Fall 2018–2019
Music 320A
Homework #5
Convolution, Impulse Response
Theory Part Due 11/4/2018 by 11:59 pm
Lab Part Due 11/4/2018 by 11:59pm

Theory Problems

1. (20 pts) [Convolution] For the signals

$$\begin{aligned} x &= [0,0,1,0,0,0,1,0] \quad \text{and} \\ h &= [1,1,1,1,1,0,0,0], \end{aligned}$$

   and denoting circular convolution by $\circledast$ and acyclic convolution (such as computed by the matlab `conv` function) by $*$, compute

   (a) $y(n) = (x * h)_n$
   (b) $y(n) = (x \circledast h)_n$

   Note that there is typically no reason to write $\circledast$ for circular convolution, because we may normally assume that the output signal space is the same as the input signal space (typically $\mathbf{C}^N$ for this class). Finite-length signals requires cyclic convolution, while infinitely long signals ($N \to \infty$) use acyclic convolution. An exception is when the length N_2 of the output signal is allowed to be greater than the length N of the input signals, as in this problem; in that case, cyclic convolution produces the same nonzero output samples is acyclic convolution when $N_2 \geq 2N - 1$. If not specified, assume $N_2 = N$.

2. (20 pts) [Convolution] For $x = [1,2,3,2]$ and $h = [3,-1,2,1]$, find the convolution $(x * h)_n$ and correlation $(x \star h)_n$. Note that they are both cyclic, not acyclic. Hint: knowing x is even can help you.

3. (15 pts) [Convolution] The *impulse* or "unit pulse" signal is defined by

$$\delta(n) \triangleq \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

   For example, $\delta = [1,0,0,0]$ for $N = 4$.

   (a) Verify that the impulse signal is the *identity element* under convolution using the impulse signal $\delta = [1,0,0,0]$ and the input signal $x = [1,-1,1,-1]$. That is, show that $x * \delta = x$.

(b) Show that $x * \text{Shift}_1(\delta) = \text{Shift}_1(x)$, where $\text{SHIFT}_{1,n}(x) \triangleq x(n-1)$.

(c) Find $(x * [1, 1, 0, 0, \ldots])_n$ using linearity of the convolution operator and the preceding results.

4. (15 pts) (a). Sketch the time-domain impulse response h(n),

$$h(n) = \delta(n) + \gamma\delta(n - \tau) \tag{1}$$

where n indexes time. Here, γ is a gain and τ is an integer number of samples of time delay. For your sketch, use $\gamma = 0.5$ and $\tau = 5$ samples.

(b). Write an expression for the DFT of h(n), H(k) and for its square magnitude $|H(k)|^2$

(c). Sketch the square magnitude, $|H(k)|^2$, for $\gamma = 1.0$ and $\gamma = 0.5$, each with $\tau = 1.0$ ms and again for $\tau = 4$ ms. Use a square magnitude axes and a linear frequency axis, and include frequencies from DC to 2 kHz.

Lab Assignments

1. (20 pts) [Impulse Response and Frequency Response]

   (a) Referring to h(n) in equation (1) from question 4 (from the theory section), form and plot on a single set of axes the square magnitude of the $DFT|H(k)|^2$ using a linear frequency axis from 0 Hz to 2 kHz, and a dB magnitude axis, for τ being the integer number of samples closest to 1 ms, 2 ms, and 3 ms delay, and γ=0.9.

   (b) Form echo impulse responses using $\gamma = 0.9$, and τ set to 1, 2, 4, ..., 512, and 1024 milliseconds. Form the impulse responses such that they appear as the columns of a matrix ntaps tall and nτ wide, where ntaps is the number of impulse response taps, and produces impulse responses that are 1.2 seconds long, and nτ = 11, the number of different time delay values. Use the Matlab function sound() to listen to the impulse responses one after the other by playing the ntaps*ntau-tall column irs(:). What is the smallest separation time at which you can hear two distinct pulses?

   (c) Use the Matlab function audioread() to read the guitar_snip.wav and drums_snip.wav signals located here [1] and here [2].Use fftfilt() to convolve the signals with the impulse responses generated above to produce a matrix of processed guitar and drum sounds. (Note that fftfilt() will separately apply impulse responses in the

[1]https://ccrma.stanford.edu/courses/320/hw5/guitarsnip.wav
[2]https://ccrma.stanford.edu/courses/320/hw5/drumsnip.wav

columns of the impulse response matrix to the signal to produce a matrix of output signals.) Process the first 2.0 seconds of the guitar signal, and the first 1.5 seconds of the drum signal. When processing the signals, add a column of zeros 0.5 seconds long to provide a little silence between segments when you listen to the signals using sound. Briefly describe the effect of convolving the different impulse responses with the guitar and drum signals.

2. (20 pts) [Reverb with Impulse Response]

   (a) Form reverberation impulse responses by applying an exponential envelope to Gaussian noise generated using the Matlab function randn(). Form two impulse responses, each 4.0 seconds long, one having an exponential decay with a 60-dB decay time of 1.0 seconds, and the other having a T60 of 4.0 seconds. Plot the impulse response absolute values on a dB scale to verify the decay times.

   (b) Like the previous question, convolve the impulse responses with snippets of the guitar and drum signals, and describe what you hear compared to the "dry" signals.