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THE ACQUISITION OF MUSICAL PERCEPTS WITH A NEW SCALE

by

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This is the text of a proposal submitted to the National Science Foundation for research to study long-term learning of high-level musical concepts using a new musical scale.

The Acquisition of Musical Percepts with a New Scale

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Abstract

New musical scales, the 3:5:7:9 Bohlen-Pierce scales, have recently been invented. We propose to use these scales as a vehicle 1) to study long-term learning of high-level musical concepts and 2) as an example of the creation of a new musical language with a rich and partly specifiable harmonic structure. The Bohlen-Pierce scales have already been shown to have perceptible low-level sensory properties such as consonant and dissonant chords and chords with high intonational sensitivity (subjects can hear whether or not they are in tune). They also have a rich structure of keys which is comparable in complexity to those of the diatonic scale, but completely different in detail. We believe these scales are unique stimuli with which to study learning of higher-level concepts because subjects exist who have no previous exposure to the scales.

We plan to give subjects extensive ear training until they can reliably transcribe music played in the new scales. Then we will test the subjects on concepts such as probe-tone profiles for the different tones in a key, similarity judgements for the 13 different keys which exist in these scales, similarities of chords (including a chord and its inversions), and sensitivities to cadences and other chord progressions.

One group of subjects will have ear training also designed to teach a specified set of concepts. A comparison group will be taught with random sequences of Bohlen-Pierce pitches. We predict that these two groups will have very different concepts.

1. PROJECT OVERVIEW

This research is concerned with 1) an exploration of learning in the context of a new musical language based on a new scale and 2) techniques for designing new musical scales which have interesting perceptual and musical properties.

One of the most intractable questions in the domain of language learning is separating the components of a language which are learned from those which are an inherent part of the structure of the organism. In the domain of natural languages, this question must be tackled very indirectly because languages are learned at such young ages that experiments are difficult to run. Also, verbal languages are complicated by semantics. Musical languages are probably less complicated than verbal languages–at least their complications are in a different arena. However, studies of learning of music which is based on diatonic scales and harmonies are equally frustrated by the fact that almost everyone has been exposed to this music from childhood. Consequently, a new musical scale which has theoretical properties that seem as complex and rich as those of the diatonic scale offers a unique opportunity to study the learning of a new musical language.

New scales are also of great interest to musicians. Contemporary composers have a great desire and need for new media, partly to express their new ideas in new ways and partly because it is hard to use a medium as successful and old as diatonic scales without there being an uncomfortabe similarity to past music.

This project is primarily focused on the effects of long-term learning and the intentional creation of a system of associations for a set of new musical scales. The Bohlen-Pierce scales will be used as the primary experimental material. Sensory theory and experiment (mostly consonance and dissonance judgements) indicate that these scales may possess harmonic properties with a richness and complexity approaching that of the diatonic scales. However, so far it has not been possible to verify this optimistic hypothesis because no one has spent much time listening to Bohlen-Pierce-scale music.

In the study, we propose to take the direct approach of giving a group of subjects a substantial ear-training course in the Bohlen-Pierce scale until they can reliably transcribe music they hear in this scale. One group of subjects will also be "brainwashed" by the training material itself to associate together the pitches in a given key, chords and their inversions, and a pattern of adjacency amongst the different keys. The associations and concepts will be taught mainly by example. The subjects will learn to transcribe melodies written entirely in one key before going on to hear modulations into another key. A particular chord and its inversions will be used to harmonize a phrase of music before changing to some other chord for the next phrase. We believe that this teaching technique resembles the way ear training for diatonic music is typically done.

Another group of subjects will receive very neutral training based on sequences of random pitches from the scale.

We anticipate the training may take up to a year working an hour a day so that at the end of the training, the subjects will have been actively exposed to the new music for many hours. In order to carry out such an extensive study, an acoustic work station based on a personal computer plus a music synthesizer will be used both to train and test the subjects.

