Fall 2018–2019
Music 320A
Homework #2
Sinusoids, Complex Sinusoids
145 points
Theory and Lab Problems Due Thursday 10/11/2018 before class

Theory Problems

1. (15 pts) [Sinusoids] Define $x(t)$ as

$$x(t) = 2\sin\left(\omega_0 t - \frac{\pi}{4}\right) + \cos(\omega_0 t)$$

   (a) Express $x(t)$ in the form $x(t) = A\cos(\omega_0 t + \phi)$, where ϕ is in radians.

   (b) Does the previous result depend on some special property of the two sinusoids combined, or can any two sinusoids be combined into a single sinusoid like this? Under what conditions can two different sinusoids be combined like this?

   (c) Find a complex valued signal $\tilde{x}$ such that $x(t) = Re\{\tilde{x}(t)\}$

2. (15 pts) The phase of a sinusoid can be related to time shift as follows:

$$x(t) = A\cos(2\pi f_0 t + \phi) = A\cos(2\pi f_0(t - t_1))$$

   In the following parts, assume that the period of the sinusoidal wave is $T_0 = 12$ sec

   (a) When $t_1 = -3$ sec, the value of the phase is $\phi = \pi/4$. Explain whether this is true or false.

   (b) When $t_1 = 3$ sec, the value of the phase is $\phi = \pi/2$. Explain whether this is true or false.

   (c) When $t_1 = 7$ sec, the value of the phase is $\phi = 5\pi/6$. Explain whether this is true or false.

3. (15 pts) [Sinusoids]

   (a) For a sinusoid with a period $T_0 = 1/10$ seconds, what is the frequency f_0 in Hz? What is the frequency ω_0 in radians per second?

(b) Define $x(t)$ as

$$x(t) = A\sin[\omega \cdot (t - \tau)].$$

Write an expression for the phase in terms of the frequency ω and time delay τ.

(c) For $x(t)$ defined as above, find the phase at $t = 0$ for a time delay of $\tau = .25$ seconds and the frequency obtained for part (a).

4. (35 pts) [Complex Sinusoids] Define the discrete-time generalized sinusoid $x[n] = X z_0^n$ for $n = 0, 1, 2, \ldots$, where

$$\begin{aligned} X &= 2\,e^{j\pi/4} \\ z_0 &= 0.9\,e^{j\pi/8} \end{aligned}$$

(a) (5 pts) What is the amplitude of this sinusoid? What is the phase in radians? What is the phase in cycles? What is the phase in degrees?

(b) (5 pts) What is the time constant τ of decay (in samples)?

(c) (5 pts) What is the 60 dB decay time T_{60} in time constants?

(d) (5 pts) What is T_{60} in samples?

(e) (5 pts) What is the 80 dB decay time T_{80} in time constants?

(f) (5 pts) What is T_{80} in samples?

(g) (5 pts) If the sampling rate is 800 Hz, what are τ and T_{60} in seconds, and what is the frequency of the sinusoid in Hz?

5. (10 pts) [AM, phasors] An amplitude modulated (AM) cosine wave is represented by

$$x(t) = [12 + 7\sin(\pi t - \pi/3)]\cos(13\pi t)$$

Use complex sinusoids to show that $x(t)$ can be expressed as

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + A_3\cos(\omega_3 t + \phi_3)$$

where $\omega_1 < \omega_2 < \omega_3$, thereby finding each A_i, ϕ_i, and ω_i.

Lab Assignments

README! VERY IMPORTANT!!

For all lab assignments, submit your M-file scripts, functions, and figures in one zip file through the corresponding HW directory on the'Assignments' page on canvas[1]. Within coursework, upload the zip file using the filename specification below.

The zip file should be named with your last name, and homework number. For example, for Gavin Harrison's zip file, the file should be titled `Gavin_Harrison_hw2.zip`. For Gavin's answer to lab problem 3 on homework 2, the file would be titled `q3.m`. Also, at the beginning of each script, include the following comment: `% Your Name / Lab # - Question #`

You should create your code in such a way that I will be able to run it from my own computer. This means that you should have the proper file references and root directories, etc.. All your plots should have appropriate titles, axis labels, and units.

For problems with question(s), include your answer(s) in the body of the script files as comments. Also, please fully comment your matlab code for readability.

1. (10 pts) Write a Matlab script that generates a sinusoidal wave.
   (a) Its length must be 5 seconds, with 44100 samples per second (CD quality).
   (b) Its amplitude level must be -6 dB, where 0 dB corresponds to peak-amplitude $A = 1$.
   (c) Use 440 Hz for its frequency.
   (d) Your script must be able to save the wave as a sound file named '`mysound.wav`' and play it. Note that you do not need to submit the sound file: just submit your script.

