AMBISONIC DECODERS

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Music 222, Sounds in Space
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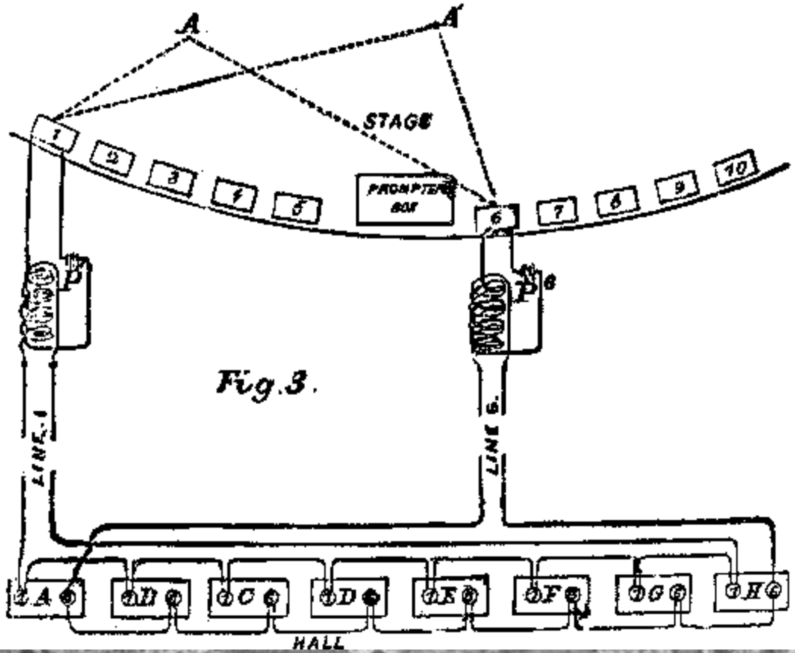
Outline

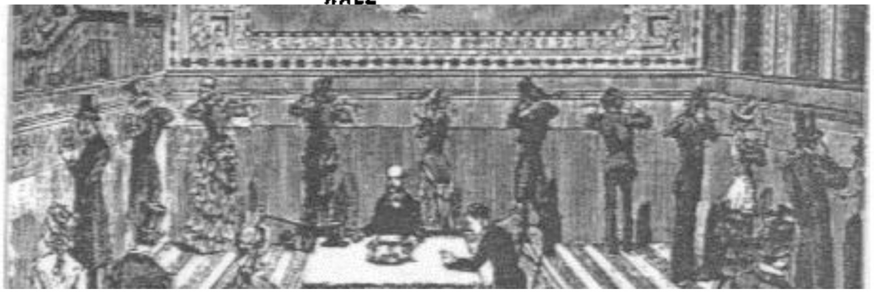
- Spatial Audio History
  - Paris Opera, Bell Labs, Blumlein/EMI, Gerzon
- Decoders
  - Psychoacoustics, rE, rV
  - Types of decoders
- ADT
  - example
  - alternatives
  - ambi2yt workflow
- Interchange
  - Coordinate systems
  - Real Spherical Harmonics
  - SymYlm, Panners, format adapters

140 YEARS OF SPATIAL AUDIO HISTORY

Clément Adler (1881)

- Paris Opera to International Exposition of Electricity





Lord Rayleigh, Duplex Theory (1907)

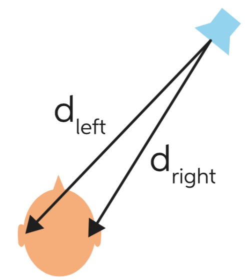
XII. *On our Perception of Sound Direction**.
By Lord Rayleigh, *O.M., Pres. R.S.*†

It is some thirty years ago since I executed a rather extensive series of experiments in order to ascertain more precisely what are the capabilities of the ears in estimating the direction of sounds ‡. It appeared that when the alternative was between right and left, the discrimination could be made with certainty and without moving the head, even although the sounds were pure tones. Nor was any difficulty introduced by the requirement that the ears should be stopped at the moment when the sounds commenced.

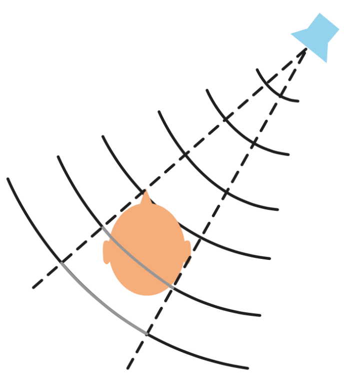
On the other hand, if the question was whether a sound were situated in front or behind the observer, no pronouncement could be made in the case of pure tones. The impossibility of distinguishing front and back carries with it further confusions relating to cases where the sound may be obliquely situated. But with sounds of other character and notably with the speaking voice, front and back could often be distinguished. It is understood, of course, that the head was kept still. A slight rotation, bringing a pure tone (originally situated exactly in front or exactly behind) to the right or the left, gives the information that was previously lacking.