After training, we will measure the subjects' perceptual spaces for the new scale using techniques that have proven to be interesting for diatonic music such as the probe-tone technique, similarity judgements of passages in different keys, and clustering judgements for chords. In addition we plan to examine higher-level percepts such as the sense of finality produced by a generalized cadence to see if the cadence concept from diatonic music can be carried over to the new scale.

As a second control group we will present equivalent diatonic training material to a group of subjects who have not had formal musical training. Where appropriate, we will compare subjective performance for the new scale with that for an equivalent task in the diatonic scale.

2. PRIOR WORK RELATED TO THIS PROPOSAL

In the musical domain, the last century has been marked by an almost continuous and sometimes frantic search by contemporary composers for new musical materials with which to express themselves or perhaps with which to escape from the long shadow of the classic style that dominated much music since the 18th century. Notable examples include the "generalization" of the rules of harmony and modulation in the late romantic period (Wagner and Mahler), twelve-tone music (Schoenberg), completely structured music (Babbitt and Boulez), tape music using either concrete sounds (Schaeffer) or electronic sounds (Stockhausen), computer music starting about 1960 (Mathews and Hiller), synthesizer music (Moog) and finally digital-synthesizer music (Yamaha).

Most of these innovations were done by musicians with little help from the scientific community. Few bridges existed between science and music. With a few important exceptions (Helmholtz), the science of psychoacoustics had not reached a state of musical usefulness.

In the last thirty years much has changed. Auditory sensory psychology has made great advances. The physiology of the ear up to the eighth nerve is much better understood. The computer has provided an ideal laboratory tool for synthesizing musically interesting sounds, for measuring subjective reactions to these sounds, and for analyzing the subjective data. Even more encouraging are the working groups which span music and science in several countries (CCRMA at Stanford, IRCAM in Paris, KTH in Stockholm). The steadily increasing vigor of these groups in both the musical and scientific domains indicate that they are producing musically useful and scientifically interesting results.

An area of long time shared interest between scientists and musicians is that of scales. Philosophers and scientists have analyzed properties of various scales since Pythagoras.

This includes diverse aspects such as pitch perception, dissonance and consonance, melodic and harmonic compositional rules, keys, and modulation. Many of the attempts to create new materials involve new scales, the most notable example being 12-tone music, but other important examples being microtonal scales, scales based on subdivision of the octave into N equal steps, where N is a number other than 12, scales based on western folk music such as the pentatonic scale, and scales based on eastern folk music. In most of these innovations the composer has had to depend mainly on his personal ear and musical intuitions to choose the pitches on which he bases his music. The ear and genius of the composer must finally, of course, determine the value of his music, but psychoacoustics and technology may now be able to help him pursue promising pitch structures and avoid dead ends.

Before computer music synthesis, the use of new scales was painfully limited by the difficulty in finding instruments to play the new scales. Often the composer had to make his own instruments (Parch). This could be exciting, but it also could be a time-consuming diversion from composing. As a result, a great body of theoretical speculation exists about scales which have not been heard, either in music or in the laboratory.

The computer has changed this picture. Using pure digital synthesis, any possible scale can be easily synthesized with a precision better than the ear's discrimination. For real-time synthesis, some synthesizers can generate any equal-tempered scale (Synclavier), others are limited to twelve-tone scales (DX-7) though these limitations will soon be removed in more recent equipment (DX-7 II D).

2.1. Bohlen-Pierce Scales

The Bohlen-Pierce Scales are an example of the kind of scales which can now be created and studied. Existing work with these scales shows both the possibilities and limitations of our present psychoacoustic knowledge applied to musical scales.

The scales were described in 1978 (Bohlen) and rediscovered by Pierce after he heard the results of listening tests (Roberts and Mathews) with two nontraditional triads in which the frequency ratios of the notes in the triads are 3:5:7 and 5:7:9. Interest by musical psychologists in the ratios of the integers 3, 5, 7, and 9 is also old (Meyer, Farnsworth).

