hz.

2.2.2 Trials

The stimuli were presented in A/B pairs. A and B alternated continually until the subject stopped them. There were 20 different stimuli and 80 different A/B pairs of stimuli. Not all possible pairs were used. Rather, every pair had at most one variable change between A and B. A and B could be the same. Each A/B pair appeared in the experiment twice, once with A presented first and once with B presented first. Thus there was a total of 160 trials in the experiment, which took approximately 40 minutes to run (the time varying according to the subject's pace).

Before doing the experiment itself, each subject did a "trial run" consisting of 25 trials chosen randomly from the 80 unique A/B pairs used in the main experiment.

2.2.3 Apparatus

The sounds had been generated on CCRMA's Systems Concepts Digital Synthesizer ("Samson Box") using additive synthesis, and were transferred beforehand to a NeXT computer. There was no danger of aliasing: the Samson Box synthesis used a sampling rate of 37707.39 Hz, making the Nyquist rate over twice that of the highest frequency in the stimuli, which was about 8370 Hz. The synthesized stimuli were converted to analog and simultaneously transferred to the NeXT computer using a Metaresearch Digital Ears device, without making an intermediary recording. The specification of the Metaresearch analog-to-digital conversion and the NeXT digital-to-analog conversion is 16-bit samples at 44100 Hz.

The subject sat at the monitor of the NeXT computer, in a soundproof room, with the NeXT CPU in another room to isolate its fan and disk noise. The NeXT audio was connected via a Hafler power amplifier to two high-quality Westlake speaker units. The speakers were symmetrically placed at about plus and minus 45 degrees from directly ahead of the subject, at a distance of about one and half meters. The sound pressure level of the stimuli at the subject's position was measured to be approximately 63 dB, using a standard USASI S1.4 (and IEC R123) sound level meter. Curve "A" (45-dB weighted) and curve "B" (flat) gave similar readings.

2.2.4 Subjects

There were eight subjects, all of whom had considerable musical experience[2] and no hearing problems. All were men under 40, and all were current or former students of computer music. In addition, some of them had previously been subjects in psychoacoustic experiments, and three of them had experience in music with nonstandard tunings. Details on the subjects, all of whom were volunteers, are given in Table 1 on page 64. The subjects are ordered in the table according to how consistent their responses were, as described under "Consistency of responses" on page 96. A ninth subject also did the experiment, but his results were omitted from the analysis because he was significantly more inconsistent in his responses than the others (see Table 6 on page 97).

2.2.5 Procedure

The task was to judge how much more in tune stimulus A was than stimulus B, or how much more in tune B was than A. The possible responses ranged from 1 to 9, with 1 being "A is much more in tune than B," 9 being "B is much more in tune than A," and 5 being "both are equally in tune." Subjects were asked to use their normal sense of musical acceptability in judging the intonation. The instructions did not mention beating; but neither did they indicate that pitch relations should be the sole criterion for judgment.

The subject's responses were entered by using a mouse to click on a graphical item on the NeXT computer's screen. The program interface (see Figure 3 below) included 9 graphical buttons for the various responses. There were also graphical buttons for replaying

2. All of my experiments studied musicians only, since the intonation judgments were likely to be too difficult for novices and it was desirable to use subjects expected to be capable of using pitch relations as well as beating in their judgments. Restriction to musically trained subjects is not atypical for experiments of this kind (see, for example, Hall and Hess [1984] or Vos [1986]).

Table 1. Subjects in Experiment One.

Subject	Consistency rating (0 to 1)	Age	Years of musical experience	Musical activity: Instruments studied for more than 3 years (including voice); Composition	Years of musical education			Highest music degree	Experience with non-standard tunings?	Years of study in psychoacoustics	Ever before been a subject in a psychoacoustic experiment?	Comments
					Lessons	Courses	Ensemble					
1	.880	29	20	Drums, piano, guitar; composition	0	0	0	None	No	0	No	Subject is self-taught
2	.865	24	15	Piano	n/a	n/a	n/a	B.A.	No	3	Yes	
3	.792	27	14	Percussion; composition	2	3	5	None	Yes	4	Yes	Composes microtonal music
4	.774	36	27	Cello; composition	9	8	12	D.M.A.	No	1	Yes	
5	.762	30	17	Clarinet; composition	10	9	10	M.M.	Yes	1/2	Yes	Experience with Chinese instruments
6	.700	31	26	Piano, voice; composition	30	12	8	M.F.A.	Yes	1/2	Yes	Has composed some microtonal music
7	.698	25	17	Trombone	15	2	10	None	No	1/2	No	
8	.687	32	22	Piano; composition	10	8	0	D.M.A.	Yes	1/2	Yes	Experience as a piano tuner

the current trial, stopping its playback, and playing the next trial. The screen also displayed the current trial number and the subject's response. While the stimuli were being played, the screen showed whether A or B was currently sounding. The subject might listen to the current stimuli as long as desired, and might stop and restart them and/or change responses as many times as desired before starting the next trial.

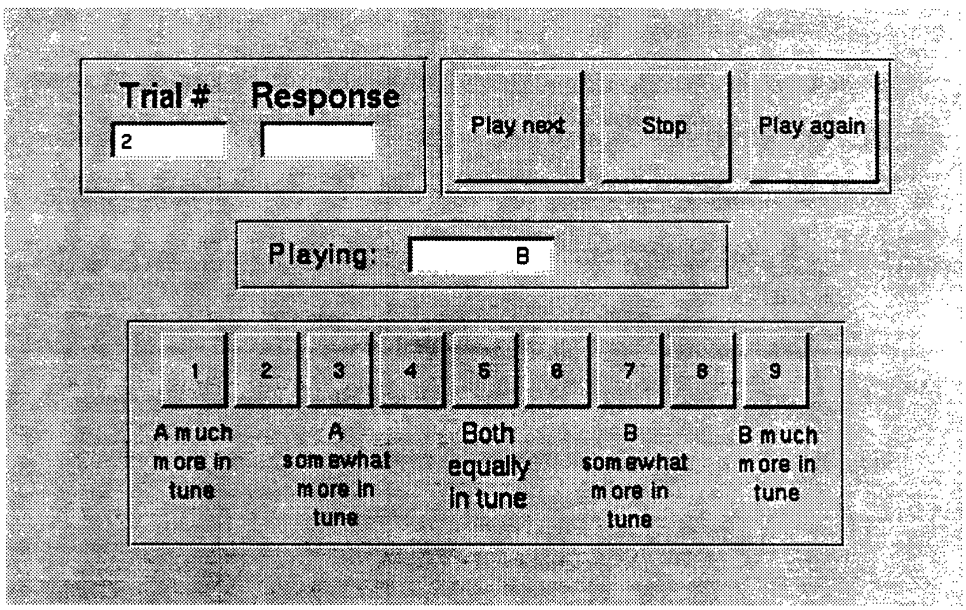

Fig. 3. User Interface for Experiment One.

2.2.6 Data Reduction

A paired-comparison matrix is a way of getting single values for a stimulus variable from judgments of pairs of stimuli. In Experiment One, subjects were asked to judge not the intonation of an isolated stimulus, but rather how much more in tune one stimulus was than another. For example, they compared a stimulus having a 2-Hz projected beat rate to a stimulus with a 10-Hz projected beat rate. But we sought a single overall value for the intonation of a 2-Hz stimulus. This overall value was obtained by averaging the responses for the pairs of stimuli 2 Hz vs. 0 Hz, 2 vs. 5, 2 vs. 10, and 2 vs. 25. Table 2 is an example of one of the paired-comparison matrices created in this way.

Each element is an average over all subjects, where 9.0 means that B is much more in tune than A and 1.0 means that A is much more in tune than B. The average of a column is shown at the bottom and represents the overall intonation value for a given projected beat rate, with 9 being the most in tune and 1 the least. (The responses to pairs containing identical stimuli—on the diagonal from upper left to lower right—are omitted when calculating the column means.)

This particular table shows only trials where the two stimuli (A and B) had differing projected beat rates but the same beat amplitude and the same method of controlling beat rate. Recall from the description of the trials that no more than one variable would change

BEAT AMPLITUDE: MAXIMUM
METHOD OF CONTROLLING BEAT RATE: RETUNED INTERVAL

A: \ B:	0 Hz	2 Hz	5 Hz	10 Hz	25 Hz
0 Hz	[4.889]	3.222	3.000	1.333	1.444
2 Hz	6.222	[5.111]	3.444	2.000	1.778
5 Hz	6.889	6.889	[5.111]	2.333	1.444
10 Hz	7.000	7.889	7.444	[4.889]	2.444
25 Hz	7.667	7.889	7.444	6.889	[5.000]
MEANS	6.944	6.472	5.333	3.139	1.778

Table 2. An example of a paired-comparison matrix from Experiment One; based on stimulus pairs with differing beat rates.

its value between stimulus A and stimulus B. Similar matrices were created for other combinations of the stimulus variables. The column means from each of these matrices were used as the input data for three sets of analyses, grouped according to which variable changed between stimulus A and stimulus B. For each of the three groups of trials, we shall first present two different types of diagram displaying the paired-comparison matrix column means averaged over subjects, followed by the results of some descriptive and comparative statistical analyses.

2.3 Results

2.3.1 Trials with Changing Projected Beat Rate

We first examine the results for all the trials in which the variable that changed between stimulus A and stimulus B was projected beat rate.

Graphs of mean responses

The mean responses,[3] averaged across subjects, are shown in the three-dimensional diagram (Figure 4). Each axis represents one of the independent variables (projected beat rate, beat amplitude, and method of controlling beat rate). It is immediately evident that the vertical dimension, representing projected beat rate, has the greatest variation of responses as well as a fairly regular pattern: the faster the projected beat rate, the more out of tune the stimulus sounds. By contrast, beat amplitude and method of controlling beat rate appear to have relatively little effect.

3. I use the term "response" loosely in the discussion of the analysis results of Experiment One to refer to the corresponding column mean from the paired-comparison matrix, as described in the previous section ("Data Reduction").

The next diagram (Figure 5) plots the same results in two dimensions, with the mean response (the judged "goodness" of the intonation, averaged over subjects) along the vertical axis and the projected beat rate along the horizontal. Four different symbols are used to represent the four combinations of the other two independent variables (beat amplitude and method of controlling beat rate). A second-order polynomial curve is fitted to each of the four conditions.

Especially in this form of graph, the very strong effect of projected beat rate is clear. For each value of projected beat rate, the four different symbols are clustered fairly close

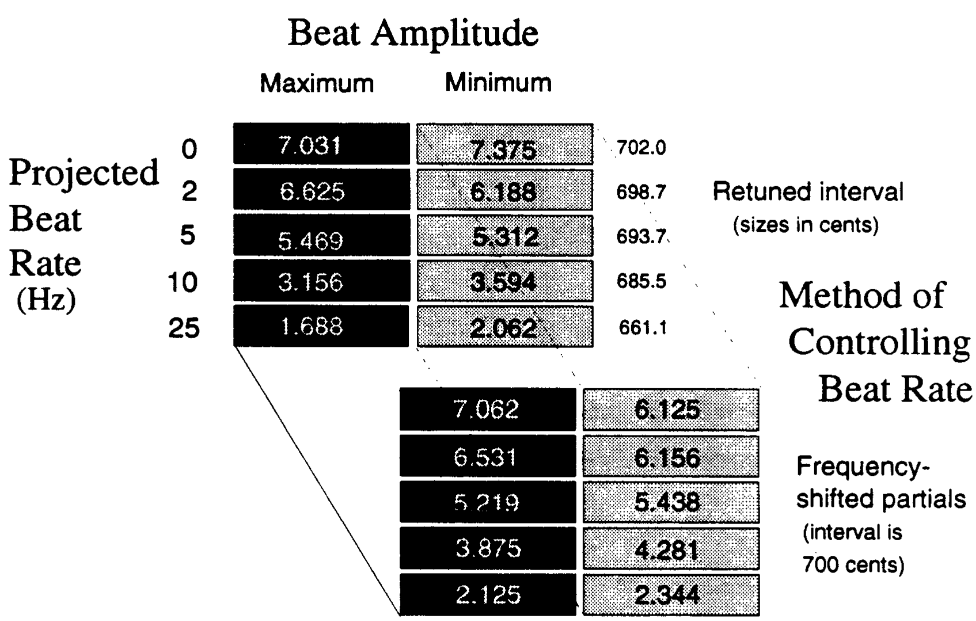

Fig. 4. Experiment One: Judgments of intonation of fifths, based on trials where the two stimuli had different beat rates. The "response" in each cell is the average of one column from the corresponding paired-comparison matrix, averaged over all subjects. 9.0 = maximally in tune, 1.0 = maximally out of tune. Based only on trials in which the variable that changed between stimulus A and stimulus B was "projected beat rate."

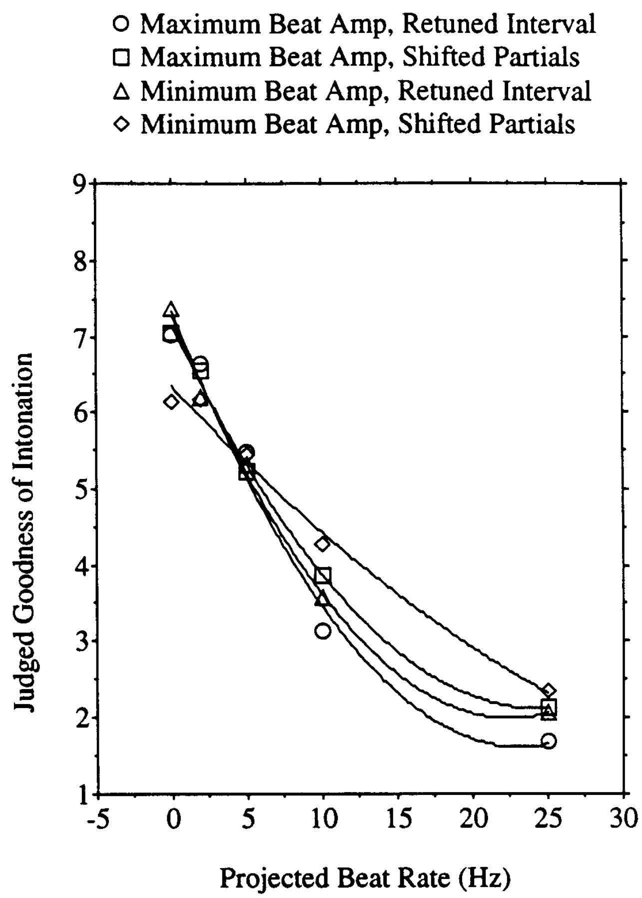

Fig. 5. Experiment One. Trials in which "projected beat rate" was the variable that changed between stimulus A and stimulus B. The other two variables create four stimulus types, given in the legend. Each is fit with a separate curve showing the second-order polynomial regression.

together, suggesting that the other two variables are much less significant in this group of trials.

Examining the curve at the 10-Hz and 25-Hz points, it appears that the inharmonic stimuli (i.e., those where beat rate was controlled by shifting only the beat partials) are somewhat more in tune than the harmonic ones (those where all the partials were shifted). Further, it appears that the "minimum beat amplitude" form of each of these is slightly more in tune than the "maximum beat amplitude," suggesting that beating at these fast rates contributes to "out-of-tuneness." (As discussed below, however, the statistical analyses did not find these differences to be significant.) For zero, two, and five Hz, the relations are less clear.

Figure 6 shows the same results with each stimulus condition in its own plot, so that the standard deviation over subjects at each point can be displayed clearly with vertical error bars.

We should perhaps not be surprised that neither beat amplitude nor method of controlling beat rate appear very significant on these plots, because these data come from trials where those two variables stayed the same between stimulus A and stimulus B. One would expect a subject, in comparing two stimuli, to base the judgment on qualities that change between the two. However, the effects of the other variables can show up if there is an interaction between variables—for example, if the difference between two projected beat rates sounds more extreme when the beating is not deleted. This is one of the purposes of the ANOVA (analysis of variance) routine.

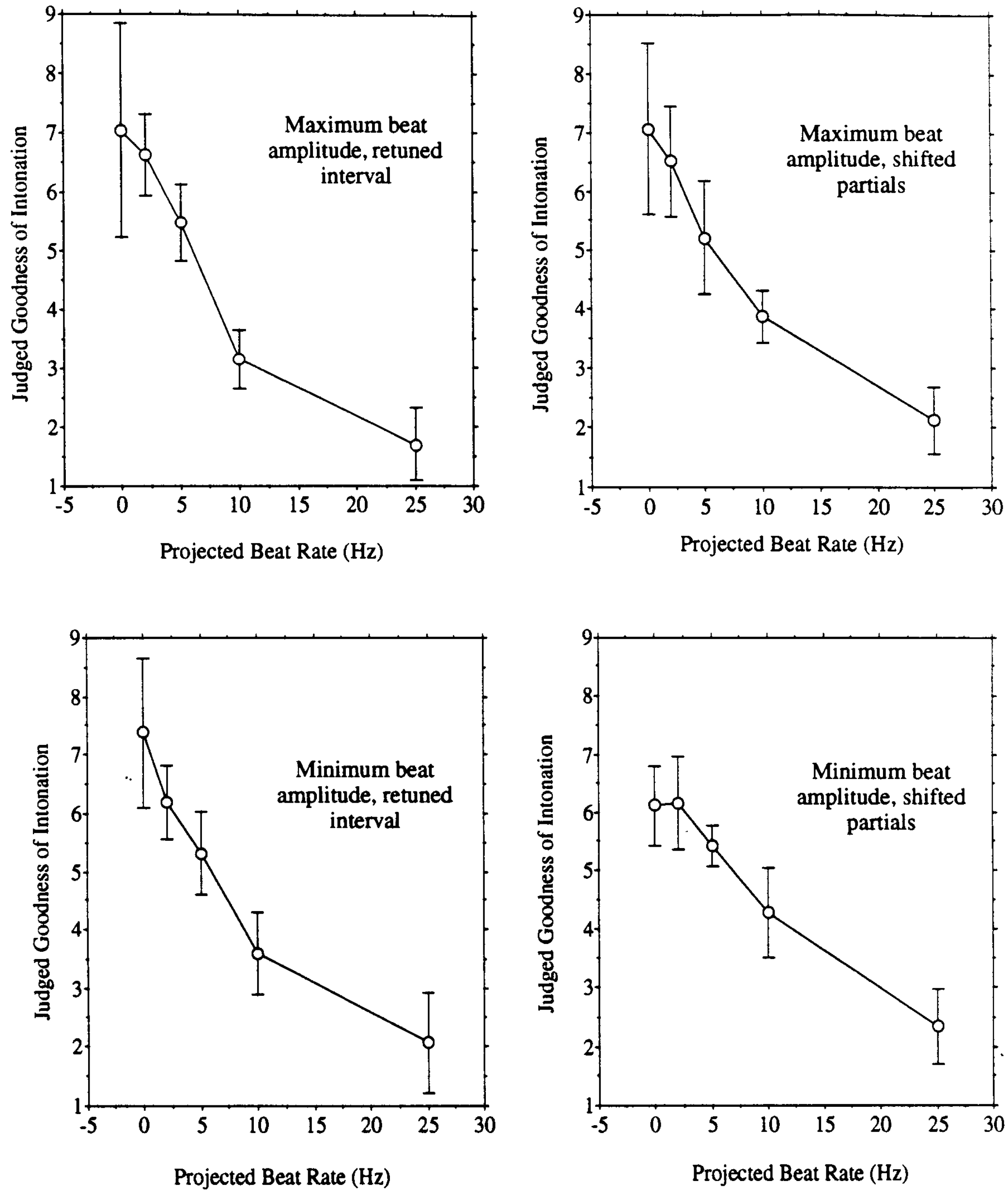

Fig. 6. Experiment One, trials in which "projected beat rate" was the changing variable. Each stimulus condition is plotted separately. Vertical error bars display the standard deviation over subjects.

Statistical analyses

The values shown in the diagrams above were subjected to some descriptive and comparative statistical routines, using the StatView II software package for the Macintosh. Table 3 summarizes the analysis results.

The first statistic, mean response, shows the average response (on a scale of 1 to 9). Since this is very close to the midpoint of the scale, 5.0, and since half the trials use the same stimuli as the other half, but in reverse order, this indicates that there is no order effect. In other words, it doesn't matter which stimulus comes first in a trial; subjects don't tend to call the first of the two stimuli any more or less out of tune than the second.

Table 3. Analysis results, Experiment One. Trials in which projected beat rate differed between stimulus A and stimulus B.

Mean Response	4.883
Standard Deviation	1.855
Correlation of Mean Responses to:	
Projected Beat Rate	–.950
Beat Amplitude	.003
Method of Controlling Rate	.018
3-way ANOVA on Mean Responses:	
Projected Beat Rate	F=158.51 (df 4,4), p<.01
Beat Amplitude	F=.004 (df 1,4), n.s.
Method of Controlling Rate	F=.22 (df 1,4), n.s.
Rate x Beat Amplitude	F=1.30 (df 4,4), n.s.
Rate x Method	F=2.45 (df 4,4), n.s.
Beat Amplitude x Method	F=.53 (df 1,4), n.s.

The second statistic, standard deviation, indicates the variation of the responses. There is a fair amount of spread; that is, the mean responses tend to fill out the available range. Since these are means from a paired-comparison matrix, the actual raw data (the responses per trial) showed more variance.

The third group of statistics show the correlation of the mean responses to the three independent variables. (Correlation values can range from –1.0 to 1.0; a value of zero indicates no correlation, and –1.0 or 1.0 both correspond to perfect correlation, with the minus sign indicating that one quantity increases as the other decreases.) The mean responses are very highly correlated to projected beat rate (–.963): the higher the projected beat rate, the lower the mean response, i.e., the more likely the stimulus is to sound out of tune. However, the judgments of intonation are not at all correlated to beat amplitude (.006) or method (.044). (For beat amplitude, "minimum" was represented by the quantity 1 and "maximum" by 2 in computing the correlation; for method of controlling beat rate, 1 designated "retuned interval" and 2 "shifted partials.")

The analysis of variance (ANOVA)[4] supports this conclusion: of the three variables, only projected beat rate is found to be significant for these trials. (The symbol "p" indicates the probability that the results could be attributed to chance, which for projected beat rate is less than one chance in a thousand—highly significant. The symbol "n.s." stands for "not significant.") Further, there is no significant interaction between variables. Recall that we had observed in the regression plot that at 10 and 25 Hz there seemed to be some effects of beat amplitude and of method of controlling beat rate. However, when the overall

4. The ANOVA results given do not include a triple interaction term, since it was used as the error term for the analysis. In this experiment, unlike Experiments Two and Three, there was only one repeat of a given trial (and in the repeat the presentation order was changed, e.g. from "ABABA..." to "BABAB..."); thus it was inadvisable to attempt to derive an error term from repeated trials.

variability of such judgments is taken into account (including the responses to all five values of projected beat rate), as it is in the analysis of variance, it appears that these differences could conceivably have occurred by chance.

Although the regression plot suggested that, at projected beat rates of 10 and 25 Hz, stimuli with the beating partials deleted might be judged more in tune, these differences turn out to be nonsignificant when considering all five values of projected beat rate, as the ANOVA routine does. On the same portion of the plot, the stimuli having only the beating partials shifted appeared to be judged more in tune than stimuli having the interval retuned; but this cannot be shown to be statistically significant either.

Per-subject results

Figures 7 and 8 show the per-subject results for trials in which projected beat rate was the variable that changed between stimulus A and stimulus B. The figures are presented in order of decreasing subject consistency: As in Table 1 (page 64), Subject 1 is the one who gave the most consistent responses from trial to trial for a given stimulus, Subject 10 the least consistent. Note that the general shape of the curves for most subjects is quite similar to those of the data averaged over subjects, as displayed in Figure 5 (page 69). With few exceptions, subjects rated the stimuli with the 25-Hz projected beat rate as having intonation values in the range of 1 to 3 (on the scale of 1 to 9). Stimuli with the zero-Hz and two-Hz rates were rated the highest.

Note that, for stimuli with a zero-Hz projected beat rate, nearly all the subjects judged the stimuli with shifted partials and a minimum beat amplitude to have worse intonation than the other types of stimuli. On would expect the "retuned interval" stimuli to sound more in tune than the inharmonic stimuli at zero Hz, because of its lack of inharmonicity.

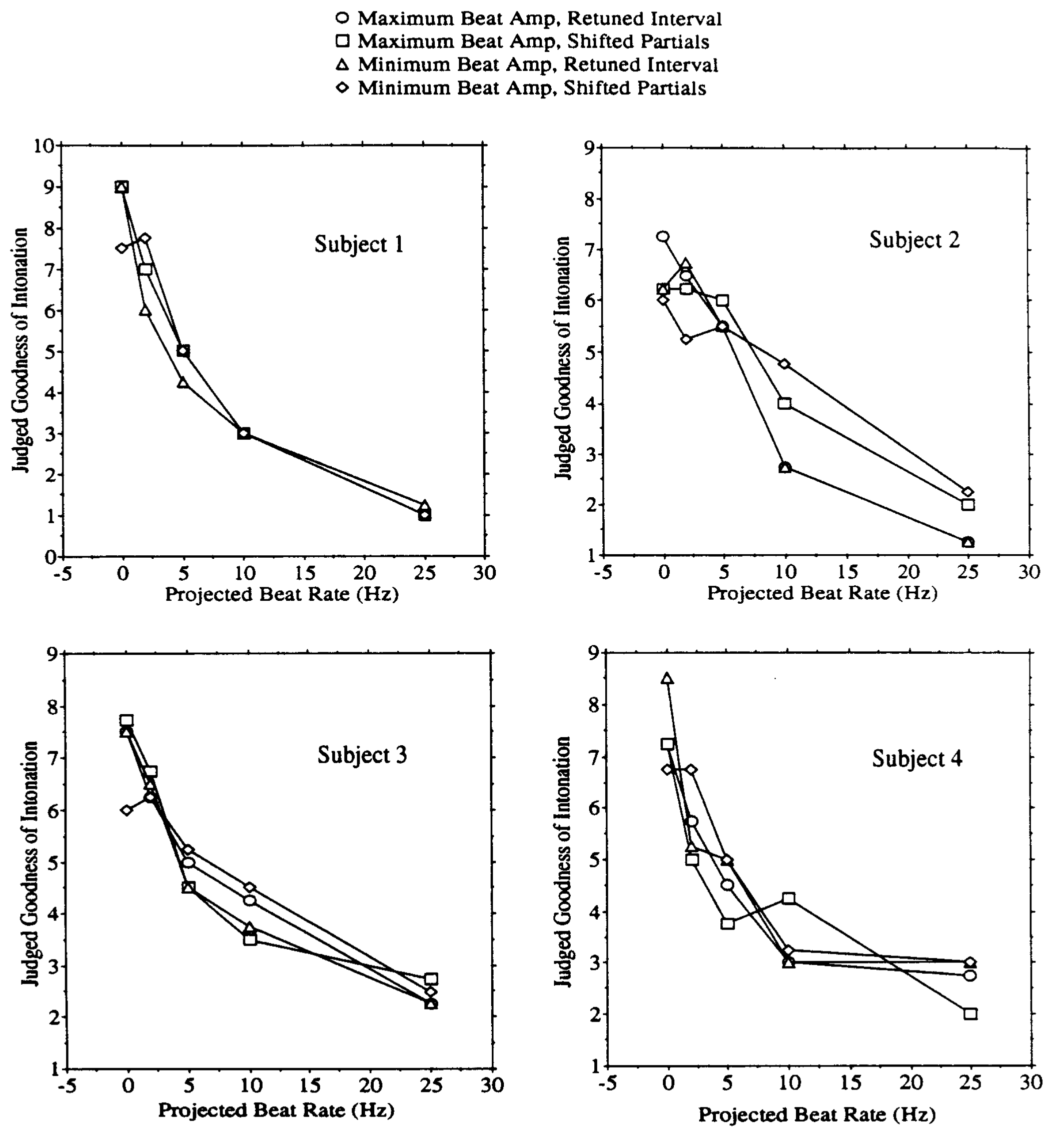

Fig. 7. Experiment One, results per subject. Trials in which "projected beat rate" was the variable that changed between stimulus A and stimulus B. (Continued in Figure 8).

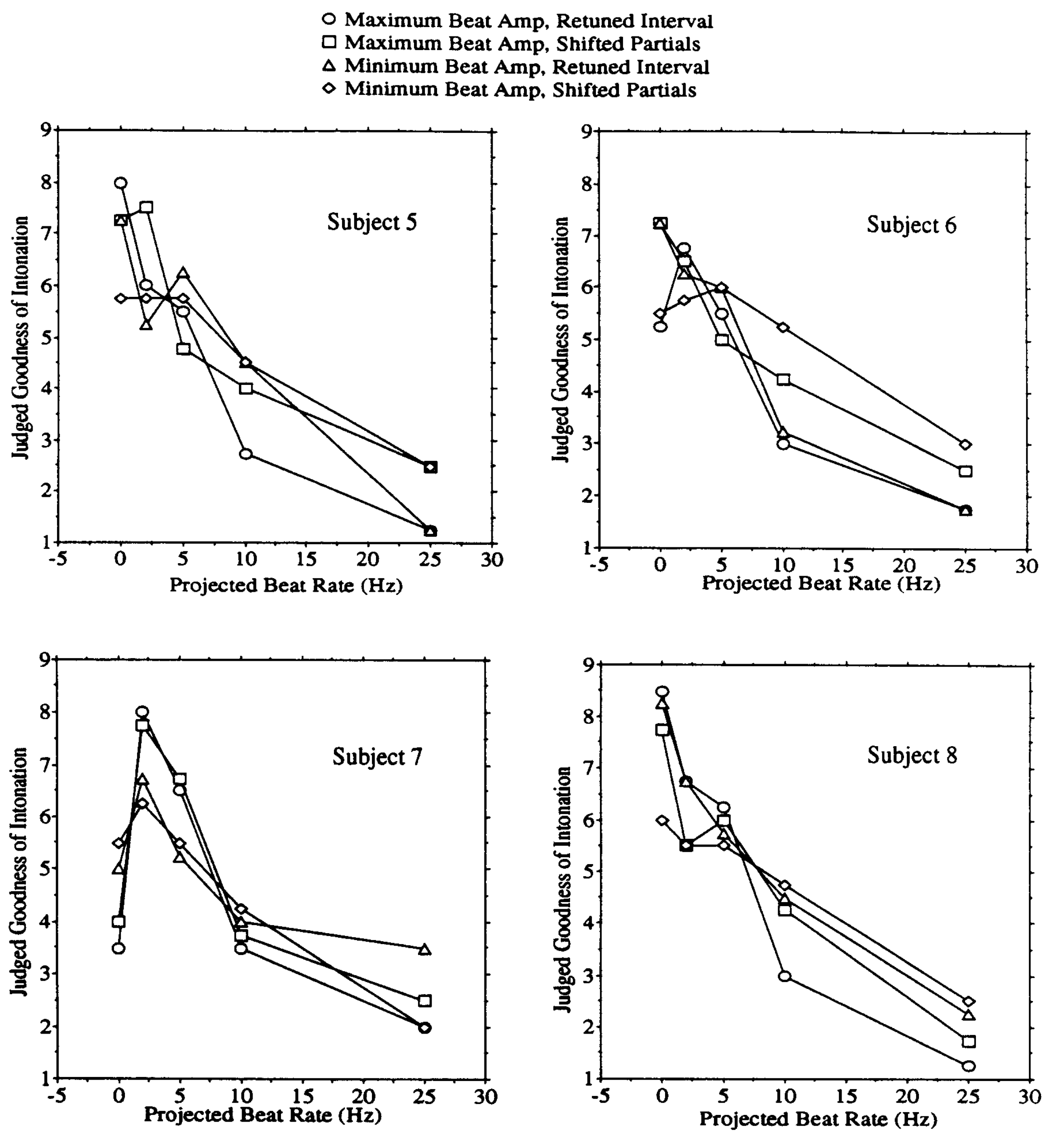

Fig. 8. Experiment One, results per subject (continued from Figure 7). Trials in which "projected beat rate" was the variable that changed between stimulus A and stimulus B.