The discrimination between right and left is usually supposed to be explicable by the greater intensity of sensation experienced by the ear which lies nearer to the sound. When

interaural time difference (ITD)



interaural intensity difference (IID)



The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 13:74, 214-232,

Bell Labs (1932)

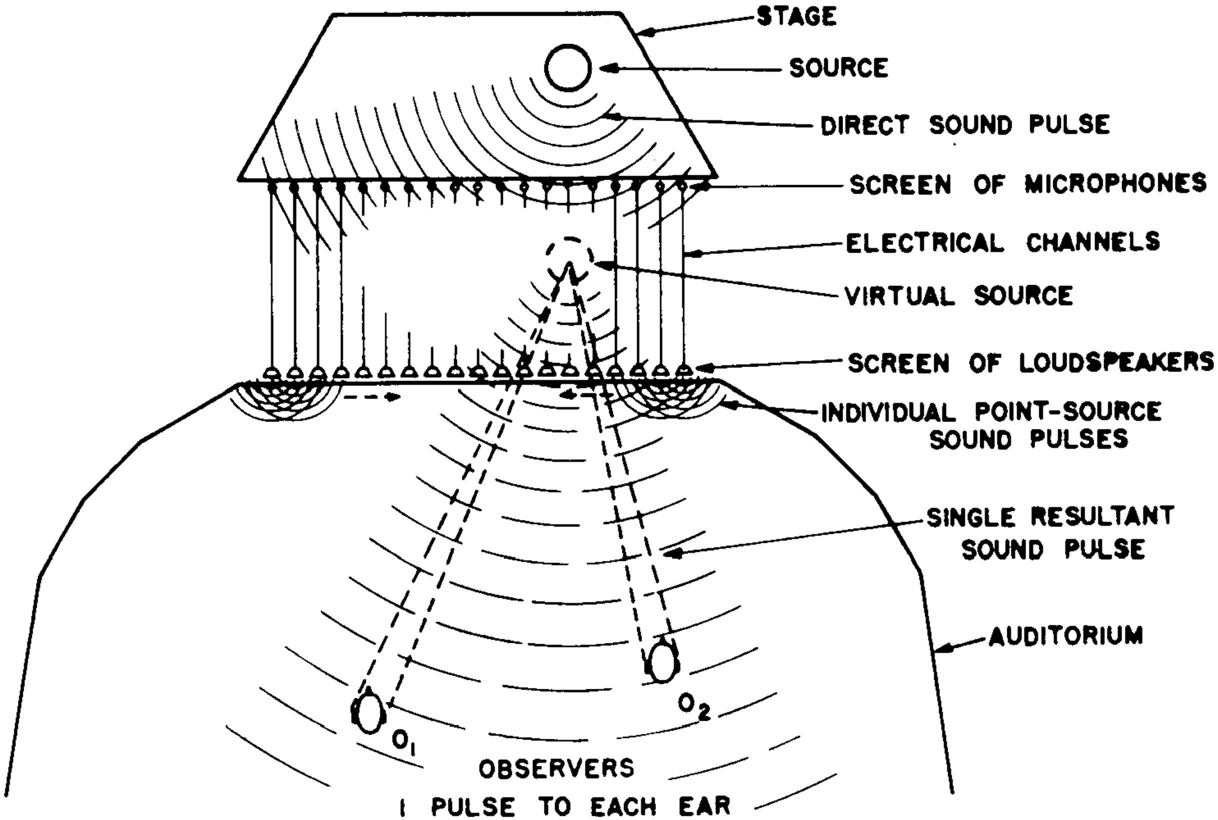


Fig. 2. Ideal stereophonic system. A very large number of very small microphones and loudspeakers would give a perfect reproduction of the original sound.

Bell Labs (1934)