Roberts and Mathews postulated that in order to be interesting in a harmonic sense, it should be possible for a listener to hear clearly whether or not a chord is in tune. They devised a simple paradigm in which the listener rated the out-of-tuneness of the chord as a function of how far the middle tone of the chord deviated from a just (integer ratio) pitch. This subjective rating function they called intonational sensitivity. Both the 3:5:7 and 5:7:9 triads exhibit intonational sensitivities similar to those of the traditional major triad which has frequency ratios 4:5:6. This similarity is hardly surprising since all these chords involve simple integer frequency ratios and slightly out-of-tune chords will have pairs of overtones which beat with each other in either a pleasant or unpleasant way (Plomp).

Pierce noted that since the diatonic scale is based on the major triad, one should be able to construct a scale with the nontraditional chords. The scale should have perceptible harmonic properties which depend on the intonational sensitivities of the chords. However, most recent music uses an equal-tempered variant of the diatonic scale which makes possible a multiplicity of keys sharing subsets of twelve pitches. Modulations between keys provide a very rich structural basis for much of Western music since the 18th century.

Pierce sought an equivalent equal-tempered set of scales using the new chords. He combined the two chords into a tetrachord with 3:5:7:9 frequency ratios, proposed a scale in which the traditional 2:1 octave is replaced by a 3:1 tritave, this ratio corresponding to the ratio of the lowest to the highest notes in the tetrachord. He suggested that timbres having only odd partials would be appropriate for this scale since the ratio of the frequencies of the first two partials is 3:1 rather than the 2:1 in normal timbres.

Pierce then proposed an equal-tempered scale step of the 13th root of 3 which gives 13 equal steps spanning the tritave. The factor 3 is a consequence of the 3:1 tritave frequency ratio. The 13th root was simply found by trial as giving an excellent approximation to the tetrachord. 3 raised to the 6/13 power is within 7 musical cents of 5/3 and 3 raised to the 4/13 power is within 3 cents of 7/5. On the average, the equal tempered Bohlen-Pierce scale approximates the just Bohlen-Pierce scale twice as well as the twelve-tone equal-tempered scale approximates the diatonic scale.

A diagram of the equal tempered scale is shown on Fig 1. Also shown are a specific 9 tone subset of these thirteen tones that make up a Bohlen-Pierce scale in a particular key. The choice of the subset and selection of the number of tones (9) is important, but is much more arbitrary than choice of the equal-tempered pitches. With the selection made, a given key has "major" and "minor" chords as shown on Fig 1 where a major chord is defined as a 6-step interval below a 4-step interval. It is the equal- tempered approximation to a 3:5:7 chord. A minor chord is a 4- step interval below a 6 step interval.

Thirteen different keys exist in the Bohlen-Pierce scales, one starting on each step of the chromatic scale. A pattern of adjacency amongst these keys exists. Inspection of Fig 1 will show that two keys are adjacent to every key in that they share all tones in common with the key except one. Thus two simple modulations from a given key to adjacent keys are possible, one being called an upward (dominant) modulation, the other a downward (subdominant) modulation. A complex and potentially rich and interesting harmonic structure may exist for the Bohlen-Pierce scales. But can listeners hear and appreciate this structure? For example, can they hear a modulation from one key to another?

2.2. Evaluation of the scales

Being given a new scale that seems to be interesting, how does one find out if it is really useful? Certain properties of the Bohlen-Pierce scales have already been studied experimentally. But a more complete answer to this question is one of the main hoped-for results from this proposed research.