(Note also that the inharmonic stimuli are equal-tempered whereas the "retuned interval" stimuli are just.) But it is curious that at a zero-Hz projected beat rate, the "maximum beat amplitude, shifted partials" stimulus would sound more in tune than the "minimum beat amplitude, shifted partials" stimulus. There should be no beating in either of these, but perhaps the lower rating given to the "minimum amplitude" version reflects the difference in timbre caused by the deletion of even partials from the upper tone. Although the "beat amplitude" variable (which is also correlated with a change in timbre) was not found to be significant overall, perhaps it is significant at 0 Hz.

The clearest pattern in the graph averaged over subjects (Figure 5 on page 69) is the fact that the four types of stimuli are rated in the same order for 10 Hz as for 25 Hz, generating four almost parallel lines between these two projected beat rates. At these points, the stimuli are rated in the following order, from best intonation to poorest: "minimum beat amplitude, shifted partials," "maximum beat amplitude, shifted partials," In the graphs for the individual subjects, this ordering is echoed the most closely in the patterns of Subjects 2 and 6. Subject 8 has a consistent but different ordering of the stimuli at these points. Subject 3 has a consistent ordering from 5 Hz to 10 Hz.

Subject 1, whose responses were the most consistent (in other words, who had the greatest correlation between the ABAB... and BABA... presentations of a given stimulus pair), also appears to have the least effect of stimulus type in his graph. In fact, it is difficult to distinguish some of the stimuli in his graph because the points coincide so frequently. It is interesting to note that this subject reported using beat rate as a cue for intonation. This would explain the closeness between the "shifted partials" and "retuned interval" types of stimuli for this subject, but it doesn't explain the closeness of the minimum and maximum beat amplitudes.[5]

A number of the subjects rate some of the stimuli as being less in tune at the zero-Hz projected beat rate than at 2 Hz. Subject 7 is remarkable in this respect, in that every one of the stimuli at zero Hz is rated as worse than any of the 2-Hz stimuli. This behavior suggests that this subject prefers stimuli with some beating, as in the group of "rich listeners" that Roberts and Mathews (1984) found. With the exception of this subject's response to the zero-Hz stimuli, the general trend is quite clear in the graphs: subjects perceive the intonation as steadily worsening as the projected beat rate increases. Again, however, it doesn't follow that beating is important, since projected beat rate is correlated with either interval tuning or inharmonicity, depending on the method of controlling beat rate. Since the ANOVA showed beat amplitude to be nonsignificant, the appropriate interpretation is that the technique for controlling beat rate is itself responsible for the change in subjective intonation.

2.3.2 Trials with Changing Method of Controlling Beat Rate

We now present the results and analyses for the trials where "method of controlling beat rate" changed between stimulus A and stimulus B. In each of these trials, one of the stimuli used the "retuned interval" method and the other used the "shifted partials" method.

Graphs of mean responses

Figure 9 is a three-dimensional diagram of the mean responses. As before, there is not much left-to-right difference; beat amplitude appears to have little effect on the intonation. There is an interesting interaction of the other two variables, however. Observe that the rear columns (those representing the "retuned interval" method of controlling beat

---

5. It is likely that Subject 1 also made use of strategies other than beat rate. As described on page 100, subjects found it difficult to hear any beating at all in the "minimum beat amplitude" stimuli.

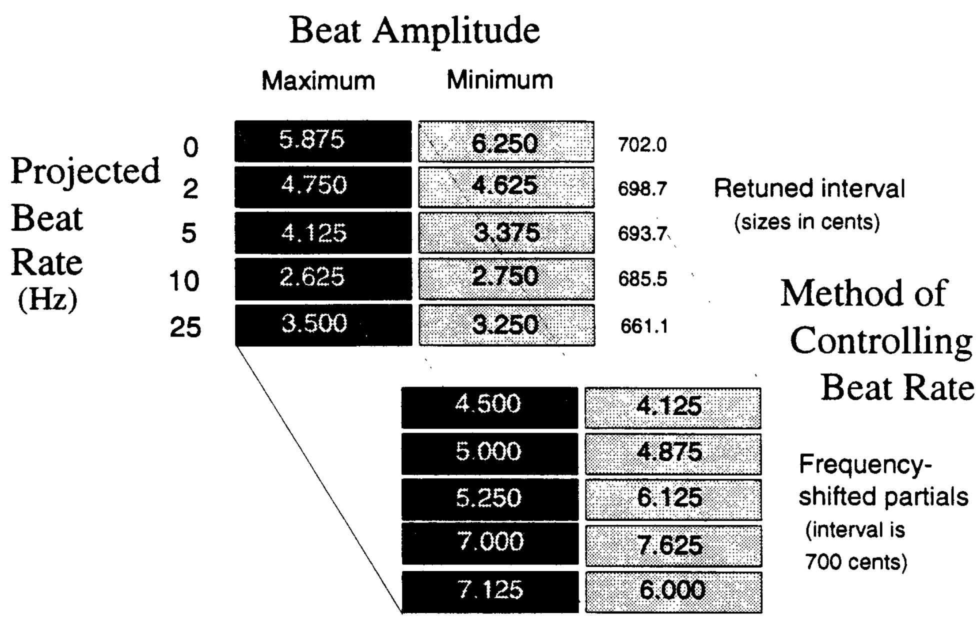

Fig. 9. Experiment One: Judgments of intonation of fifths, for trials in which the variable that changed between stimulus A and stimulus B was "method of controlling beat rate." 9.0 = B is much more in tune than A; 1.0 = A is much more in tune than B. The value of a given cell represents stimulus B; stimulus A uses the opposite method of controlling beat rate.

rate) have the pattern we found in the previous group of trials: the mean responses get lower as the projected beat rate gets higher. For the forward columns (the "shifted partials" method), however, the reverse pattern holds: the responses increase as the projected beat rate increases. These opposite effects are even clearer on the regression plot (Figure 10), in which the two upper curves are roughly mirror images of the two lower curves.

In order to understand this result, we must recall that the values come from paired-comparison matrices in which the columns and rows represented degrees of projected beat rate. In the last set of trials, where projected beat rate was the variable that changed between A and B, we could use the mean response shown in a given cell in the three-dimensional

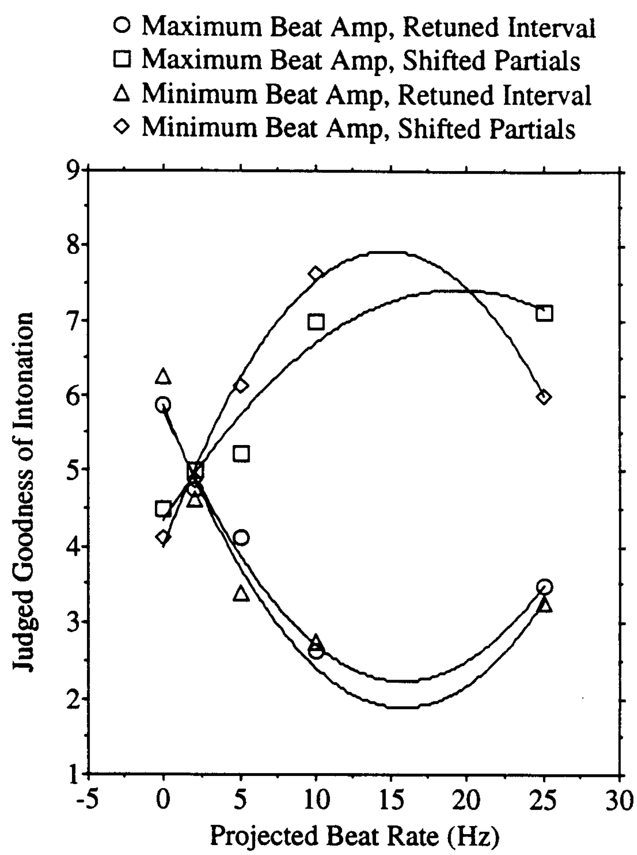

Fig. 10. Experiment One. Trials in which "method of controlling beat rate" was the variable that changed between stimulus A and stimulus B. As before, each stimulus condition is fit with a separate curve showing the second-order polynomial regression. 9.0 = B is much more in tune than A; 1.0 = A is much more in tune than B. The symbols in the legend represent stimulus B; stimulus A uses the opposite method of controlling beat rate. (To tell which curve belongs with each symbol, see the points at 25 Hz.)

diagram as being equivalent to the goodness of intonation of the corresponding stimulus. However, in these trials, it is the method of controlling beat rate that changes between A and B. The value shown in any cell of Figure 9 is not an absolute measure of the corresponding stimulus, but rather indicates the mean response if stimulus B is the corresponding stimulus and stimulus A has the opposite method of controlling beat rate. As an example, take the mean response of 7.111 for the cell with a 25-Hz rate, maximum amplitude, and the "shifted partials" method of controlling rate. We cannot take this as an absolute measure of the intonation of this stimulus as compared to, for example, the 4.556 value of the stimulus at the top of the same column. To do so would suggest that the faster-beating, more inharmonic stimulus sounds more in tune. Instead, the mean response given in a cell really denotes the intonation of that stimulus, relative to the stimulus with the opposite method of controlling beat rate. In the current example, the 7.111 means that this stimulus (25 Hz, maximum amplitude, shifted partials) is judged to be much more in tune than the stimulus directly behind it in the diagram (25 Hz, maximum amplitude, retuned interval).

To state this interaction in simple terms: At a beat rate of 0 Hz or thereabouts, it doesn't matter much whether one controls the beat rate by retuning the interval or by making the beating partials inharmonic. At 25 Hz, it matters a great deal; retuning sounds much worse than making the partials inharmonic.

If a pair of stimuli with the same projected beat rate but different methods of controlling beat rate are widely separated in judged intonation value, we may conclude that the method of controlling beat rate is important at that beat rate. In Figure 10 the curves cross at 2 Hz, indicating that the method of controlling beat rate is not important at this projected beat rate.

Note that in Figure 10 the two lower curves, which represent the stimuli "maximum beat amplitude, retuned interval" and "minimum beat amplitude, retuned interval." have a trend that is similar to that found in the trials for which beat rate was the variable that changed between stimulus A and stimulus B. (Compare Figure 5 on page 69.) The trend seems to suggest that increasing the projected beat rate by the method of retuning the interval makes the interval sound more out of tune. Here, however, the appropriate interpretation is that increasing the rate makes the interval sound more out of tune than it does when one uses the method of frequency-shifting the potentially beating partials.

Also note in this graph that the curves for opposite methods of controlling beat rate are most widely separated at 10 Hz, not 25 Hz. The interpretation is that although the "retuned interval" stimuli sound much more out of tune than the "shifted partials" stimuli at 10, at 25 Hz the inharmonicity of the "shifted partials" stimuli makes them sound somewhat out of tune as well, so there is less difference in perceived intonation between the "shifted partials" and "retuned interval" stimuli at 25 Hz.

Figure 11 shows the stimulus conditions separately, with the standard deviation for each stimulus type depicted by error bars.[6]

6. It is curious that the variability is greatly diminished at 2 Hz in three of the four plots. It is conceivable that subjects were most consistent at this beat rate because it is the most distinctive. Studying the relative difference limen for intensity, Riesz (1928) found that subjects were most sensitive to beating at a rate of about 3 Hz. Similarly, a pilot study I conducted indicated that when two different beat rates are presented simultaneously, the one closest to 2.5 - 3 Hz dominates. This effect would only explain the behavior for the "maximum beat amplitude" stimuli, however. A better explanation is that at 2 Hz the "retuned interval" and "shifted partials" stimuli have the most similar tuning (698.7 and 700 cents, respectively). Since in these trials one stimulus used the "retuned interval" method and the other the "shifted partials" method, stimulus A and B would be very similar at the 2 Hz rate, leading subjects to give them a rating consistently close to 5, "both equally in tune." Looking at the graphs, we see that this is indeed the case.

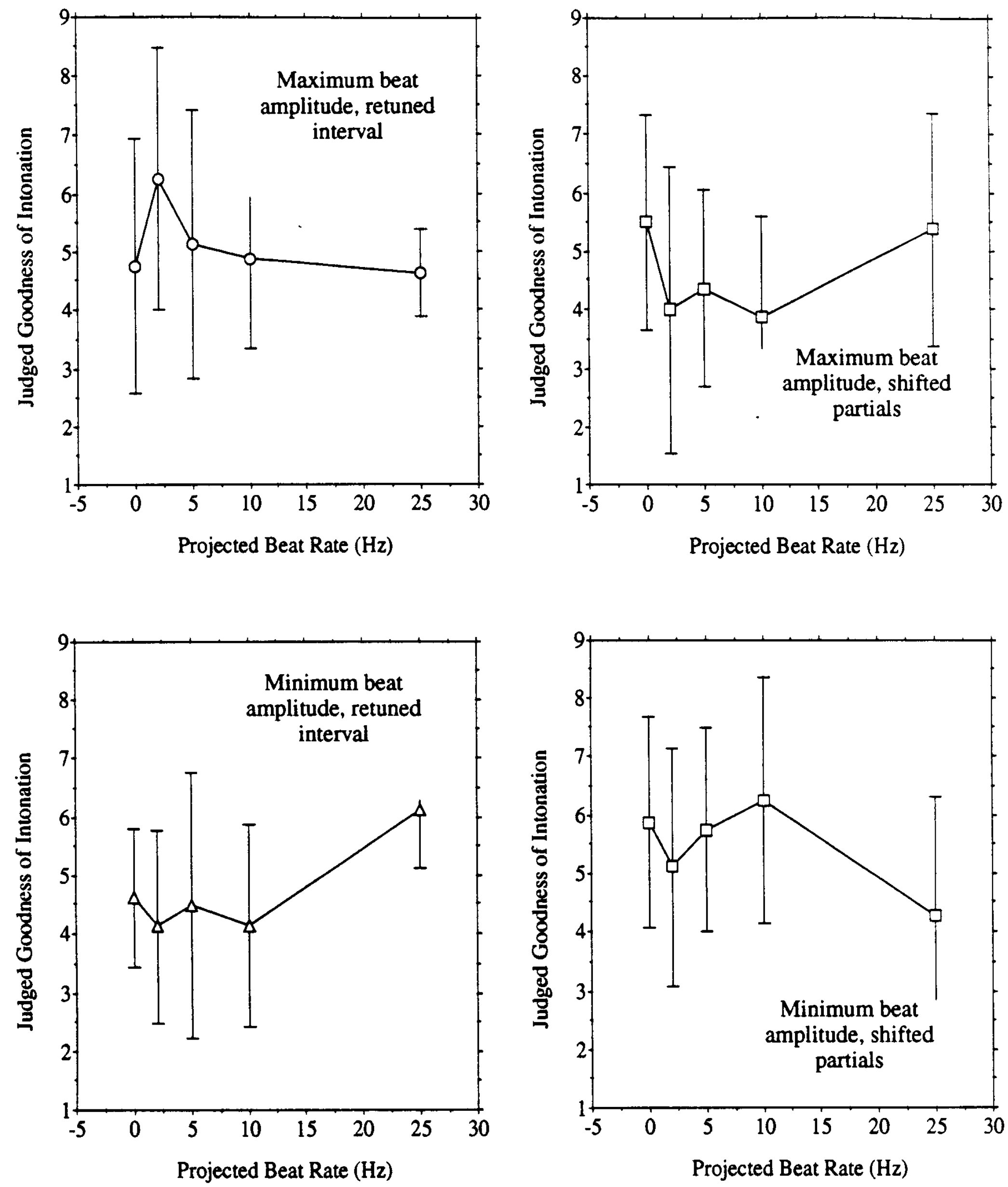

Fig. 11. Experiment One, trials in which "method of controlling beat rate" was the changing variable. Each stimulus condition is plotted separately. Vertical error bars display the standard deviation over subjects.

Statistical analyses

The results of the statistical analyses for the group of trials where "method of controlling beat rate" was the changing variable are listed in Table 4.

The mean is similar to the previous group of trials; the standard deviation is somewhat smaller. (Compare Table 3 on page 72.) The judged goodness of intonation is correlated to the method of controlling beat rate (+.579). The plus sign signifies that retuning an interval is more likely to make it sound out of tune than is shifting only the beating partials.[7] It is understandable that the method of controlling beat rate was

Table 4. Analysis results, Experiment One. Trials in which "method of controlling projected beat rate" differed between stimulus A and stimulus B.

Mean Response	4.938
Standard Deviation	1.463
Correlation of Mean Responses to:	
Projected Beat Rate	.001
Beat Amplitude	–.026
Method of Controlling Rate	.579
3-way ANOVA on Mean Responses:	
Projected Beat Rate	F=.50 (df 4,4), n.s.
Beat Amplitude	F=.11 (df 1,4), n.s.
Method of Controlling Rate	F=52.15 (df 1,4), p<.01
Rate x Beat Amplitude	F=.58 (df 4,4), n.s.
Rate x Method	F=23.77 (df 4,4), p<.01
Beat Amplitude x Method	F=.05 (df 1,4), n.s.

7. In computing the correlation, method (a) was assigned a value of 1 and method (b) a value of 2. Since a greater value for the response meant the stimulus was judged more in tune, a positive correlation between "method" and "response" means that method (b) tends to sound more in tune.

important, since that is the variable that changed between the two stimuli; subjects apparently made judgments on the basis of the variable that changed.

As in the first group of trials, the mean responses were uncorrelated with beat amplitude (–.026). It is surprising, however, that projected beat rate receives a similarly low correlation coefficient (.001). This can best be explained by referring back to the combined plot (Figure 10 on page 80). As previously explained, the fact that the judgments were comparisons between the two opposite methods of controlling beat rate results in a mirror-image sort of graph. The top two curves really are presenting basically the same information as the bottom two curves—each point represent trials with the same two stimuli as the corresponding point in the bottom curve, but with stimulus A and stimulus B switched with each other. However, when the correlation to projected beat rate is computed, these curves cancel each other out, and the resulting coefficient is very close to zero. If the mirror forms had been combined, the plot would have borne more resemblance to that of the first group of trials (Figure 5 on page 69), and the correlation would have been much larger, especially if the coefficient were computed taking account of the curvilinear nature of the relation.

Similarly, the analysis of variance (ANOVA) finds only the method of controlling beat rate to be significant. If the mirror forms had been combined, projected beat rate would probably also be significant. The ANOVA's finding of a significant interaction between projected beat rate and method of controlling beat rate is again an artifact of the mirroring observable in the plot. These problems were solved in Experiment Two.

Per-subject results

Figures 12 - 13 show the per-subject results for these trials, in which "method of controlling beat rate" was the variable that changed between stimulus A and stimulus B. Recall from the discussion on pages 79 - 82 that such graphs do not show absolute judged intonation. Rather, they show the judged intonation of a stimulus relative to the analogous stimulus with the opposite method of controlling beat rate. The graphs tend to be symmetrical around the judged intonation value of 5 Hz because of this property. Typically a curve will be roughly mirrored by another that is approximately its reflection about the 5-Hz midpoint line, although the mirroring is less perfect than in the graph of the results averaged over subject (Figure 10 on page 80).

With the exception of Subject 5, all eight subjects clearly echo the average pattern, favoring the "shifted partials" stimuli over the "retuned interval" stimuli at projected beat rates greater than 2 Hz. The patterns at zero Hz and 2 Hz are less clear. Most of the subjects also echo the averaged data in showing a greater differentiation between the two methods of controlling beat rate at 10 Hz than at 25 Hz.[8]

8. The individual subjects' data are integers here, unlike in the first group of trials. The reason is that each point here actually represents one trial. Since "method of controlling beat rate" has only two levels, the paired-comparison matrices were 2x2 squares, whereas in the previous group they were 5x5 squares (see page 66). The diagonals of these matrices represent trials in which stimulus A is the same as stimulus B. As before, these trials were omitted when computing the column totals, thus each column total was based on only one trial. The only analyses that included trials where A and B were the same were the calculations of subject reliability (see page 96).

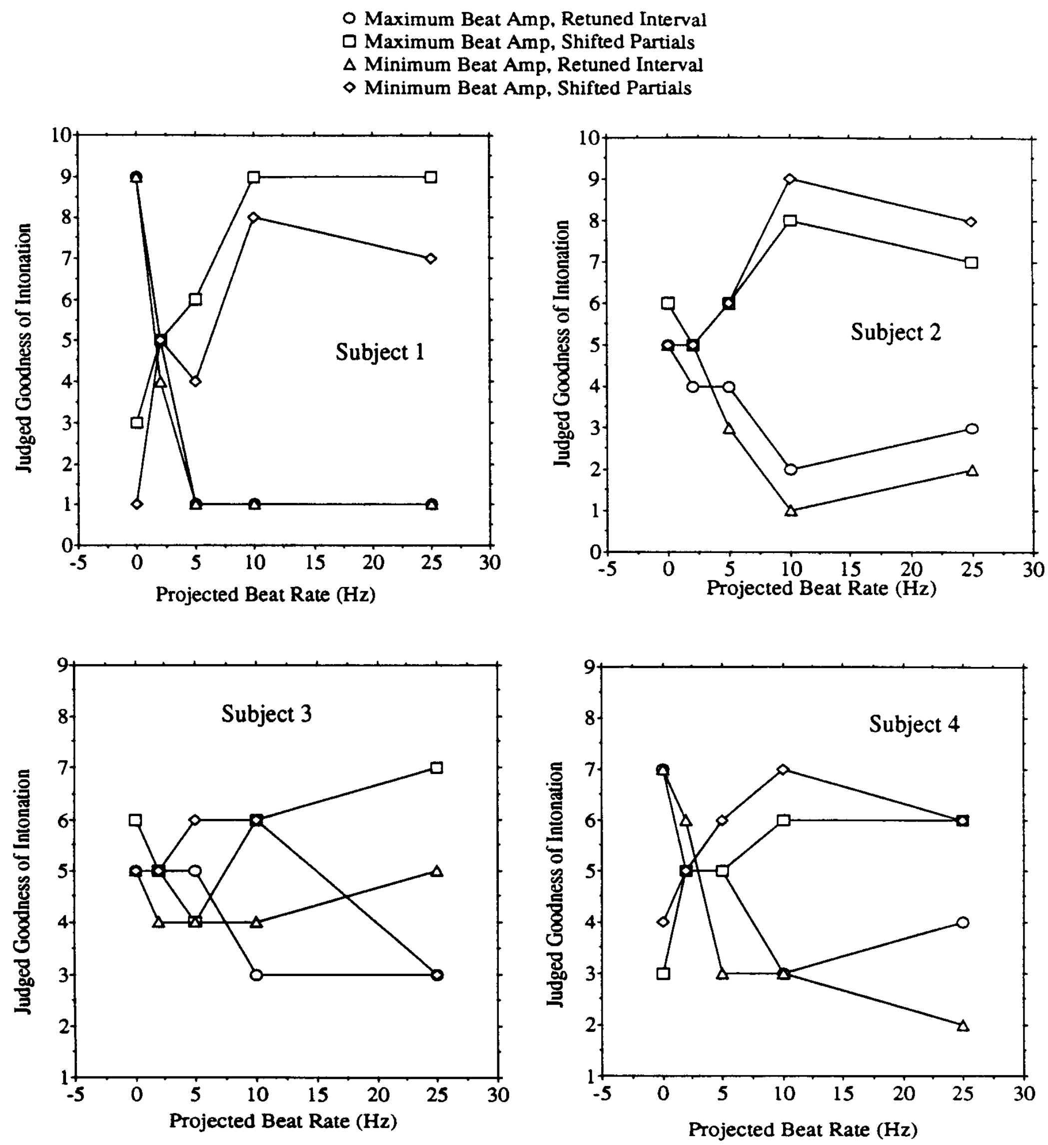

Fig. 12. Experiment One, results per subject. Trials in which "method of controlling beat rate" was the variable that changed between stimulus A and stimulus B. (Continued in Figure 13.)

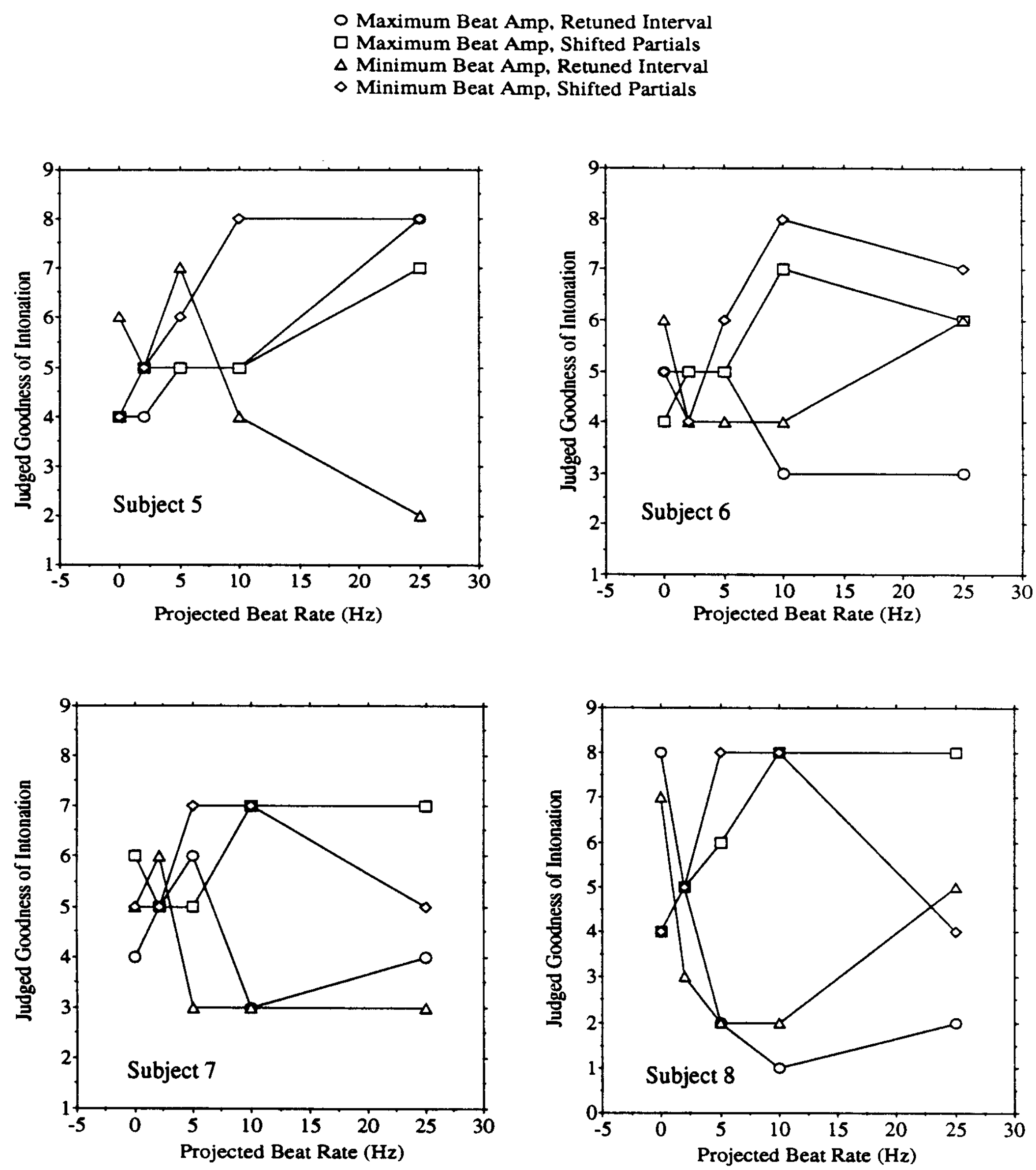

Fig. 13. Experiment One, results per subject (continued from Figure 12). Trials in which "method of controlling beat rate" was the variable that changed between stimulus A and stimulus B.

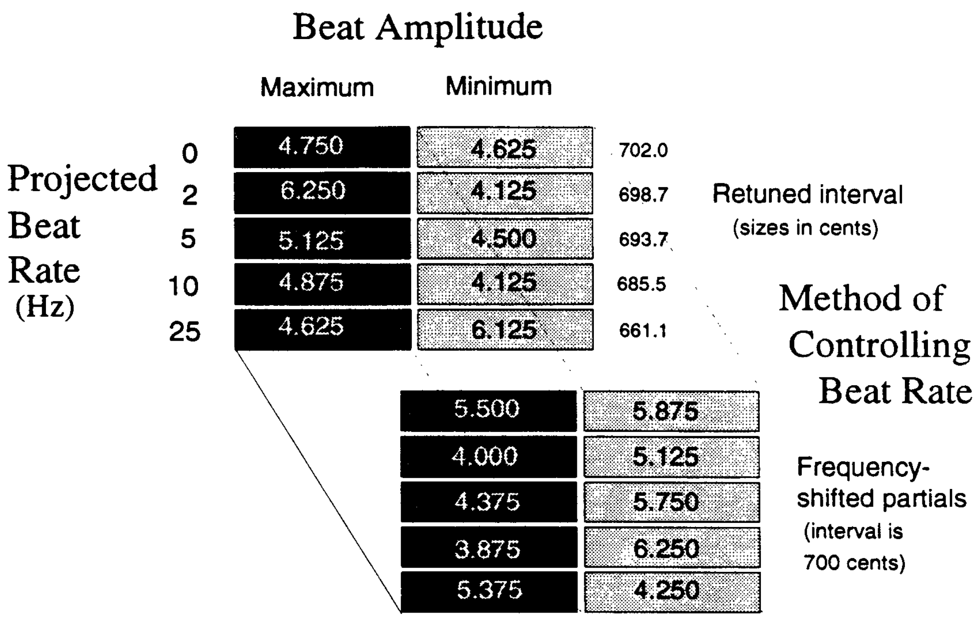

Fig. 14. Experiment One: Judgments of intonation of fifths, for trials in which beat amplitude was the variable that changed between stimulus A and stimulus B. 9.0 = B is much more in tune than A; 1.0 = A is much more in tune than B. The value of a given cell represents stimulus B; stimulus A uses the opposite method of controlling beat rate.

2.3.3 Trials with Changing Beat Amplitude

The third group of trials has beat amplitude as the variable that changed between stimulus A and stimulus B. (Note that these groups are separate only for purposes of analysis; in the experiment, trials from the three groups were intermingled.)