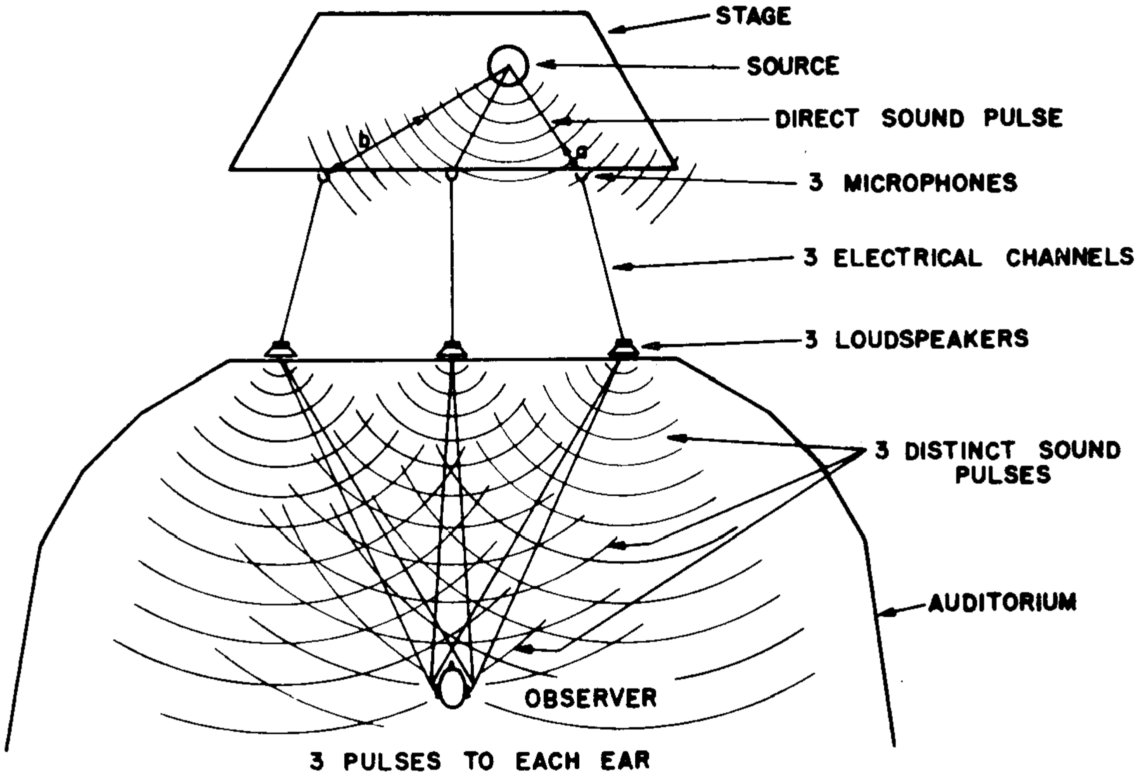
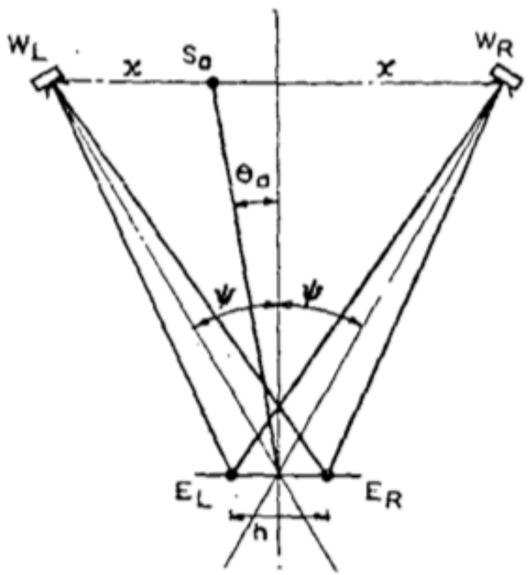
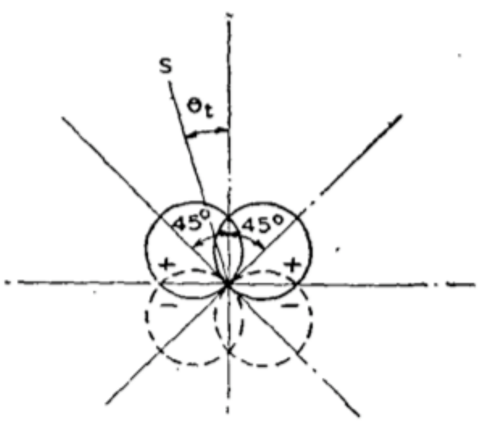
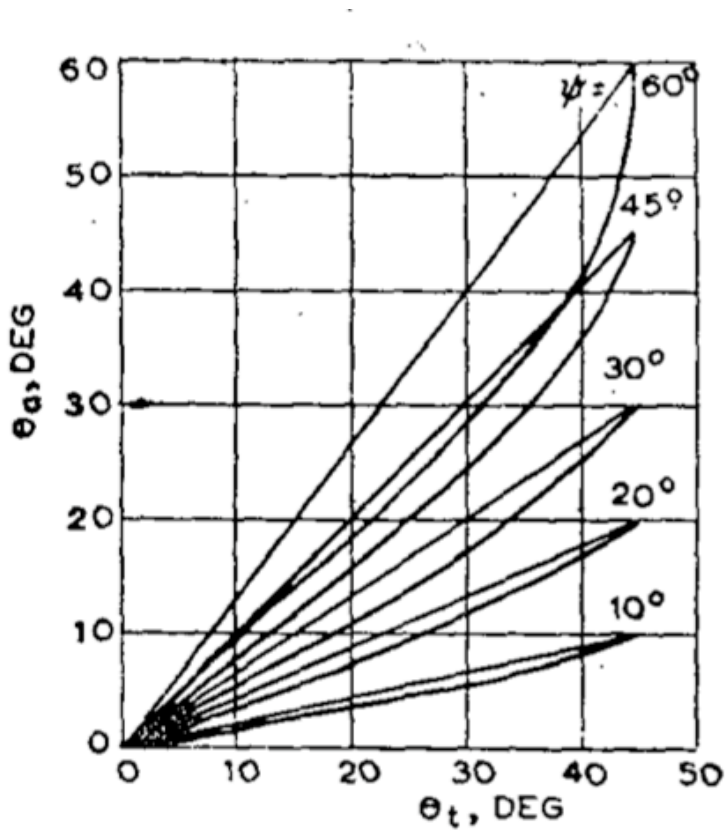