In the case of the Bohlen-Pierce scale, a number of listening tests were carried out which gave useful, but certainly not sufficient results, to validate the scale. Most of the clear results were consistent with known properties of the peripheral auditory system. Most of the unanswered questions involved long- term learning or more cognitive activities. One particular test, the judged similarity of chords and their inversions is particularly interesting with respect to this proposal. The evaluations can be summarized as follows:

1. Because of their construction, the major triads have strong intonational sensitivity.

2. The dissonance of all possible dyads that can be formed from the chromatic scale was judged. The results are consistent with a theory of dissonance based on beating partials (Plomp etc). A big range in dissonance from the most consonant to the most dissonant dyad was observed, indicating that dissonance would probably be a musically useful variable.

3. The dissonance of all possible triads that can be formed from the chromatic scale within one tritave was judged. The results are again consistent with a theory of dissonance based on beating partials. Again, a big and musically useful range of dissonance was observed. There was good agreement among observers on what are the most dissonant triads. There was less agreement on the identity of the most consonant triads. The major and minor triads were among the more consonant ones, but, alas, not exceptionally so.

4. The similarity of triads and their inversions (inversions about the tritave) were judged. This is a very provocative experiment. In order to harmonize an upper melodic voice with major chords, inversions of certain chords must be used. Consequently it is important that people hear triads and their inversions as similar. In the diatonic scale, inversions of traditional diatonic chords must also be used for the same reason. The experiment was designed to compare traditional chord judgements and Bohlen-Pierce-scale chord judgements. Two groups of subjects, one musically trained, the other without special musical training, judged both kinds of chords. The results can be summarized as follows:

a) Neither group of subjects found Bohlen-Pierce-scale chords and their inversions to be similar. Instead their similarity ratings were correlated to the average pitch of the chords.

b) The musically trained subjects rated traditional chords and their inversions as similar. The musically untrained subjects DID NOT. Instead their similarity ratings were again correlated with average pitch of the chords.

These results are consistent with the hypothesis that these similarity judgements are learned and that with training subjects might judge Pierce scale chords and music quite differently.

Several experiments were not done because preliminary informal listening tests indicated that they were unlikely to yield interesting results. These include:

1. Probe-tone experiments (Krumhansl and Shepard). These have proven to be dramatically successful in establishing a hierarchy of tonal functions in diatonic music. None of our subjects achieved enough experience with the Bohlen-Pierce scale to be likely to produce interesting patterns of probe-tone ratings.

2. Judgements of distances between keys (Krumhansl, Bharucha, Castellano). Again, although there is a strong theoretical structure relating Bohlen-Pierce scale keys, it seemed unlikely that the relatively inexperienced subjects would be able to perceive the structure. In short musical examples, some subjects thought they could hear the places where modulations occur, but they had little confidence in their perception.

3. Judgements of perceived finality of cadences (Mathews, Pierce). Earlier work with stretched scales showed that although stretching does not prevent subjects from perceiving the key of a given passage, it does destroy all sense of finality in a cadence. Thus, stretching destroys a very important higher level judgement. So far no Bohlen-Pierce scale cadences have been found which give much of a sense of finality.

4. Identification of key and judgement of modulation. Several short example compositions were made in the Bohlen-Pierce scale which involved key changes and modulations. Again, although one could believe that the key change could be heard, the perception was so tenuous that no systematic studies were done.

In addition to formal experiments, one composition (Appleton) and a number of exercises in well-known styles–canons, fugues, minuets–were composed with the Bohlen-Pierce scale. The composition is expressive and the exercises are interesting. But most listeners felt that they did not have sufficient experience hearing the scale to appreciate properly any subtleties in the music.

How can one hope to proceed further in evaluating a new scale? The answer would appear to center around getting a group of subjects who have spent enough time being exposed to various samples from the new scale to have built up some long-term memories and concepts about the scale. This proposal is a reasonably straightforward way to create some experienced subjects by an ear-training course.

3. PROJECT DESCRIPTION

The work we have described demonstrates that for new scales, present psychoacoustic techniques are effective in studying low- level perceptions such as the acoustic dissonance of chords with unusual patterns of pitches or partials. The results supply necessary, but not sufficient, conditions for the understanding and creation of scales that are musically useful.