Graphs of mean responses

The three-dimensional diagram is given in Figure 14 (above) and the plot of mean responses versus projected beat rate in Figure 15. Note that all the values are surprisingly similar. Whereas in the previous two groups of trials, the variable that changed between

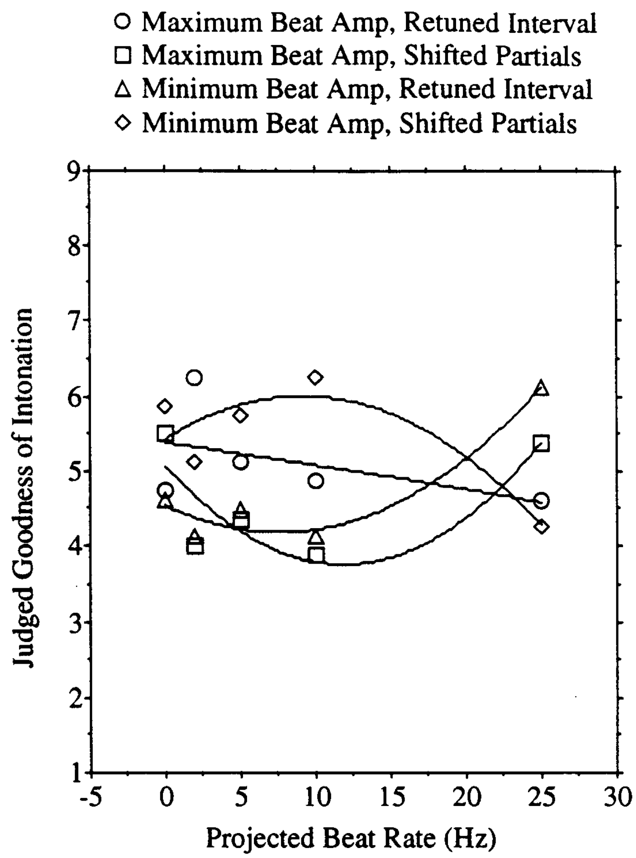

Fig. 15. Experiment One. Trials in which beat amplitude was the variable that changed between stimulus A and stimulus B.9.0 = B is much more in tune than A; 1.0 = A is much more in tune than B. The symbols in the legend represent stimulus B; stimulus A uses the opposite beat amplitude. (To tell which curve belongs with each symbol, see the points at 25 Hz. Although curves have been fitted to each set of points, the variability shown in Figure 16 makes it apparent that one cannot really know whether different functions are represented.)

stimulus A and stimulus B was significant, here it appears that no variable is very significant.

Figure 16 shows the results by stimulus type, with standard deviations over subjects. as before.

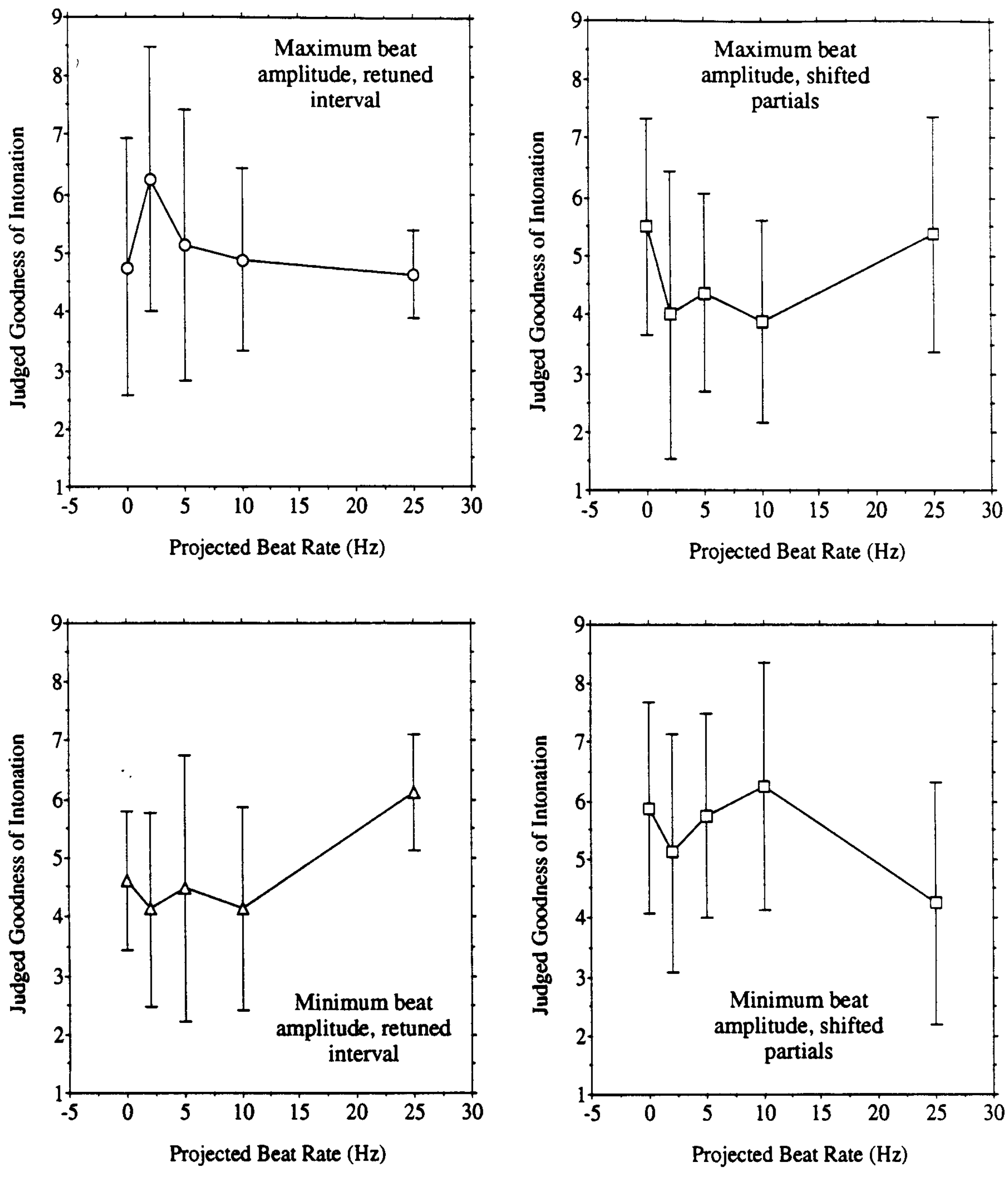

Fig. 16. Experiment One, trials in which beat amplitude was the changing variable. Each stimulus condition is plotted separately. Vertical error bars display the standard deviation over subjects.

Statistical analyses

Table 5 lists the results of the statistical analyses for the group of trials with beat amplitude as the changing variable.

Once again, the mean is very close to 5.0. As we would expect from having seen the three-dimensional diagram, the standard deviation is smaller than in the other two groups of trials. Both the correlation and the ANOVA show that no variable is significant. We shall consider the implications of the nonsignificance of beat amplitude under the "General Discussion" below.

Table 5. Analysis results, Experiment One. Trials in which beat amplitude differed between stimulus A and stimulus B.

Mean Response	4.975
Standard Deviation	.778
Correlation of Mean Responses to:	
Projected Beat Rate	.019
Beat Amplitude	.132
Method of Controlling Rate	.082
3-way ANOVA on Mean Responses:	
Projected Beat Rate	F=.07 (df 4,4), n.s.
Beat Amplitude	F=.14 (df 1,4), n.s.
Method of Controlling Rate	F=.05 (df 1,4), n.s.
Rate x Beat Amplitude	F=.15 (df 4,4), n.s.
Rate x Method	F=.34 (df 4,4), n.s.
Beat Amplitude x Method	F=1.32 (df 1,4), n.s.

Per-subject results

Figures 17 and 18 show the individual subjects' results for this group of trials. Recall from the discussion of the averaged results that there was little differentiation in judged intonation between stimuli with maximum beat amplitude and those with minimum beat amplitude. The per-subject graphs show a wider range of points than does the averaged data (compare Figure 15 on page 90), where the data all fell in the middle range between 3.5 and 6.5. The fact that the individual subjects' plots in Figures 17 and 18 scarcely resemble each other suggests that the averaged data in Figure 15 should not be given too much importance. It is difficult to discern any meaningful overall pattern by visual inspection of the per-subject graphs.

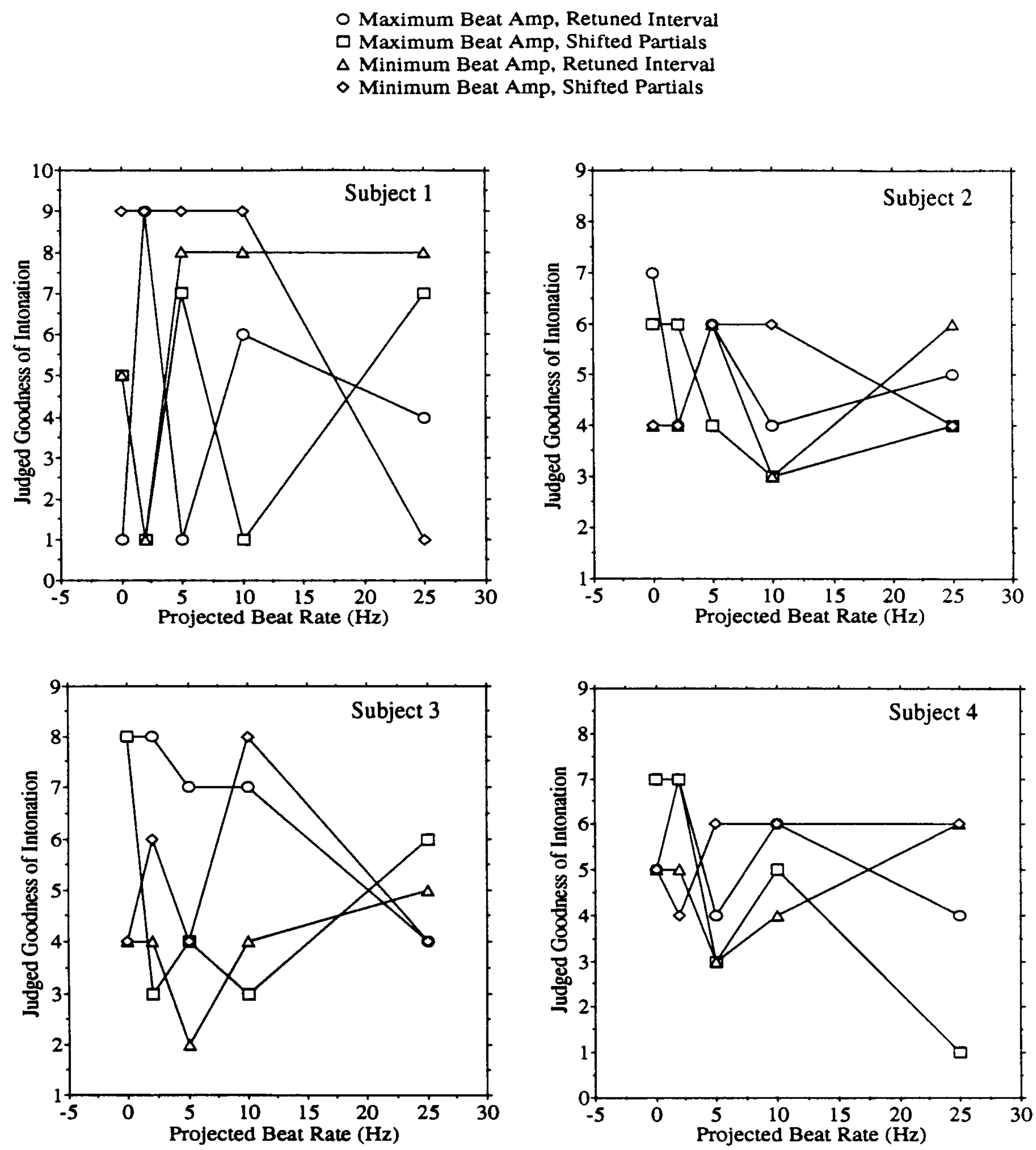

Fig. 17. Experiment One, results per subject. Trials in which beat amplitude was the variable that changed between stimulus A and stimulus B. (Continued in Figure 18.)

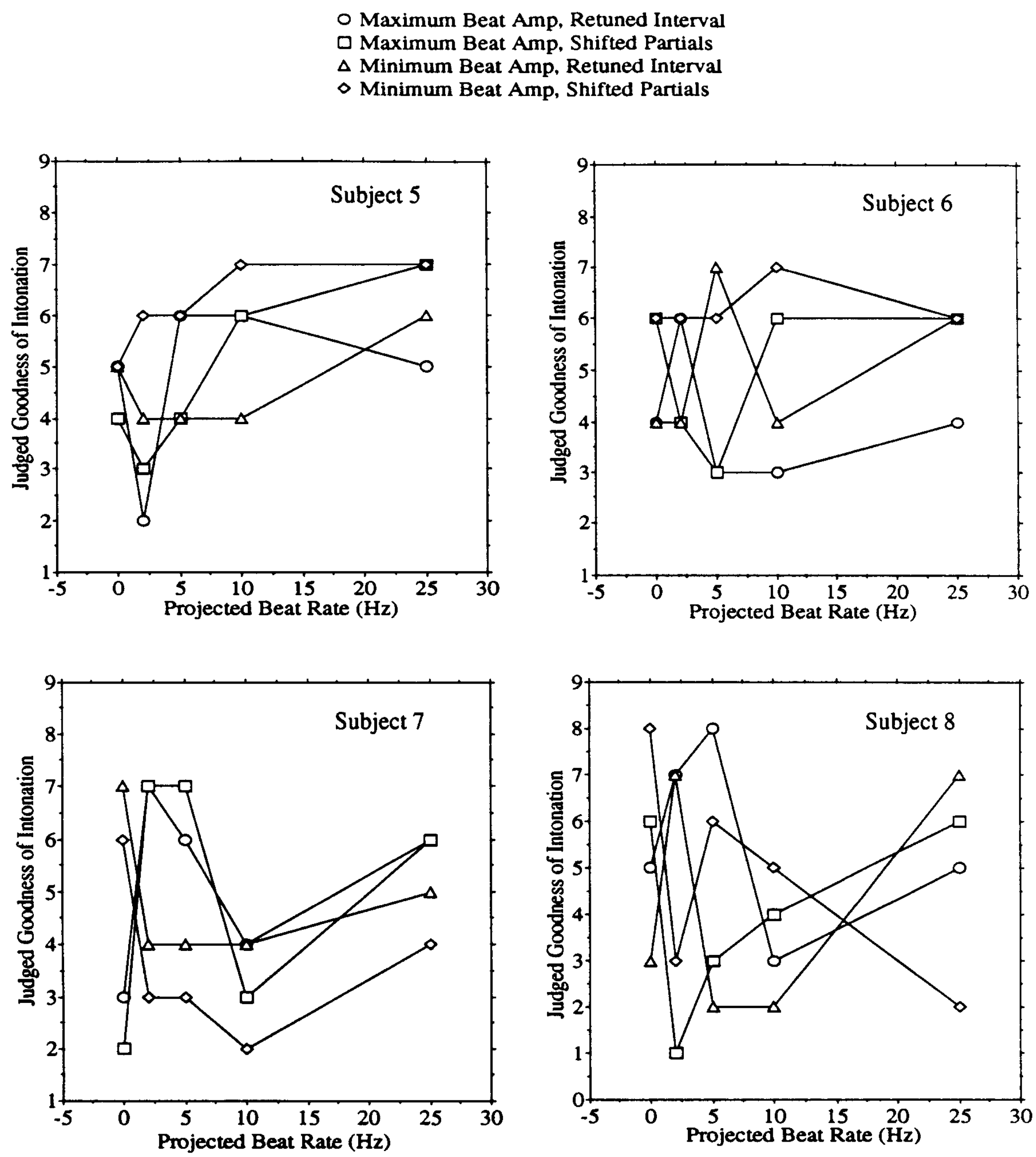

Fig. 18. Experiment One, results per subject (continued from Figure 17). Trials in which beat amplitude was the variable that changed between stimulus A and stimulus B.

2.3.4 Reliability of Responses

The reliability of the subjects' responses was tested by examining responses to pairs of trials in which the stimuli were the same, but not presented in the same order. For this purpose, it is not necessary to divide the trials into groups based on the variable that changed between stimulus A and stimulus B. In fact, this analysis also includes the trials—excluded from the paired-comparison matrices that generated the input to the previous analyses—where stimulus A and stimulus B are identical.

Order effects

As we saw above, the mean responses are very close to 5.0 for all three groups. This indicates that the order of presentation has no effect on the response, for if stimulus A tended to be heard as more in tune, the mean would be smaller than 5, and if B were heard as more in tune, it would be larger than 5.

Consistency of responses

Whether or not there is an order effect, one can test for subjects' consistency by finding the correlation of responses to the ABAB... presentation with those for BABA... (The forward and reverse forms of a trial were used to measure consistency, since exact repeats were unavailable in this experiment.) As shown in Table 6, the correlation over all the trials in the experiment is –0.729. One subject performed much more inconsistently than the others (1.95 standard deviations below the mean, at about the 2.5% confidence level). This subject was therefore omitted from all the analyses, since his results were not representative of the group of trained musicians under study.[9]

Table 7 shows correlations, per subject and per projected beat rate, between (1) responses to trials presented in the order "ABAB..." and (2) responses to the corresponding trials having the same stimuli but presented as "BABA...". Only trials in which the

Table 6. Experiment One. Correlation of responses to "ABAB..." presentation with responses to "BABA..." presentation.

Subject	Correlation (r)	z-score
1	−0.880	1.376
2	−0.865	1.313
3	−0.792	1.074
4	−0.774	1.029
5	−0.762	1.000
6	−0.700	0.867
7	−0.698	0.863
8	−0.687	0.842
9	−0.409	0.435
Mean	−0.729	0.978

Mean z-score: 0.978
Std. Dev. of z-scores: 0.279

#9's score is 1.95 standard deviations below the mean z-score.

9. z-score values for this and subsequent tables taken from Table D, p. 388 in McCall (1970), *Fundamental Statistics for Behavioral Sciences*, 4th ed. Increment for r in table is .005; intermediary values above computed by linear interpolation.

projected beat rate did not change between A and B were included. This comprises exactly half of the trials in the experiment.

We find that subjects are somewhat more consistent when the projected beat rate is higher. This makes sense, for at the higher beat rates, there is more likely a noticeable difference between the two beat amplitudes or the two methods of controlling beat rate. The grand mean correlation for these trials is –0.494; by comparison, the previously found correlation for all the trials in the experiment was –0.729. This means that the correlation for the trials in which projected beat rate does change is –0.965. Again, projected beat rate seems to be a much more important variable than beat amplitude or method of controlling beat rate, since subjects' responses are much more consistent across trials that are identical (except for order of presentation) when projected beat rate is the variable that changes between the two stimuli.

Table 7. Experiment One. Correlation of Responses to "ABAB..." Presentation with Responses to "BABA..." Presentation, per Projected Beat Rate.

Subject: \ Rate:	0	2	5	10	25
1	-0.533	-0.979	-0.036	-0.834	-0.866
2	-0.905	0.000	-0.561	-0.866	-0.829
3	-0.697	-0.788	-0.701	-0.980	-0.629
4	-0.696	-0.606	-0.269	-0.835	-0.680
5	-0.120	0.048	0.104	-0.324	0.000
6	0.709	-0.391	-0.927	-0.261	-0.087
7	-0.856	-0.660	-0.906	-0.127	-0.671
8	-0.398	0.875	-0.830	-0.668	-0.733
9	-0.577	-0.585	-0.454	-0.237	-0.857
MEAN	-0.453	-0.343	-0.509	-0.570	-0.595
GRAND MEAN: -0.494					

2.4 General Discussion of the Results of Experiment One

For purposes of comparison, we again present the results of the statistical analyses, combining the tables for each of the three groups into one table (Table 8). We can be more confident about the results in the first column than those in the second and third, because as was detailed above under "Consistency of responses," subjects were much more consistent in their responses to trials in which projected beat rate changed between stimulus A and stimulus B (column 1) than trials in which beat amplitude (column 2) or method of controlling beat rate (column 3) changed.

Table 8. Analysis results, Experiment One. Summary of Table 3, Table 4, and Table 5.

	Variable that changed between stimulus A and stimulus B:		
	Rate	Beat Amp	Method
Mean Response	4.883	4.975	4.938
Standard Deviation	1.855	.778	1.463
Correlation of Mean Responses to:			
Projected Beat Rate	−.950	.019	.001
Beat Amplitude	.003	.132	−.026
Method of Controlling Rate	.018	.082	.579
3-way ANOVA on Mean Responses:			
Projected Beat Rate	p<.01	n.s.	n.s.
Beat Amplitude	n.s.	n.s.	n.s.
Method of Controlling Rate	n.s.	n.s.	p<.01
Rate x Beat Amplitude	n.s.	n.s.	n.s.
Rate x Method	n.s.	n.s.	p<.01
Beat Amplitude x Method	n.s.	n.s.	n.s.

The correlations and ANOVAs show that projected beat rate is the primary variable influencing judgments of intonation. The greatest variability in responses, as shown by the standard deviations, occurs for trials in which projected beat rate changes. This is not unexpected; we might predict the most important variable to cause the greatest contrasts in the perceived intonation.

The most striking feature of the correlations and ANOVAs is the discrepancy between columns—the results differ depending on which variable changes between stimulus A and stimulus B. Subjects appear to respond mainly on the basis of the changing variable, which makes sense, since their task is to perform a relative judgment (comparing two stimuli) rather than an absolute judgment on a single stimulus.

The most surprising result, given the previous literature on this topic, is that beat amplitude appears to be nonsignificant in all three groups of trials, even the group where beat amplitude was the changing variable. One would expect a spectrum with all 16 partials, some of which beat, to sound less in tune than the corresponding spectrum with the beating partials deleted. One possible hypothesis for why deletion of the beating partials didn't improve the perceived intonation is that the deletion had a negative side effect. For example, the deletion creates gaps in the spectrum of one of the tones, which conceivably could make its timbre sound less pleasing. However, if this were the case, one would expect the zero-amplitude, zero-Hz stimulus to sound less in tune than the full-amplitude, zero-Hz stimulus, which was not borne out by the responses.

Another explanation is that other mechanisms (such as combination tones) maintain the beating even when the partials are deleted. To check this, an informal study was run in which three subjects listened carefully to the stimuli with deleted partials and attempted to tap out or sing the rhythm of any periodic phenomenon they heard.[10] Only one reported hearing any temporal variation at all, and he described it as being much more subtle than the beating in the stimuli with full-amplitude beats. The rhythms he produced in response to these minimum-amplitude stimuli did not reliably match their "projected beat rates." By contrast, all three subjects easily identified the beat rates in stimuli with full-amplitude beats. This result conforms with the literature, which reports such secondary forms of beating as being much weaker than the beats of nearly coinciding partials. It is therefore unlikely that alternate sources of beating, such as combination tones, can explain why subjects in Experiment One did not rate the maximally beating stimuli as more out of tune than the minimally beating ones.

It is also possible that beat amplitude would have been significant had there been more values of projected beat rate in the vicinity of 10 to 25 Hz. Some of the plots gave the impression that, in this region, deleting the beating partials tended to improve the perceived intonation.

Although it may seem paradoxical that projected beat rate is significant but beat amplitude is not, this seeming contradiction can be explained by the fact that the variable "projected beat rate" doesn't necessarily refer to real beating. Rather, it refers to some physical change that was made in order to potentially affect the anticipated perceived beat rate by a certain amount. This physical change is either a mistuning of the interval or a frequency-shifting of certain partials. In the case where beating partials are deleted, we expect "projected beat rate" to describe only the amount of mistuning or frequency-shifting, not an actual beat rate. This is why I refer to the variable as "projected beat rate" rather than

10. All three subjects had a great deal of musical training and also participated in at least one of the three main experiments.

"beat rate," to emphasize the fact that it describes potential beating, not necessarily actual beating. Since projected beat rate has about as strong an effect when the beat amplitude is zero as when it is maximum, one can hypothesize that the importance of projected beat rate comes not from beating but from the other factors that covary with projected beat rate—interval mistuning on the one hand and inharmonicity on the other.[11]

The high significance of "method of controlling beat rate," in the trials where method was the variable that changed within a trial, means that the two techniques for manipulating beat rate are not equivalent. As could be seen from the regression plot for these trials, controlling the beat rate by mistuning the interval tends to make it sound more out of tune than does shifting the beating partials to inharmonic positions.

Summary: Conclusions from Experiment One

Increasing the projected beat rate tends to make a stimulus more out of tune, as expected. Increasing the beat rate by shifting the beating partials of one note to inharmonic positions appears to introduce less "out-of-tuneness" than does moving all partials equally (i.e., retuning the interval). Projected beat rate is a very strong cue for intonation, since subjects are very consistent when projected beat rate changes between A and B, but less so when another variable changes. Although projected beat rate is highly significant, changing the beat amplitude did not have a significant effect in this experiment. This suggests that the effect of projected beat rate derives largely from the covarying interval mistuning or inharmonicity, rather than from beating partials. Thus, in terms of the factors which we originally set out to investigate—interval tuning versus "real" (rather than "projected") beat rate—we can tentatively conclude that interval tuning is a much more important cue for

11. Another factor might be a shift of periodicity pitch in the "shifted partials" stimuli, but this phenomenon would be less important than the perceived inharmonicity, as discussed on page 113.

intonation than is beat rate. However, let us first examine the results of the other two experiments.

Chapter

3 *Experiment Two: Single Fifths*

In Experiment One, subjects had judged the relative intonation of a pair of stimuli. Each stimulus in the pair was a perfect fifth. The two stimuli were identical except for one of three variables (projected beat rate, beat amplitude, or method of controlling beat rate) whose value might be different between stimulus A and stimulus B.

The results of Experiment One indicated that projected beat rate was highly significant for intonation, but that beat amplitude had no effect. This seeming paradox could be explained by the interval mistuning and inharmonicity that covaried with projected beat rate. Even so, the nonsignificance of beating contradicts the findings of some previous researchers, such as Vos (1986). In order to verify that these results were not some sort of artifact of the experimental design—for example, a result of the use of paired comparisons rather than single stimuli—a second experiment was run in which each trial consisted of a single stimulus.

3.1 Method

3.1.1 Stimuli and Apparatus

The stimuli and apparatus were identical to those of Experiment One, but the stimuli were presented one per trial, rather than in A/B pairs. See the description of Experiment One for details (pages 60 and 62). The user interface to the computer program presenting the stimuli was accordingly modified (see "Procedure" below.)

3.1.2 Trials

There were 20 unique stimuli and 160 trials. Each stimulus therefore occurred eight times in the experiment. The order of the stimuli was randomized within each of the eight groups of 20 stimuli. Before doing the experiment itself, each subject did a "trial run" consisting of 40 trials, with each stimulus occurring twice.

3.1.3 Subjects

There were 10 subjects, who had an average of 19.5 years of musical experience, and all of whom were students in a graduate course in computer music. None of the subjects had been a subject in Experiment One. Details about the subjects, who are ordered by the consistency of their responses, are given in Table 9 on pages 106 and 107.[1] An eleventh subject also did the experiment, but his results were omitted from the analysis because he was significantly more inconsistent in his responses than the others (see Table 11 on page 116). The subjects were volunteers.

1. Most of the subjects were male, except where noted in the table. The subjects' sex is reported in light of O'Keeffe's (1975) finding of sex differences in intonation judgments. (See page 51 of this dissertation.)

Table 9. Subjects in Experiment Two.

Subject	Consistency rating (0 to 1)	Age	Years of musical experience	Musical activity: Instruments studied for more than 3 years (including voice); Composition	Years of musical education			Highest music degree	Experience with non-standard tunings?	Years of study in psychoacoustics	Ever before been a subject in a psychoacoustic experiment?	Comments
					Lessons	Courses	Ensemble					
1	.892	26	20	Organ, piano; composition	15	4	15	Diploma	Yes	2	Yes	Experience with piano tuning
2	.873	22	10	Piano; composition	10	8	8	B.M., B.A.	No	1/4	No	Nearly absolute pitch; female
3	.872	35	25	Trumpets	16	8	16	D.Mus.	No	1/2	No	
4	.872	26	20	Piano; composition	10	5	2	B.M., B.M.	Yes	1	No	Poss. irregularity with hearing in left ear; composing piece in nonstandard tuning
5	.856	27	15	Saxophone; composition	4	5	3	B.M.	No	1	No	
6	.842	20	10	Voice	7	1	0	None	No	0	No	
7	.840	24	8	Piano; composition	8	0	0	B.A.	Yes	0	No	Microtonal composer; experience with non-Western tunings
8	.839	40	37	Piano; composition	n/a	n/a	n/a	Ph.D.	No	0	No	Absolute pitch; female

Table 9, continued. Subjects in Experiment Two.

Subject	Consistency rating (0 to 1)	Age	Years of musical experience	Musical activity: Instruments studied for more than 3 years (including voice); Composition	Years of musical education			Highest music degree	Experience with non-standard tunings?	Years of study in psychoacoustics	Ever before been a subject in a psychoacoustic experiment?	Comments
					Lessons	Courses	Ensemble					
9	.833	35	25	Violin	n/a	5	n/a	M.A.	No	0	No	Experience with Chinese music
10	.806	36	25	Piano	15	15	2	M.A.	No	2	Yes	

3.1.4 Procedure

The task was to judge how in tune the single stimulus was. As in Experiment One, the possible responses ranged from 1 to 9. Subjects were asked to judge the intonation with respect to an ideal perfect fifth, so that no stimulus would be judged as a somewhat out-of-tune tritone, for example.The trials were presented by a computer program with a user interface very similar to the one illustrated for Experiment One (Figure 3 on page 65). The only functional differences were: (1) the display of the current stimulus label (A or B) was now omitted, since each trial consisted of only one stimulus; (2) the "Stop" button was omitted, since the stimulus was only played once (although the subject could repeat the playback by clicking the "Play again" button); and (3) the legend below the response buttons now read "Very out of tune" (under button 1), "Somewhat out of tune" (under button 5), and "Exactly in tune" (under button 9).

3.2 Results

3.2.1 Graphs of Mean Responses

The raw responses were averaged over subjects and over the eight repeats of each stimulus. These mean responses are displayed in the three-dimensional diagram of Figure 19. We can see that the pattern is very similar to that of the group of trials in Experiment One where the changing variable was projected beat rate (Figure 4 on page 68). The perceived intonation worsens as projected beat rate increases, and there appears to be little effect of the other two variables (beat amplitude and method of controlling beat rate). The

Beat Amplitude

Method of Controlling Beat Rate: Retuned interval (sizes in cents)

Projected Beat Rate (Hz)	Maximum	Minimum	
0	8.325	7.637	702.0
2	6.850	7.350	698.7
5	5.250	5.750	693.7
10	3.737	3.375	685.5
25	1.538	1.362	661.1

Method of Controlling Beat Rate: Frequency-shifted partials (interval is 700 cents)

Projected Beat Rate (Hz)	Maximum	Minimum
0	8.100	7.637
2	7.062	7.575
5	6.125	6.825
10	4.988	5.875
25	2.575	2.912

Fig. 19. Experiment Two: Judgments of intonation of fifths. The response in each cell is the average over 10 subjects and 8 trials. 9.0 = exactly in tune, 1.0 = very out of tune.

graph of mean response versus projected beat rate (Figure 20), is also very similar to the analogous graph in Experiment One (Figure 5 on page 69).

Figure 21 shows the same data as Figure 20, but with separate plots for each of the four stimulus conditions. The vertical bars display the standard deviation over subjects for each stimulus.

