A New Approach to Digital Reverberation using Closed Waveguide Networks

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Abstract

This paper presents a new type of digital reverberator which is based on closed networks of intersecting *waveguides*. Each waveguide is a bi-directional delay line, of arbitrary length, and each intersection (of any number of waveguides) produces lossless signal scattering. By creating a closed network of waveguides, the total signal energy in the structure is preserved. A reverberator is constructed by introducing small losses in the network to achieve a desired reverberation time. The inputs and outputs can be chosen anywhere in the structure.

There are many reasons to construct reverberators (or any recursive digital filter for that matter) from lossless waveguide networks: (1) the scattering junctions can be made time varying without altering stored energy, (2) an "erector set" for lossless networks is obtained, allowing any number of branches to be fitted together in any desired configuration (with changes allowed in real time), (3) limit cycles and overflow oscillations are easily eliminated, regardless of interconnection, (4) an exact physical interpretation exists for all signals in the structure, and (5) the implementation is computationally efficient.

Introduction

Digital reverberation has been a standard post-processor for digital music synthesis since Schroeder's original papers in '61 and '62 [4,9]. The basic acoustics of reverberation and the design of concert halls has a long history covering many different approaches [1-24]. The basic goal of digital reverberation is to arrive at a digital filtering operation which simulates the effect of a good concert hall or "listening space" on the source sound. This goal is made difficult by the fact that typical listening spaces are inherently large-order systems which cannot be precisely simulated in real time using commonly available computing techniques. In architectural acoustics, the study of digital reverberation aids in the design of concert halls with good "acoustics." In digitally synthesized music, the reverberator is a part of the instrumental ensemble, providing a direct enrichment to the sound quality. This paper is concerned with the latter application; while there is no need for a detailed physical model, it is desired to capture all musically important qualities of natural reverberation.

Digital room simulation has been implemented by simulating specular reflection in actual concert-hall geometries or some approximation thereto [17,10,16,8,12]. It has been found that the diffusive scattering of sound by natural listening environments cannot be neglected in high-quality models [16]. However, models which accomodate diffusing reflections are beyond the reach of present computing power when applied to listening spaces of nominal size over the audio frequency band.

Another implementation of digital reverberation is to record an approximation to the impulse response between two spatial points in a real hall. The effect of the hall on sound between these two points can be very accurately simulated by convolving the measured impulse response with the desired source signal [35]. Again this leads to a prohibitive computational burden (two to three orders of magnitude out of real time for typical mainframes).

We can easily summarize the current state of high-quality digital reverberation: it is well understood, but too expensive to compute. It would seem that much progress is possible, because there is much detail in natural reverberation that is not important perceptually. For example, it has been noted that convolving an unreverberated sound with exponentially decaying white noise gives the best known artificial reverberation [16]. The key to a successful digital reverberator design is to replace the details of a quantitative physical model by simple computations which retain all important qualitative behavior.

Some basic building blocks of presently pervasive digital reverberators, introduced by Schroeder [6], include cascaded allpass networks, recursive and non-recursive comb filters, tapped delay lines, and lowpass filters. The early reflections can be exactly matched [8,16] for a fixed source and listener position using a tapped delay line, and the late reverberation can be qualitatively matched using a combination of allpass chains, comb filters, and lowpass filters [8,16]. Using a lowpass filter in the feedback loop of a comb filter is used to simulate air absorption and non-specular reflection [16]. This overall strategy for reverberation, or some subset of it, has been the basis for reverberation design at CCRMA for more than a decade [13,15]. While these elements do not provide reverberation on par with excellent natural listening environments, they do a good job at providing some of the most essential aspects of reverberation—especially for smoothly varying sounds at low reverberation levels.

A New Approach

The proposed technique is to build digital reverberators as closed networks of lossless digital waveguides [46,47]. Such a network can be constructed from any given number of multiplies, additions, and delay elements. The available multiplies and additions determine how many signal-scattering nodes can be implemented, and the available delay elements determine the total delay which can be distributed among the branches interconnecting the various nodes. By choosing the number of intersecting branches and scattering coefficients appropriately, multiplies can be eliminated completely [46,47] (the scattering coefficients are reduced to a power of two or a simple function of powers of two such as $3/4 = 1/2 + 1/4$). There are simple rules for connecting branches to nodes in a way which preserves signal energy. The design variables (in the lossless prototype) are branch-connection topology, delay lengths, and the characteristic impedances of the individual waveguides.

The lossless prototype reverberator is augmented by one or more simple loss factors (of the form $1 - 2^{-n}$, typically) to set the reverberation decay time to any desired value. That is, T_{60} (the time over which the reverberation decays 60 dB) is infinite in the prototype, but arbitrary in the final network. This decoupling of reverberation time from structural aspects incurs no loss of generality.

