Fall 2018–2019
Music 320A
Homework #3
Complex Sinusoids, Geometric Signal Theory
160 points
Theory Part Due Friday 11/19/2018 by 11:59 pm
Lab Part Due Fri 11/19/2018 by 11:59 pm

Theory Problems

1. (10 points) Show that the z-tranform is a *linear* operator. Specifically, show that $\mathcal{Z}_z\{\alpha x_1(\cdot) + \beta x_2(\cdot)\} = \alpha X_1(z) + \beta X_2(z)$ for any two (real or complexs) signals x_1 and x_2, and any two (real or complex) scalars (constant gains) α and β.

2. (10 pts) [Orthogonality of sinusoids] Show that if two length N sampled sinusoids $s_k(n)$ and $s_l(n)$ are orthogonal, *i.e.*, $s_k \perp s_l$, then $\mathcal{A}s_k \perp \mathcal{B}s_l$ for all complex constants $\mathcal{A}$ and $\mathcal{B}$ where $\mathcal{A} = Ae^{j\alpha}$ and $\mathcal{B} = Be^{j\beta}$. That is, the orthogonality of two sinusoids is independent of their phases and (nonzero) amplitudes.

3. (10 pts) Find the following sum for any integer $N > 0$:

$$\sum_{k=0}^{N-1} e^{\frac{j2\pi k}{N}}$$

   You may find it helpful to remember

$$S(r) = \sum_{i=n_1}^{n_2} r^i = \frac{r^{n_1} - r^{(n_2+1)}}{1-r}$$

   which can be quickly rederived by solving for $S(r)$ in the expression $S(r) - rS(r)$.

4. (15 pts) [Inner Products] For the set of N-sample-long complex sinusoids defined as:

$$S_k[n] = e^{j\frac{2\pi kn}{N}}$$

   Compute the following inner products for $N = 6$:

   (a) $\langle S_0, S_1 \rangle$

   (b) $\langle S_1, S_1 \rangle$

(c) $\langle S_1, S_2 \rangle$

5. (10pt) [Inner Products] For the complex vectors $x = (1, j, 1 - j)$ and $y = (1 + j, -1 + j, -j)$ in the 3-D complex space $\mathbf{C}^3$, find the inner products

   (a) $\langle x, x \rangle$

   (b) $\langle y, y \rangle$

   (c) $\langle x, y \rangle$

   (d) $\langle y, x \rangle$

Lab Assignments

1. (30 pts) [Additive synthesis] Using your `additive` function (from the previous homework), synthesize the following sounds:

   (a) Square wave (`square.wav`)

   (b) Triangle wave (`triangle.wav`)

   (c) Sawtooth wave (`sawtooth.wav`)

   Write a matlab script which specifies

   (a) a vector harmonic frequencies `f` and complex-amplitudes `Z`) for each sound,

   (b) utilizes your additive-synthesis function to generate the sounds and save the results as `.wav` files using the names given above.

   (c) plots every waveform for the first five periods.

   Please note:

   (a) Your sounds should be 2 seconds long.

   (b) Use $f = 441$ Hz for the fundamental frequency and $f_s = 44100$ Hz for your sampling rate.

   (c) Make sure your function works fast enough, and your sounds are not clipped. If necessary, modify your function.

   (d) Verify your result with plot. Put the three waveforms in one figure, and name them appropriately.

   (e) Submit your script and function (no plots). Include your name, lab and problem number. For each sound, how many partials do you need to obtain a faithful production of the waveform? How do they sound? Also, describe the characteristics of your frequency components.

2. (20 pts) [Signal Metrics Lab] In matlab, experiment with computing signal metrics for various signals. Create two sinusoids ($f_1 = 440$, $f_2 = 880$), two saw waves ($f_1 = 440$, $f_2 = 880$), and two noise signals (rand and randn) all one second long, where $fs = 44100$. Normalize all signals to have a max amplitude of 1. For the saw waves, please compute the total number of harmonics using floor(fs/2/freq), where freq is the fundamental frequency of the signal. See homework 2 solutions if needed.

   (a) What is the mean, variance, and total energy of each signal?

   (b) Sum each complementary signal together (sine + sine, saw + saw, noise + noise) and recompute the signal metrics as above. What is the mean, variance, and total energy of each signal?

(c) For each complementary signal, find the angle θ between each pair of vectors. Hint: recall vector cosine from the textbook.

(d) Comment on the various signal metrics. Specifically:

  i. Comment on the individual variance and the variance of the sum for each signal

  ii. Comment on the vector cosine results