Theory Problems

1. (15 pts) [Even and odd parts] A sequence \(x[n]\) is said to be **even** if \(x[-n] = x[n]\), and **odd** if \(x[-n] = -x[n]\), \(n = 0, 1, 2, \ldots, N - 1\), where indexing is carried out modulo \(N\). Find the even and odd parts of the following sequences:
   
   (a) \([0, 1, 2, 1]\)
   
   (b) \([0, 1, 0, -1]\)
   
   (c) \([0, 1, 2, 3]\)
   
   (d) \(x(n) = n^2, n = 0, 1, 2, \ldots, N - 1\)

2. (30 pts) If \(Y(k)\) denotes the \(k\)th element of the length \(N\) DFT of \(y\), show that:

   (a) \(\text{im}\{y\} = 0 \iff Y(k) = \overline{Y[N - k]}\) (DFT\{real\} is Hermitian)
   
   (b) \(\text{re}\{y\} = 0 \iff Y(k) = -\overline{Y[N - k]}\) (anti-Hermitian)
   
   (c) \(y\) even \(\iff Y\) even
   
   (d) \(y\) odd \(\iff Y\) odd
   
   (e) \(y\) real, even \(\iff Y\) real, even
   
   (f) \(y\) real, odd \(\iff Y\) imag, odd
   
   (g) \(y\) imag, even \(\iff Y\) imag, even
   
   (h) \(y\) imag, odd \(\iff Y\) real, odd

3. (30 pts) [DFT] Define \(\omega_k T \triangleq 2\pi k/N\). Find the length \(N = 8\) DFTs \(X_i(k), k = 0, 1, \ldots, 7\), for the following sequences (without using Matlab):

   (a) \(x_1 = [1, 0, 0, 0, 0, 0, 0, 0]\)
   
   (b) \(x_2 = [0, 1, 0, 0, 0, 0, 0, 0]\)

   Use the shift operator to express \(x_2(n)\) in terms of \(x_1(n)\), and then compute the DFT of \(x_2(n)\) in terms of the the DFT of \(x_1(n)\).

   (c) \(x_3 = [0, 0, 0, 0, 0, 0, 0, 1]\)

   Use the flip operator to express \(x_3(n)\) in terms of \(x_2(n)\), and then compute the DFT of \(x_3(n)\) in terms of the DFT of \(x_2(n)\).
(d) \( x_4 = [2, 1, 0, 0, 0, 0, 1] \)

Use linearity to express \( x_4(n) \) in terms of \( x_1(n) \), \( x_2(n) \), and \( x_3(n) \), and then compute the DFT of \( x_4(n) \) in terms of \( x_1(n) \), \( x_2(n) \), and \( x_3(n) \).

(e) Express \( X_4(k) \) in terms of \( X_1(k) \) and \( X_2(k) \).

4. (20 pts) [Signal Metrics] For signals \( y_1 = [1, 2, -3] \) and \( y_2 = [1, j, 1 - j] \), find the

(a) mean \( \mu_y \)
(b) total signal energy \( E_y \)
(c) average signal power \( P_y \)
(d) sample variance \( \sigma_y^2 \)
(e) Euclidean \((L^2)\) norm \( ||y||_2 \)
(f) Chebyshev \((L^\infty)\) norm \( ||y||_\infty \)

Lab Assignments

1. (40 pts) [Short-time Fourier transform] Write a function that generates a spectrogram of an input signal.

    function myspecgram (x, fs, frameSize, hopSize, fftSize)
    
    % function myspecgram (x, fs, frameSize, hopSize, fftSize)
    % A function to plot the spectrogram of an input signal x
    % using the Hann window
    %
    % x: input signal (row or column vector) - assume real
    % fs: sampling rate of x
    % frameSize: frame size (in samples) = window length (make it odd)
    % hopSize: time between start-times of successive windows (in samples)
    % fftSize: sets the zero-padding factor - defaults to length(x)
    %
    % Your Name

    Plot the spectrogram image using the imagesc function (see ‘help imagesc’). Plot the spectrogram in dB, with time on the x-axis, frequency on the y-axis, and with a dB range of \([-60 0]\). Try with various colormap settings and choose one for your plot.

    Test your function on a two sinusoidal signals, one tuned to a DFT frequency \( \omega_k T = 2\pi k/N \), for some integer \( k \), and the other half-way between FFT bins, e.g., \( k + 1/2 \). It might be helpful to review the textbook on this topic\(^1\). Make sure your spectrogram function correctly handles parameters \texttt{fftSize}, \texttt{frameSize}, and the length of your signal. Turn in the two resulting spectrogram plots. State the signal and analysis parameters you used (e.g., in the figure title—see ‘help sprintf’).

\(^1\)https://ccrma.stanford.edu/~jos/st/Frequencies_Cracks.html
2. (20 pts) Use the following Matlab command sequence to generate a chirp signal:

\[
\begin{align*}
\text{w1} &= 100; \quad \text{w2} = 3000; \quad \% \text{(Hz)} \\
\text{T} &= 3; \quad \% \text{(sec)} \\
\text{fs} &= 8000; \quad \% \text{(Hz)} \\
\text{dT} &= 1/\text{fs}; \\
\text{t} &= (0: \text{dT}; \text{T}); \\
\text{up} &= \text{chirp}(\text{t}, \text{w1}, \text{T}, \text{w2});
\end{align*}
\]

Using your function from the previous problem, plot the spectrum of the \text{up} signal. Turn in your plot as a Matlab figure file. Do they verify that your function works as expected?

3. (20 pts) [Playing around with the Spectrogram] Use \text{myspecgram} to plot the spectrum of \text{helloMystery.wav} \[.\] In all cases, use a rectangular window with hop size = DFT size = frame size (no frame overlap and no zero padding).

(a) Create a plot with good time resolution using frame size = 88.
(b) Create a plot with good frequency resolution using frame size = 2800.
(c) Describe what you see in the spectrogram. Play around with the frame size to see what value seems to give the best time-frequency resolution trade-off. Also consider your normalization, colormap, and whether or not to use dB, etc. Turn in your best-tuned plot. [Hint: Halloween was not long ago.]
(d) (optional) Experiment with the effects of using different windows and overlap factors, such as 50% overlap with the Hann window.
(e) (optional) Experiment with the effects of using zero padding, such as a factor of 8 zero-padding in the time domain.

\[\text{http://ccrma.stanford.edu/~jos/wav/helloMystery.wav}\]