Some branches can be fixed to give specific early reflections, while other branches may be chosen to provide a desirable texture in the late reverberation. An optimality criterion for the late reverberation is to maximize homogeneity of the impulse response (make it look like exponentially decaying white noise). Waveguide networks allow every signal path to appear as a feedback branch around every other signal path. This connectivity richness facilitates development of dense late reverberation. Furthermore, the energy conserving properties of the waveguide networks can be maintained in the time-varying case [46,47]; this allows the breaking up of "patterns" in the late reverberation by subtly changing the reverberator in a way that does not modulate the reverberation decay profile. Finally, the explicit conservation of signal energy provides an easy way to completely suppress limit cycles and overflow oscillations.

Lossless Networks

A *network* is a closed interconnection of bi-directional signal paths. The signal paths are called *branches* and the interconnection points are called *nodes*. An example diagram of a simple network is shown in Fig. 1.

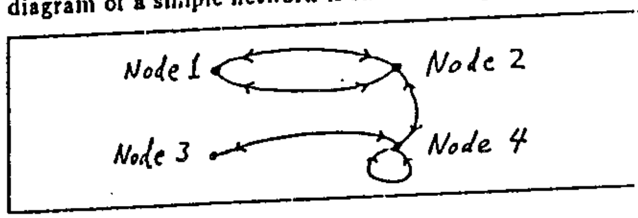

Figure 1. An example network diagram

Each signal path is *bi-directional*, meaning that in each branch there is a signal propagating in one direction and an independent signal propagating in the other direction. When a signal reaches a node, it is partially reflected back along the same branch, and partially transmitted into the other branches connected to the node. The relative strengths of the pieces of the "scattered" signal are determined by the relative characteristic impedances of the intersecting waveguides.

A *waveguide* is defined as a lossless bi-directional signal branch. In the simplest case, each branch in a waveguide network is merely a *bi-directional delay line*. The only computations in the network take place at the branch intersection points (nodes). More generally, a waveguide branch may contain any chain of cascade allpass filters. For practical reverberator design, we also introduce losses in the form of gain factors less than 1 and/or lowpass filters with frequency response strictly bounded by 1.

A lossless network preserves total stored signal energy. Energy is preserved if at each time instant the total energy stored in the network is the same as at any other time instant. The total energy at any time instant is found by summing the instantaneous power throughout the network. Each signal sample within the network contributes to instantaneous power. The instantaneous power of a stored sample is the squared amplitude times a scale factor, say g. If the signal is in units of "pressure," or equivalent, then $g = 1/Z$, where Z is the characteristic impedance of the waveguide medium. If the signal sample instead represents a "flow" variable, such as volume-velocity, then $g = Z$. In either case, the stored energy is a weighted sum of squared values of all samples stored in the digital network memory.

An N-*port* is a network in which N branches, called *ports*, leave the network to provide inputs and outputs. Figure 2 gives an example of a network with one port designated for input and two ports designated for output.

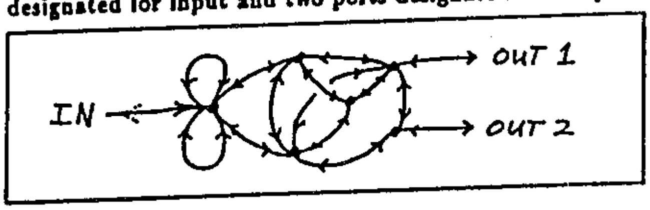

Figure 2. Example 3-port

Such a structure is suitable, for example, for providing stereo reverberation of a single channel of sound. Note, however, that really there are three inputs and three outputs. In an N-port, each branch leaving the network provides both an input and an output (because it is bi-directional). It is common practice in digital filtering applications [36] to use only one direction on a port branch as an input or output and ignore the other direction.

An N-port is lossless if at any time instant, the energy lost through the outputs (so far), equals the total energy supplied through the inputs (so far), plus the total stored energy. A lossless digital filter is obtained from a lossless N-port by using every port as both an input and output.

An N-port is *linear* if *superposition* holds. Superposition holds when the output in response to the sum of two input signals equals the sum of the outputs in response to each individual input signal. A network is linear if every N-port derived from it is linear. Only linear networks can be restricted to a large and well-understood class of energy conserving systems.

Lossless Scattering

Consider a parallel junction of N lossless waveguides of characteristic impedance Z_i (characteristic admittance $\Gamma_i = 1/Z_i$) as depicted in Fig. 3.

