Fall 2018–2019
Music 320A
Homework #7
Digital Filters, Linearity, Time Invariance
137 points
Theory part due Thurs 11/29/2018 11:59pm
Lab part due Thurs 11/29/2018 11:59pm

Theory Problems

1. (12 pts) Classify the following filters as either FIR or IIR (Finite or Inifinite Impulse Response):
   (a) $y(n) = x(n) + 2\,x(n-1) + x(n-2)$
   (b) $y(n) = x(n) + y(n-1)$
   (c) $y(n) = x(n) - x(n-1) + y(n-1)$
   (d) $y(n) = \sum_{m=0}^{\infty} 2^{-m} x(n-m)$
   (e) $y(n) = x(n) + \sqrt{y(n-1)}$
   (f) $y(n) = x(n) + \frac{1}{n} y(n-1)$

2. (20 pts) For the filter

$$y(n) = x(n) + 2\,x(n-1) + x(n-2)$$

   Find the
   (a) impulse response
   (b) frequency response
   (c) amplitude response
   (d) phase response
   (e) dc gain
   (f) Nyquist limit gain (gain at frequency $f_s/2$)
   (g) gain at frequency $f_s/4$
   (h) time delay at frequency $f_s/4$

   [Hint: you'll need to bring the filter into the z domain]

3. (15 points) [System Diagram]. Draw both Direct Form I and Direct Form II signal flow graphs for the filters determined by the difference equations below:

(a) (All-zero FIR case)

$$y(n) = x(n) + \frac{1}{3}x(n-1) - \frac{1}{7}x(n-2)$$

(b) (All-pole IIR case)

$$y(n) = \frac{1}{2}x(n) + \frac{1}{3}y(n-1) + \frac{1}{4}y(n-2)$$

(c) (Pole-zero IIR case)

$$y(n) = x(n) + 0.5x(n-1) - 0.8y(n-1) - 0.5y(n-2)$$

(d) What is the *order* of each of the above two filters?

4. (20 points) [IIR Filters] Consider the filter

$$y(n) = 0.5x(n) + 0.5x(n-1) + 0.8y(n-1)$$

Find and sketch the first 10 samples of

(a) the impulse response.

(b) the output when the input is given by

$$x(n) = [1, -1, 1, 1, -1, -1, 0, 0, 0, 0]$$

(c) the output when the input is given by

$$x(n) = [1, 1, 1, 1, 1, 1, 0, 0, 0, 0]$$

Lab Assignments

For all lab assignments, submit your M-file scripts, functions, and figures in one zip file through Canvas.[1] Within Canvas, upload the zip file using the DropBox menu.

The zip file should be named with your last name, and homework number. For example, for Brian Eno's zip file, the file should be titled `Brian_Eno_hw2.zip`. For Brian's answer to lab problem 3 on homework 2, the file would be titled `q3.m`. Also, at the beginning of each script, include the following comment:

```
% Your Name / Lab # - Question #
```

1. (20 pts) Verify the *linearity* of the filter in problem 1(a) by the following steps:
   (a) Generate 60 samples of the sinusoid $x_0(n) = 0.5\cos(0.2\pi n)$
   (b) Zero-pad to length $N = 100$, forming the signal $x_1(n)$ using the Matlab statement "`x1 = [x0(:)',zeros(1,40)]`" or equivalent.
   (c) Using Matlab's `filter` function, filter the input signal $x_1(n)$ to obtain $y_1(n)$.
   (d) Plot x_1 in `subplot(2,1,1)` and y_1 in `subplot(2,1,2)`.
   (e) What kind of convolution has been performed (cyclic or acyclic)?
   (f) Would all output samples have been returned if we did not zero pad?
   (g) Filter the input signal $x_2(n) = 2\cos(0.1\pi n)$ to obtain $y_2(n)$.
   (h) Filter the input signal $x_3(n) = 0.5x_1(n) + 2x_2(n)$ to obtain $y_3(n)$.
   (i) Compare $y_3(n)$ to $0.5y_1(n) + 2y_2(n)$ by plotting both signals on the same plot using `hold on` command in Matlab.

   Turn in your Matlab code and answers.

2. (20 points) Verify the *time-invariance* of the filter in problem 1(b) using the following steps:
   (a) Filter the input signal $x(n) = \cos(0.25\pi n)$ to obtain $y(n)$.
   (b) Time shift (delay) the original input signal $x(n)$ by 3 samples to get $x_{s,3}(n) = \text{SHIFT}_3\{x\}$.
   (c) Filter $x_{s,3}(n)$ to obtain $y_{s,3}(n)$.

[1] http://canvas.stanford.edu

(d) Compare $y_{s,3}(n)$ to $y(n)$ by plotting both signals on the same plot using `hold on` command in Matlab.

(e) What would you have to do to make them the same?

Turn in your Matlab code and answers.

3. (10 pts) Form the spectrogram of the signal 'Uncle Sam, almost.wav' [2] using various DFT window lengths.

   (a) Turn in the spectrogram formed using the largest power of two frame size that doesn't have sufficient frequency resolution to read the embedded message

   (b) and the spectrogram formed using the shortest power of two that doesn't have sufficient time resolution to read the embedded message.

   (c) Have a Happy Thanksgiving!

[2]found here: https://ccrma.stanford.edu/courses/320/hw7/UncleSamAlmost.wav