Many musically important perceptions involve higher level cognitive processes, which are much harder to study in the laboratory. In particular, effects which depend on long-term learning must be dealt with in very different experimental ways. We propose here to study directly some effects of long-term exposure to new musical materials by the straightforward method of actively exposing subjects to these materials for many hours and then testing for changes in some of their higher-level concepts. The Bohlen-Pierce scales will be used as the test material.

The principal active exposure will consist of an abbreviated ear- training course which will attempt to train the subjects to a reasonable degree of proficiency in transcribing melodic and harmonic passages of music written in the Bohlen-Pierce scales. Subjects will then be evaluated by probe-tone procedures and by other methods that give interesting results for diatonic music. We will test several groups of subjects who have been given different training experiences and who, we believe, will have different concepts.

3.1. Ear training and brainwashing

Traditional ear training is by no means an unbiased teaching process in which musicians learn to identify and transcribe arbitrary sequences of pitches. Instead, the training material consists of fragments that either are or could be music. The fragments obey the compositional rules for the style from which they have been chosen. As a result, the student not only learns to transcribe notes, but he learns the rules of the style so that he can hear whether or not a given piece of music follows these rules. We conjecture that he may even learn to like music that follows the rules and reject music that does not.

We propose to teach one group of subjects a set of reasonable, but rationally and arbitrarily created, rules governing music written in the Bohlen-Pierce scales. The details of the rules will have to be worked out as part of the experimental design, but we would hope to include:

1) The concept of a key as a specific subset of 9 pitches selected from the 13 possible chromatic pitches. Within a key the subject will be biased toward a hierarchy of tones with 0,3,and 10 being most important, these being the tonic of the key and of the tonics of the two most closely related other keys.

2) The importance of major and minor chords. Harmonic ear training will start from passages entirely harmonized with major and minor triads to emphasize their importance.

3) Cadences. Cadences will be used as the first systematic chord progressions to be studied. They will be used as terminators of passages and as terminators of a key followed by a modulation into another key. Before using cadences in the Bohlen-Pierce scale, we will have to create an appropriate definition for them. The definition of a generalized cadence is an interesting question in its own right. Understanding the sense of finality and resolution of a cadence is a question that has interested psychologists at least as far back as 1900 (Meyer, Lipps). Our initial conjectures as to the necessary properties are: a) the cadence consists of a relatively dissonant chord followed by a relatively consonant chord. b) the two chords should differ only slightly in the pitches of their notes in two voices. If possible two voices in the first chord should move to the second chord by one

step pitch changes with contrary motion. c) the chords should uniquely define the key in which they are written. These conjectures will have to be checked and possibly be revised in the course of the research.

4) Modulation will be used to establish the concept of adjacent keys. In the Bohlen-Pierce scale, one can modulate through all possible keys by single-step key changes in two ways–one always moving to the next higher key, the other to the next lower key. These changes correspond to modulating in the sharp direction or in the flat direction in diatonic music. We would try to build up a sense of adjacency of keys in the subjects by emphasizing single-step modulations.

5) Similarity of a chord and its inversions. This perception is both the most interesting and most problematic result of previous work with the Bohlen-Pierce scale. Considering the inversions of a chord as being other forms of that chord rather than being different chords is a great simplification introduced into harmony in the 17th century (Rameau). The use of inversions is essential to harmonize an upper voice with proper voice leading.

When asked about a diatonic chord and its inversions, a typical trained musician will say that he has no trouble distinguishing them, but that they are very similar because they "contain the same notes". Thus he makes the tacit assumption that pitchs an octave apart "are the same notes". The concept of a spiral perceptual space for pitch (Shepard) supports his assumption. It is not clear whether the tritave can serve the same function as the octave for this purpose. Attempts to construct a spiral pitch paradox using intervals other than the octave have had limited success.