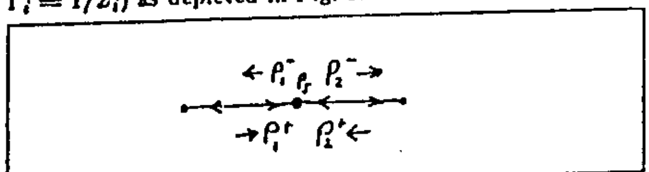

Figure 3. A junction of 2 waveguides.

If the incoming traveling pressure waves are denoted by P_i^+, $i = 1, \ldots, N$, the outgoing pressure waves are given by [46,47]

$$P_i^- = P_J - P_i^+ \qquad (1)$$

where P_J is the *resultant junction pressure*,

$$P_J = 2\frac{\sum_{i=1}^{N} \Gamma_i P_i^+}{\sum_{i=1}^{N} \Gamma_i} \qquad (2)$$

The series flow-junction is equivalent to the parallel pressure-junction. The series pressure-junction or the parallel flow-junction can be found by use of duality [46,47].

Equation (1) is a *computationally efficient* way to implement an N-port scattering junction. In the case $N = 2$, the well-known *one-multiplier lattice* filter section (minus its unit delay) is obtained immediately from (1). More generally, an N-way intersection requires N multiplies and $N - 1$ additions to obtain P_J, and one addition for each outgoing wave, for a total of N multiplies and $2N - 1$ additions.

Normalized Waves

We can normalize the pressure and flow variables by the square root of the characteristic impedance to obtain propagation waves in units of root power:

$$\begin{array}{ll} \tilde{P}_i^+ \triangleq P_i^+ \sqrt{\Gamma_i} & \tilde{P}_i^- \triangleq P_i^- \sqrt{\Gamma_i} \\ \tilde{U}_i^+ \triangleq U_i^+ \sqrt{Z_i} & \tilde{U}_i^- \triangleq U_i^- \sqrt{Z_i} \end{array} \qquad (3)$$

By restricting all waveguides to normalized waves, we obtain a generalization of the *normalized ladder* structure for digital filters [34,36,39]. The stored power in each section is unchanged if the characteristic impedance is changed (the pressure and flow variables are scaled in a complementary fashion). The use of normalized waves yields digital filter structures whose signal energy is not modulated by time-varying coefficients [46,47]. Signal power and "coefficient power" are decoupled.

Energy and Power

The *instantaneous power* in a waveguide containing instantaneous pressure P and flow U is defined as the product of pressure and flow:

$$\mathcal{P} = PU = (P^+ + P^-)(U^+ + U^-) = \mathcal{P}^+ + \mathcal{P}^- \qquad (4)$$

where

$$\begin{aligned} \mathcal{P}^+ &= P^+U^+ = Z(U^+)^2 = \Gamma(P^+)^2 \\ \mathcal{P}^- &= P^-U^- = -Z(U^-)^2 = -\Gamma(P^-)^2 \end{aligned} \qquad (5)$$

define the *right-going* and *left-going* power, respectively.

For the N-way waveguide junction, we have, using Kirchoff's node equations [26,27,46,47],

$$\mathcal{P}_J \triangleq \sum_{i=1}^{N} P_i U_i = \sum_{i=1}^{N} P_J U_i = P_J \sum_{i=1}^{N} U_i = 0 \qquad (6)$$

Thus, the N-way junction is *lossless;* no net power, active or reactive, flows into or away from the junction.

Quantization Effects

While the ideal waveguide junction is lossless, finite wordlength effects can make exactly lossless networks unrealizable. In fixed-point arithmetic, the product of two numbers requires more bits (in general) for exact representation than either of the multiplicands. If there is a feedback loop around a product, the number of bits needed to represent exactly a circulating signal grows without bound. Therefore, some sort of round-off rule must be included in a finite-precision implementation. The guaranteed absence of limit cycles and overflow oscillations is tantamount to ensuring that all finite-wordlength effects result in power *absorption* at each junction, and never power creation. If *magnitude truncation* is used on all outgoing waves, then limit cycles and overflow oscillations are suppressed [32]. Magnitude truncation results in greater losses than necessary to suppress quantization effects. More refined schemes are possible. In particular, by saving and accumulating the low-order half of each multiply at a junction, energy can be exactly preserved in spite of finite precision computations [46,47].

Conclusions

A construction was presented parametrizing all lossless linear networks. The construction is free of overflow oscillations and limit cycles, and a valuable energy decoupling property is obtained for time-varying networks. The added complexity relative to the best pre-existing recursive filter architectures is negligible. Therefore, these structures are likely to become standard in the near future.

In addition to implementing robust reverberation and digital filtering, waveguide structures can provide accurate models of coupled vibrating strings, wind instruments, reed instruments, and many other physical systems. In these applications, the signals propagating in a waveguide are coupled to a nonlinear "excitation element," such as a reed, bow, switching air-jet, or lips [41]. On the other side of the excitation element, another waveguide network can be used to model the player windway, bow assembly, or other interacting resonating system.