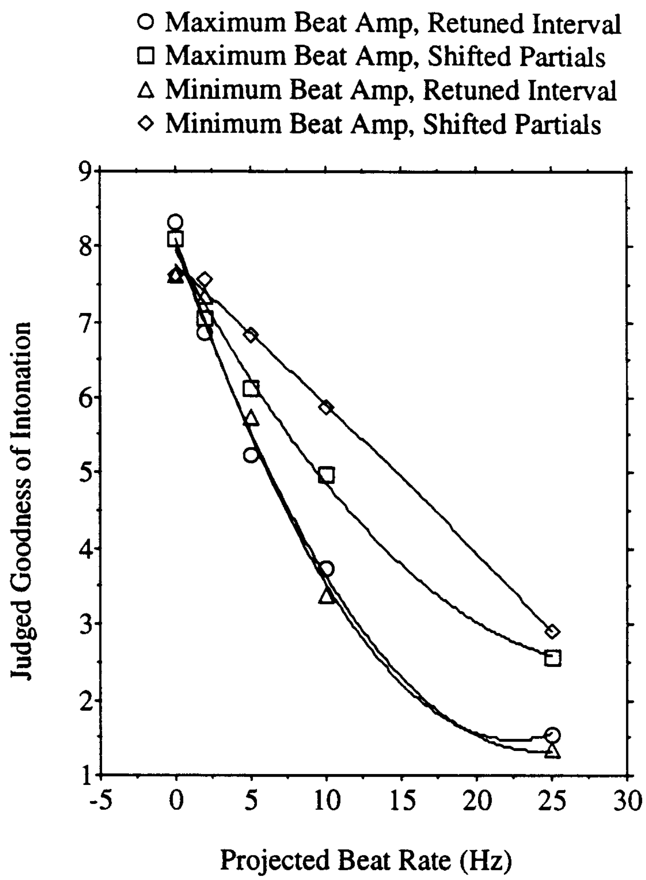

Fig. 20. Experiment Two: Judgments of intonation of fifths. Each of the four combinations of "beat amplitude" and "method of controlling beat rate" is fitted with a second-order polynomial regression curve. Each point is the average of 80 raw data points (10 subjects and 8 trials).

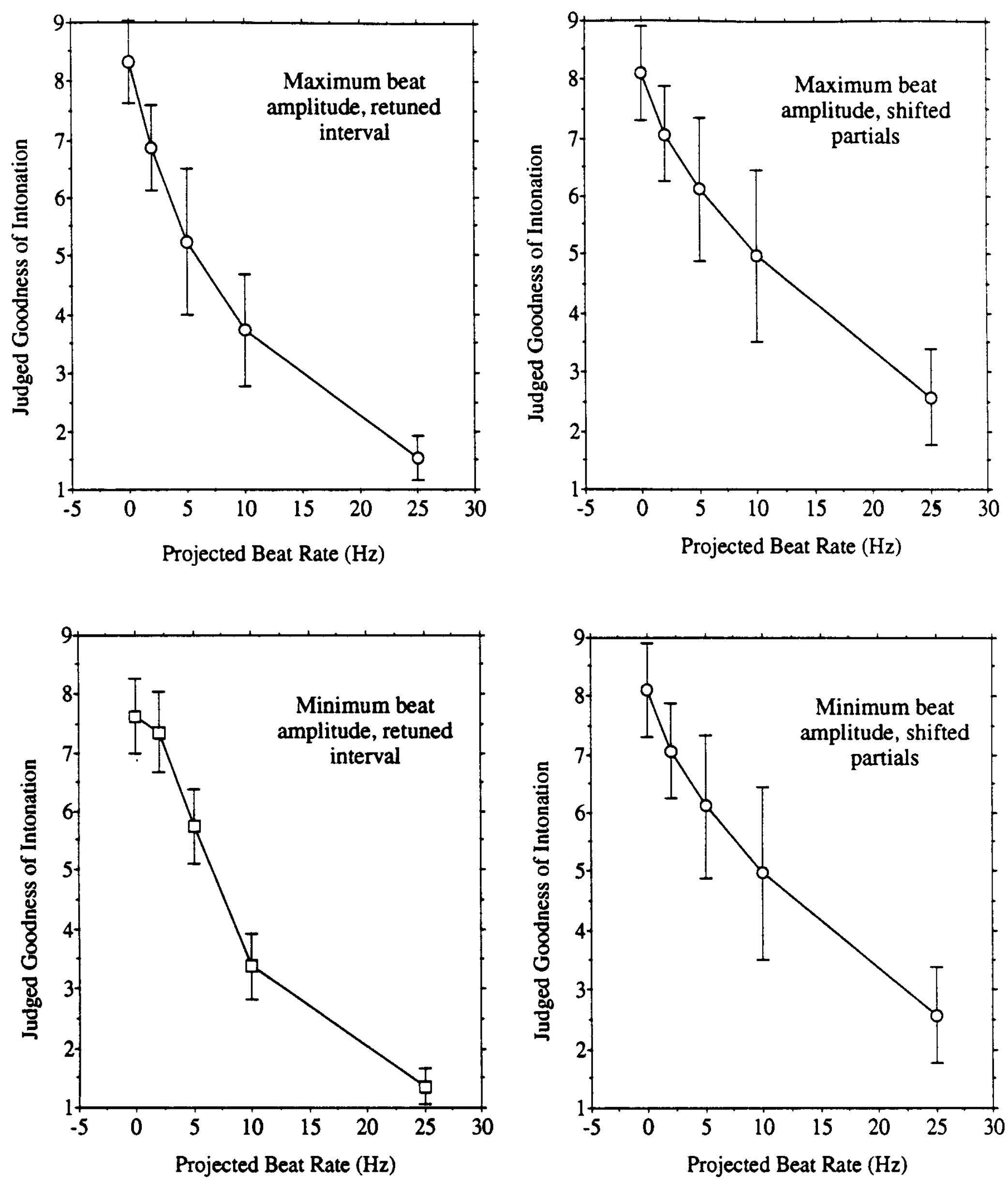

Fig. 21. Experiment Two. Same data as Figure 20, but each stimulus condition is plotted separately. Vertical error bars display the standard deviation over subjects for each stimulus.

3.2.2 Statistical Analyses

The mean responses were run through the same descriptive and comparative statistical routines as in Experiment One. The main results are given Table 10.

The most interesting result is that, as in Experiment One, the deletion of beating partials didn't tend to make the stimuli sound any more in tune. The implication is that beating, *per se*, is unimportant for the perceived intonation of these stimuli. Neither of two alternate explanations—the possibility of a negatively perceived change in timbre due to the deletion of partials, or a possible persistence of other forms of beating such as beating combination tones—seems adequate to explain this result.

Table 10. Analysis results, Experiment Two.

Mean Response	5.542
Standard Deviation	2.227
Correlation of Mean Responses to:	
Projected Beat Rate	–.937
Beat Amplitude	.04
Method of Controlling Rate	.196
3-way ANOVA on Mean Responses:	
Projected Beat Rate	F=21.22 (df 4,1400), p<.01
Beat Amplitude	F=.15 (df 1,1400), n.s.
Method of Controlling Rate	F=3.55 (df 1,1400), n.s.
Rate x Beat Amplitude	F=.21 (df 4,1400), n.s.
Rate x Method	F=.64 (df 4,1400), n.s.
Beat Amplitude x Method	F=.24 (df 1,1400), n.s.
Rate x Beat Amplitude x Method	F=.06 (df 4,1400), n.s.

The conclusion, then, is that beating is unimportant for perceived intonation, at least for these stimuli. Clearly, "projected beat rate" is important, which means simply that the interval sounds more out of tune when either the interval is mistuned or inharmonicity is increased.

Neither is there much effect of the method of controlling projected beat rate. Recall that the method is either (a) mistuning the entire interval or (b) frequency-shifting only certain partials. The positive correlation (.196) shown in Table 10 seems to indicate that mistuning the interval tends to make it sound slightly more out of tune than does introducing the inharmonicity needed to achieve the same projected beat rate.[2] If this were true, it would mean that for a given projected beat rate, shifting only a few partials introduces less "out-of-tuneness," in spite of the fact that it also introduces inharmonicity. However, "method of controlling beat rate" doesn't show up in the ANOVA averaged over subjects as being significant at the .05 confidence level, so the correlation of .196 (which is a relatively low correlation) can probably just be attributed to chance.[3]

Shifted periodicity pitch in inharmonic stimuli

It is possible that some of the effect I have been attributing to inharmonicity is instead due to a change in the periodicity pitch caused by the shifted partials. However, this potential contributor seems insufficient to explain the magnitude of the effect. To achieve beat rates of 0, 2, 5, 10, and 25 Hz, the third partial of the lower note and its multiples (6, 9, 12, 15) were equally shifted up by –2.0, 1.3, 6.3, 14.5, and 38.9 cents, respectively. Recall from the discussion on page 46 that Moore, Glasberg, and Peters (1985) found that only the

2. See the footnote on page 84 regarding the meaning of the direction of correlation.
3. It does appear, on the other hand, that some of the individual subjects judged the two methods of controlling beat rate differently, as will be discussed below.

first six harmonics affect the pitch, and a single partial shifts the pitch by about one sixth of the partial's frequency shift. If we assume an additive relation, our stimuli would have a pitch shift of about one third of each partial's frequency shift, since there are two effective partials (the third and the sixth). This means that in the worst case (the 25 Hz projected beat rate) the perceived size of the fifth might change by about 13 cents—which is somewhat less than the JND for frequency ratio reported in the literature (discussed on page 39). The average shift would be about 4 cents. Note that the shift in cents of the shifted partials is the same as the shift of the entire tone in the "retuned interval" stimuli with the corresponding projected beat rates (see page 61). Thus if periodicity pitch shift were the major factor in the intonation of the "shifted partials" stimuli, we might expect them to sound only about one third as out of tune as the "retuned interval" stimuli at the corresponding projected beat rates. But since the ANOVA did not find "method of controlling beat rate" to be significant, it seems that the inharmonic stimuli are heard as being approximately as out of tune as the "retuned interval" stimuli. This discrepancy indicates that some factor other than pitch shift is dominant, pointing to the inharmonicity itself. Some of the subjects in fact reported hearing inharmonicity in the stimuli, but the inharmonicity could also have affected the judgments of any subjects who may not have recognized it as such.

The potential shift of periodicity pitch could have been reduced with an alternate design that permitted different partials to be shifted in opposite directions, in an attempt to cancel each other out and yield a net periodicity pitch equal to the fundamental. This approach was not taken for two reasons. First, it would result in a set of simultaneous beat rates that would not necessarily be integrally related. Not only would these differ from the "harmonic series" of beat rates in the "retuned interval" stimuli, but the overall perceived beat rate might be ambiguous, making it difficult to classify such stimuli according to projected beat rate. Secondly, Moore, Glasberg, and Peters (1985) found that the exact contribution of each partial differed widely across individuals, so there is no guarantee that periodicity pitch shifts could be eliminated with such a strategy.

It should be noted that even if shifted periodicity pitch accounted for some of the effect that I have attributed to perceptual inharmonicity, this would not take away from the finding that beat amplitude was nonsignificant, a finding which applied to strictly harmonic stimuli as well.

3.2.3 Reliability of Responses

The reliability of the subjects' responses was tested by examining responses to trials containing the same stimulus. This was quantified by arranging the data in 8 columns of 20 rows, where each row corresponded to one stimulus and each column to a different group of trials. (Thus, the first column contained responses to the first 20 trials, the second column contained responses to trials 21 - 40, etc.) A correlation matrix was computed based on this data, and the mean correlation computed for each subject.[4]

Overall, the subjects were very consistent. (See Table 11.) However, subject 11, who had the least musical training, was significantly more inconsistent, and therefore his results were excluded from the analysis (as had been done for Subject 9 in Experiment One). The mean correlation over subjects was 0.843 (or 0.853, excluding #11's data). By comparison, the mean correlation in Experiment One was –0.729.[5] In Experiment One, the correlations were computed between only two trials per stimulus pair, and the two trials had stimulus A

4. The diagonals of the correlation matrices (containing correlations of a column to itself, hence scores of 1.0) were omitted when computing the mean correlation.

Subject	Correlation (r)	z-score
1	.892	1.432
2	.873	1.346
3	.872	1.341
4	.872	1.341
5	.856	1.278
6	.842	1.228
7	.840	1.221
8	.839	1.218
9	.833	1.198
10	.806	1.116
11	.749	.971
Mean	0.843	1.245

Mean z-score: 1.245
Std. Dev. of z-scores: 0.127
#11's score is 2.158 standard deviations below the mean z-score.

Table 11. Experiment Two. Subject consistency: correlation between repeated trials.

and B presented in reverse order. Experiment Two, with its 8 trials per stimulus, provides a more stable check on the subjects' reliability.

5. All that need concern us here is the absolute value of the correlations. The correlation value in Experiment One is negative simply because stimulus A and stimulus B were switched in the two trials considered to be "repeats." The response scale compared the two stimuli and thus was inverted depending on which of the two stimuli was presented first. In Experiment Two the trials contained only one stimulus, and so the response scale had the identical meaning in the repeated trials, which in this case really were exact repeats.

3.2.4 Per-Subject Results

Figures 22 - 24 display the per-subject results for Experiment 2. Each data point is the average of eight trials for that stimulus. As before, the figures are presented in order of decreasing subject consistency: Subject 1 is the one who gave the most consistent responses from trial to trial for a given stimulus, Subject 10 the least consistent.

The general pattern of results is similar to Experiment One: intonation is judged to be poorer as the projected beat rate increases, with a fairly consistent ordering by stimulus type at 10 and 25 Hz. Most subjects agree with the ordering of the stimuli seen in the average over subjects (Figure 20 on page 110). That is, for the points at 10 and 25 Hz, and to a lesser extent for the point at 5 Hz, most subjects' curves appear for the most part in the same order from top to bottom as the curve for the data averaged over subjects—namely, the top curve (best intonation) is "minimum beat amplitude, shifted partials," followed by "maximum beat amp, shifted partials," then "minimum beat amplitude, retuned interval," and finally "maximum beat amplitude, retuned interval." In other words, subjects clearly preferred the inharmonicity engendered by the "shifted partials" method of controlling beat rate to the pitch mistuning that accompanies the "retuned interval" method. Subjects 1, 2, and 6, however, rated "maximum beat amplitude, shifted partials" as sounding more in tune than "minimum beat amplitude, shifted partials." A few of the subjects rate the "maximum beat amplitude, retuned interval" stimuli as worse than "minimum beat amplitude, retuned interval." This would be the expected pattern if beating were detrimental to intonation, but the averaged data shows the reverse order (in contrast with the average from Experiment 1). However, the differences here are not large.

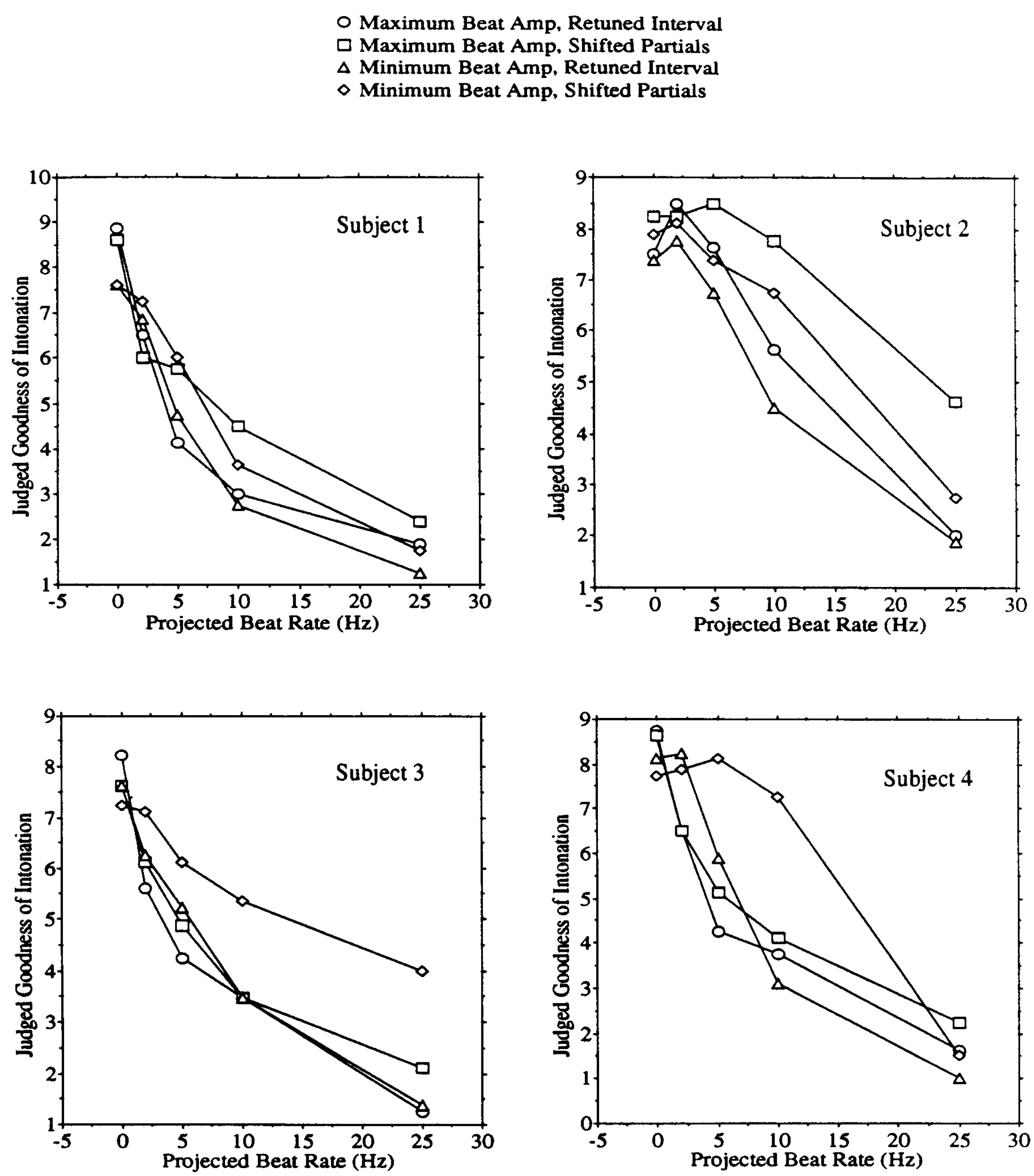

Fig. 22. Experiment Two, results per subject. (Continued in Figure 23.)

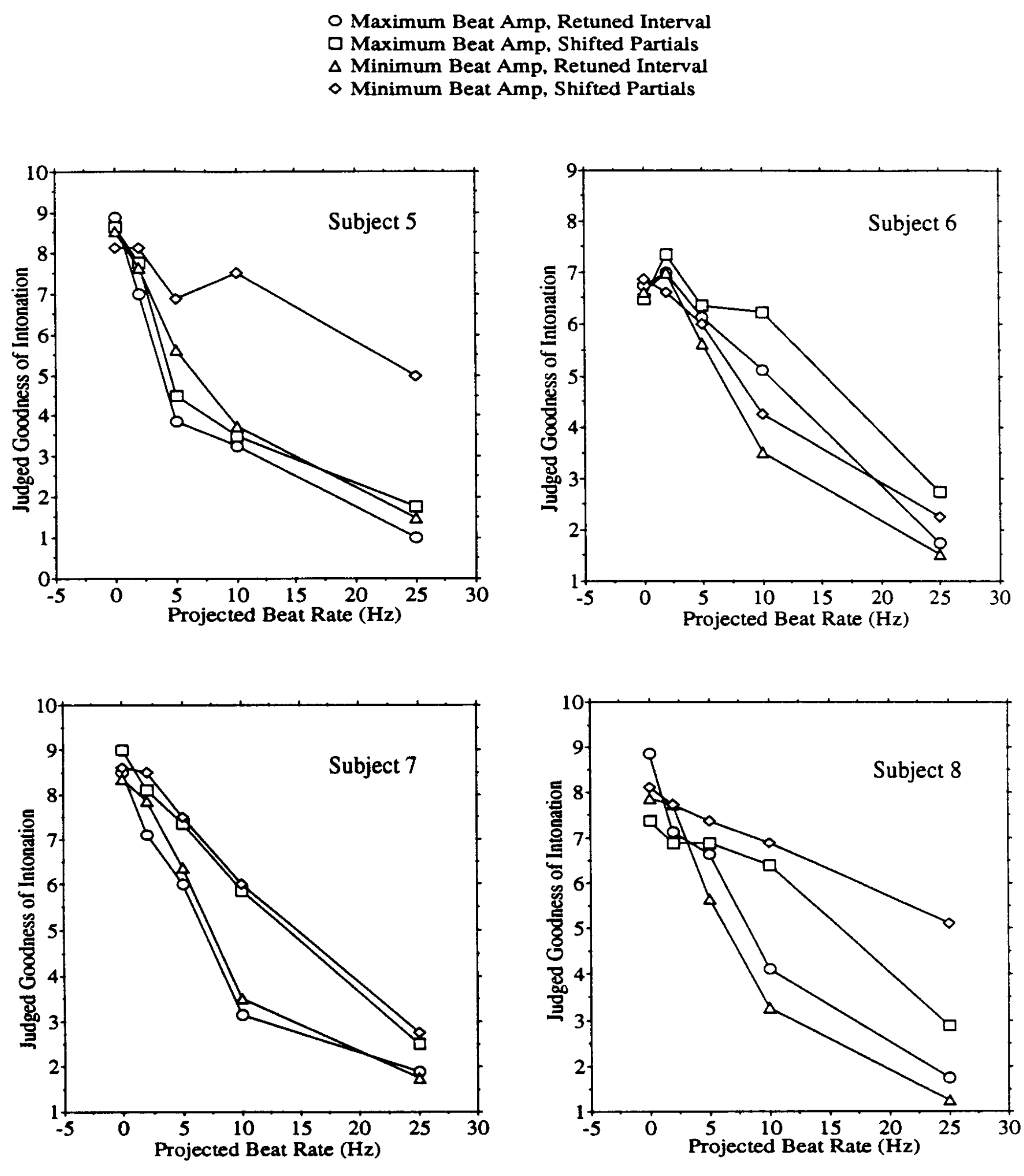

Fig. 23. Experiment Two, results per subject. (Continued from Figure 22 and continued in Figure 24.)

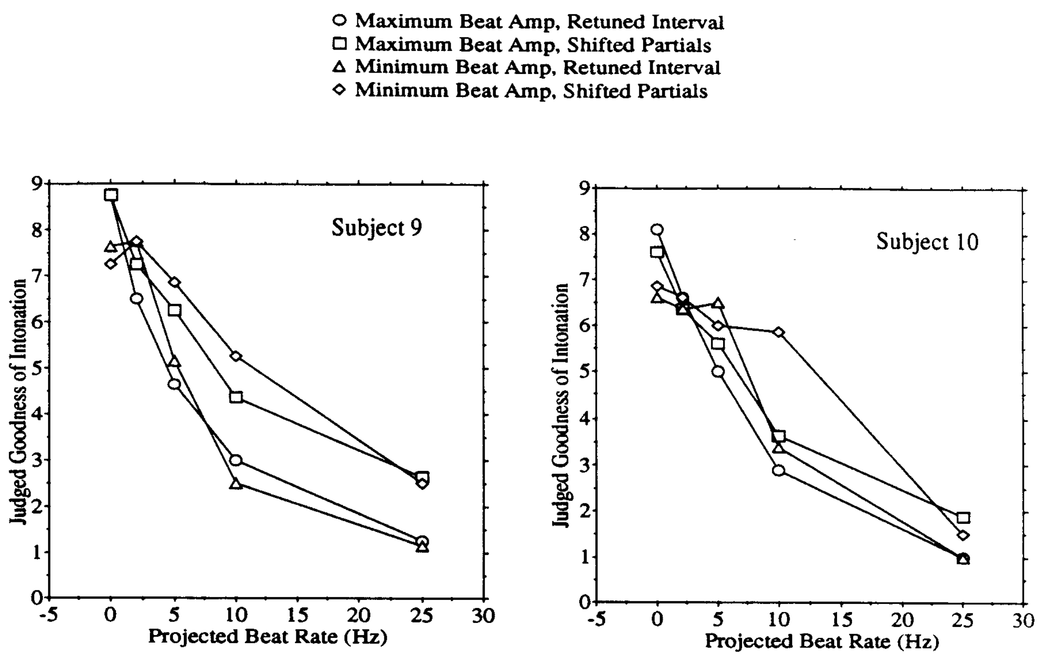

Fig. 24. Experiment Two, results per subject. (Continued from Figure 23.)

At all rates, the pattern of Subject 9's data is close to that of the group average. A few of the subjects judged the stimuli at zero Hz to be somewhat more out of tune than the corresponding stimuli at 2 Hz, again bringing to mind the "rich listener" interpretation of Roberts and Mathews (1984); however, none exhibits the extreme behavior in this regard of Subject 7 in Experiment 1.

Subject 8 has absolute pitch. She seems to perceive the "shifted partials" stimuli as somewhat more in tune than the other subjects do, suggesting she pays more attention to pitch in judging intonation (since the inharmonic stimuli all have the standard equal-tempered tuning). Subject 2, who has "close to perfect pitch," also shows this tendency. However, both subjects rate the 25-Hz "shifted partials" stimulus with maximum beat amplitude as quite out of tune, similarly to the other subjects, suggesting that the fast beating of this stimulus, perhaps added to its inharmonicity, overrides its fundamental frequency ratio as a determinant of perceived intonation.

3.3 Comparison with Results of Experiment One

The results of Experiment One were somewhat unclear, since the data was divided into three groups of trials, depending on which of the three main variables changed between the two stimuli in each trial. In Experiment One, projected beat rate was significant ($p < .01$) for trials in which the projected beat rate changed between stimulus A and stimulus B, but not significant in trials where another variable changed. Similarly, the method of controlling rate was significant ($p < .01$) for trials in which the method changed between the two stimuli, but not in other trials.

In Experiment Two, the ambiguity caused by the pairing of stimuli is gone. We see that projected beat rate is again the strongest factor, while "method of controlling beat rate" is much less important, and beat amplitude is completely nonsignificant. In other words, these results indicate that increasing either the mistuning or the inharmonicity of a perfect fifth tends to make it sound increasingly out of tune. While it appears that mistuning the interval might have a slightly more detrimental effect on the perceived goodness of intonation than does increasing the inharmonicity (when both are measured in terms of their effect on projected beat rate), the differences could just be due to chance. The differences in judgments occasioned by changing from minimum to maximum beat amplitude are no greater than the inherent variability of the judgments. Thus it would seem that the effect of "projected beat rate" cannot be attributed to beating itself, but to the changes in interval tuning or inharmonicity that accompany any changes in "projected beat rate."

The results of this experiment clearly reinforce those of Experiment One, validating the use of the paired-comparison technique in the earlier experiment's design, and supporting the conclusions we made on page 102.

Chapter

4 *Experiment Three: Major Thirds*

Experiment Two confirmed the results of Experiment One. Both experiments showed that beating *per se* was not important for the perception of the intonation of the stimuli. However, both experiments used the same interval: the perfect fifth. To help generalize these results, it was desirable to study another musical interval for comparison. Since in tuning theory perfect fifths and major thirds are often considered the most important building blocks of musical scales,[1] and since these are the two intervals that Vos (1986) used, an additional experiment was run using major thirds.

1. Of course, octaves are also extremely important, but most tuning theory has considered the 2:1 octave a virtually inviolable given. There are interesting exceptions: for example, Kolinski (1959) proposed an equal-tempered tuning with the octave slightly stretched in order to make the perfect fifth just, Wyshnegradsky (1972) described "non-octavian" microtonal scales, and John Pierce has explored a scale based on the idea that the 3:1 ratio can replace the octave as the unit of transposition under which pitch class is preserved (Mathews and Pierce [1989]). The practice of piano tuning also makes frequent use of slightly stretched scales, constructed empirically rather than theoretically. Several psychological studies show evidence that the subjective melodic octave is slightly larger than a 2:1 ratio (Burns and Ward [1982]), in contradiction to the postulates of most tuning theory, for which harmonic relations are central.

Besides historical precedent, another reason I did not study the octave is the difficulty of creating stimuli analogous to those of the other experiments, since in an octave all the harmonics of the upper tone beat with harmonics of the lower.

4.1 Method

4.1.1 Stimuli

The stimuli were similar to those of the previous experiments, except that the intervals were major thirds instead of perfect fifths. As before, there were two simultaneous notes, each consisting of 16 equal-amplitude partials, with certain partials deleted in some of the conditions. The temporal characteristics (amplitude envelope and duration) were identical to the previous experiments. The upper note was now A4 (440 Hz) and the lower note, which could be retuned, was again approximately F4. A complete specification of the stimuli is given in the Appendix.

Constants

Harmonic musical interval (two simultaneous notes)
Pitch of upper note: A4 (440 Hz)
Pitch of lower note: approximately F4 (could be retuned; see variables below)
Time-invariant frequency spectrum
Flat frequency spectrum (0 dB/octave rolloff)
16 partials per note (1st 16 of harmonic series, though some may be deleted or made inharmonic in some stimuli)
Duration: 1.5 seconds
Trapezoidal amplitude envelope
Attack portion of amplitude envelope: .05 seconds
Decay portion of amplitude envelope: .5 seconds
Beginning phase of all partials is 0.0 (not randomized phase), sine phase

Variables

(1) Beat amplitude (2 levels):

(a) maximum (beating partials have equal amplitude)[2]

---

2. In certain cases, a high partial of the upper tone was deleted, in order to remove a beat rate that wasn't a multiple of the projected beat rate. See the Appendix for details.

(b) minimum (partial of upper note deleted, for each beating pair. See the stimulus tables in the Appendix for details.)

(2) Projected beat rate (5 levels): 0, 2, 5, 10, 25 Hz

Note that this variable is present even when beat amplitude is zero. This is because we want to test other effects of the methods used to control beat rate.

Also, a sixth level of –13.9 Hz is added for Method of Controlling Beat Rate (a), in order to obtain an equal-tempered third for comparison with the equal-tempered third used in Method of Controlling Beat Rate (b). Its projected beat rate is given a minus sign to indicate that its direction of mistuning from the just third is opposite to the direction used for the stimuli with rates of 2 Hz and larger.

(3) Method of controlling beat rate (2 levels):

(a) Retuned interval

The entire lower tone (F4) is transposed (i.e., all its partials are shifted an equal amount in log frequency). This method preserves harmonicity but changes the interval tuning. The projected beat rates of –13.9, 0, 2, 5, 10, and 25 Hz yield intervals of 400, 386.3, 384.3, 381.4, 376.5, and 361.9 cents, respectively.

(b) Frequency-shifted partials

The partials in the lower tone that beat with partials of the upper tone (or that would beat if the partials of upper tone were not deleted) are shifted in frequency. This method introduces inharmonicity but preserves the fundamental frequency ratio.

(4) Tuning (2 levels, for Method of Controlling Beat Rate (b) only):

(1) Just intonation = 386.3 cents. F4 = 352.000 Hz.
(2) Equal temperament = 400 cents. F4 = 349.228 Hz.

For Method of Controlling Beat Rate (a), tuning is not an independent variable.

Note that we have introduced a new variable, tuning, that applies only to the stimuli where the method of controlling beat rate was "shifted partials." This is because the just and equal-tempered thirds are noticeably different in size (14 cents' difference), making it difficult to determine which of these two tunings should be used for the stimuli in which tuning does not change with beat rate (namely, the "shifted partials" stimuli). By including both tunings, we can avoid making any assumptions about which tuning would be considered the standard. This also enables us to reach direct conclusions about subjects' preference for just intonation versus equal temperament, independent of beating and inharmonicity.