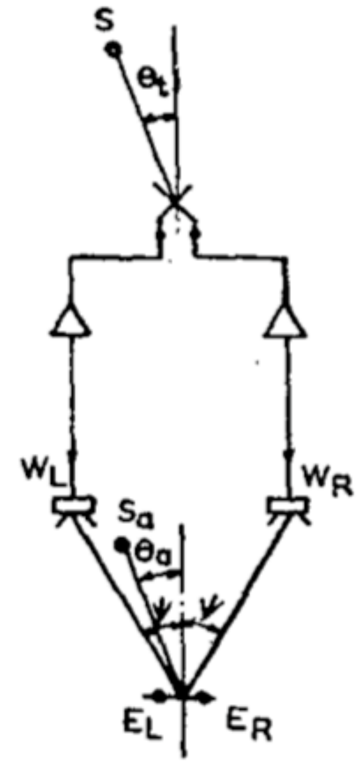
Fig. 3. Actual 3-channel stereophonic system. A practical stereophonic system gives a multiple reproduction of the original sound which the observer interprets as coming from a single source.

Blumlein/EMI (1931)









PREPRINT NO. 1571 (A6)

PRACTICAL PERIPHONY: THE REPRODUCTION OF FULL-SPHERE SOUND

Michael A. Gerzon, Mathematical Institute, University of Oxford, England

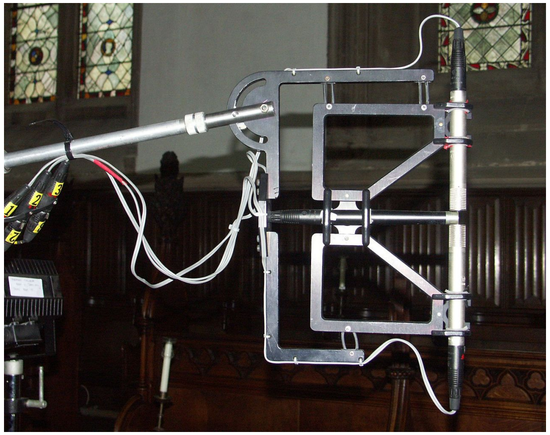
Presented at the 65th Convention 1980 February 25 through 28 London

AES

AN AUDIO ENGINEERING SOCIETY PREPRINT

What is Ambisonics?

- Extensible, hierarchical system for representing sound fields
  - Says how something should sound, rather than specific speaker signals.
- Capture or creation – "Encoding"
  - Microphone arrays
    - 2-D or 3-D
    - Native B-format (Nimbus),
    - Tetrahedral, Spherical arrays, …
  - Ambisonic panners, reverbs



- Transmission – Interchange Conventions
- Reproduction – "Decoding"
  - 2-D, "horizontal" or 3-D "with height" loudspeaker arrays
  - "Any" size or shape array of loudspeakers

DECODERS

What is an Ambisonic Decoder?

- In Ambisonics, the program format is independent of the reproduction layout (vs. Dolby Surround, Auro-3D)

- The decoder's task is to create the best *perceptual impression* possible that the sound field is being reproduced accurately, given the resources available
  - Bandwidth, number of speakers, configuration of speakers …
  - This makes a hierarchical ("coarse-to-fine") representation desirable

- Terminology
  - Channels are denoted by degree and order of the spherical harmonic (l,m)
  - Ambisonic order of a signal set is the highest degree channel in use
  - Mixed-order signal sets have different horizontal and vertical orders

Heller, Benjamin, *Design and implementation of Filters for Ambisonic Decoders, First International Faust Conference, 2018, Mainz*, Germany

