Fall 2018–2019
Music 320A
**Homework #4**
Signal Metrics, DFT, Fourier Theorems
Theory Part Due 10/26/2018 by 11:59 pm
Lab Part Due 10/26/2018 by 11:59 pm

# Theory Problems

1. (15 pts) [Even and odd parts] A sequence $x[n]$ is said to be *even* if $x[-n] = x[n]$, and *odd* if $x[-n] = -x[n], n = 0, 1, 2, \ldots, N-1$, where indexing is carried out modulo $N$. Find the even and odd parts of the following sequences:
   (a) $[0, 1, 2, 1]$
   (b) $[0, 1, 0, -1]$
   (c) $[0, 1, 2, 3]$
   (d) $x(n) = n^2,\ n = 0, 1, 2, ..., N-1$

2. (30 pts) If $Y(k)$ denotes the $k$th element of the length $N$ DFT of $y$, show that:
   (a) $\text{im}\{y\} = 0 \Longleftrightarrow Y(k) = \overline{Y[N-k]}$ (DFT{real} is *Hermitian*)
   (b) $\text{re}\{y\} = 0 \Longleftrightarrow Y(k) = -\overline{Y[N-k]}$ (*anti-Hermitian*)
   (c) $y$ even $\Longleftrightarrow Y$ even
   (d) $y$ odd $\Longleftrightarrow Y$ odd
   (e) $y$ real, even $\Longleftrightarrow Y$ real,even
   (f) $y$ real, odd $\Longleftrightarrow Y$ imag, odd
   (g) $y$ imag, even $\Longleftrightarrow Y$ imag, even
   (h) $y$ imag, odd $\Longleftrightarrow Y$ real, odd

3. (30 pts) [DFT] Define $\omega_k T \triangleq 2\pi k/N$. Find the length $N = 8$ DFTs

$$X_i(k), \quad k = 0, 1, \ldots, 7,$$

   for the following sequences (without using `Matlab`):
   (a) $x_1 = [1, 0, 0, 0, 0, 0, 0, 0]$
   (b) $x_2 = [0, 1, 0, 0, 0, 0, 0, 0]$
   Use the shift operator to express $x_2(n)$ in terms of $x_1(n)$, and then compute the DFT of $x_2(n)$ in terms of the the DFT of $x_1(n)$.
   (c) $x_3 = [0, 0, 0, 0, 0, 0, 0, 1]$
   Use the flip operator to express $x_3(n)$ in terms of $x_2(n)$, and then compute the DFT of $x_3(n)$ in terms of the DFT of $x_2(n)$.

(d) $x_4 = [2, 1, 0, 0, 0, 0, 0, 1]$
Use linearity to express $x_4(n)$ in terms of $x_1(n)$, $x_2(n)$, and $x_3(n)$, and then compute the DFT of $x_4(n)$ in terms of $x_1(n)$, $x_2(n)$, and $x_3(n)$.

(e) Express $X_4(k)$ in terms of $X_1(k)$ and $X_2(k)$.

4. (20 pts) [Signal Metrics] For signals $y_1 = [1, 2, -3]$ and $y_2 = [1, j, 1 - j]$, find the
   (a) mean $\mu_y$
   (b) total signal energy $\mathcal{E}_y$
   (c) average signal power $\mathcal{P}_y$
   (d) sample variance $\sigma_y^2$
   (e) Euclidean ($L^2$) norm $\|y\|_2$
   (f) Chebyshev ($L^\infty$) norm $\|y\|_\infty$

## Lab Assignments

1. (40 pts) [Short-time Fourier transform] Write a function that generates a spectrogram of an input signal.

```
function myspecgram (x, fs, frameSize, hopSize, fftSize)

% function myspecgram (x, fs, frameSize, hopSize, fftSize)
% A function to plot the spectrogram of an input signal x
% using the Hann window
%
% x: input signal (row or column vector) - assume real
% fs: sampling rate of x
% frameSize: frame size (in samples) = window length (make it odd)
% hopSize: time between start-times of successive windows (in samples)
% fftSize: sets the zero-padding factor - defaults to length(x)
%
% Your Name
```

Plot the spectrogram image using the `imagesc` function (see '`help imagesc`'). Plot the spectrogram in dB, with time on the x-axis, frequency on the y-axis, and with a dB range of `[-60 0]`. Try with various colormap settings and choose one for your plot.

Test your function on a two sinusoidal signals, one tuned to a DFT frequency $\omega_k T = 2\pi k/N$, for some integer $k$, and the other half-way between FFT bins, *e.g.*, $k + 1/2$. It might be helpful to review the textbook on this topic.[1] Make sure your spectrogram function correctly handles parameters `fftSize`, `frameSize`, and the length of your signal. Turn in the two resulting spectrogram plots. State the signal and analysis parameters you used (*e.g.*, in the figure title—see '`help sprintf`').

[1] `https://ccrma.stanford.edu/~jos/st/Frequencies_Cracks.html`

2. (20 pts) Use the following Matlab command sequence to generate a chirp signal:

```
w1=100; w2=3000; %(Hz)
T=3; %(sec)
fs=8000; %(Hz)
dT=1/fs;
t=(0:dT:T);
up = chirp(t,w1,T,w2);
```

Using your function from the previous problem, plot the spectrum of the `up` signal. Turn in your plot as a Matlab figure file. Do they verify that your function works as expected?

3. (20 pts) [Playing around with the Spectrogram] Use `myspecgram` to plot the spectrum of `helloMystery.wav`.[2] In all cases, use a rectangular window with hop size = DFT size = frame size (no frame overlap and no zero padding).

   (a) Create a plot with good time resolution using frame size = 88.

   (b) Create a plot with good frequency resolution using frame size = 2800.

   (c) Describe what you see in the spectrogram. Play around with the frame size to see what value seems to give the best time-frequency resolution trade-off. Also consider your normalization, colormap, and whether or not to use dB, etc. Turn in your best-tuned plot. [Hint: Halloween was not long ago.]

   (d) (optional) Experiment with the effects of using different windows and overlap factors, such as 50% overlap with the Hann window.

   (e) (optional) Experiment with the effects of using zero padding, such as a factor of 8 zero-padding in the time domain.

[2] `http://ccrma.stanford.edu/~jos/wav/helloMystery.wav`