In Experiments One and Two, we had no "tuning" variable for the "shifted partials" stimuli; only the equal-tempered interval was used. That was because the difference in size between the equal-tempered and just perfect fifths is very small—only two cents, which is about an order of magnitude smaller than the JND (just noticeable difference) for frequency ratio.[3] When musicians can discriminate between just and equal-tempered fifths, they no doubt use other cues such as beating, rather than the actual size of the interval. (If such a statement seems contradictory in a study whose primarily result downplays the importance of beating, consider that we addressed only intonation judgments, not *discrimination* between beating and non-beating stimuli nor *identification* of just intonation versus equal temperament. But it seems likely that with stimuli such as ours, in which beat rate is made independent of tuning, subjects would find it more difficult to identify whether a stimulus was just or equal-tempered. This prediction is supported by Vos's (1982) finding that discrimination thresholds for tuning are determined primarily by beat rate.)

3. See the discussion of ratio JND's on page 39.

4.1.2 Experimental Design

There were 30 unique stimuli and 120 trials. Each stimulus therefore occurred four times in the course of the experiment. The order of the stimuli was randomized within each of the four groups of 30 stimuli. Before doing the experiment itself, each subject did a "trial run" consisting of 60 trials, with each stimulus occurring twice.

Note that although there are six levels of projected beat rate for the condition where the method of controlling beat rate was "retuned interval," there are still just 5 projected beat rates x 2 beat amplitudes x 3 "methods" = 30 unique stimuli. This is because the stimulus with a zero-Hz projected beat rate and the method "retuned interval" is actually the same as the stimulus with a zero-Hz projected beat rate and the method "frequency-shifted partials with just intonation tuning." Since these conditions use the same stimulus, the additional four trials were omitted, and the same judgments were used for both conditions in the analyses. This explains why the results at zero Hz are identical for the methods "retuned interval" and "frequency-shifted partials with just intonation tuning" (see Figure 25 on page 131). Similarly, there are cells in the figure for –13.9 Hz and the method "frequency-shifted partials with equal-tempered tuning," but these are just the same trials as the "retuned interval" method at –13.9Hz. In both cases, the numbers are shown in parentheses in Figure 25, to indicate the redundancy.

4.1.3 Apparatus

The apparatus was identical to Experiment Two's, including the computer program's user interface. (See page 62 and page 105 for the apparatus of Experiment One and Two, respectively.)

Table 12. Subjects in Experiment Three.

Subject	Consistency rating (0 to 1)	Age	Years of musical experience	Musical activity: Instruments studied for more than 3 years (including voice); Composition	Years of musical education			Highest music degree	Experience with non-standard tunings?	Years of study in psychoacoustics	Ever before been a subject in a psychoacoustic experiment?	Comments
					Lessons	Courses	Ensemble					
1	.790	31	26	Piano, voice; composition	30	12	8	M.F.A.	Yes	1/2	Yes	Has composed some microtonal music
2	.747	35	26	Trombone, voice, piano	10	17	26	B.A.	Yes	1	Yes	Some experience with gamelan & 1/4-tones
3	.676	36	20	Piano, re-hu; composition	8	8	4	M.A.	No	0	No	Re-hu is a Chinese stringed instrument
4	.619	18	11	Violin, voice	11	8	8	None	No	0	Yes	Absolute pitch; female
5	.557	35	10	Piano; composition	10	10	0	M.M.	No	0	Yes	Some listening to 1/4-tone music
6	.466	41	15	Piano; composition	12	7	3	M.A.	Yes	0	Yes	Gamelan; composing w. alternate tunings; female
7	.458	29	7	Recorder, brass insts.	2	1	13	None	Yes	0	No	Pythagorean tuning in early music; female
8	.439	38	26	Guitar, piano	8	8	15	M.A.	No	1	Yes	Harpsichord tuning
9	.396	20	3	[Guitar]	1	1/2	0	None	No	1/2	Yes	
10	.381	29	9	Clarinet	4	0	0	None	No	2	Yes	Possible tinnitis

continued below

Table 12 (continued). Subjects in Experiment Three.

Subject	Consistency rating (0 to 1)	Age	Years of musical experience	Musical activity: Instruments studied for more than 3 years (including voice); Composition	Years of musical education			Highest music degree	Experience with non-standard tunings?	Years of study in psychoacoustics	Ever before been a subject in a psychoacoustic experiment?	Comments
					Lessons	Courses	Ensemble					
11	.346	26	20	Piano; composition	10	5	2	B.M., B.M.	Yes	1	No	Poss. irregularity with hearing in left ear; composing piece in nonstandard tuning
12	.341	36	25	Piano	15	15	2	M.A.	No	2	Yes	

4.1.4 Subjects

There were twelve subjects, who had an average of 16.5 years of musical experience. Table 12 (pages 128 and 129) gives details on the subjects, who are ordered by the consistency of their responses.[4] All the subjects were volunteers.

Subject 1 had been a subject in Experiment One. (He is called Subject 6 in the chapter describing that experiment.) Subjects 11 and 12 had been subjects in Experiment Two (in the description of which they are Subjects 4 and 10, respectively).

4.2 Results

4.2.1 Graphs of Mean Responses

Figure 25 shows the responses to the stimuli, averaged over subjects and over repeated trials.

Immediately evident from this diagram is the homogeneity of responses along the different dimensions. Almost all the stimuli have mean responses in the neighborhood of 5 to 7. It is only the stimuli with the "retuned interval" method of controlling beat rate that show a strong pattern of decreasing "in-tune-ness" with increasing projected beat rate. Since there appears to be little effect of projected beat rate for the "shifted partials" method of controlling rate, inharmonicity may not be a strong factor for the perceived intonation of the major third stimuli. And as in the other experiments, beat amplitude—the horizontal

4. The subjects were male, except where noted in the table. (See page 105, footnote 1.)

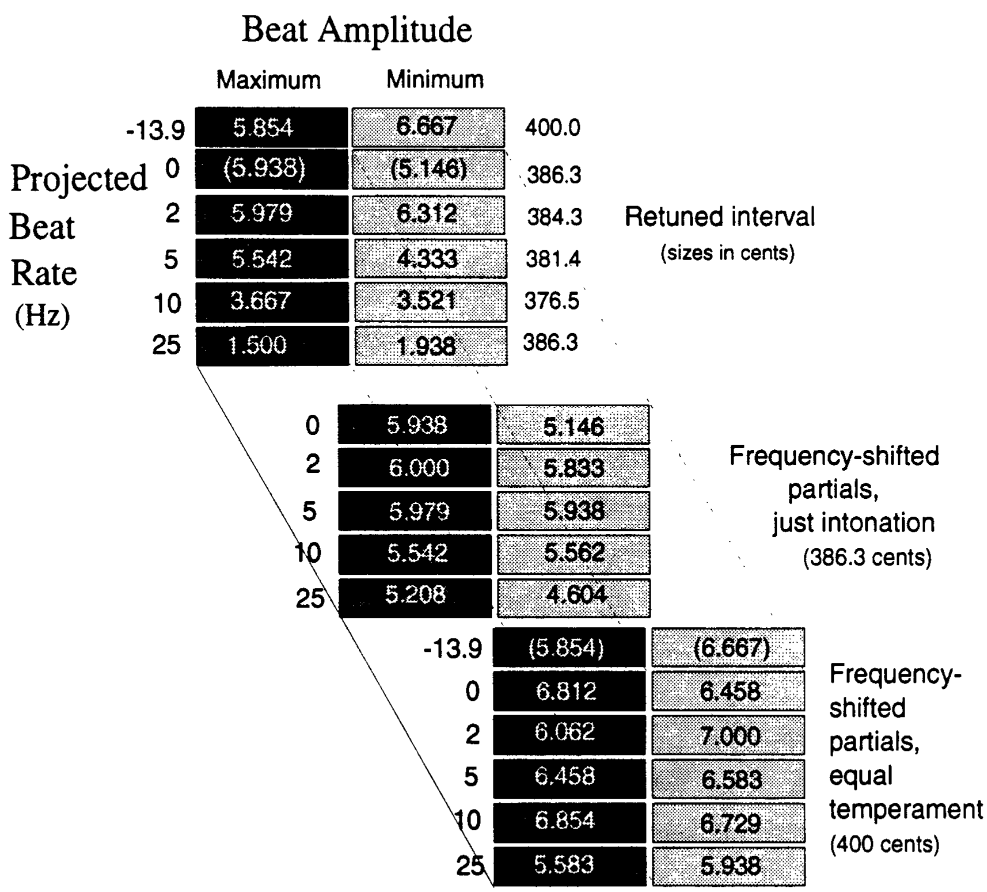

Fig. 25. Experiment Three: Judgments of intonation of major thirds. The response in each cell is the average over 12 subjects and 4 trials. 9.0 = exactly in tune, 1.0 = very out of tune. The cells at 0 Hz with the "retuned interval" method refer to the same trials as those at 0 Hz with the method "frequency-shifted partials, just intonation," since these stimuli can be interpreted either way. (No shifting is necessary to achieve a zero-Hz beat rate for a just interval.) Similarly, the cells at -13.9 Hz with the method "frequency-shifted partials, equal temperament" are the same as those at -13.9 Hz with the method "retuned interval."

dimension in the figure—appears to have very little effect. Each column on the left is similar to its neighbor on the right.

It can be seen in Figure 25 that it makes sense to use a minus sign for the projected beat rate of –13.9 Hz. First, this preserves the correct order of the resultant interval sizes in cents (shown on the right-hand side of the columns labeled "retuned interval"). If we had instead used +13.9 and kept the projected beat rates in order, the 400-cent interval would come between 376.5 and 361.9. Second, and more important, the subjects' responses clearly show that the perceptual ordering is the one with –13.9. It makes little sense to imagine any perceptual difference between a positive and negative beat rate, however. If beat rate were the dominant criterion for intonation, we would expect the responses for the –13.9 Hz interval to be in the neighborhood of the 10-Hz one. That they are instead in the neighborhood of the zero-Hz tone is further evidence that interval tuning is the dominant factor for the perception of these stimuli, since the pattern of judgments fits the ordering according to interval tuning, and not according to the absolute value of the beat rate.

The plot of mean response versus projected beat rate (Figure 26 on page 133) is also interesting. Here it is immediately evident that the stimuli with the "retuned interval" method of controlling beat rate are judged differently from those with the "shifted partials" method. The former follow the diagonal pattern that was seen in the graphs of the perfect fifth experiments, whereas the latter are essentially horizontal, indicating no effect of projected beat rate. As described under "Experimental Design" on page 127, the two stimuli at –13.9 Hz are the equal-tempered intervals with harmonic partials, which can be interpreted either as "retuned intervals" or as "shifted partials, equal temperament" (where the partials are shifted zero cents to achieve the –13.9 Hz projected beat rate). Note that these two values fall on the diagonal connecting the stimuli with the "retuned interval" method, but they also can be grouped horizontally with the stimuli having the "shifted partials" method. Since they have about the same judged goodness of intonation as the other

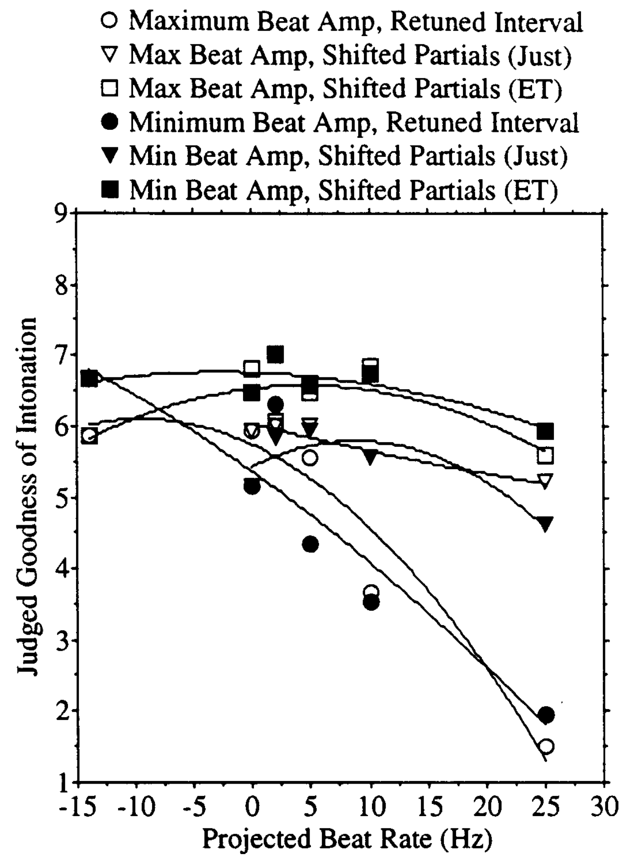

Fig. 26. Experiment Three: Judgments of intonation of major thirds. The various combinations of beat amplitude, method of controlling beat rate, and tuning are fitted with second-order polynomial regression curves. Each point is the average of 48 raw data points (12 subjects and 4 trials). To tell which curve belongs with each symbol, see the points at 25 Hz.There are four symbols at −13.9 Hz: a white circle superimposed upon a white square, and a black circle superimposed upon a black square (the black circle is indistinguishable). The circles and squares coincide here because there were really only two stimuli at −13.9 Hz, but they could each be interpreted two ways. (See the text.)

equal-tempered stimuli, this is further evidence that interval tuning is the dominant factor in intonation for major thirds, just as it was found to be for fifths.

Figures 27 and 28 on pages 134 and 135 show the results separated by stimulus condition, with vertical error bars depicting the standard deviation over subjects.

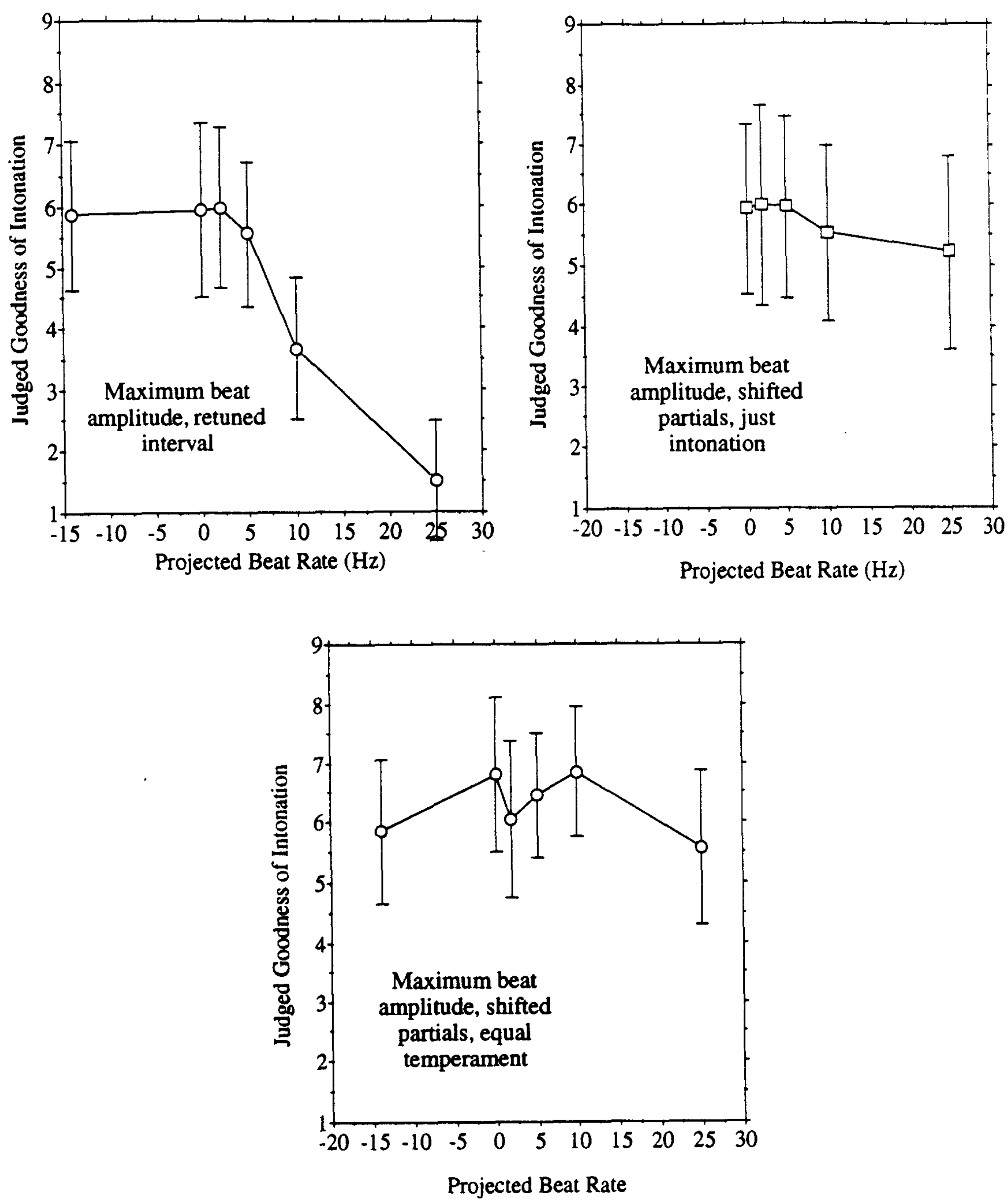

Fig. 27. Experiment Three. Same data as Figure 26, but each stimulus condition is plotted separately. Vertical error bars display the standard deviation over subjects for each stimulus. Stimuli with maximum beat amplitude. (Continued in Figure 28).

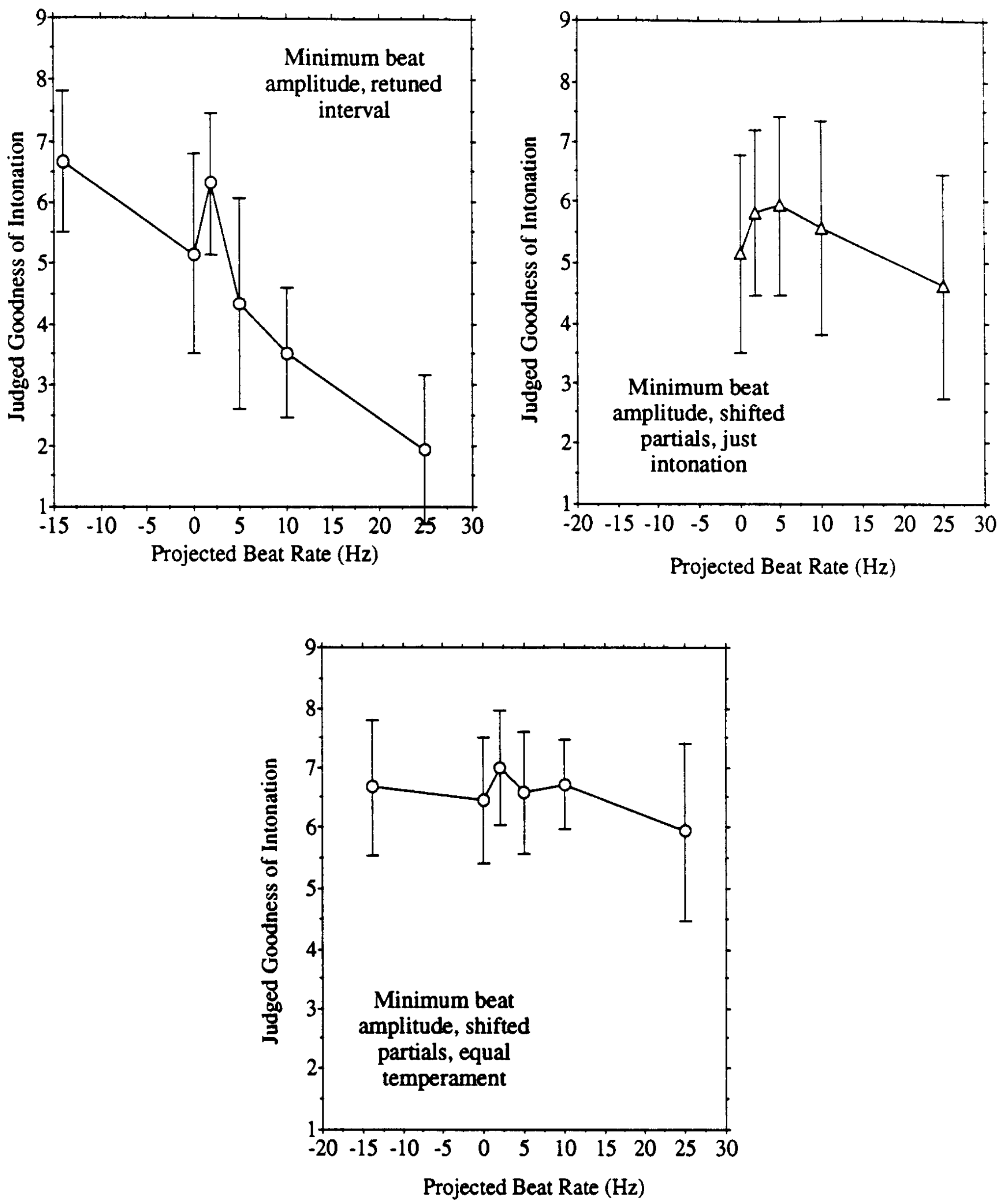

Fig. 28. Experiment Three. Continuation from Figure 27 of graphs with error bars showing standard deviation over subjects. Stimuli with minimum beat amplitude.

Mean Response	5.518
Standard Deviation	1.346
Correlation of Mean Responses to:	
Projected Beat Rate	-.545
Beat Amplitude	-.01
Method of Controlling Rate	.591

Table 13. Analysis results, Experiment Three. Descriptive statistics. "Method of Controlling Rate" here is considered to have 3 values: "retuned interval," "shifted partials, just intonation tuning," and "shifted partials, equal temperament tuning."

4.2.2 Statistical Analyses

Table 13 shows some of the analytical results. The average response is slightly higher than it was for perfect fifths, suggesting that subjects are less particular about the intonation of major thirds. One might advance the alternate explanation that, for a given beat rate, the major thirds were mistuned less—or their partials frequency-shifted less—in terms of cents than were the fifths, since the beating partials are higher in frequency for the major third and thus a smaller mistuning (or frequency shift) is required to yield a given beat rate. However, the overall range of tunings was about the same, since the thirds included the extra equal-tempered stimuli, with a beat rate of –13.9 Hz. The thirds thus had a range of 38.1 cents in comparison to the fifths' 40.9 cents.[5]

5. Note that the range in terms of beat rate was also the same—if one considers *perceptually meaningful* beat rate, since –13.9 Hz would be heard as 13.9, making the range of the thirds and the fifths both 0 - 25 Hz. We have more to say about the amount of frequency shift relative to fifths on page 149.

The mean responses were about equally correlated to "projected beat rate" and "method of controlling rate," and completely uncorrelated to beat amplitude. Notice that the correlation of responses to projected beat rate is rather lower than it was for perfect fifths, and the correlation of responses to method of controlling beat rate is higher. (The values for fifths in Experiment Two were –.937 and .196, respectively). This reflects the horizontal nature of the regression curves for the inharmonic stimuli on the one hand, and the greater separation between the curves for the inharmonic and the harmonic stimuli on the other.[6]

The analysis of variance routine was run on three separate groups of trials, because of the asymmetry introduced by the two different values of interval tuning (just intonation and equal temperament) for the stimuli with the "shifted partials" method of controlling beat rate. Also, the stimuli with the –13.9-Hz projected beat rate were omitted from the ANOVAs. Table 14 (page 138) displays the results.

Surprisingly, there is only one significant variable: method of controlling beat rate, for the analysis of the "retuned interval" method versus the "shifted partials" method with equal-tempered tuning. These correspond to the highest and lowest pairs of curves on the regression plots. Given the plots, it is somewhat surprising that, according to the ANOVA results, there is no significant interaction between rate and method—i.e., the pattern of changing judgments of intonation with changing projected beat rate is not reliably different for the different methods of controlling beat rate. We might also have expected the inharmonic stimuli tuned in just intonation to be significantly different from the harmonic stimuli (i.e., those with the "retuned interval" method of controlling beat rate). The lack of significance in these cases may have to do with the overlap of the curves at the lower

6. See the footnote on page 84 regarding the method of computing correlation.

Table 14. Analysis results, Experiment Three. Analyses of variance (ANOVAs).

Retuned Interval vs. Shifted Partials, Just Intonation:	
Projected Beat Rate	F=2.127 (df 4,720), n.s.
Beat Amplitude	F=.203 (df 1,720), n.s.
Method of Controlling Rate	F=3.270 (df 1,720), n.s.
Rate x Beat Amplitude	F=.069 (df 4,720), n.s.
Rate x Method	F=.932 (df 4,720), n.s.
Beat Amplitude x Method	F=.001 (df 1,720), n.s.
Rate x Beat Amplitude x Method	F=.079 (df 4,720), n.s.
Retuned Interval vs. Shifted Partials, Equal Temperament:	
Projected Beat Rate	F=1.814 (df 4,720), n.s.
Beat Amplitude	F=.004 (df 1,720), n.s.
Method of Controlling Rate	F=9.037 (df 1,720), p<.01
Rate x Beat Amplitude	F=.127 (df 4,720), n.s.
Rate x Method	F=.978 (df 4,720), n.s.
Beat Amplitude x Method	F=.114 (df 1,720), n.s.
Rate x Beat Amplitude x Method	F=.034 (df 4,720), n.s.
Shifted Partials; Just Intonation vs. Equal Temperament:	
Projected Beat Rate	F=.244 (df 4,720), n.s.
Beat Amplitude	F=.009 (df 1,720), n.s.
Method of Controlling Rate	F=1.575 (df 1,720), n.s.
Rate x Beat Amplitude	F=.049 (df 4,720), n.s.
Rate x Method	F=.036 (df 4,720), n.s.
Beat Amplitude x Method	F=.132 (df 1,720), n.s.
Rate x Beat Amplitude x Method	F=.029 (df 4,720), n.s.

projected beat rates, and also with the greater error term, i.e., less consistency, for major thirds (as discussed below).

4.2.3 Reliability of Responses

Table 15 (on page 140) shows the correlations between repeated trials of the same stimulus, averaged across stimuli. The average correlation across all stimuli and all subjects was 0.518. This is quite a bit lower than with perfect fifths; the correlations in Experiment One and Two were .729 and .843 respectively.[7] In other words, subjects judge the intonation of major thirds less reliably than that of perfect fifths. This may be related to the fact that equal-tempered thirds differ significantly from just thirds, making it harder to imagine an unambiguous reference point, whereas the just and equal-tempered fifths are very close. This result is not particularly surprising, since not only much of the tuning theory literature, but also previous psychoacoustic research, has indicated that listeners are more sensitive to mistuning of the fifth than of the third. The range of the correlations is also much larger than for fifths; some subjects judged the thirds more consistently than others did.

Each correlation score is the mean of that subject's correlation matrix, where the columns of the data input to the correlation routine were that subject's responses to the 4 different repeats of the 30 stimuli. (The input data had 4 columns and 30 rows.) The

7. Note that the correlation was computed on the basis of two repeated trials per stimulus in Experiment One as opposed to eight in Experiment Two and four in Experiment Three. (Actually, in Experiment One it was two trials per *pair* of stimuli, since a trial consisted of two stimuli in alternation. The two "repeated trials" were slightly different in that one presented the stimuli as "ABAB..." and the other as "BABA..." Thus the correlation value there was actually –.729 rather than .729, since the response scale compared the two stimuli and thus was inverted depending on which of the two stimuli was presented first.)

Subject	Correlation (r)	Z-score
1	.790	1.071
2	.747	.966
3	.676	.822
4	.619	.723
5	.557	.629
6	.466	.505
7	.458	.495
8	.439	.471
9	.396	.419
10	.381	.401
11	.346	.361
Mean	0.518	0.602

Mean z-score: 0.602
Std. Dev. of z-scores: 0.243
2 standard deviations below the mean z: .116
2 standard deviations above the mean z: 1.088

Table 15. Experiment Three. Subject consistency: correlation between repeated trials. All subjects are within 2 standard deviations of the mean z-score.

diagonals of the correlation matrices (containing correlations of a column to itself, hence scores of 1.0) were omitted when computing the mean correlation.

4.2.4 Per-Subject Results

The per-subject results for Experiment Three are shown in Figures 29 - 31 (pages 142 - 144). Each data point is the average of four trials for that stimulus. (Recall from page 127 that there are four cases where a given stimulus is considered part of two different "curves." As seen more clearly in the three-dimensional diagram [Figure 25 on page 131], at −13.9 Hz the "retuned interval" and "shifted partials, equal-tempered" curves share the same stimuli, as do the "retuned interval" and "shifted partials, just intonation" curves at zero Hz.) Again, the figures are presented in order of decreasing subject consistency. See Table 12 on page 128 for the consistency ratings and details about each subject's musical experience.

Inspecting the plots, we notice that for the "retuned interval" stimuli, all subjects exhibit the same trend observed in the averaged results (Figure 26 on page 133), as well as in the results of the previous two experiments: intonation is judged progressively poorer as the projected beat rate increases to 25 Hz. Many but not all subjects also show this effect to some extent for the intervals with shifted partials (whether just or equal-tempered), but the range of responses is much more limited than for the retuned intervals.

Subject 8 is unusual in that he employed a rather limited extent from the range of possible responses, giving the most out-of-tune stimulus an average rating of only 4.5. (Recall that the range was 1 to 9, and a rating of 5 was labelled "somewhat out of tune.") This subject's average rating was the highest, 7.21, just about halfway between exactly in tune (9) and somewhat out of tune (5). He also used less of the range than any of the others, with a standard deviation of .90. Most of the subjects made use of the available range from 1 to 9, but this subject judged nearly all the stimuli to be at least somewhat in tune.

Preference for tuning system

It is interesting to examine the individual data for indications of preference between just intonation and equal temperament. Recall that in the averaged data (see Figure 25 on page 131), the equal-tempered thirds are rated higher in every case than the corresponding

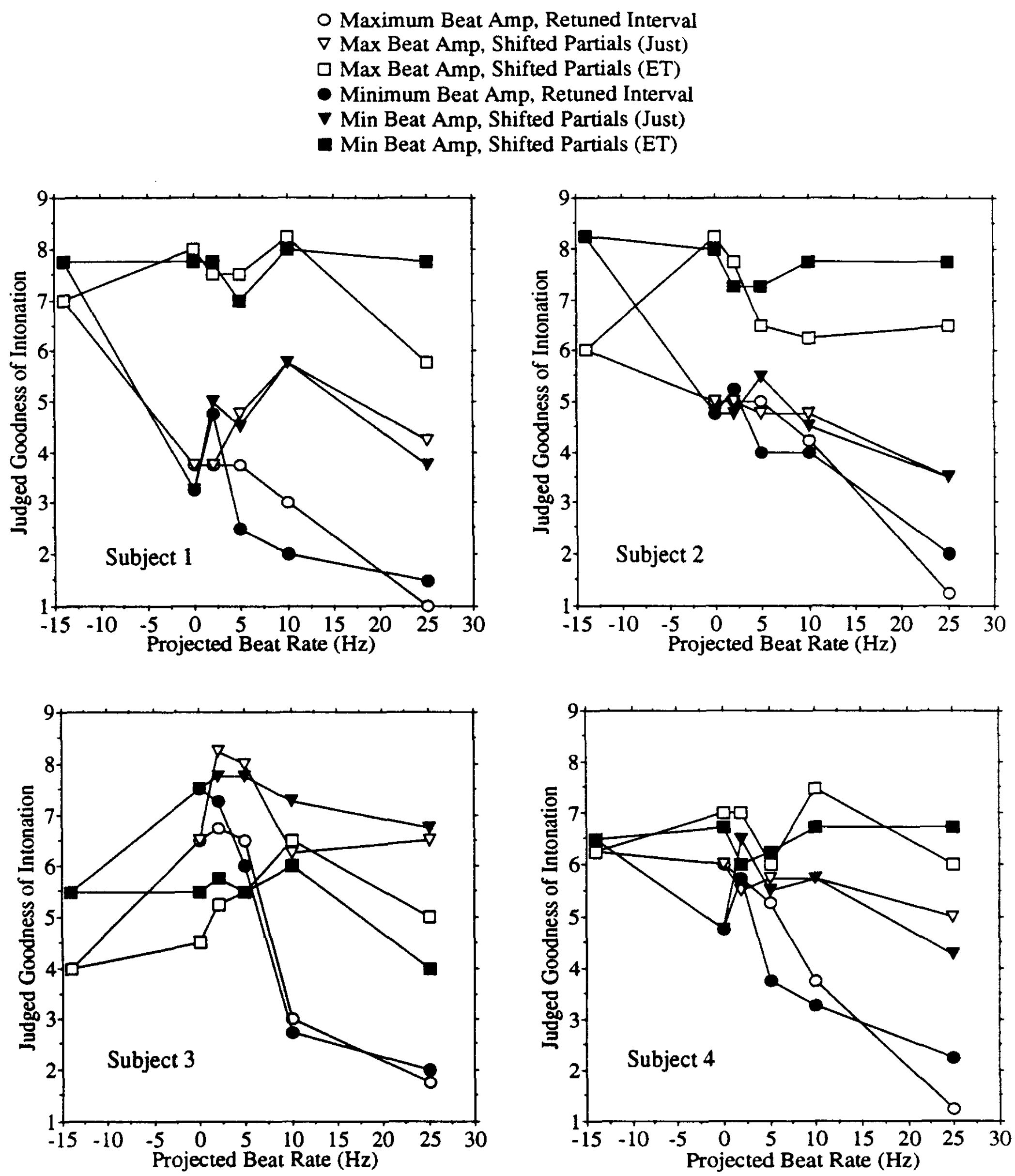

Fig. 29. Experiment Three, results per subject. (Continued in Figure 30.)