Spatial Audio Psychoacoustics

- Human directional hearing operates over three frequency ranges
  - Low (< 800 Hz) using time-of-arrival cues (interaural time differences, ITD, rV)
  - High (800-5000 Hz) using relative intensity cues (internal level differences, ILD, rE)
  - Very high (> 5 kHz) where pinna effects dominate
- First two are listener independent and are based on Rayleigh's *Duplex Theory* of directional hearing
  - Ambisonics incorporates different criteria for LF and HF reproduction
- (Third is satisfied by listener's head in the soundfield – interesting implications for binaural renderer)

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Gerzon's Theory of Auditory Localization

- Early workers in stereo did theoretical analysis showing how stereo did (or didn't) provide proper localization cues
- Gerzon's contribution was to integrate those theories and came up with a theory that defined
  - $\mathbf{r}_V$, the vector sum of the signals from the loudspeakers
  - $\mathbf{r}_E$, the vector sum of the squares of the signals from the loudspeakers.
- By providing a simple mathematical encapsulation, we can use these to
  - design decoders
  - prove theorems, e.g., polygonal decoder theorem
  - help understand what various spatial sound reproduction systems can and cannot do

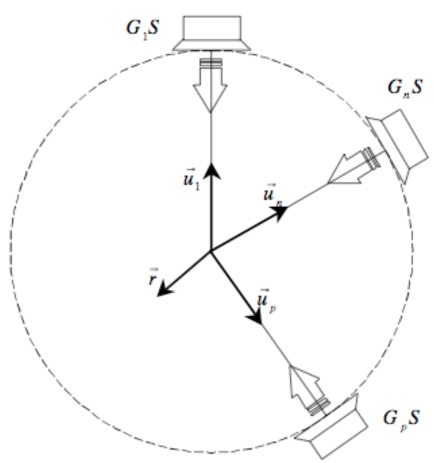
Localization Models

- Two primitive models
  - Velocity localization vector
    - *ITD -- Blumlein, Clark, et al.*

$$r_V \hat{\mathbf{r}}_{\mathbf{V}} = \mathrm{Re} \frac{\sum_{i=1}^{n} G_i \hat{\mathbf{u}}_{\mathbf{i}}}{\sum_{i=1}^{n} G_i}$$

  - Energy localization vector
    - *ILD -- Fransen, Mertens, Makita…*

$$r_E \hat{\mathbf{r}}_{\mathbf{E}} = \frac{\sum_{i=1}^{n} (G_i G_i^*) \hat{\mathbf{u}}_{\mathbf{i}}}{\sum_{i=1}^{n} (G_i G_i^*)}$$



Localization Vector Theory

- $\mathbf{r}_V$ predicts low-frequency localization almost perfectly.
  - If $\mathbf{r}_V=1$, then low-frequency sounds will be precisely located.
- $\mathbf{r}_E$ predicts mid-frequency localization moderately well.
  - If $\mathbf{r}_E=1$, then mid-frequency localization will be good
  - BUT… $\mathbf{r}_E$ is always less than 1, unless the sound is coming from a single point source.
  - At best $\mathbf{r}_E = \cos(\theta/2)$, where θ is the angle between the loudspeakers, so for a square array $\mathbf{r}_E \leq 0.707$.
  - In general, $\mathbf{r}_E$ is low in directions with few loudspeakers
  - Best we can do is have it change smoothly in performance from dense areas to sparse areas.

Energy Localization Vector

- Maximizing $\mathbf{r}_E$ *and* getting it to point in the right direction is the crux of the decoder design problem.
  - Easy with regular arrays
  - Irregular arrays always involve tradeoffs
  - Virtually all real-world arrays are irregular!
    - Arrays need to fit in real rooms
    - ITU 5.1 is the dominant domestic standard, rear speakers 120° apart.

Goals for decoder design

- Mimic conditions of natural hearing
  - Constant amplitude gain for all source directions (*P*)
  - Constant energy gain for all source directions (*E*)
  - At low frequencies, correct reproduced wavefront direction and velocity (r_V)
  - At high frequencies, maximum concentration of energy in the source direction (r_E)
  - Matching high- and low-frequency perceived directions

- Getting r_E correct is the most difficult aspect
- Recent work shows that it is also the most important!

Near-Field Compensated High-Order Ambisonics

- "Full-featured decoder" -- supports all the signal processing needed for *Near-Field Compensated High-Order Ambisonics (NFC-HOA)* [1,2]
- Requirements:
  (1) Phase-matched, band-splitting filters
    - enable the use of separate low- and high-frequency decoding matrices, supporting the duplex theory of human directional hearing (known as dual-band decoding or Vienna decoding)

  (2) Per-speaker, near-field compensation
    - most Ambisonic decoder design techniques assume the superposition of plane waves, whereas the loudspeakers are actually at a finite distance and radiating spherical waves (known as NFC)

  (3) Delay and level compensation
    - accommodate loudspeakers at varying distances from the center of the array

- 1 & 2 require frequency-dependent processing
- 3 is broadband delay and gain adjustment

[1] J. Daniel, "Spatial Sound Encoding Including Near Field Effect: Introducing Distance Coding Filters and a Viable, New Ambisonic Format ," *Preprints 23rd AES International Conference, Copenhagen*, 2003.