In the ear training, we will try to teach the concept of identity of notes a tritave apart by various strategies–by giving the notes the same name, by using tritave transformations of phrases, by presenting subjects with chord progressions which emphasize a chord and its inversions, and by naming the inversions of a chord after the name of the root position chord.

3.2. Control subjects

To compare with the brainwashed subjects, we envision two control groups. The first will be trained with random pitches and chords which do not have any restrictions other than they must be pitches from the Bohlen-Pierce chromatic scale. We anticipate that these subjects will have a much more difficult task than the first group and we are not sure what level of performance they can achieve.

A second possible control group will be trained with diatonic scales but with simple rules that are as close an analog as possible to the Bohlen-Pierce rules heard by the first group of subjects. We would like to compare Bohlen-Pierce results with diatonic results, though we realize that the diatonic subjects cannot avoid being affected by their past experience.

3.3. Evaluation of the trained subjects

After training, some of the predictions we make about the subject's perceptual spaces are:

1. Probe-tone tests. We predict these will reveal significant probe-tone patterns with peaks on pitches 0, 3, and 10 only for the brainwashed subjects.

2. Relatedness of various keys. The subjects who have been systematically exposed to a sequence of modulations from one key to an adjacent key will show this pattern of adjacency in their perceptions. We hesitate to predict how the direction of modulation in the training will affect the pattern and we are most curious about this point. The control subjects will not show a pattern or will show a pattern simply based on pitch height.

3. Cadences. The subjects who have heard the cadences as terminators will give these chord progressions a higher finality rating than the randomly trained subjects.

4. Similarity of chords study. Here, we optimistically predict that the brainwashed subjects will perceive chords and their inversions as similar and the randomly trained subjects will not.

3.4. Equipment and procedure

The substantial amount of ear-training work is feasible only because of computer automatization. The course will be administered by an acoustic work station consisting of a personal computer plus a synthesizer. This equipment is sufficiently inexpensive so we can hope to run a reasonable number of subjects. Many ear training-programs, both commercial and experimental, already exist and have been shown to be effective. We expect to be able to adapt one of the existing programs to our needs. We will simplify the programs to focus the training on pitch effects; thus we hope to shorten the training time. We anticipate that several months training of an hour a day will bring the subjects to a state of proficiency to be able to transcribe simple, slowly presented, random melodies and chords to an almost 100 percent accuracy.

We hope the computer will do almost all the tedious work of training. It can compute the pitches of the training materal according to the rules we wish to study, read the subject's responses, score the subject's performance, pace the difficulty of the material to produce effective learning, and keep track of the subject's level of proficiency. If desired, it can measure the subject's reaction times.

An acoustic work station appears to be very useful for musical psychoacoustic research. In previous studies obtaining sufficiently accurate measurements of the subjective perceptions even of simple low-level quantities such as consonance or dissonance has been difficult. We hope the acoustic work stations will make practical an improvement in the accuracy of these measurements.

Measurement of the dissonance of chords is a good example. The method of paired comparisons is generally an accurate and neutral way to make this kind of assessment. However for the large number of chords about which we need information, paired comparisons is very tedious and expensive. Consequently, most of the results discussed above were obtained by asking the subjects to rate individual chords on a 1 to 10 numerical bipolar scale. This method inherently depends on the subject's memory to compare various chords and as a result a lot of "memory" noise is introduced into the data.

The acoustic work station can be programmed to implement a "computer sort board" technique in which a group of stimuli are represented by small circles on the terminal screen. By pointing to one of the circles with a mouse-pointer, the subject can hear the associated chord. Thus the subject can listen to the stimuli in any order he desires. He can make repeated comparisons between very similar stimuli and avoid spending time comparing very different stimuli. The subject can also move the circles till the arrangement on the screen corresponds to his subjective space for these chords. The screen can directly represent a two dimensional subjective space. We believe the computer sort board will provide a practical way to get more accurate opinions about the large number of chords that must be understood in music.

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