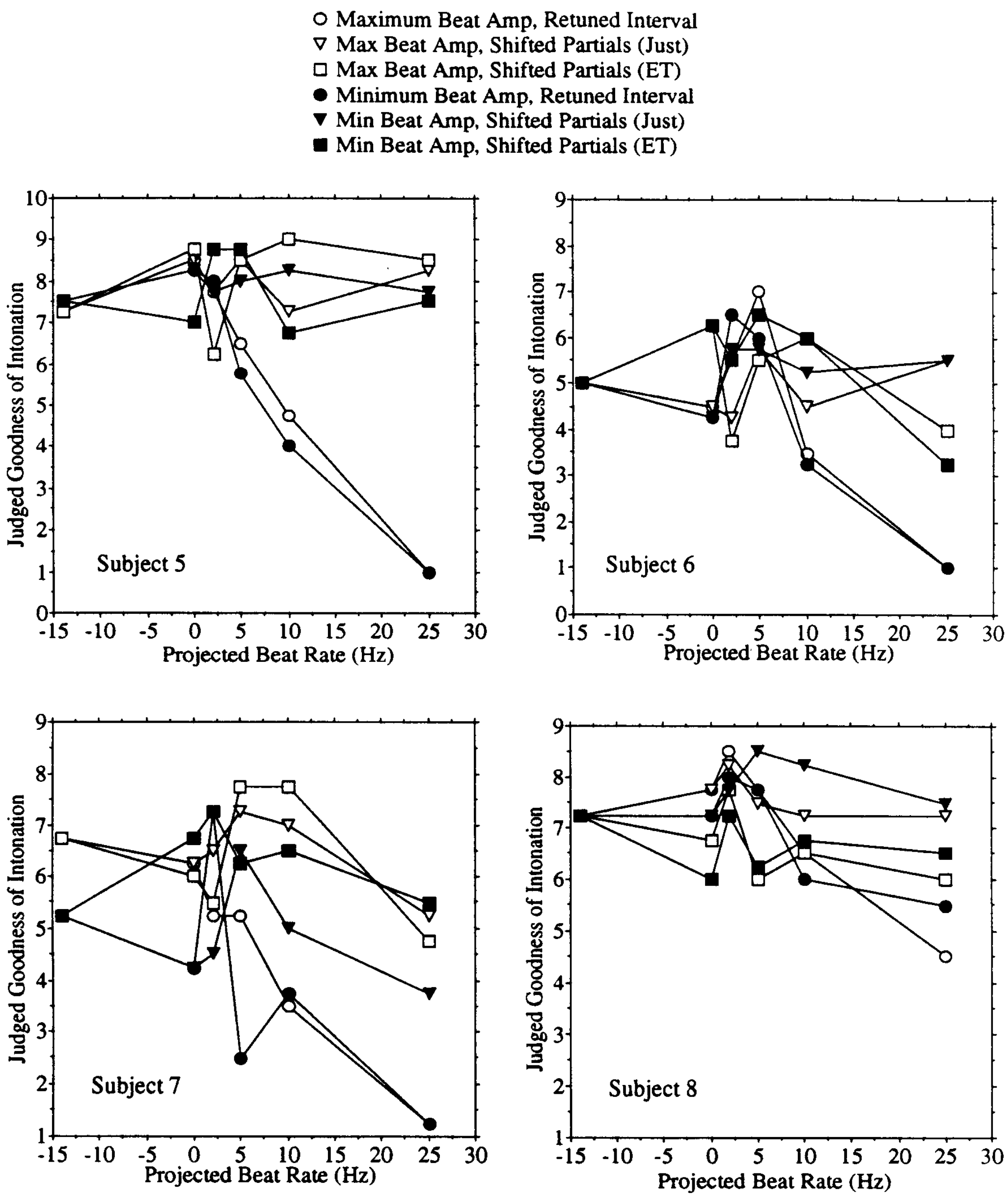

Fig. 30. Experiment Three, results per subject. (Continued from Figure 29 and continued in Figure 31.)

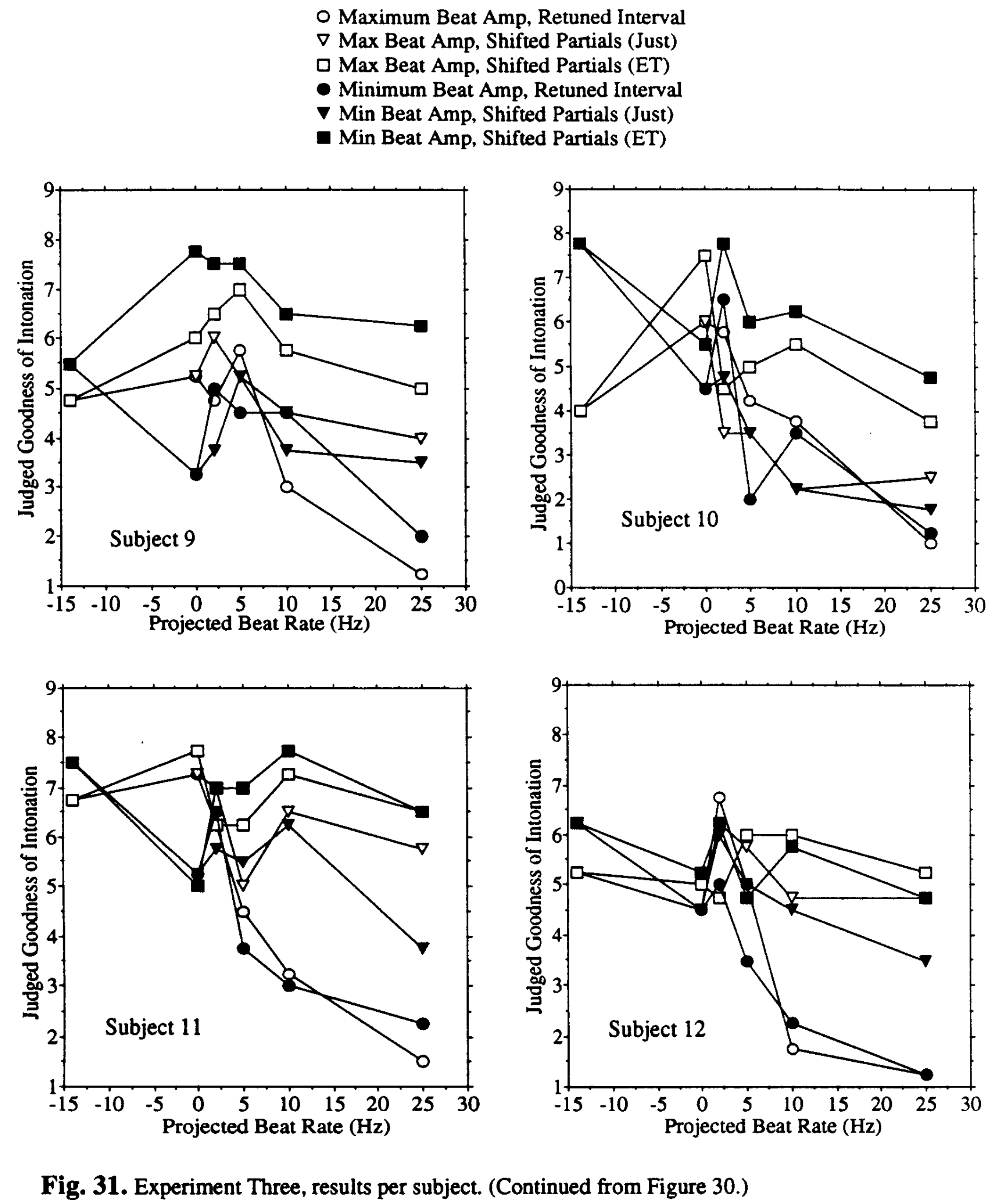

Fig. 31. Experiment Three, results per subject. (Continued from Figure 30.)

just stimuli, although the ANOVA for shifted partials (just intonation versus equal temperament) did not show this difference to be significant (Table 14 on page 138). The plots of Subjects 3 and 8 make it appear that they prefer just thirds. Subject 3 judged the stimuli at –13.9 Hz, which are equal-tempered, to be more out of tune than did the other subjects, making his curves more arch-shaped than the others'.

Subjects 5, 6 and 7 don't display much visible difference between the two tunings in their plots. Subjects 1, 2, 9, and 10 have a visibly greater than average separation between the two pairs of "shifted partials" curves, suggesting a greater preference for equal-tempered thirds.

Subjects 1, 6, and 11 have composed music in nonstandard tunings. Subjects 1 and 11's response patterns are similar to the average, although subject 1 has a greater than average preference for equal temperament over just intonation. Subject 6, however, shows no clear pattern suggesting a preference for just intonation or equal temperament. Unlike Subject 1, Subject 6 works primarily with just intervals in her music with nonstandard tunings, which probably accounts for the fact that she judges the just thirds to be no less in tune than the equal-tempered ones. Her response to the equal-tempered thirds with the –13.9 Hz projected beat rate (which correspond to the "normal" equal-tempered third, having no inharmonicity) is lower than average.

It must be remembered that these data represent only one interval of the tuning system, presented in isolation, so caution is advised in drawing conclusions about an individual's preference for an entire tuning system. However, the major third is a very important interval in differentiating just intonation from equal temperament, since there is 14 cents' difference between the two tunings, while for perfect fifths and fourths the difference between the two tunings is only 2 cents. (With the exception of the minor third, which has a 16-cent difference, all other harmonic intervals are less frequent in traditional music. For our purposes, the inversions of both thirds, namely the minor and major sixths, can be considered equivalent to the corresponding thirds. Because they are inversions of the thirds, they have the same differences between the two tunings as the respective thirds.) Thus individuals' judgments of major thirds are likely to be very influential in determining their preference for just intonation vis-à-vis equal temperament.

Beat amplitude

In the data averaged over subjects (Figure 26 on page 133), the curves for the two beat amplitudes are not greatly separated for a given "method." In the individual plots, though, it seems that where a subject has a clear separation of the two curves, it is usually the one with minimum beat amplitude that is higher, suggesting that the subject prefers minimally beating intervals for those particular stimuli. For example, subjects 2 and 10 have rated the minimally beating equal-tempered stimuli in the range from 5 to 25 Hz as more in tune than the maximally beating ones, and subject 8 has a similar response to the corresponding just stimuli. In this regard it is interesting to note the responses of subjects 7 and 9. Subject 9 is unique in that the two highest curves never cross—every single equal-tempered stimulus is rated (on the average) to be more in tune when it has a minimum beat amplitude. This is in accordance with what one might expect from the literature, if there is to be a difference at all. However, this same subject shows exactly the opposite pattern with the just intervals! Here, the upper curve is the one with maximal beating, and again every point on the upper curve is higher than the corresponding point on the lower curve, with the exception of the point at 5 Hz, where they coincide.

Subject 7 also shows this sort of unexpected pattern for the just intervals: all points in the curve for the maximally beating just stimuli lie above the corresponding points with minimum beat amplitude, and the separation of the two curves is even greater than Subject 9's. We may speculate on why a subject would seem to prefer beating in just intervals. If a subject is accustomed to equal temperament, with its beating thirds, he or she might find that beating in a just interval makes it more similar to the familiar equal-tempered third, and thus rate it higher than the corresponding stimulus with minimum beat amplitude. Subject 7 is accustomed to Pythagorean intonation from her performance of early music, and Pythagorean thirds beat even faster than equal-tempered thirds. So for this subject, the beats might be effective in rendering just thirds, and even equal-tempered thirds, more acceptable. Note that this subject also rates the beating equal-tempered thirds higher than the equal-tempered thirds having minimum beat amplitude, with two exceptions. (All of this speculation is unsupported by analyses showing the differences to be significant, of course. Also, the patterns we observe visually tend to be limited to the 5 - 25 Hz range, and don't occur for all the stimuli.) For the retuned intervals, Subject 7's curves for minimum and maximum beat amplitude have more overlap. Since these intervals are tempered in the direction away from the Pythagorean third—increasingly smaller than equal-tempered as the projected beat rate increases—one would expect this subject to hear them as having poor intonation, just as the other subjects do.

Absolute pitch

Subject 4 has absolute pitch. However, her responses are not unusual; to the contrary, they are perhaps the closest of any subject's to the responses averaged over subjects. (Compare her graph [page 142] with the plot of the average responses on page 133.) Recall that the two subjects in Experiment Two with absolute pitch showed some tendency to rate the "shifted partials" stimuli (which were equal-tempered) as more in tune than did the other subjects, at least at the higher projected beat rates. (See page 121.) We supposed that this suggests that the subjects with absolute pitch tended to use pitch relations (rather than secondary cues such as inharmonicity) to evaluate the intonation. In this experiment, however, subjects in general didn't show much effect of projected beat rate for the "shifted partials" stimuli (as illustrated by the flatness of the curves for both equal temperament and just intonation), so Subject 4 does not stand out.

We probably do not have enough data from these experiments to generalize about the performance of subjects with absolute pitch, but there is no evidence here that their performance in judging intonation is exceptional. Subject 4 in Experiment Three had a consistency rating that was only 0.5 standard deviations above the mean z-score, whereas Subject 1 (who does not have absolute pitch) had the highest consistency, nearly two standard deviations above the mean z-score. Similarly, in Experiment Two, the two subjects with absolute pitch had z-scores 0.8 standard deviations above the mean and 0.2 below the mean, respectively. By comparison, the subject with the highest consistency in Experiment Two did not have absolute pitch and his z-score was 1.5 standard deviations above the mean.

4.3 Comparison of Results of Experiment Three with the Other Experiments

The results of Experiment Three are somewhat different from those of the previous two experiments. The pattern of increasing "out-of-tuneness" with increasing projected beat rate, found in both Experiment One and Experiment Two, is duplicated in Experiment

Three only for the stimuli with the "retuned interval" method of controlling beat rate. For stimuli with the "shifted partials" method, projected beat rate has little effect. The latter phenomenon apparently contributes to the lack of an overall significant effect for projected beat rate, as measured by the three ANOVAs. However, it does probably contribute to the significance of the "method of controlling beat rate," since the patterns are so different for "retuned interval" and "shifted partials." In Experiment Two, the difference was no greater than might have happened by chance variation alone.[8]

The divergent patterns for the harmonic and inharmonic stimuli (that is, the "retuned interval" and "shifted partials" stimuli, respectively) can be explained by examining which partials are frequency-shifted. In the perfect fifth stimuli, the third partial of the lower tone and its multiples (6, 9, 12 and 15) were candidates for frequency shifting. In the major third stimuli, by contrast, the candidate partials were multiples of the fifth partial of the lower tone. Thus there were fewer of them—only three in the group of 16 partials. So if the amount of shift were equal, the overall inharmonicity would be less than with perfect fifths, lessening the contribution of inharmonicity to any potential effect of projected beat rate among the stimuli with the "shifted partials" method of controlling beat rate.

There is an additional reason why the inharmonic major thirds were not judged to be as out of tune as the corresponding perfect fifths. Not only are the inharmonic partials fewer, but their amount of shift is less. Recall from page 113 that for the fifths, the partials were shifted –2.0, 1.3, 6.3, 14.5, and 38.9 cents for the projected beat rates of 0, 2, 5, 10, and 25 Hz, respectively. The average shift of these stimuli is 12.6 cents (taking the absolute values

8. Recall that in Experiment One, a significant difference for this variable was found only for the trials in which "method of controlling beat rate" changed between stimulus A and stimulus B. As mentioned earlier, the analyses of these trials was problematic and thus should be given less weight.

of each shift). The inharmonic major thirds in just intonation were shifted somewhat less: 2.1, 4.8, 9.7, and 24.4 cents, for 2, 5, 10, and 25 Hz, with an average of 10.3 cents. The equal-tempered thirds were shifted 13.7, 11.7, 8.8, 3.8, and –11.1 cents, for the corresponding rates of 0, 2, 5, 10, and 25 Hz. The average shift here is 9.8 cents, so as a group, the major thirds had an average shift of about 10 cents. Combining this fact with the smaller number of shifted partials (three, versus five for the fifths), the total inharmonicity of the thirds is about 75% that of the fifths, if we simply multiply these numbers. (These calculations exclude the stimuli that can be considered members of both the "shifted partials" and the "retuned interval" categories—namely, the just third with the zero-Hz projected beat rate, and the equal-tempered third with the –13.9-Hz rate. Including them, the relative inharmonicity would be only 40%.).

In terms of potential shift of periodicity pitch, the thirds also have an advantage. Since only the first six partials are said to affect periodicity pitch (Moore, Glasberg and Peters [1985]), the major third has only one inharmonic partial affecting pitch, whereas the fifth has two. If we accept the approximation that the pitch is shifted by one-sixth of the partial's frequency shift, the maximum periodicity pitch shift in the "shifted partials" thirds is 4.1 cents for the just third at the 25 Hz rate, and the average pitch shift is only 1.4 cents (compared to 4.2 cents for the fifths).

Notice that with the equal-tempered thirds, the inharmonicity decreases as beat rate increases (the shift is 13.7, 11.7, 8.8, 3.8, and –11.1 cents for the five rates from 0 to 25 Hz). This is because the natural, unshifted equal-tempered third beats fairly rapidly compared to either the just third or the equal-tempered fifth; thus more inharmonicity is required to achieve a projected beat rate of zero Hz, and decreasing the inharmonicity increases the rate. The reversed direction was a consequence of the shifting algorithm, given in the

Appendix, and has the virtue of minimizing inharmonicity for these stimuli. It also probably weakens the correlation between judged goodness of intonation and projected beat rate in the analytical results.

The harmonic stimuli, by contrast, were severely mistuned for the higher values of projected beat rate, just as they had been for perfect fifths. (In fact the ranges of mistuning from equal temperament are similar—for the extreme mistuning of 25 Hz, the retuned perfect fifth had a value of 661 cents and the retuned major third had a value of 362 cents, placing each about 40 cents below the equal-tempered interval.) Thus the stimuli with the "retuned interval" method of controlling beat rate would be strongly affected by projected beat rate, for both fifths and thirds.

Where the results of Experiment Three unequivocally support those of the earlier two experiments is in the nonsignificance of beat amplitude. Again, it made virtually no difference in judgments of mistuning whether the beating partials were present or not. Since this is probably the most interesting result of the present research, in light of earlier studies which it seems to contradict, the consistency across the two intervals is a welcome strengthening of the result.

Chapter 5 Conclusions

This final chapter presents the overall conclusions of this research, and suggests directions for future work. First, however, a comparison of the present results with those of Vos (1986) will be instructive, since the present study is more closely related to that work than to any of the other literature. (Review Chapter 1, page 30 for a summary of Vos's study.)

5.1 Comparison with Results of Vos (1986)

5.1.1 Experiments with Perfect Fifths

We first examine Vos's results in terms of the tempering of the interval in cents. Figure 32 shows the results of the experiment in which subjects rated the "subjective purity" of perfect fifths; these data were obtained by visual inspection of Figures 2a, 2b, 4a and 4b of Vos (1984). In Vos's figures, negative values on the x-axis represent intervals smaller than the just version, and positive values represent intervals larger than just. (This is true of Vos's plots whether the x-axis measures tempering in cents or beat rate in Hz.)

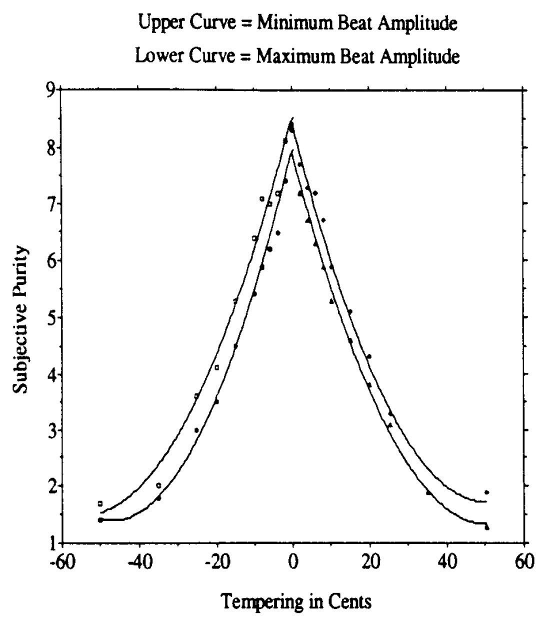

Fig. 32. After Vos (1984). Subjective purity of perfect fifths, plotted vs. tempering in cents.

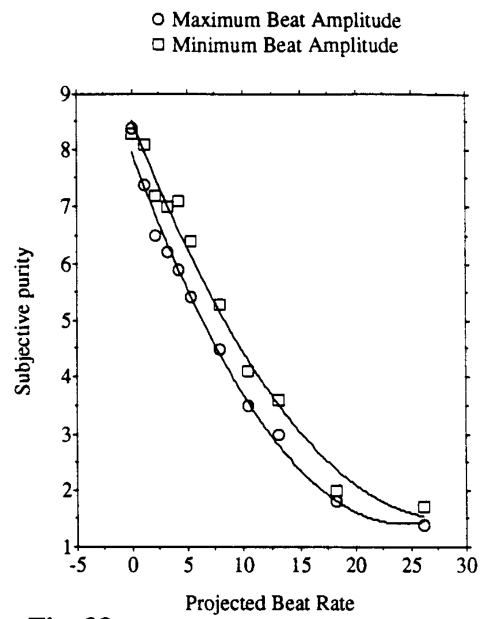

Fig. 33. Left-hand side of Figure 32, flipped horizontally for plotting vs. projected beat rate in Hz. (See text.)

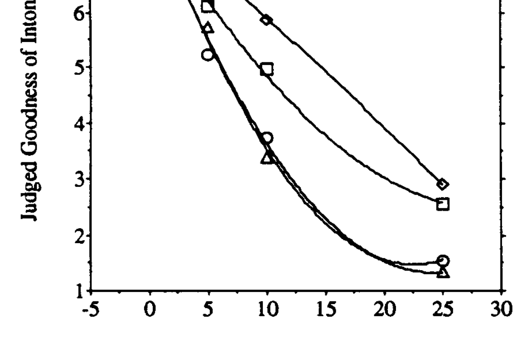

Fig. 34. Intonation of fifths, from Experiment 2 of the present study.

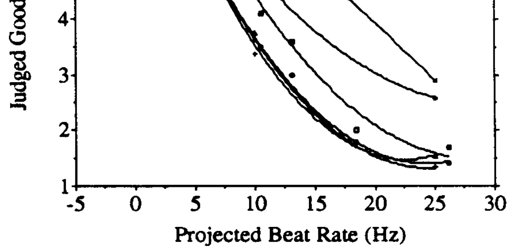

Fig. 35. Comparison of present results with Vos's (superposition of Figure 33 and Figure 34).

Note that Vos used more values for interval tuning than the present study, and included an equal number of temperings on each side of the just interval.[1] Figure 32 exhibits a clear separation between the upper and lower curves, showing that the removal of beating (by the use of odd-numbered partials) increased the subjective purity. Vos's analyses demonstrate that the difference between these curves is statistically significant.

In order to better compare Vos's results to the present results, we next plot the same results in terms of projected beat rate rather than tempering in cents (Figure 33)[2]. Although Vos studied fifths that were both larger and smaller than the just fifth, here we show only the negative temperings (i.e., tunings of the fifth equal to or smaller than the just fifth) from Figure 32, since the present study only used negative temperings. Note that although my stimuli were tempered in the direction smaller than the just interval[3] (with the exception of the equal-tempered third), I normally referred to the projected beat rates as positive, since from a perceptual point of view, negative beat rates are meaningless. This posed no problem, since almost all my stimuli were tempered in the same direction from the just interval. However, since Vos used both negative and positive temperings, for purposes of comparison we need to flip the orientation of his graph such that a positive beat rate corresponds to a negative tempering in cents and vice versa.[4]

1. Vos's subjects actually rated the subjective purity of intervals on a scale from 1 to 10; I have transformed the values to a range of 1 to 9 for purposes of comparison with my data. One reason I had chosen an odd number of possible responses for my experiments was to have an exact midpoint, corresponding to the value 5.
2. Vos also plotted his results in terms of beat rate; see his Figures 4 and 5, for example. Unlike the present study, the stimulus variable was tempering in cents (0, ±2, ±4, ±6, ±8, ±10, ±15, ±20, ±25, ±35, and ±50 cents), and the fundamental frequencies were varied such that the midpoint of the interval was any of ten frequencies symmetrically surrounding 370 Hz. This means that stimuli with a given tempering in cents actually had differing beat rates. However, the variation is not large, so Vos's change of x-axis variable seems justifiable, and a given point on the x-axis can be interpreted as the mean beat rate for that group of stimuli (or very close to it).
3. They were made smaller than just in order that the direction of increasing beat rate be the same as the direction away from the standard versions of the major third, including the just, equal-tempered and Pythagorean versions.

Figure 34 recapitulates the main results of Experiment 2 of the present study. Note the similarity of the lower two curves—corresponding to the stimuli where the interval was retuned—to the lower of Vos's curves in Figure 33. Since Vos's stimuli also had retuned intervals rather than shifted partials, this similarity makes sense. Figure 35 is a superposition of the data in Figure 33 and Figure 34, and shows just how closely the curves for the retuned intervals match between the two studies. Note, however, that in Figure 34 the two bottom curves are almost identical, showing the lack of a significant difference between the stimuli with maximally beating partials and those in which the beating was minimized. In contrast, in Figure 33 one can see that the two curves are clearly separated—in Vos's study beating made a difference. After examining the plots for major thirds, we shall discuss possible explanations for the differences between the results of the two studies.

5.1.2 Experiments with Major Thirds

The data for major thirds in the two studies are illustrated in Figure 36 through Figure 39. Figure 36 shows Vos's data, plotted with tempering in cents on the x-axis.[5] Again, note the increase in subjective purity when beating is removed. Figure 37 recapitulates the results from Experiment 3 of the present study. (As before, positive beat rates in this study correspond to intervals smaller than just, which is the opposite orientation from Vos's.) Figure 38 superimposes the data from the two studies for negative projected beat rates (according to the convention used in this study), corresponding to thirds equal to or larger

4. Vos himself kept the orientation consistent when translating from cents to beat rate.
5. The regression for the left-hand side of the plot includes the points at –50 cents. Vos eliminated the points post hoc from his regression computation, since the interval here is closer to a just minor third than to a just major third. The rise in the curve suggests that indeed subjects were hearing it as a sharp minor third. Vos does not mention whether the instructions indicated that the stimuli were mistuned major thirds and to be judged as such.

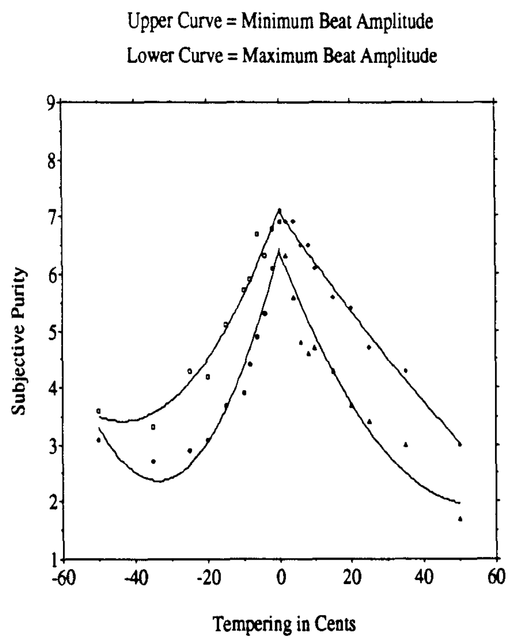

Fig. 36. After Vos (1984). Subjective purity of major thirds, plotted vs. tempering in cents.

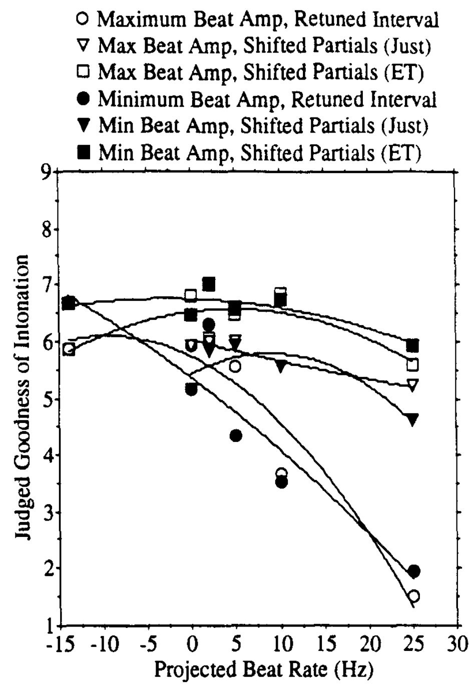

Fig. 37. Intonation of thirds, from Experiment Three of the present study. Positive rates correspond to negative temperings (smaller than just).

than the just third—in other words, the right side of Figure 36 and the left side of Figure 37. Figure 39 shows the superimposition for positive projected beat rates (which, for the retuned intervals, corresponds to negatively tempered thirds—but not for my stimuli with shifted partials, which were always just or equal-tempered.)

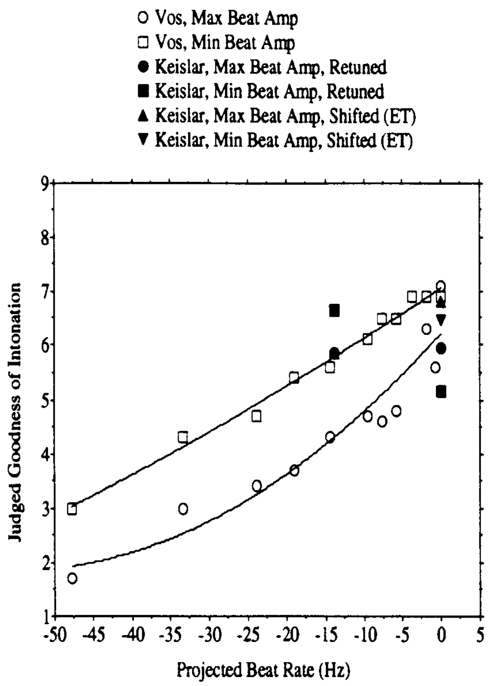

Fig. 38. Comparison, thirds larger than just. Right-hand side of Figure 36, flipped horizontally, with points at 0 and –13.9 Hz from Figure 37 added.