[2] J. Daniel, "Représentation de champs acoustiques, application à la transmission et à la reproduction de scènes sonores complexes dans un contexte multimédia (*Acoustic field representation, application to the transmission and the reproduction of complex sound environments in a multimedia context)*," PhD Thesis, Université Paris 6, Paris, 2001.

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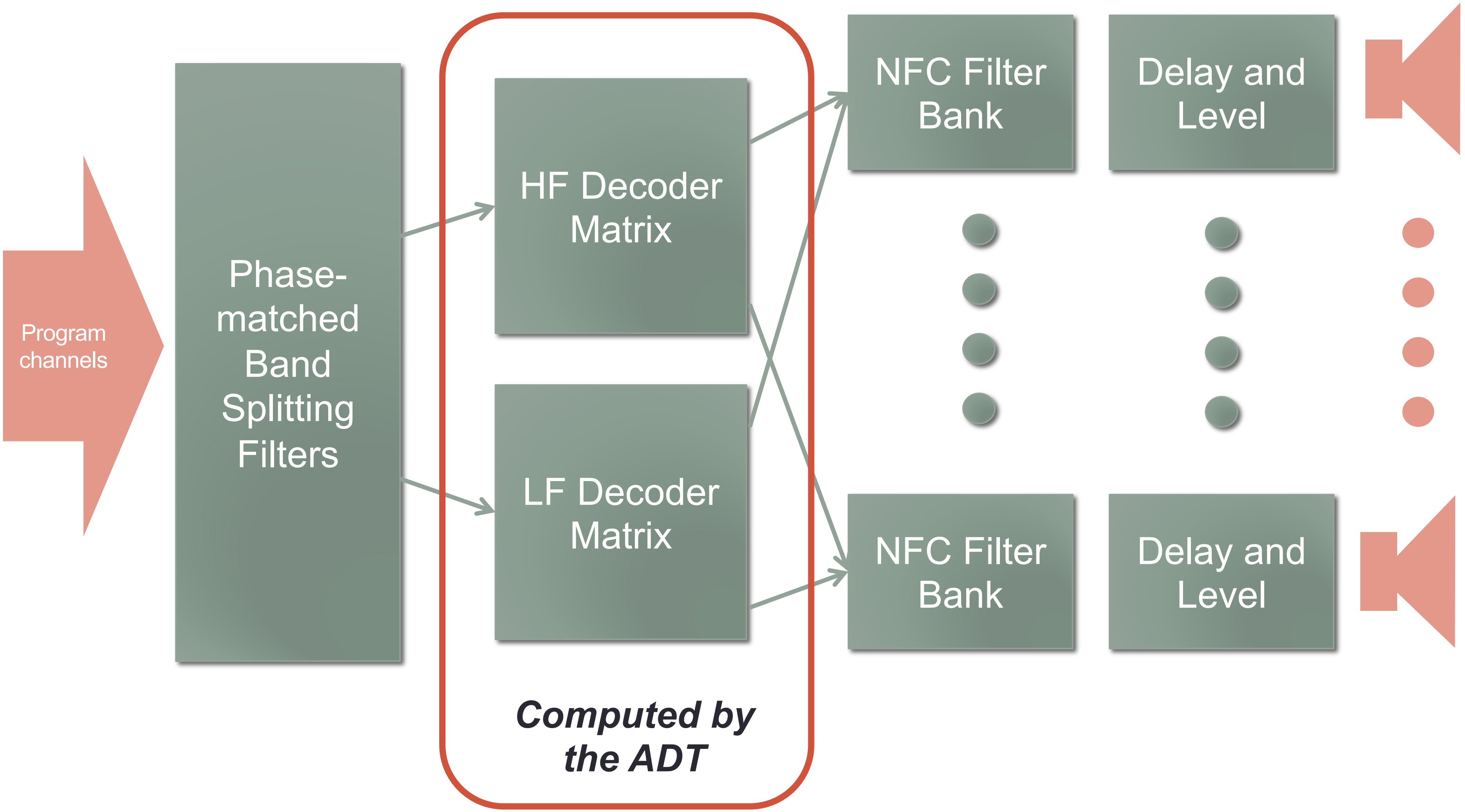
Math formalism

- Ambisonics is based on a spherical-coordinate representation of the sound field
  - Solve the Helmholtz wave equation in spherical coordinates
  - Solution is the Fourier-Bessel description of the soundfield

$$p(kr, \theta, \phi) = \sum_{l=0}^{+\infty} j^l j_l(kr) \sum_{m=-l}^{+l} B_{lm} Y_{lm}(\theta, \phi)$$

  - Solutions decompose into *angular* part (the spherical harmonics) and a *radial* part (the spherical Bessel functions)
- In spherical waves, the pressure gradient arises from both the direction of propagation (real) and the spreading of energy across the expanding wavefront (reactive) ➔ LF boost
- Most decoder design techniques ignore the radial part by assuming the loudspeakers produce plane waves
  - Good decoders compensate for this with NFC filters
  - Not-so-good decoders just ignore the effect.