Appendix—Application Notes

In this appendix, some practical tips are listed for obtaining good reverberation.

- 1000 echoes per second is considered sufficiently dense for late reverberation [6, p. 219].
- Because air absorption increases with spatial frequency, lowpass filters should be used here and there in the waveguides to give qualitatively the correct relative time constants of decay versus frequency.
- The scattering coefficients can be randomly modulated to better approach an exponentially decaying white noise impulse response. This places the signal in a closed, randomly changing maze.
- Modulating the scattering coefficients with sinusoids, FM, or other complex waveforms, produces an appealing "undulating" reverberator. A physical analogy is time-varying absorption coefficients in the walls of a concert hall (plus magic tunneling of absorbed energy into vibrational modes elsewhere). Increasing the modulation frequency to audio rates causes a kind of "sideband" generation in the reverberated sound, corresponding to weak amplitude modulation. This can be understood by considering that as a signal is reflecting from a junction, the amplitude of the reflection is directly proportional to the reflection coefficient. Therefore, all scattering coefficient modulation (random or not) should occur at frequencies below audible AM modulation rates in order to avoid this effect. Random switching should occur at subaudio rates and employ proper audio fade-in/fade-out (e.g. 100ms fade time).
- The reverberation is generally less "colored" [6] when each output of the reverberator is taken to be a resultant pressure P_J at a junction of multiple waveguides.
- Setting branch delays to an interval of a Fibonacci series has given good results [45].
- Choosing equal reflection coefficients at a junction leads to an even energy distribution throughout the network. (A beam incident on the junction is scattered equally in all directions.) If the scattering coefficients are too disparate, "hot spots" or nearly lossless sub-paths may appear in the network. A uniform energy distribution helps to minimize the probability of overflow.
- Duality can be used to improve the dynamic range. In high-impedance waveguides, pressure tends to be large, while in low-impedance waveguides, flow is large (for a given signal power). Therefore, switching between pressure and flow for the propagating variable in each waveguide allows maximum use of the available dynamic range. This is the same thing as choosing the sign parameters in standard lattice filters [36]. Use of normalized waves eliminates the need to decide on pressure versus flow, and the signal level is always proportional to the root-power, independent of the characteristic impedance.
- Choosing a power of two for the number of branches of equal characteristic impedance intersecting at a node yields a multiplier-free realization.
- Physical analogies can give considerable insight into the operation of a waveguide network. For example, placing a finger on the midpoint of a freely vibrating string (making the tone rise one octave) is a physical analog to introducing a junction with rising reflection coefficient in the middle of a single waveguide with reflecting terminations. Another analogy is an optical waveguide containing beam-splitters in the form of partially silvered partitions. Visualizing more than two intersections is less easy; one example is to imagine waves along many taut wires of varying thickness welded together at a common point.
- It is possible to create the effect of moving walls by smoothly varying the delay-line lengths as well as the scattering coefficients. The basic technique for this is described in [44]. One way to avoid energy modulation is to effectively "slide" a junction along a line formed by two waveguides meeting at that junction. The delay lost by one waveguide is given to the other.
- A desirable reverberator property is that the density of resonant modes between any input/output point grow as the square of frequency [1]. The number of complex modes in any nondegenerate digital filter is equal to the total number of delay elements. Thus, a closed waveguide network always has as many complex resonances as there are stored samples. Finding exactly where the resonances are tuned as a function of the interconnection topology and scattering coefficients seems to be a difficult problem. Whenever a new input or output point is chosen, the zeros of transmission are changed. The poles of the point-to-point transfer function, however, are invariant under general conditions. Con-

sequently, if a realistic mode distribution is found, it can be used with a wide variety of input and output ports.

- Picking n_i in the branch loss factors $g_i = 1 - 2^{-n_i}$ as a function of T_{60} appears hard to do exactly. An approximate formula is to choose n_i so that $g_i^{T_{60}/D_i}$ close to 0.001, where D_i is the delay of the ith waveguide in seconds.
- Reverberation is realistically diffuse if the steady-state reverberator response to a sinusoidal input signal has a Rayleigh distributed amplitude throughout the delay elements of the reverberator [20]. Equivalently, the intensity is exponentially distributed, phase is uniformly distributed, and the real and imaginary parts of the sinusoidal response phasor are Gaussian distributed [20]. These distributions correspond physically to the excitation of many modes of vibration in the hall, yielding plane waves traveling in "all directions" with independent random phases. See also [1,7].
- Assuming a Rayleigh amplitude distribution allows calculation of probability of overflow as a function of the number of guard bits provided.

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