   (hint: `audiowrite` and `sound` can come handy)

2. (20 pts) Define the discrete-time generalized sinusoid $x(n) = X z_0^n$ for $n = 0, 1, 2, ...$, where

$$X = 2\, e^{j\pi/4}$$

$$z_0 = 0.9\, e^{j\pi/8}$$

[1] http://canvas.stanford.edu

(a) Plot $\text{re}\{Xz_0^n\}$ and $\text{im}\{Xz_0^n\}$ versus n.

(b) Plot Xz_0^n as a collection of points in the complex plane (imaginary part versus real part).

(c) Mark circles on your plot, with the kth circle having radius $|X|e^{-k}$, where $k = 0, 1, 2, 3$ indicating the amplitude of the signal after k time constants have passed.

(d) What is the number of time steps (samples) that it takes for the signal to traverse between successive circles? Does it take the same amount of time to go from the $k = 0$ circle to the $k = 1$ circle as it does to go from the $k = 1$ circle to the $k = 2$ circle?

(e) Find the time constant τ of decay in samples.

(f) Mark the 60 dB decay time T_{60} on the plots.

3. (25 pts) Additive synthesis, the sum of K cosine waves, can be expressed as

$$y(t) = \sum_{k=1}^{K} A_k \cos(2\pi f_k t + \phi_k)$$

where A_k, f_k, and ϕ_k are the peak amplitude, frequency (Hz), and initial phase (rad) of kth sinusoidal component. Also, K is the number of sinusoidal components.

Write a Matlab function that implements this synthesis method and saves the result as an audio file. The syntax of your function should be as follows:

```
function y = additive(f, Z, fs, dur, name)

% function y = additive(f, Z, fs, dur, name)
% f: vector of frequencies in Hz
% Z: vector of complex amplitudes A*exp(j*phi)
% fs: sampling rate in Hz
% dur: total duration of the signal in seconds
% name: name of the output audio file
% f and Z must be of the same length:
% Z(1) corresponds to f(1) and so on.
% Your Name / Lab 2-2
```

Remember:

(a) Your function must be able to take any length of `f` and `Z`, as long as they are of the same length.

(b) Note that `Z` is a vector of complex amplitudes (that is, phasors), not real numbers.

(c) Try to make it run as fast as possible: can you implement this without using any loop in your code?

(d) Use your *additive* function to generate a one-second-long, unit-amplitude, zero-phase sine wave at $f_0 = 200$ Hz.

(e) Use your *additive* function to generate the sum of four sinusoids with frequencies $[220, 660, 1100, 1540]$, and amplitudes $[1, 1/3, 1/5, 1/7]$, all with zero phase.

(f) Using your function (or creating another one), create a signal that is comprised of each frequency (starting with 220Hz) presented in a sequence 1 second apart with 1 second periods of silence in between (e.g. 220Hz for 1 second, silence for 1 second, 660Hz for 1 second, silence for 1 second, ..., 1540 Hz for 1 second).

(g) Listen to all three of these signals, and plot the first 10 milliseconds of each on the same axes, labelling the time axis in milliseconds.

(h) Use your *additive* function to generate the sum of sinusoids with frequencies $f_0 \cdot [1{:}7]$ and amplitudes $1/[1{:}7]$. What waveform does this approximate?

(i) Generate the same, but with randomized phase. Does it sound the same? Does it look the same?

Optional: Use the Dual-tone multi-frequency signaling (DTMF) table on the Wikipedia DTMF page[2], generate a sequence of tones for your own telephone numbers, each tone last around 300ms, comment your own telephone number in the matlab script, and wavwrite your number into a wave file.

(General Hints: Obtain the real sinusoid by taking the real part of a complex sinusoid. For faster implementation, think about vector and matrix multiplication in place of loops over samples.)

4. Plotting Generalized Complex Sinusoids

(a) (5 pts) For each of the following complex signals $x[n]$, plot the first $10,000$ samples in the time domain. Plot the real and the imaginary part of the signal separately.

i. $x[n] = e^{j\frac{\pi}{4}} \left(0.999\, e^{j\frac{\pi}{N}}\right)^n$
(N = 512)

ii. $x[n] = e^{j\frac{\pi}{2}} \left(0.999\, e^{-j\frac{\pi}{N}}\right)^n$
(N = 2048)

iii. $x[n] = \left(e^{j\frac{\pi}{N}}\right)^n + \left(0.9995\, e^{-j\frac{4\pi}{N}}\right)^n$
(N = 1024)

(b) (5 pts) Repeat the previous problem, plotting the complex amplitudes of the first $10,000$ samples of each of the signals $x[n]$ in the z plane.

(c) (10 pts) Discuss the two different representations of the signals. What are the advantages of one plot over the other? How are they related? [Hint: Consider Euler's identity and circular motion as depicted on the cover of the textbook (see the gif here: `http://en.wikipedia.org/wiki/Phasor`)]

[2] http://en.wikipedia.org/wiki/Dual-tone_multi-frequency_signaling#Keypad