Ambisonic Decoder Architecture



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Designing Decoders

- Inversion problem
  - Know the desired output. What inputs produce this?
- Decoders for regular polygon and polyhedra loudspeaker arrays are easy to design
  - Build the speaker encoding matrix, K, by sampling the spherical harmonics at the speaker directions
  - Use pseudoinverse to find the basic decoding matrix M
  - rE guaranteed to point in same direction as rV

- However…
  - Room geometry or visual considerations often limit speaker placement
  - 3-D HOA requires placing more speakers above and *below* the listener

Tradeoffs

- Once we deviate from regular geometry
  - we must trade off localization accuracy for uniform loudness
  - Directions of rE and rV are not the same
- Localization degrades outside the area with a high density of loudspeakers
- Gerzon used nonlinear optimization for this
  - Many implementations: Wiggins, Moore & Wakefield, Tsang, BLaH
- Works well for small arrays (e.g., ITU 5.1)
- Convergence is slow for large HOA arrays (hrs)

- IDHOA (Scaini and Arteaga)
  - Better objective function and zero out small coefficients
- New implementation for the Python ADT
  - Based on automatic differentiation to get analytic expressions for partial derivatives

Ambisonic Decoder Toolbox

- Open-source MATLAB/Octave toolbox
  - computes decoder configurations
  - given the geometry of the loudspeaker array and the set of Ambisonic transmission channels in use
- Implements three major design techniques and variants, including techniques well suited for “difficult” (but common!!) configurations such as domes and stacked rings.
- Includes a “full-featured” decoder engine written completely in Faust
  - Key features
    - Support of a wide variety of plugin architectures and hosts
    - Focus on math, not DSP coding tricks, target system, …
    - (about 240 lines w/o configuration section)
- Git Repository
  - https://bitbucket.org/ambidecodertoolbox/adt

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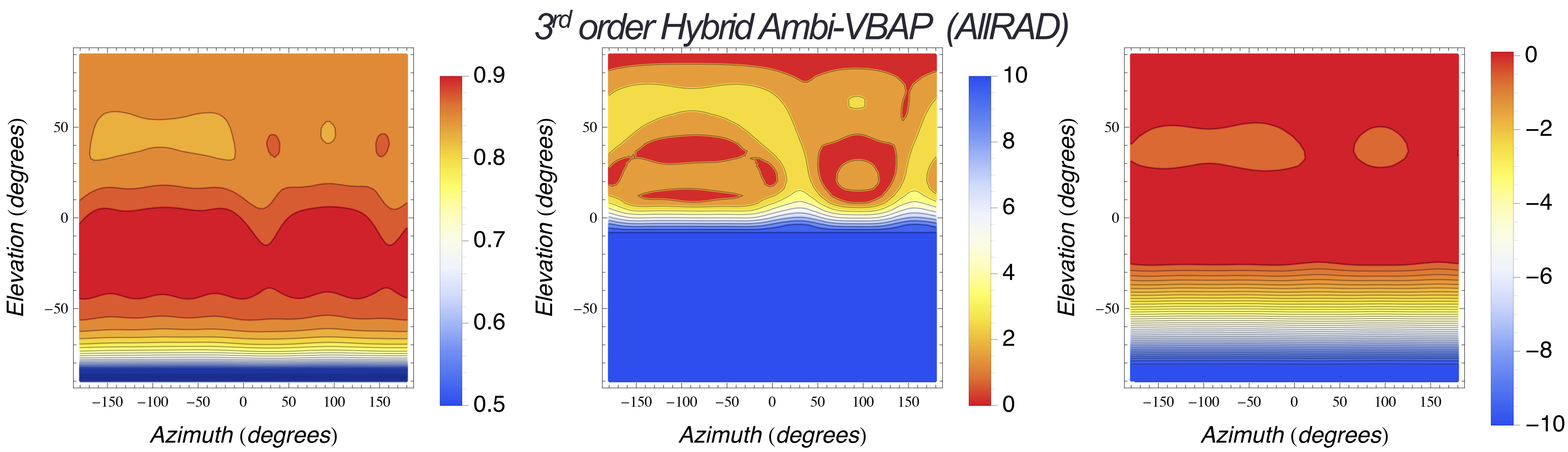
Strategies in ADT

