Fall 2018–2019
Music 320A
**Homework #2**
Partial Fraction Expansion
Theory part due 12/7/2018 by 23:59 pm
Lab part due 12/7/2018 by 23:59 pm

# Theory Problems

1. (50 pts) Consider the following first-order transfer functions:

$$L(z) = g \cdot \frac{1 + z^{-1}}{1 - pz^{-1}}$$

$$H(z) = g \cdot \frac{1 - z^{-1}}{1 - pz^{-1}}$$

$L(z)$ is a simple type of lowpass filter, and $H(z)$ is highpass. Assume $g$ and $p$ are real numbers.

(a) (5 pts) Give the *difference equations* for these filters in the (discrete) time domain.

(b) (5 pts) Find the gain $g$ that normalizes the dc gain of $L(z)$ to 1, and the Nyquist-limit gain of $H(z)$ to 1.

(c) (10 pts) Derive analytic expressions for the squared magnitude frequency responses $|L(e^{j\omega T})|^2$ and $|H(e^{j\omega T})|^2$.

(d) (10 pts) Given a desired "cut-off frequency" $\omega_c = 2\pi f_c$, derive a formula for $p$ to set the lowpass filter gain to -3 dB at frequency $\omega_c$. [The $-3$ dB point $f_c$ may be defined for any low-pass filter $L(z)$ as the smallest frequency $f_c$ such that $|L(e^{j2\pi f_c/f_s})/L(1)|^2 = 1/2$]. State any restrictions on your result. [Hint: Be sure that the resulting filter is *stable*.]

(e) (5 pts) What is the corresponding formula for the high-pass filter $H(z)$? [Hint: You can simply adapt the low-pass result to obtain this case.]

(f) (5 pts) For the sampling rate $f_s = 44.1$ kHz, find a value for $p$ that puts the $-3$ dB frequency $f_c$ of each filter at $f_c = 1$ kHz.

(g) (5 pts) Plot the pole and zero locations of $L(z)$ and $H(z)$ in the $z$-plane (showing the unit circle) for $f_c = 1$ and $f_s = 44.1$.

(h) (5 pts) Plot the filter frequency-response magnitudes in dB (Normalize to 0dB peak) as a function of frequency for $f_c = 1$ and $f_s = 44.1$. [Hint: Use `freqz()`].

2. (65 points) [Turkey Dinner] You wake up on Saturday morning, put your turkey into a 350 degree oven ($T_{oven} = 350$), go back to sleep, wake up again, and...oh no! You forgot to write down what time you put the turkey in the oven! You must figure out when ($t_{cooked}$) the center of the turkey will reach the desired temperature $T_{cooked} = 165$. To do so, you measure the current temperature $T_1 = 100$ at $t_1 = 0$ and a later temperature $T_2 = 110$ at $t_2 = 30$ where $T$ is in degrees Farenheit and $t$ is in minutes.

   Using these measurements, we can now solve for $t_{cooked}$ by approximating the system as a leaky integrator (one-pole) filter $y[n] = (1 - \lambda)x[n] + \lambda y[n - 1]$. The input $x[n]$ is the oven temperature over time (assumed constant), while the output $y[n]$ is the turkey temperature over time. Here, $n$ denotes quantized time $t$ in minutes.

   (a) (5 points) Draw the signal flow diagram of the turkey/oven system.

   (b) (5 points) Write the transfer function of the turkey/oven system.

   (c) (5 points) Plot the poles and zeros of the turkey/oven system.

   (d) (15 points) Write the turkey temperature trajectory—the measured turkey temperature as a function of time after the first measurement at $t_1 = 0$, written in terms of the forgetting factor $\lambda$, the initial turkey temperature $T_1 = 100$, and the oven temperature $T_{oven} = 350$.

   (e) (15 points) Solve for the "forgetting factor" $\lambda$.

   (f) (5 points) Solve for the turkey time constant $\tau$. [Hint: Write $\lambda$ as $e^{-T/\tau}$ where $T$ is the sampling interval in minutes and solve for $\tau$ in terms in $\lambda$.]

   (g) (5 points) Solve for the time $t_{cooked}$ when the turkey will reach $T_{cooked}$.

   (h) (5 pts) Given that the turkey was in the refrigerator at 40 degrees Farenheit, and assuming you immediately put the turkey in the oven when you woke up, what time did you wake up?

3. (15 pts) [Symmetric Filter Chain]

   Given a filter of the form

$$y(n) = x(n) + 2\,x(n-1) + 3\,x(n-2) + 2\,x(n-3) + x(n-4)$$

   (a) (5 pts) Find an expression for the group delay. [Hint: think about general time-domain vs frequency-domain features (i.e symmetry, real or complex) that describe this system].

   (b) (5 pts) Find an expression for the phase delay

   (c) (5 pts) Find an expression for the phase and group delay for a chain (series combination) of $N$ of these filters. [Hint: Try looking at the angle of frequency-response returned by `freqz` for some example coefficients. How does the phase behave/look when you chain several of these filters?]

4. Create a cascade of N 1st order all-pass filters, G[z],

$$G[z] = \frac{(\rho + z^{-1})}{(1 + \rho z^{-}1)}$$

(a) Find an analytic expression for group delay, G($\omega$)

$$[hint : G(\omega) = -\frac{d}{d\omega}\Theta(\omega) = -imag(\frac{d}{d\omega}ln(|G(\omega)|\ e^{j\angle(G(\omega))}))] \quad (1)$$

(b) Make a plot of the group delay as a function of frequency, $\omega$

(c) For a single element in the cascaded filter, state whether the all pass parameter, $\rho$ was positive or negative.

(d) Consider the filter:

$$\frac{(\rho + z^{-3})}{1 + \rho z^{-3})} \quad (2)$$

Find an expression for the group delay
[hint:think about what happens to z and $\omega$ on the real-imaginary plane (unit circle)]

(e) Consider a filter above with fs = 8 kHz, an all-pass parameter $\rho$ = 0.9 and $\rho$ = -0.5. How many all-pass filters would you need in cascade (N) to have a 0.5 second delay between the DC arrival and fs/2 ? for a 2 second delay?

(f) MATLAB: for the case where $\rho$ = 0.9 and $\rho$ = -0.5, plot the impulse response by cascading the N from part (d) above. Listen to the response using soundsc(). Downsample the guitarsnip.wav and drumssnip.wav from HW 5 to 8kHz and apply the filter to the samples. Listen to the result and comment on what you hear.