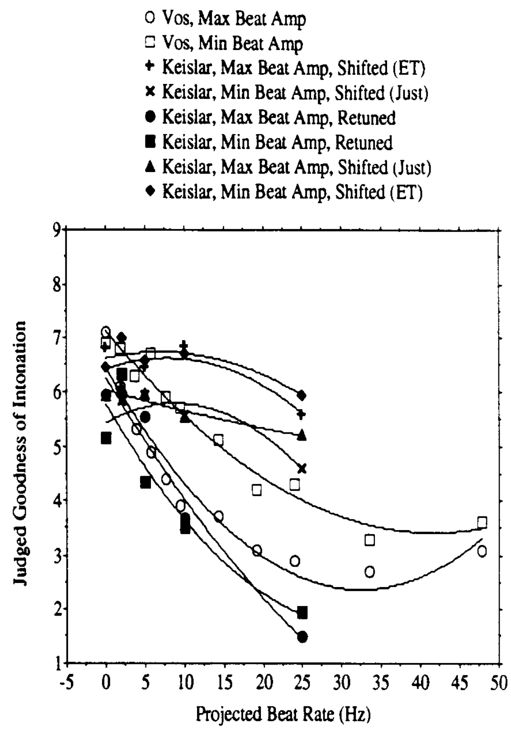

Fig. 39. Comparison, thirds smaller than just. Left-hand side of Figure 36, flipped horizontally, with non-negative points from Figure 37 added.

5.1.3 Discussion: Comparison with Vos's Results

Some of the data exhibit a very good agreement between the present study and Vos (1986), particularly in the judged intonation of fifths with traditional beating (the "retuned interval, maximum beat amplitude" sort of stimulus). However, a major difference between the two studies is that Vos's data show an effect of beating for both major thirds and perfect fifths, while mine do not. We can speculate about the reasons for this contradiction. First it should be noted that the stimuli in the two studies did not have identical spectra (even leaving aside my stimuli with shifted partials). Vos used a 6dB/octave rolloff, while the stimuli in the present study had equal-amplitude partials. In Vos's experiments the lower tone had the first 20 harmonics of the harmonic series, and the upper had either the first 20 harmonics or the first 10 odd harmonics. In my experiments, on the other hand, the lower tone had the first 16 harmonics, and the upper tone had either the first 16, or the first 16 minus whichever partials happened to be part of a beating pair.[6] Whereas Vos's stimuli with minimal beating had all the even partials deleted, I only deleted partials that actually were in a beating pair. In the case of the fifths, this turns out to include all the even partials through the tenth, except in the case of the retuned interval with the more extreme mistunings. For thirds, however, typically only every other even partial was deleted. It is conceivable that the greater number of deleted partials in Vos's study could account for some of the difference between the two studies, but the difference in the spectral envelopes is probably more important.

Secondly, there may be a subtle linguistic difference in the tasks. Vos asked his subjects to evaluate the purity of the stimuli, while in the present study the subjects were asked to judge how in tune they were. In Dutch the word for "pure" has some of the connotations of "pure" in English, but it also is the word used to mean "in tune."[7] In English, "pure" can mean "in tune," but the term "in tune" is used much more commonly for this purpose, and is relatively free of other connotations. One might wonder whether subjects asked for judgments of "purity" would be more likely to respond to differences of timbre (caused for example by the removal of partials) and roughness (such as that caused by beating) than would subjects asked to judge how "in tune" an interval was. The latter

6. See the Appendix for a fuller description of my stimuli.
7. Rudolf Rasch, personal communication (1990).

group of subjects would probably be more likely to respond to the pitch relation. A timbre can certainly be evaluated as more or less "pure," but the notion of an "in-tune" timbre is peculiar.

A third possibility is that the presence in my experiments of stimuli in which beat rate was uncorrelated with interval tuning—namely, the stimuli with inharmonic partials—served to discourage the use of beat rate as a cue for judging intonation. If so, the subjects might or might not have been conscious of their suppression of this cue.

A fourth difference in the experiments—and perhaps the most important—is that Vos varied the fundamental frequency independently of the interval tuning, so that both tones' fundamental frequencies changed from stimulus to stimulus. (Recall that in the present study only the lower note's fundamental frequency varied, as determined by the desired interval tuning; the upper note's frequency was kept constant.) The midpoint of Vos's intervals varied plus or minus a whole step in increments of a quarter-tone; this fact combined with the many possible sizes of the intervals means that the lower and upper notes each moved to many different frequencies, and typically the distance by which they moved from one stimulus to the next would be a nonstandard interval. In a pilot study in which subjects were asked to estimate the exact size of an interval, I found that subjects' performance was better when the lower tone's frequency was kept constant than when the frequency of both tones was changed by microtonal amounts. This is not surprising; it is much easier to evaluate pitch relations in the context of a stable reference pitch. (We shall leave aside the question of whether this is a cause or an effect of tonal music.) So it is likely that it was easier for subjects to evaluate the intonation in terms of pitch relations in my experiments; and it is possible that when the pitch relations are more confusing, as in Vos's study, subjects are more easily swayed by other factors such as spectrum and beating. This is not necessarily a criticism of Vos's method; for it can be argued that a constant pitch reference, as in my experiments, allows subjects to make judgments on the basis of the changing pitch of the lower tone rather than according to the tuning of the interval. In other words, they might be using frequency discrimination more than frequency ratio discrimination. It seems that arguments can made for either approach. We cannot rule out the possibility that the subjects in the present study preferred equal-tempered thirds simply because the lower note was always flatter than that of the other thirds. However, this condition may be more relevant to real musical situations, in which one typically has a stable pitch reference. (Even for atonal music, a trained musician remembers the pitches of the chromatic scale.) So if equal-tempered thirds are preferred to just thirds simply because the lower note is flatter, this is still useful information. In any event, pitch height cannot be the only factor; although intervals larger than the equal-tempered third were not tested, clearly after some amount of increased mistuning a large major third would be judged to be worse than the equal-tempered third.

To sum up this comparison between the present results and those of Vos (1986): although there is a very good match in the judged intonation of mistuned fifths, and a reasonably good match for mistuned thirds, Vos found beating to be important for both thirds and fifths, while the present study does not. Differences in the stimuli's spectra and in the stimulus presentation (fixed versus variable pitch of the upper note) seem likely to have contributed to this discrepancy. The difference in the tasks (judging "purity" versus judging how "in tune" the stimuli were) might have had some effect. It is also possible that the presence among my stimuli of rapidly beating but ostensibly in-tune intervals confounded any strategy that attempted to use beating as a cue for intonation. Only in this study was beat rate independent of the interval's fundamental frequency ratio.

5.2 General Conclusions And Discussion

The primary result of this research is the finding that, for these stimuli, beating partials appear to be unimportant for perceived intonation. The dominant factor is the tuning of the interval, in other words, the pitch relation of the two notes. Since it is unlikely that beats would be more audible in real musical situations than under these laboratory conditions, these results suggest that the perception of intonation in music is dependent on the actual interval tuning rather than the concomitant beat rate. This conclusion does not disprove Helmholtz' theory that beating partials were responsible for the historical origin of the consonances and dissonances in music, but it indicates that such factors are not necessarily paramount for contemporary listeners. Subjects apparently made reference to learned interval sizes instead of relying on acoustic cues such as beating, indicating that cognitive processes play a crucial role in intonation judgments.

A secondary finding is that inharmonicity can have a strong effect on perceived intonation, since in the perfect fifth stimuli the inharmonic partials apparently created a percept of "out-of-tuneness," even when the beating partials were deleted. This effect was almost as strong as the corresponding interval mistuning for the same projected beat rate.

Finally, we noted that major thirds appear to be more ambiguous than perfect fifths, in that subjects are less consistent in judging them, and the effects of inharmonicity and interval mistuning are weaker. This is in line with much of what is reported in the literature about the relative sensitivity to mistuning of major thirds and perfect fifths.

It is particularly interesting that beat amplitude was nonsignificant in these experiments, in contrast to the results of other researchers, notably Vos (1986). As noted above, the discrepancy might be explained by differences in the stimulus, the presentation, and the task. Also, it should be noted that only in the present research has beat rate been controlled independently of interval tuning.

Common sense could be invoked to support the finding that beats are not very important, for when we listen to music we are normally unaware of beating partials. Usually when we hear something that sounds out of tune, we are more likely to notice a mistuned pitch than any beating. If beating partials really are unimportant, and interval tuning is the primary determinant of intonation, this supports the notion of a cultural basis for musical scales, as opposed to the acoustical basis set forth by Helmholtz and others. It suggests that acoustical phenomena such as beats are not terribly worrisome, and that we can perhaps be accustomed to any interval given enough exposure. This idea is a double-edged sword for composers interested in experimenting with nonstandard tunings. On the one hand, anything is possible; on the other, the potential acoustical foundations for a new tuning system are weaker. Certainly many composers of microtonal music or music with other nonstandard tunings have cited acoustical principles as a justification for their efforts. If the principles are not very relevant perceptually, it behooves these composers to do what great composers have always done and follow the dictates of their aural imagination, unfettered by scientific theoretical systems.

5.3 Suggestions for Future Research

There are a number of basic psychoacoustic studies related to intonation which still need to be conducted. For example, a thorough study of frequency ratio JNDs among musicians and nonmusicians would be very useful. Such a study should include the standard chromatic intervals as well as nonstandard intervals (such as those of the quarter-tone scale). For musical relevance, the stimuli should be complex tones instead of (or in addition to) sinusoids. Another example is the study of the pitch of inharmonic sounds, still in its infancy. Extensive data on the precise pitch of slightly inharmonic sounds would have shed further light on the stimuli used in this study. We have mentioned the work of Moore, Glasberg and Peters (1985), but that study is just the start of what needs to be accomplished before one can predict the perceived pitch of arbitrary sets of inharmonically related frequencies.

Over the years there have been quite a few experiments based on adjustments of musical intervals, but it appears that none of these has tested beating. It would be interesting to see how removal of beating harmonics affects interval adjustment. There also seems to have been no work done on estimation of the exact sizes of intervals. In the latter sort of experiment, the subjects would probably have to be musicians, and they would report the size of various arbitrary frequency ratios by marking points along a line having tick marks labeled with the chromatic intervals. In a small pilot study with such a task, I found some evidence for categorical perception effects: there was some tendency by subjects to hear nonstandard intervals as being closer to the nearest standard interval than they really were, until the interval approached a quarter-tone (the category boundary), around which point the judgments were more accurate. Mirroring effects were also found: a subject might correctly identify the approximate amount of mistuning, but err in the direction of mistuning (mistaking sharp for flat and vice versa).[8] I also created a computer program for carrying out such a task using an adaptive paradigm, and found that I could begin to identify

8. None of these data were subjected to statistical analysis, however.

nonstandard intervals to a greater accuracy than one might be led to expect from the psychoacoustic literature. Such a tool could be used as an ear-training aid in music education. The extension of traditional ear-training to the microtonal case could be useful not only for experimental music, but also as a means of sharpening intonational sensitivity among performers of traditional music.

In the course of discussing the present experiments, we have mentioned several alternative experimental approaches that were rejected, but any of which might have made an interesting supplementary study. For example, the net inharmonicity could have been reduced, at the expense of the natural "harmonic series" of beat rates (see the footnote on page 58). Likewise, the net periodicity pitch shift could probably have been reduced by allowing simultaneous beat rates that were not multiples of the lowest beating pair's beat rates (see page page 114). These options were rejected since the central phenomenon of interest for this study was beating, and these alternative stimuli would have had ambiguous or unnaturally emphasized beat rates.

Another variation on the experimental design, mentioned above, would have been to make both pitches of the interval variable in frequency, as in Vos (1986); this approach was rejected since the pitch relations would have become more complicated and alien, making musical judgment more difficult and rendering the task less similar to judging intonation in a musical context. As mentioned on page 159, however, there are advantages to both approaches.

An objection could be raised that in the present study, and others like it, not all interaction of partials within a critical bandwidth was eliminated. Partials separated by more than 50 Hz were not deleted in this study. Even within a single tone, the higher harmonics are within a critical bandwidth of their neighbors. One might wonder whether these pairs of partials increase the total roughness of the stimulus, thereby helping to disguise the beating between tones. Perhaps beating would be more significant if the only pairs of partials within a critical band were the beating pairs, or if the potential roughness was reduced by attenuating the higher partials. Such stimuli deserve further study.[9]

Rather than frequency-shift the beating partials to decouple beat rate from interval tuning, one could replace the beating pair with an amplitude-modulated tone whose modulation rate was equal to the pair's beat rate (i.e., their frequency separation in Hz), and whose frequency was the mean of the pair's.[10] Amplitude modulation (AM) has a somewhat different envelope shape from that of beating pairs; in the one case the envelope is a rectified sine wave, and in the other, a normal sine wave.[11] Another reason I chose frequency-shifted partials in place of amplitude modulation is that the former technique allows more convenient manipulation of beating in the case where the partials are of unequal, time-varying amplitude. As it turned out, the stimuli of the present study did have only equal-amplitude partials, but I had envisioned an extension of the study to the case of natural instrument sounds, in which the partials' amplitudes are not only different, but time-

9. This problem has been implicitly treated in predictions of dissonance based on Plomp and Levelt's model. Vos (1986), for example, has a graph showing predicted dissonance for different numbers of equal-amplitude partials, including n=1 through 6, and n=5, 10, 15, and 20. The overall predicted dissonance does increase as the number of partials increases. However, even at n=15 and n=20, the consonance peaks are still very prominent around the 3:2 and 5:4 ratios, indicating that the beating partials are still important when comparing stimuli that have the same number of partials.
The musical relevance of these predictions is brought into question by observing the drastic effect of the number of partials. For example, a perfect fifth with 15 partials appears on these graphs to be more dissonant than a minor second with five partials.

10. I studied such an approach at IRCAM in 1987, where I used phase-vocoder analyses of recorded instrumental tones as a basis for resyntheses with dynamic control over the amplitudes of various partials. With extreme amplitude modulation, even a single note could sound out of tune. One of the sounds synthesized was a piano tone; the extreme modulation sounded like an exaggerated version of the beating when the three strings of a single piano note are mistuned from each other.

11. AM also lacks the subtle pitch shift in slow beats studied by Feth (see page 22).

varying in a complex way. With frequency-shifting, one simply retains the original amplitude functions regardless of the separation of the two partials, but to model the beating accurately with amplitude modulation, one must use a complicated modulation function that changes with beat rate. (Often, natural instruments also have time-varying frequency, but it is not always as critical for realism as the amplitude variations. Adding frequency variations further complicates the design when intonation is being studied.) Such an extension of the current study to synthesized instrument sounds would be musically interesting. My expectation is that beating would be even less important in such stimuli, since the higher partials would typically be attenuated, the beating partials would have unequal amplitudes, and other temporal variations might help disguise the beating.

Besides extending the stimuli in terms of timbre, augmenting them with more musical intervals would be useful. Most other intervals have fewer beating pairs than do perfect fifths and major thirds, but the perfect fourth lies between the fifth and the third in terms of the number of its beating partials. However, all the chromatic intervals within the octave could profitably be examined.

In the review of the literature in Chapter 1, we touched briefly upon the question of musical context. It seems likely that psychoacoustical phenomena such as beating would receive even less attention in a musical context, where cognitive factors are probably of increased importance, than they do in isolated intervals. If the results of the present study had shown beating to be important, a subsequent study using musical passages would logically have been the next test of the musical relevance of those results. There are a great many interesting complications that arise when considering intonation in musical passages—for example, the relative weight of melodic intervals vis-à-vis harmonic intervals, the importance of tonal relations (as in the example cited earlier of a perfect fifth

F#-C# embedded in a cadential passage in the key of C), and so on. Questions about intonation in musical passages could certainly fuel many dissertations.

Appendix: Specification of Stimuli

The tables beginning on page 173 specify the frequencies present in all the stimuli used in the three experiments. These tables are necessary for a full description of the stimuli, since a computer algorithm determined which partials would be frequency-shifted or deleted, and the precise pattern of these modifications cannot be succinctly expressed. First the algorithm is discussed, and then a summary of the patterns in the tables is given.

A.1 Algorithm for Frequency-shifting or Deleting Partials

The algorithm[1] considered any pair of partials to be potentially beating if their frequency separation was less than 50 Hz and their frequency ratio was less than 1.2. (These are reasonable figures, since 50 Hz is too fast to hear as beats, and the ratio 1.2 is larger than a critical bandwidth for the range of frequencies in question.) If the stimulus was to have a beat amplitude of "minimum," the partial of the upper note was deleted. If the technique for controlling beat rate was "frequency-shifted partials" rather than "interval tuning," the partial of the lower note was shifted to the position that resulted in the correct beat rate. In addition, if the pair of beating partials were not multiples of the lowest beating pair, the upper partial was deleted. This was to ensure that the beat rates of all the pairs of beating partials were multiples of the projected beat rate, since that is the simple case that has been shown in the literature to yield an overall perceived beat rate equal to the beat rate of the lowest beating pair (Vos 1984).

1. For clarity, the algorithm is given below as "pseudocode." The original code was written in Pla, a compositional language developed at Stanford (Schottstaedt 1983).

A.1.1 Part One (Algorithm to find all beating partials and process appropriately)

```
FOR the 1st through the 16th partial of the lower note
   BEGIN "lower note loop"
      FOR the 1st through the 16th partial of the upper note
         BEGIN "upper note loop"
         IF the frequency separation between the upper note's partial and the lower
            note's partial is less than 50 Hz and the frequency ratio of the two partials
            is less than 1.2
            THEN BEGIN "this pair beats"
            We found a pair of beating partials. If it's the first pair we've found,
               remember the harmonic number of the lower note (call it "a").
            If the desired beat amplitude is "minimum," delete the partial of the upper
               note.
            If the method of controlling beat rate is "frequency-shifted partials," move
               the partial of the lower note as follows:
               The new beat rate (call it "x") of this pair of partials will be the projected
               overall beat rate times i/a, where "i" is the harmonic number of the lower
               note's partial and "a" is the harmonic number of the lowest beating
               partial in the lower note. Move the lower note's partial to be x Hz higher
               or lower than the upper note's partial (lower if it already was lower,
               higher if it was already higher or equal).
            END "this pair beats"
         If this pair doesn't beat, but we've already found a partial of the upper note that
            beats with this partial of the lower note, we don't need to check any higher
            partials of the upper note, so continue to the next partial of the lower note.
         END "upper note loop"
   END "lower note loop"
```

A.1.2 Part Two (Find anomalous beat rates; done by hand)

Inspect for pairs of beating partials, found by the algorithm above, whose beat rates aren't multiples of the overall projected beat rate. In each such case, delete the partial of the upper note. See "Descriptive Summary of Stimulus Tables," below, for a list of these anomalous cases and a specification of which partials were deleted. There was only one anomalous case among the fifths, and five cases among the thirds. In a few instances, this subsequent deletion of the upper tone's partial from the anomalous pair meant that the partial of the lower tone had in effect been frequency-shifted unnecessarily.

A.2 Descriptive Summary of the Tables

A.2.1 Format of the Tables

Each stimulus is identified by the values of the main variables: Projected Beat Rate, Method of Controlling Beat Rate, and (for the major thirds with shifted partials) Tuning. (The Beat Amplitude variable is not listed, because each table applies to both the "maximum beat amplitude" and the "minimum beat amplitude" stimuli.) Then the "interesting" component frequencies are listed in ascending order. *To save space, not all the harmonics are shown in these tables.* The omitted harmonics can be easily calculated by the reader; in every case they are strictly harmonic and are not members of a beating pair. Recall that every stimulus used the first sixteen harmonics, except for those of these sixteen that were deleted. Any partial that is shifted to an inharmonic position is shown, as are any pair of harmonics that beat (or that would beat if the harmonic of the upper tone were not deleted), as well as any pair of partials that are integer multiples of the lowest beating pair. The fundamental frequencies are also shown in each case.

The following format is used in the tables:

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	357.000			
		440.000	1.000	
		1760.000	4.000	
5.000	1785.000			25.000
11.000	3927.000			
		[3960.000]	[9.000]	

Five columns are given: two for the partial number and frequency of each partial in the lower note, two analogous columns for the upper note, and a fifth column for the beat rate. The beat rate is given following any partial that is within 50 Hz of the preceding partial in the other tone. Observe that for the upper note, the "freq" column comes before the "partial #" column, in order to make it easier to compare the frequencies in Hz of neighboring partials (for instance, 1760.000 and 1785.000 above).

For stimuli that have partials deleted in order to eliminate beating, the frequencies are the same, except that for every beating pair (including those where the beat rate is zero), the partial of the upper note is deleted. Also, as noted under "Part Two" of the algorithm, there were some cases where partials were deleted even in the stimuli with maximum beat amplitude. In the tables these anomalous cases are indicated by putting the deleted partial in brackets.

Although the tables are the final authority on the composition of the stimuli, the following paragraphs will be helpful as a summary.

A.2.2 Summary of Tables for Fifths

The first group of tables list the stimuli for the first two experiments, which used perfect fifths. These tables are ordered as follows:

(1) Method of Controlling Beat Rate: shifted partials; Projected beat rate: 0, 2, 5, 10, 25 Hz

(2) Method of Controlling Beat Rate: retuned interval; Projected beat rate: 0, 2, 5, 10, 25 Hz

Beat rates

In no case do the stimuli that were intended to have minimum beat amplitude have any conventional beating. (For the present purposes, beating is considered to occur when any two partials have a frequency ratio less than 1.2 and a frequency difference less than 50 Hz.)

The stimuli with maximum amplitude (i.e., those with no deleted partials) all fall into the expected pattern, namely, that the only beating partials are multiples of the third partial of the lower tone and the second partial of the upper tone. Thus, the beat rates all form a harmonic series whose fundamental is the beat rate of the lowest beating pair. There is one exception:

The stimulus with a projected beat rate of 25 Hz and the "Retuned interval" method has, in addition to the beating of the third and second partials, a beat rate of 41.080 between the 16th partials of the lower tone and the 11th of the higher. This is the only stimulus in all the experiments that has beating at a rate that isn't a multiple of the projected beat rate. This was an oversight; human error was responsible, rather than computer error, since this task was done by hand, as described under "Part Two" of the algorithm given above. However, this single anomaly doesn't seem very problematic, since (1) its beat rate is so high, (2) the overall projected beat rate of the stimulus is the highest possible (25 Hz), and (3) these partials are very high, and the literature indicates that only the lowest partials affect the perceived beat rate. It's unlikely that a separation of 41 Hz between these very high partials would affect the overall projected rate of 25 Hz. Even 25 Hz itself is probably too fast to perceive the individual beats.

Deletion of partials

The stimuli with "Beat Amplitude: maximum" all have sixteen equal-amplitude partials. The stimuli with "Beat Amplitude: minimum" all have partials #2, 4, 6, 8, and 10 deleted, with the following two exceptions among the stimuli with the "Retuned interval" method:

The stimulus with a projected beat rate of 10 Hz has partials 2, 4, 6, and 8 deleted, but not 10 (because the frequency separation is 50 Hz).

The stimulus with a projected beat rate of 25 Hz has partials 2 and 11 deleted, but not 4, 6, 8, or 10 (because the frequency separation is 50 Hz or greater). The 11th partial was deleted because it was 41.080 Hz away from the 16th partial of the lower note.

A.2.3 Summary of Tables for Thirds

The major third stimuli are listed in the second group of tables, ordered as follows:

(1) Method of Controlling Beat Rate: shifted partials; Tuning: just intonation; Projected beat rate: 0, 2, 5, 10, 25 Hz

(2) Method of Controlling Beat Rate: shifted partials; Tuning: equal temperament; Projected beat rate: 0, 2, 5, 10, 25 Hz

(3) Method of Controlling Beat Rate: Retuned interval; Projected beat rate: –13.9, 2, 5, 10, 25 Hz

Beat rates

As with the fifths, the stimuli with minimum beat amplitude (i.e., with beating partials deleted) never have any beating (as defined above).

The expected pattern for the stimuli with maximum amplitude is that the fourth partial of the upper note will beat with the fifth partial of the lower note, and similarly for the multiples of these partials. This was true for all the major third stimuli. However, there were additional pairs of partials that would have beat had corrective measures not been taken. To ensure that all the beat rates of the upper partials formed a harmonic series upon the beat rate of the third and second partials, certain partials had to be deleted in some of the stimuli. (Recall from the discussion above that among the fifths, there was only one stimulus with an anomalous beating pair, and that in that case no corrective measure was taken since the beat rate was so fast.)

The anomalous cases are listed below. In every case but the last, the interval tuning was equal temperament and the partial that had to be deleted was the 11th partial of the upper note, which beat with the 14th partial of the lower note.

Stimuli with Beat Amplitude: maximum; Method of Controlling Beat Rate: shifted partials; Tuning: equal temperament; Projected beat rate: 0, 2, 5, 10 Hz.

The corresponding problematic rates (had the 11th partial not been deleted) were: 0, 5.6, 14.0, and 28 Hz, respectively.

Stimulus with Beat Amplitude: maximum; Method of Controlling Beat Rate: retuned interval; Projected beat rate: –13.9 Hz (which yields an equal-tempered third)

Problematic rate (had the 11th partial not been deleted): 49.195 Hz.

Stimulus with Beat Amplitude: maximum; Method of Controlling Beat Rate: retuned interval; Projected beat rate: 25 Hz

Partials that had to be deleted: 9th and 13th of upper note (A). The 9th partial of A would have beat with the 11th partial of F at 33 Hz. The 3th partial of A would have beat with the 16th partial of F at 8 Hz.

Deletion of partials

As just mentioned, some partials had to be deleted among the stimuli with maximum beat amplitude. For the stimuli with minimum beat amplitude, we would expect the fourth, eighth, and twelfth partials of the upper tone to be deleted. In fact, the eighth and twelfth partials did not get deleted in the case with a retuned interval and a 25 Hz projected rate, because in this case they were 50 Hz or more away from the nearest partials. In addition, the same partials were deleted as in the anomalous cases just discussed under Beat rate. The last of these cases (maximum beat amplitude, retuned interval, 25 Hz projected rate) also had the 11th partial deleted.

A.3 Stimulus Tables

A.3.1 Perfect Fifths

```
Projected Beat Rate: 0 Hz
Method of Controlling Beat Rate: shifted partials
```

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.230			
		523.240	1.000	
2.997	1046.480			
		1046.480	2.000	.000
5.993	2092.960			
		2092.960	4.000	.000
		3139.440	6.000	
8.990	3139.440			.000
11.986	4185.920			
		4185.920	8.000	.000
14.983	5232.400			
		5232.400	10.000	.000

Projected Beat Rate: 2 Hz
Method of Controlling Beat Rate: shifted partials

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.230			
		523.240	1.000	
		1046.480	2.000	
3.002	1048.480			2.000
		2092.960	4.000	
6.005	2096.960			4.000
		3139.440	6.000	
9.007	3145.440			6.000
		4185.920	8.000	
12.009	4193.920			8.000
		5232.400	10.000	
15.011	5242.400			10.000

Projected Beat Rate: 5 Hz
Method of Controlling Beat Rate: shifted partials

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.230			
		523.240	1.000	
		1046.480	2.000	
3.011	1051.480			5.000
		2092.960	4.000	
6.022	2102.960			10.000
		3139.440	6.000	
9.033	3154.440			15.000
		4185.920	8.000	
12.043	4205.920			20.000
		5232.400	10.000	
15.054	5257.400			25.000

Projected Beat Rate: 10 Hz
Method of Controlling Beat Rate: shifted partials

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.230			
		523.240	1.000	
		1046.480	2.000	
3.025	1056.480			10.000
		2092.960	4.000	
6.050	2112.960			20.000
		3139.440	6.000	
9.076	3169.440			30.000
		4185.920	8.000	
12.101	4225.920			40.000
		5232.400	10.000	
15.126	5282.400			

Projected Beat Rate: 25 Hz
Method of Controlling Beat Rate: shifted partials

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.230			
		523.240	1.000	
		1046.480	2.000	
3.068	1071.480			25.000
		2092.960	4.000	
6.136	2142.960			
		3139.440	6.000	
9.204	3214.440			
		4185.920	8.000	
12.272	4285.920			
		5232.400	10.000	
15.341	5357.400			

Projected Beat Rate: 0 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	348.827			
		523.240	1.000	
		1046.480	2.000	
3.000	1046.481			.001
		2092.960	4.000	
6.000	2092.962			.002
		3139.440	6.000	
9.000	3139.443			.003
		4185.920	8.000	
12.000	4185.924			.004
		5232.400	10.000	
15.000	5232.405			.005

Projected Beat Rate: 2 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.493			
		523.240	1.000	
		1046.480	2.000	
3.000	1048.479			1.999
		2092.960	4.000	
6.000	2096.958			3.998
		3139.440	6.000	
9.000	3145.437			5.997
		4185.920	8.000	
12.000	4193.916			7.996
		5232.400	10.000	
15.000	5242.395			9.995

Projected Beat Rate: 5 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	350.493			
		523.240	1.000	
		1046.480	2.000	
3.000	1051.479			4.999
		2092.960	4.000	
6.000	2102.958			9.998
		3139.440	6.000	
9.000	3154.437			14.997
		4185.920	8.000	
12.000	4205.916			19.996
		5232.400	10.000	
15.000	5257.395			24.995

Projected Beat Rate: 10 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.160			
		523.240	1.000	
		1046.480	2.000	
3.000	1056.480			10.000
		2092.960	4.000	
6.000	2112.960			20.000
		3139.440	6.000	
9.000	3169.440			30.000
		4185.920	8.000	
12.000	4225.920			40.000
		5232.400	10.000	
15.000	5282.400			

Projected Beat Rate: 25 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	357.160			
		523.240	1.000	
		1046.480	2.000	
3.000	1071.480			25.000
		2092.960	4.000	
6.000	2142.960			
		3139.440	6.000	
9.000	3214.440			
		4185.920	8.000	
12.000	4285.920			
		5232.400	10.000	
15.000	5357.400			
16.000	5714.560			
		5755.640	11.000	41.080

A.3.2 Major Thirds

Projected Beat Rate: 0 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = just intonation

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.000			
		440.000	1.000	
5.000	1760.000			
		1760.000	4.000	.000
10.000	3520.000			
		3520.000	8.000	.000
15.000	5280.000			
		5280.000	12.000	.000

Projected Beat Rate: 2 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = just intonation

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.000			
		440.000	1.000	
		1760.000	4.000	
5.006	1762.000			2.000
		3520.000	8.000	
10.011	3524.000			4.000
		5280.000	12.000	
15.017	5286.000			6.000

Projected Beat Rate: 5 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = just intonation

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.000			
		440.000	1.000	
		1760.000	4.000	
5.014	1765.000			5.000
		3520.000	8.000	
10.028	3530.000			10.000
		5280.000	12.000	
15.043	5295.000			15.000

Projected Beat Rate: 10 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = just intonation

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.000			
		440.000	1.000	
		1760.000	4.000	
5.028	1770.000			10.000
		3520.000	8.000	
10.057	3540.000			20.000
		5280.000	12.000	
15.085	5310.000			30.000

Projected Beat Rate: 25 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = just intonation

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.000			
		440.000	1.000	
		1760.000	4.000	
5.071	1785.000			25.000
		3520.000	8.000	
10.142	3570.000			
		5280.000	12.000	
15.213	5355.000			

Projected Beat Rate: 0 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = equal temperament

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.228			
		440.000	1.000	
5.040	1760.000			
		1760.000	4.000	.000
10.079	3520.000			
		3520.000	8.000	.000
		[4840.000]	[11.000]	
13.859	4840.000			
		5280.000	12.000	
15.119	5280.000			.000

Projected Beat Rate: 2 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = equal temperament

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.228			
		440.000	1.000	
5.034	1758.000			
		1760.000	4.000	2.000
10.068	3516.000			
		3520.000	8.000	4.000
		[4840.000]	[11.000]	
13.875	4845.600			
15.102	5274.000			
		5280.000	12.000	6.000

Projected Beat Rate: 5 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = equal temperament

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.228			
		440.000	1.000	
5.025	1755.000			
		1760.000	4.000	5.000
6.000	2095.369			
10.051	3510.000			
		3520.000	8.000	10.000
		[4840.000]	[11.000]	
13.899	4854.000			
15.076	5265.000			
		5280.000	12.000	15.000

Projected Beat Rate: 10 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = equal temperament

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.228			
		440.000	1.000	
5.011	1750.000			
		1760.000	4.000	10.000
10.022	3500.000			
		3520.000	8.000	20.000
		[4840.000]	[11.000]	
13.939	4868.000			
15.033	5250.000			
		5280.000	12.000	30.000

Projected Beat Rate: 25 Hz
Method of Controlling Beat Rate: shifted partials
Tuning = equal temperament

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.228			
		440.000	1.000	
4.968	1735.000			
		1760.000	4.000	25.000
9.936	3470.000			
		3520.000	8.000	
		4840.000	11.000	
14.060	4910.000			
14.904	5205.000			
		5280.000	12.000	

Projected Beat Rate: -13.9 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	349.228			
		440.000	1.000	
5.000	1746.141			
		1760.000	4.000	13.859
10.000	3492.282			
		3520.000	8.000	27.718
		[4840.000]	[11.000]	
14.000	4889.195			
15.000	5238.424			
		5280.000	12.000	41.576

Projected Beat Rate: 2 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	352.400			
		440.000	1.000	
		1760.000	4.000	
5.000	1762.000			2.000
		3520.000	8.000	
10.000	3524.000			4.000
		5280.000	12.000	
15.000	5286.000			6.000

Projected Beat Rate: 5 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	353.000			
		440.000	1.000	
		1760.000	4.000	
5.000	1765.000			5.000
		3520.000	8.000	
10.000	3530.000			10.000
		5280.000	12.000	
15.000	5295.000			15.000

Projected Beat Rate: 10 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	354.000			
		440.000	1.000	
		1760.000	4.000	
5.000	1770.000			10.000
		3520.000	8.000	
10.000	3540.000			20.000
		5280.000	12.000	
15.000	5310.000			30.000

Projected Beat Rate: 25 Hz
Method of Controlling Beat Rate: Retuned interval

LOWER NOTE		UPPER NOTE		
PARTIAL #	FREQ (HZ)	FREQ (HZ)	PARTIAL #	BEAT RATE
1.000	357.000			
		440.000	1.000	
		1760.000	4.000	
5.000	1785.000			25.000
		3520.000	8.000	
10.000	3570.000			
11.000	3927.000			
		[3960.000]	[9.000]	
		5280.000	12.000	
15.000	5355.000			
16.000	5712.000			
		[5720.000]	[13.000]	

References

Ayres, Thomas, Susan Aeschbach, and Edward L. Walker (1980). Psychoacoustic and experiential determinants of tonal consonance. *Journal of Auditory Research* 20:31-42.

