Theory Problems

1. (20 pts) [Convolution] For \( x = [1, 2, 3, 2] \) and \( h = [3, -1, 2, 1] \), find \((x * h)_n\) and \((x \star h)_n\). Note that they are both cyclic, not acyclic. Hint: knowing \( x \) is even can help you.

2. (20 pts) [Correlation] You are given a signal \( y(n) = [1, 0, -0.75, 0, 0.5, 0, -0.25, 0] \), which corresponds to the (circular) autocorrelation of an 8-sample long signal \( x(n) \) zero-padded to a zfp of 2 (note that you are given only one half of the autocorrelation).

   (a) Assuming that \( x(n) = \alpha \cos(\omega_0 n) \), find \( \alpha \) and \( \omega_0 \)

   (b) Can you find another signal (other than \(-x(n)\) or \(x(-n)\)) with the same autocorrelation? If you answer yes, give an example. If you answer no, explain why.

   (c) Without explicitly computing it, can you find the autocorrelation of \( x_{1/2}(n) = \cos(\frac{\omega_0}{2} n) \)? And \( x_2(n) = \cos(2\omega_0 n) \)? Explain your reasoning.

   (d) Compute the circular autocorrelation of \( x(n) \) (without zero-padding).

3. (15 points) [Convolution] The *impulse* or “unit pulse” signal is defined by

\[
\delta(n) \triangleq \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}
\]

For example, \( \delta = [1, 0, 0, 0] \) for \( N = 4 \).

   (a) Verify that the impulse signal is the *identity element* under convolution using the impulse signal \( \delta = [1, 0, 0, 0] \) and the input signal \( x = [1, -1, 1, -1] \). That is, show that \( x * \delta = x \).

   (b) Show that \( x * \text{Shift}_1(\delta) = \text{Shift}_1(x) \), where \( \text{SHIFT}_1(x) \triangleq x(n - 1) \).

   (c) Find \((x * [1, 1, 0, 0, \ldots])_n\).

      (Hint: use linearity of convolution and the preceding results)
4. (10 pts) [Fourier Theorems] Show that \( \text{DFT}(\text{DFT}(y)) = N \cdot \text{FLIP}(y) \),
where \( \text{FLIP}_n(y) \triangleq y[-n] = y[N-n] \). What does this say about \( \text{DFT}(\text{DFT}(\text{DFT}(\text{DFT}(y)))) \)\

**Lab Assignments**

**Note:** follow the naming conventions of the last Homework

1. (20 pts) [Impulse Response and Frequency Response]

\[
h(n) = \delta(n) + \gamma \delta(n - \tau) \quad (1)
\]

(a) Referring to \( h(n) \) in equation (1), form and plot on a single set of axes the square magnitude of the DFT of \( h(n) \), \( |H(k)|^2 \) using a linear frequency axis from 0 Hz to 2 kHz, and a dB magnitude axis, for \( \tau \) being the integer numbers of samples closest to 1 ms, 2 ms, 3 ms delay, and \( \gamma = 0.9 \).

(b) Form echo impulse responses using \( \gamma = 0.9 \), and \( \tau \) set to \([1, 2, 4, \ldots, 512, 1024]\) milliseconds. Form the impulse responses such that they appear as the columns of a matrix \( \text{irs} \) that is \( \text{ntaps} \) tall and \( \text{ntau} = 11 \) columns wide, one column for each value of \( \tau \). Set \( \text{ntaps} \) so that the impulse responses are 1.2 seconds long. Use the Matlab function \( \text{sound()} \) to listen to the impulse responses one after the other by playing the \( \text{ntaps*ntau}-\text{tall column} \) \( \text{irs}(:, : ) \). What is the smallest separation time at which you can hear two distinct pulses?

(c) Use the Matlab function \( \text{audioread()} \) to read the \text{guitar snip.wav} \text{ and} \text{drums snip.wav} signals located here\(^1\) and here\(^2\). Use \( \text{fftfilt()} \) to convolve the signals with the impulse responses generated above to produce a matrix of processed guitar and drum sounds. (Note that \( \text{fftfilt()} \) will separately apply impulse responses in the columns of the impulse response matrix to the signal to produce a matrix of output signals.) Process the first 2.0 seconds of the guitar signal, and the first 1.5 seconds of the drum signal. When processing the signals, append a column of zeros 1.5 seconds long to the input guitar and drum signals to provide silence for the increased length of the convolution compared to the original input signal, and to provide a little silence between signals when listening. Briefly describe the effect of convolving the different impulse responses with the guitar and drum signals.

2. (20 pts) [Reverb with Impulse Response]

\(^1\)https://ccrma.stanford.edu/courses/320/hw5/guitarsnip.wav
\(^2\)https://ccrma.stanford.edu/courses/320/hw5/drumsnip.wav
(a) Form reverberation impulse responses by applying an exponential envelope to
Gaussian noise generated using the Matlab function `randn()`. Form two impulse
responses, each 4.0 seconds long, one having an exponential decay with a 60-dB
decay time of 1.0 seconds, and the other having a T60 of 4.0 seconds. Plot the
impulse response absolute values on a dB scale to verify the decay times.

(b) Like the previous question, convolve the impulse responses with snippets of the
guitar and drum signals making sure to zeropad the input to accommodate the
impulse response length, and describe what you hear compared to the "dry"
signals.