Barbour, J. Murray (1953). *Tuning and Temperament: A Historical Survey*, 2nd ed. East Lansing: Michigan State College.

Beeckman, Isaac (1604-1634/1939-1953). *Journal tenu par Isaac Beeckman de 1604 à 1634* (4 vols). Ed. C. de. Waard. Den Haag.

Biock, Hans-Reinhard (1975). *Zur Intonationsbeurteilung kontextbezogener sukzessiver Intervalle*. Kölner Beitrage zur Musikforschung (Heinrich Hüschen, ed.), vol. 82 (Akustische Reihe Vol. 6). Regensburg: Gustav Bosse.

Blackwood, Easley (1985). *The Structure of Recognizable Diatonic Tunings*. Princeton: Princeton Univ.

de Boer, E. (1956). Pitch of inharmonic signals. *Nature* (London) 178:535-536.

de Boer, E. (1976). On the "residue" and auditory pitch perception, in W. D. Keidel and W. D. Neff (eds.), *Handbook of Sensory Physiology*, vol. V(3). Vienna: Springer-Verlag.

Boomsliter, P., and Creel, W. (1961). The long pattern hypothesis in harmony and hearing. *Journal of Music Theory* 5:2-31.

Boomsliter, P., and Creel, W. (1963). Extended reference—an unrecognized dynamic in melody. *Journal of Music Theory* 7: 2-22.

Brues, A. M. (1927). The fusion of non-musical intervals. *American Journal of Psychology* 38:624-638.

Burns, E. M. and Ward, W. D. (1978). Categorical perception—phenomenon or epiphenomenon: Evidence from experiments in the perception of musical intervals. *Journal of the Acoustical Society of America* 63:456-468.

Burns, E. and Ward, W. D. (1982). Intervals, scales and tuning. In D. Deutsch (ed.), *The Psychology of Music*. New York: Academic, 1982.

Cazden, Norman (1962). Sensory theories of musical consonance. *Journal of Aesthetics and Art Criticism* 20(3):301-319.

Cohen, Elizabeth A. (1984). Some effects of inharmonic partials on interval perception. *Music Perception* 1(3):276-295.

Cohen, H. F. (1984). *Quantifying Music: The Science of Music at the First Stage of the Scientific Revolution, 1580–1650.* Dordrecht, Netherlands: Reidel.

Cuddy, Lola L., A. J. Cohen, and D. J. K. Mewhort (1981). Perception of structure in short melodic sequences. *Journal of Experimental Psychology: Human Perception and Performance* 7:869-882.

Danner, Gregory (1985). The use of acoustic measures of dissonance to characterize pitch-class sets. *Music Perception* 3(1):103-122.

DeWitt, Lucinda A., and Robert G. Crowder (1987). Tonal fusion of consonant musical intervals: The oomph in Stumpf. *Perception and Psychophysics* 41(1):73-84.

Eggen, J. H., and A. J. M. Houtsma (1986). The pitch perception of bell sounds. *IPO Annual Progress Report* 21:15-23.

Esbroeck, Guy van, and Franz Montfort Jr. (1946). *Qu'est-ce que jouer juste?* Brussels: Manteau.

Feth, Lawrence L. (1974). Frequency discrimination of complex periodic tones. *Perception and Psychophysics* 15(2): 375-378.

Feth, Lawrence L., Honor O'Malley, and J. W. Ramsey, Jr. (1982). Pitch of unresolved, two-component complex tones. *Journal of the Acoustical Society of America* 72(5):1403-1412.

Fritz, Barthold (1780) *Anweisung wie man Claviere, Clavecins, und Orgeln, nach einer mechanischen Art, in allen zwölf Tönen gleich rein stimmen könne,...*, 3rd ed. Leipzig.

Galilei, Vincenzo (1589). *Discorso intorno alle opere de Gioseffo Zarlino et altri importanti particolari attenenti alla musica.* Venezia. (Facs. ed. Milano, 1933.)

Geary, J. M. (1980). Consonance and dissonance of pairs of inharmonic sounds. *Journal of the Acoustical Society of America* 67(5):1785-1789.

Geringer, John M. and Clifford K. Madsen (1981). Verbal and operant discrimination—preference for tone quality and intonation. *Psychology of Music* 9:26-30.

Goldstein, J. L. (1973). An optimum processor theory for the central formation of pitch of complex tones. *Journal of the Acoustical Society of America* 54: 1496-1516.

Greene, P. C. (1937). Violin intonation. *Journal of the Acoustical Society of America* 9:43-44.

Greer, R. D. (1970). The effect of timbre on brass-wind intonation. *Experimental Research in Music: Studies in the Psychology of Music* 6:65-94.

Hagerman, B., and Sundberg, J. (1980). Fundamental frequency adjustment in barbershop singing. *Quarterly Progress and Status Report* 1980(1):28-42. Speech Transmission Laboratory (Royal Institute of Technology, Stockholm).

Hall, D. E., and J. T. Hess (1984). Perception of musical interval tuning. *Music Perception* 2:166-195.

Helmholtz, Hermann von (1877/1954). *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*, 6th ed. (Braunschweig: Vieweg, 1913); translated as *On the Sensations of Tone as a Physiological Basis for the Theory of Music* by Alexander J. Ellis (1885). Rpt. New York: Dover, 1954.

Houtsma, A. J. M. (1968). Discrimination of frequency ratios. *Journal of the Acoustical Society of America* 44:383 (A).

Houtsma, A. J. M. and J. L. Goldstein (1971). Perception of musical intervals: Evidence for the central origin of the pitch of complex tones. Technical Report 484, Research Laboratory of electronics, MIT.

Houtsma, A. J. M., T. D. Rossing, and W. A. Wagenaar (1987). *Auditory Demonstrations* [phonodisc with booklet]. Institute for Perception (Eindhoven, Netherlands).

Husmann, H. (1953). *Vom Wesen der Konsonanz*. Heidelberg: Müller-Thiergarten-Verlag.

Hutchinson, W. and Knopoff, L. (1978). The Acoustic Component of Western Consonance. *Interface* 7:1-29.

Hutchinson, W. and Knopoff, L. (1979). The Acoustic Measurement of Consonance and Dissonance for Common Practice Triads. *Journal of Musicological Research* 3:6.

Huygens, Christiaan. *Oeuvres Complètes de Christiaan Huygens* (22 vols.), Den Haag, 1888-1950 (editorial committee).

Irvine, D. B. (1946). Toward a theory of intervals. *Journal of the Acoustical Society of America* 17:350-355.

Kameoka, A. and Kuriyagawa, M. (1969a). Consonance theory Part I: Consonance of dyads. *Journal of the Acoustical Society of America* 45:1451-1459.

Kameoka, A. and Kuriyagawa, M. (1969b). Consonance theory Part II: Consonance of complex tones and its calculation method. *Journal of the Acoustical Society of America* 45:1460-1469.

Keislar, Douglas (1987). History and principles of microtonal keyboards. *Computer Music Journal* 11(1):18-28.

Keislar, Douglas (1991). Six American composers on nonstandard tunings. *Perspectives of New Music* 29(1).

Kirnberger, Johann Philipp (1779), *Die Kunst des reinen Satzes in der Musik.* Berlin.

Kok, W. (1954) Experimental study of tuning problems. *Acustica* 4:229-230.

Kok, W. (1955). *Harmonische Orgels.* Technological University, Delft (Netherlands).

Kolinski, Mieczyslaw (1959). A new equidistant 12-tone temperament. *Journal of the American Musicological Society* 12:210-214.

Locke, S., and Kellar, L. (1973). Categorical perception in a non-linguistic mode. *Cortex* 9:355-368.

Loman, A. D. (1929). *De logische grondslagen der muziek.* Amsterdam: Alsbach.

Macran, H. S. (1902), trans. *The Harmonics* by Aristoxenus. London: Oxford.

Maher, Timothy F. (1980). A rigorous test of the proposition that musical intervals have different psychological effects. *American Journal of Psychology* 93(2):309-327.

Makeig, Scott (1979-80). The Affect of Musical Intervals. *Interval* 2(1):18-19, 24-26; 2(2-3):I12-I16; 2(4):11-13.

Makeig, Scott (1982). Affective versus analytic perception of musical intervals. In M. Clynes (ed.), *Music, Mind and Brain: the Neuropsychology of Music.* New York: Plenum.

Marcuse, Sybil (1975). *Musical Instruments: A Comprehensive Dictionary.* New York: Norton.

Martin, D. W., and W. D. Ward (1961). Subjective evaluation of musical scale temperament in pianos. *Journal of the Acoustical Society of America* 33:582-585.

Mason, J. A. (1960). Comparison of solo and ensemble performances with reference to Pythagorean, Just, and equi-tempered intonations. *Journal of Research of Music Education* 8:31-38.

Mathews, Max and John Pierce (1980). Harmony and nonharmonic partials. *Journal of the Acoustical Society of America* 68(5):1252-1257.

Mathews, Max and John Pierce (1989). The Bohlen-Pierce scale. In Mathews and Pierce (eds.), *Current Directions in Computer Music Research.* Cambridge, Massachusetts: MIT Press.

Mersenne, Marin (1636/37). *Harmonie universelle.* Paris.

Montani, A. (1947). Outline of a physiological theory of musical consonance. *Riv. Musicale Ital.* 49:168-176.

Moore, Brian, Brian Glasberg, and Robert Peters (1985). Relative dominance of individual partials in determining the pitch of complex tones. *Journal of the Acoustical Society of America* 77(5):1853-1860.

Moore, Brian, Brian Glasberg, and Robert Peters (1986). Thresholds for hearing mistuned partials as separate tones in harmonic complexes. *Journal of the Acoustical Society of America* 80(2):479-483.

Moore, Brian, Robert Peters, and Brian Glasberg (1985). Thresholds for the detection of inharmonicity in complex tones. *Journal of the Acoustical Society of America* 77(5):1861-1867.

Moran, H., and Pratt, C. C. (1926). Variability of judgments on musical intervals. *Journal of Experimental Psychology* 9:492-500.

Nordmark, Jan, and Lennart E. Fahlén (1988). Beat theories of musical consonance. *Quarterly Progress and Status Report* 1988(1):111-112. Speech Transmission Laboratory (Royal Institute of Technology, Stockholm).

Ogden, R. M. (1909). A contribution to the theory of tonal consonance. *Psychol. Bull.* 6: 297-303.

O'Keeffe, Vincent (1975). Psychophysical preference for harmonized musical passages in the just and equal tempered systems. *Perceptual and Motor Skills* 40:192-194.

Partch, Harry (1974). *Genesis of a Music*, 2nd ed. New York: Da Capo.

Pierce, John R. (1966). Attaining consonance in arbitrary scales. *Journal of the Acoustical Society of America* 40:249.

Pikler, Andrew G. (1966). History of Experiments on the Musical Interval Sense. *Journal of Music Theory.*

Piszczalski, Martin and Bernard A. Galler (1979). Predicting musical pitch from component frequency ratios. *Journal of the Acoustical Society of America* 66(3):710-720.

Platt, John R., and Ronald J. Racine (1985). Effect of frequency, timbre, experience, and feedback on musical tuning skills. *Perception and Psychophysics* 38(6):543-553.

Plomp, Reinier (1967). Beats of mistuned consonances. *Journal of the Acoustical Society of America* 42:462-474.

Plomp, Reinier (1987). General Introduction to "A Carillon of Major-Third Bells." *Music Perception* 4(3):243.

Plomp, R. and Levelt, W. (1965). Tonal Consonance and Critical Bandwidth. *Journal of the Acoustical Society of America* 38:548-560.

Pressing, Jeff (1980). Tuning System Design. *Interval* 2(2-3):I19-I24.

Rakowski, Andrej and Andrej Miskiewicz (1985). Deviations from equal temperament in tuning isolated musical intervals. *Archives of Acoustics* 10(2):95-104.

Rameau, Jean-Philippe (1722/1971). *Traité de l'harmonie*. Paris: Ballard. Trans. as *Treatise on Harmony* by Philip Gossett. New York: Dover.

Rameau, Jean-Philippe (1737). *Génération harmonique.* Paris: Prault fils. Trans. Deborah Hayes, diss. Stanford Univ. (1968).

Rasch, Rudolf (1983). Description of regular twelve-tone tunings. *Journal of the Acoustical Society of America* 73(3):1023-1035.

Rasch, Rudolf (1984). Theory of Helmholtz-beat frequencies. *Music Perception* 1:308-322.

Rasch, Rudolf and Reinier Plomp (1982). The Perception of Musical Tones. In D. Deutsch (ed.), *The Psychology of Music.* Academic Press, New York.

Rasch, Rudolf and Vincent Heetvelt (1985). String inharmonicity and piano tuning. *Music Perception* 3(2):171-190.

Resnick, L. (1981). Psychophysical basis for consonant musical intervals. *American Journal of Physics* 49:579-580.

Riesz, R. R. (1928). Differential intensity sensitivity of the ear for pure tones. *Physical Review* 31:867-875.

Roberts, Linda (1983). Consonance and dissonance: a review of the literature. Unpublished manuscript.

Roberts, Linda (1986). Consonance judgments of musical chords by musicians and untrained listeners. *Acustica* 62(2):163-171.

Roberts, Linda and Max Mathews (1984). Intonation sensitivity for traditional and nontraditional chords. *Journal of the Acoustical Society of America* 75:952-959.

Salzberg, Rita S. (1980). The effects of visual stimulus and instruction on intonation accuracy of string instrumentalists. *Psychology of Music* 8(2):42-49.

Sauveur, Joseph (1700/01). *Principes d'acoustique et de musique.* In *Histoire de l'Académie Royale des Sciences*, Paris. Rpt. Geneva: Minkoff, 1973.

Scheibler, H. (1834). *Der physikalische und musikalische Tonmesser.* Essen: Bädeker.

Schlick, Arnolt (1511). *Spiegel der Orgelmacher und Organisten.* Mainz. Facs. ed. with English trans. by E. B. Barber. Buren (Netherlands), 1980.

Schottstaedt, Bill (1983). Pla: a composer's idea of a language. *Computer Music Journal* 7(1):11-20.

Schouten, J. F. (1938). The perception of subjective tones. *Proceedings of the Koninklijke Nederlandse Akademie van Wetenschappen* 41:1083-1093.

Shankland, R. S. and J. W. Coltman (1939). The departure of the overtones of a vibrating wire from a true harmonic series. *Journal of the Acoustical Society of America* 10:161-166.

Shepard, R. and Jordan, D. (1984). Auditory illusions demonstrating that tones are assimilated to an internalized musical scale. *Science* 226:1333-1334.

Shuck, O. H,. and R. W. Young (1943). Observations on the vibrations of piano strings. *Journal of the Acoustical Society of America* 15:1-11.

Siegel, J. A., and Siegel, W. (1977a). Absolute identification of notes and intervals by musicians. *Perception and Psychophysics* 21:143-152.

Siegel, J. A., and Siegel, W. (1977b). Categorical perception of tonal intervals: Musicians can't tell sharp from flat. *Perception and Psychophysics* 21:399-407.

Slaymaker, F. H. (1970). Chords from tones having stretched partials. *Journal of the Acoustical Society of America* 47:1569-1571.

Smith, Robert (1749). *Harmonics, or the Philosophy of Musical Sounds.* Cambridge.

Sorge, Georg Andreas (1745-47). *Vorgemach der musicalischen Composition.* Lobenstein: Verlag des Autoris.

Stumpf, C. (1898). Konsonanz und Dissonanz. *Beitr. Akust. Musikwiss.* 1:1-108.

Sundberg, Johan (1982). Perception of singing. In D. Deutsch (ed.), *The Psychology of Music.* New York: Academic, 1982.

Szende, O. (1977). *Intervallic Hearing: Its Nature and Pedagogy.* Trans. Mária Baranyai, ed. Eva Pálmai. Budapest: Akadémiai Kiadó.

Terhardt, E. (1974a). Pitch, consonance, and harmony. *Journal of the Acoustical Society of America* 55:1061-1069.

Terhardt, E. (1974b) On the perception of period sound fluctuations (roughness). *Acustica* 20:215-224.

Terhardt, Ernst (1977). The two-component theory of musical consonance. In E. F. Evans and E. P. Wilson (eds.), *Psychophysics and Physiology of Hearing.* London: Academic Press.

Terhardt, E. and Zick, M. (1975). Evaluation of the tempered tone scale in normal, stretched, and contracted intonation. *Acustica* 32:268-274.

Viemeister, Neal and Deborah Fantini (1987). Discrimination of frequency ratios. In W. Yost and C. Watson (eds.), *Auditory Processing of Complex Sounds.* Hillsdale, New York: Erlbaum.

Vos, Joos (1982). The perception of pure and mistuned musical fifths and major thirds: Thresholds for discrimination, beats, and identification. *Perception and Psychophysics* 32:297-313.

Vos, Joos (1984). Spectral effects in the perception of pure and tempered intervals: Discrimination and beats. *Perception and Psychophysics* 35(2):173-185.

Vos, Joos (1986). Purity ratings of tempered fifths and major thirds. *Music Perception* 3:251-257.

Vos, Joos (1987). *The perception of pure and tempered musical intervals.* Diss., Univ. of Leiden.

Vos, Joos and Ben G. van Vianen (1985a). Thresholds for discrimination between pure and tempered intervals: The relevance of nearly coinciding harmonics. *Journal of the Acoustical Society of America* 77(1):176-187.

Vos, Joos and Ben G. van Vianen (1985b). The effect of fundamental frequency on the discriminability between pure and tempered fifths and major thirds. *Perception and Psychophysics* 37:507-514.

Vos, Joos and B. G. van Vianen (1986). Thresholds for discrimination between pure and tempered melodic unisons, major thirds, and fifths. Report No. IZF 1986-3, Institute for Perception TNO (Netherlands).

Wapnick, Joel, Gary Bourassa, and Joanne Sampson (1982). The perception of tonal intervals in isolation and in melodic context. *Psychomusicology* 2(1):21-37.

Ward, W. Dixon (1970). Musical perception. In J. Tobias (ed.), *Foundations of modern auditory theory.* New York: Academic Press.

Ward, W. D., and D. W. Martin (1961). Psychophysical comparisons of just tuning and equal temperament in sequences of individual tones. *Journal of the Acoustical Society of America* 33:586-588.

Wightman. F. L. (1973a). Pitch and stimulus fine structure. *Journal of the Acoustical Society of America* 54:397-406.

Wightman. F. L. (1973b). The pattern-transformation model of pitch. *Journal of the Acoustical Society of America* 54:407-416.

Wundt, W. (1880). *Grundzüge der physiologischen Psychologie*, 2nd ed. Leipzig: Verlag W. Engelmann.

Wyshnegradsky, Ivan (1972). Ultrachromatisme et les espaces non octaviants. *Revue Musicale* 290-91:73-130.

Young, R. W. (1952). Inharmonicity of plain wire piano strings. *Journal of the Acoustical Society of America* 24:267-273.

Zatorre, Robert J. (1983). Category-boundary effects and speeded sorting with a harmonic musical-interval continuum: evidence for dual processing. *Journal of Experimental Psychology: Human Perception and Performance* 9(5):739-752.

Zatorre, Robert J. and Andrea R. Halpern (1979). Identification, discrimination, and selective adaptation of simultaneous musical intervals. *Perception and Psychophysics* 26(5):384-495.

Zwicker, E., C. Flottorp, and S. S. Stevens (1957). Critical bandwidth in loudness summation. *Journal of the Acoustical Society of America* 29:548-557.

Additional Reading

Bharucha, Jamshed J., and Keiko Stoeckig (1987). Priming of chords: spreading activation or overlapping frequency spectra? *Perception and Psychophysics* 41(6):519-524.

Boomsliter, Paul C. and W. Creel (1962). Ratio relationships in melody. *Journal of the Acoustical Society of America* 34:1276.

Butler, J. W., and P. G. Daston (1968). Musical consonance as musical preference: a cross-cultural study. *Journal of General Psychology* 79: 129-142.

Buttram, Joe B. (1969). Perception of musical intervals. *Perceptual and Motor Skills* 28:391-394.

Cumar, Raffaele and Lucian Frusi (1978). *Prospettive sperimentali e metodi di ricerca per lo studio degli intervalli musicali.* Pisa: Giardini.

Cuddy, Lola L. (1982). On hearing pattern in melody. *Psychology of Music* 10(1):2-10.

Edmonds, E. M., and M. E. Smith (1923). The phenomenological description of musical intervals. *American Journal of Psychology* 34: 287-291.

Frances, R. (1958). *La Perception de la Musique*. Paris: Librairie Philosophique J. Vrin.

Fyk, Janina (1982a). Perception of mistuned intervals in musical context. *Psychology of Music*, special issue (Proceedings of the Ninth International Seminar on Research in Music Education):36-41.

Fyk, Janina (1982b). Tolerance of intonation deviation in melodic intervals in listeners of different musical training. *Archives of Acoustics* 7(1):13-28.

Geringer, John, and Anne C. Witt (1985). An investigation of tuning performance and perception of string instrumentalists. *Bulletin, Council for Research in Music Education* 85:90-101. [Tenth International Seminar: Research in music education (1984, Victoria, Canada).]

Guernsey, M. (1928). The role of consonance and dissonance in music. *American Journal of Psychology* 40:173-204.

Gut, Serge (1976). La notion de consonance chez les théoriciens du moyen age. *Acta Musicologica* 48:20.

Hall, Donald (1973). The objective measurement of goodness-of-fit for tunings and temperaments. *Journal of Music Theory* 17:275-289.

Hesse, Horst-Peter (1982). The judgment of musical intervals. In Manfred (ed.), *Music, Mind, and Brain: The Neuropsychology of Music*. New York: Plenum.

Hinton, D. (1982) *The Effect of Different Musical Timbres on Students' Identification of Melodic Intervals*. Ed.D. thesis, University of British Columbia. Abstract in *Dissertation Abstracts International* A 45(5), Nov. 1984.

Houtsma, A. J. M. and J. L. Goldstein (1972). The central origin of the pitch of complex tones: Evidence from musical interval recognition. *Journal of the Acoustical Society of America* 51(2):520-529.

Ives, Charles (1925). Some "quarter-tone" impressions. Reprinted in Howard Boatwright (ed.), *Essays Before a Sonata, The Majority, and Other Writings by Charles Ives*. New York: Norton, 1961.

Iwamiya, Shin-ichiro, Kyohei Kosugi, and Otoichi Kitamura (1983). Perceived principal pitch of vibrato tones. *Journal of the Acoustical Society of Japan* (E) 4(2):73ff.

Jordan, Daniel S. (1987). Influence of the diatonic tonal hierarchy at microtonal intervals. *Perception and Psychophysics* 41(6):482-488.

Kellar, Lucia A., and Thomas G. Bever (1980). Hemispheric asymmetries in the perception of musical intervals as a function of musical experience and family handedness background. *Brain and Language* 10:24-38.

Killam, R. N., Lorton, P. V., and Schubert, E. D. (1975). Interval recognition: Identification of harmonic and melodic intervals. *Journal of Music Theory* 19:212-234.

Levelt, W., Van de Geer, J., and Plomp, R. (1966). Triadic comparisons of musical intervals. *British Journal of Mathematical and Statistical Psychology* 19:163-179.

Lundin, R. W. (1947). Toward a cultural theory of consonance. *Journal of Psychology* 23:45-49.

Maher, Timothy F. and D. E. Berlyne (1982). Verbal and exploratory responses to melodic musical intervals. *Psychology of Music* 10(1):11-27.

McAdams, Stephen (1984). *Spectral Fusion, Spectral Parsing and the Formation of Auditory Images*. Diss., Stanford University.

Meyer, M. F. (1962). Listeners can be seduced to perceive the paradoxic ratio 51:87 as either one or another truly melodic interval. *Journal of the Acoustical Society of America* 34:1277.

Meyer, Max (1903). Experimental studies in the psychology of music. *American Journal of Psychology* 14:456-478.

Nickerson, J. F. (1949). Intonation of solo and ensemble performance of the same melody. *Journal of the Acoustical Society of America* 21:593.

Noorden, Leon van (1982). Two channel pitch perception. In Manfred Clynes (ed.), *Music, Mind, and Brain: The Neuropsychology of Music.* New York: Plenum.

Pierce, John R., and Max Mathews (1969). Control of consonance and dissonance with nonharmonic overtones. In Heinz von Forster and James Beauchamp (eds.), *Music by Computers*. New York: Wiley.

Pikler, A. G., and J. D. Harris (1961). Measurement of the musical interval sense. *Journal of the Acoustical Society of America* 23:862.

Plomp, Reinier (1976). *Aspects of Tone Sensation: A Psychophysical Study.* Academic Press, London.

Plomp, R., W. A. Wagenaar, and A. M. Mimpen (1973). Musical interval recognition with simultaneous tones. *Acustica* 29:101-109.

Rakowski, Andrej (1976). Tuning of isolated musical intervals. *Journal of the Acoustical Society of America* 59:S50(A).

Rakowski, Andrej (1985). The perception of musical intervals by music students. *Council for Research in Music Education Bulletin* 85:175-186.

Rasch, Rudolf (1983). Perception of melodic and harmonic intonation of two-part musical fragments. *Journal of the Acoustical Society of America* 74(S1):S22.

Schügerl, K. (1970). On the perception of concords. In R. Plomp and G. Smoorenburg (eds.), *Frequency analysis and periodicity detection in hearing.* Leiden: Sijthoff.

Seashore, C.E. (1938). *Psychology of Music.* New York: McGraw-Hill.

Sergeant, Desmond, and J. David Boyle (1980). Contextual influences on pitch judgement. *Psychology of Music* 8(2):3-15.

Shackford, C. (1962a). Some aspects of perception. Part II. *Journal of Music Theory* 6:66-90.

Shackford, C. (1962b). Some aspects of perception. Part III. *Journal of Music Theory* 6:295-303.

Siegel, Jane A. (1974). Sensory and verbal coding strategies in subjects with absolute pitch. *Journal of Experimental Psychology* 103(1):37-44.

Sundberg, Johan (1987). *The Science of the Singing Voice.* DeKalb, Illinois: Northern Illinois University Press.

Tanner, R. (1971). Le phénomène d'identification par tolérance; le préjugé de la gamme juste. *Acustica* 25:158.

Tanner, R. (1972). Le problème des deux tierces (Pythagore ou Zarlino): Sa solution psycharithmétique. *Acustica* 27:335.

Tanner, R. (1976). La fonction et la justesse mélodiques des intervalles. *Acustica* 34(5):259.

Tenney, J. (1988). *A History of "Consonance" and "Dissonance."* New York: Gordon and Breach.

Terhardt, Ernst (1984). The concept of musical consonance: a link between music and psychoacoustics. *Music Perception* 1(3):276-295.

Terhardt, Ernst (1988). Intonation of tone scales: psychoacoustic considerations. *Archives of Acoustics* 13(1-2):147-156.

Ternström, Sten, and Johan Sundberg (1988). Intonation precision of choirsingers. *Journal of the Acoustical Society of America* 84(1):59-69.

Voigt, Wolfgang (1985). *Dissonanz und Klangfarbe: Instrumentationsgeschichtliche und experimentelle Untersuchungen.* [*Orpheus*, vol. 41.] Bonn: Verlag für systematische Musikwissenschaft.

Walker, Robert (1990). *Musical beliefs: Psychoacoustic, Mythical, and Educational Perspectives.* New York: Teachers College Press.

Warren, R. M. (1978). Complex beats. *Journal of the Acoustical Society of America* 64(S1): S38.

Watkins, Anthony J. (1985). Scale, key, and contour in the discrimination of tuned and mistuned approximations to melody. *Perception and Psychophysics* 37(4):275-285.

Wier, C. C., W. Jesteadt, and D.M. Green (1977). Frequency discrimination as a function of frequency and sensation level. *Journal of the Acoustical Society of America* 61:178-184.