- Use an inversion technique suited to ill-conditioned matrices
  - Constant energy decoder
  - Truncated SVD
  - Energy limited
- Invert a well-behaved full-sphere virtual speaker array, map to a real array
  - Hybrid Ambisonic-VBAP
  - AllRAD (Zotter and Frank)
- Derive a new set of basis functions for which inversion is well behaved
  - Spherical Slepian Functions
  - EPAD (Zotter, Pomberger, Noisternig)

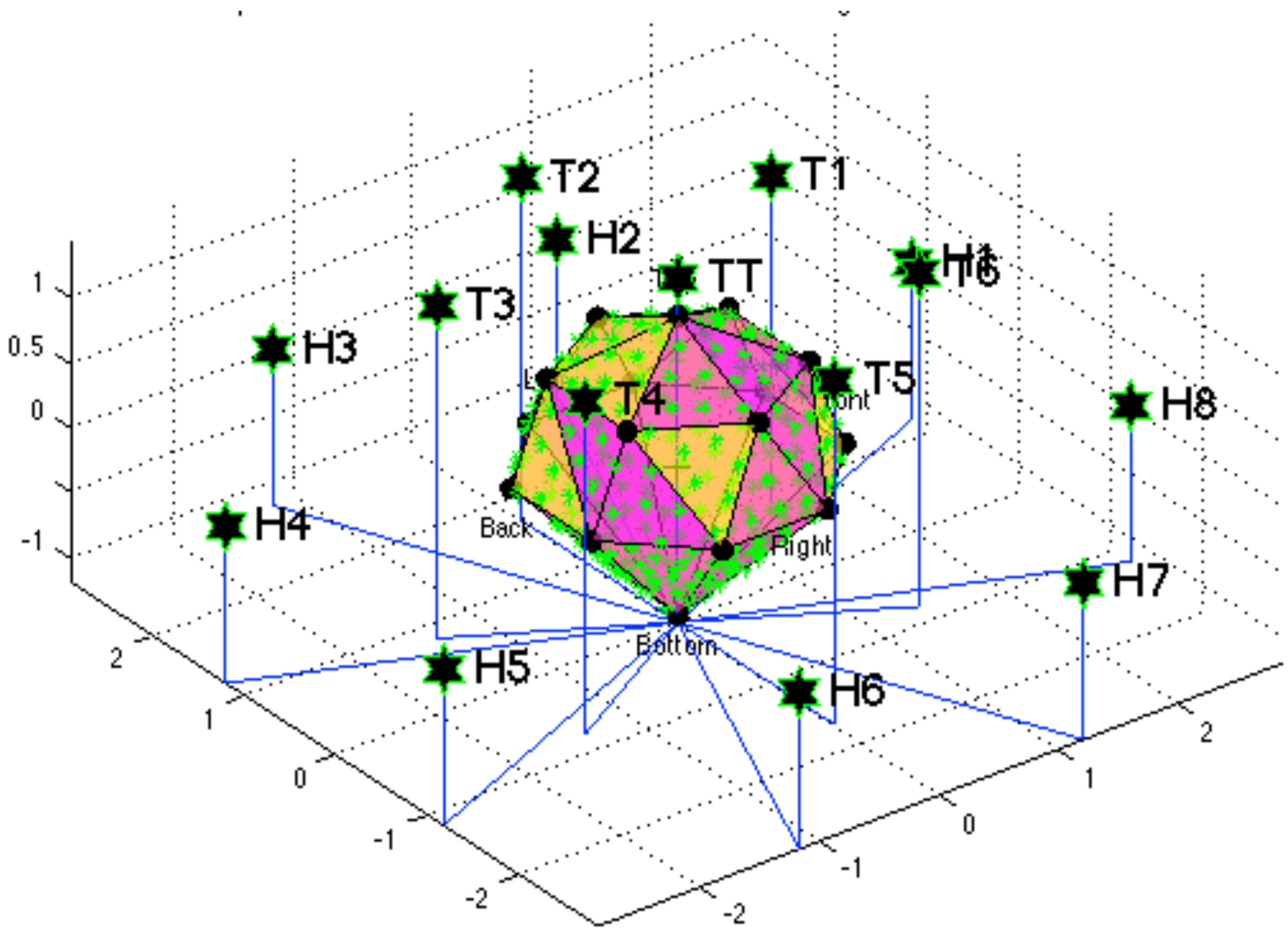
Are these decoders Ambisonic?

- Ambisonic theory specifies performance goals, not how to design a decoder
- We use the same criteria for these decoders
- But…
  - Apply them only to source directions in the covered part of the sphere
  - Require them be "well behaved" in other directions



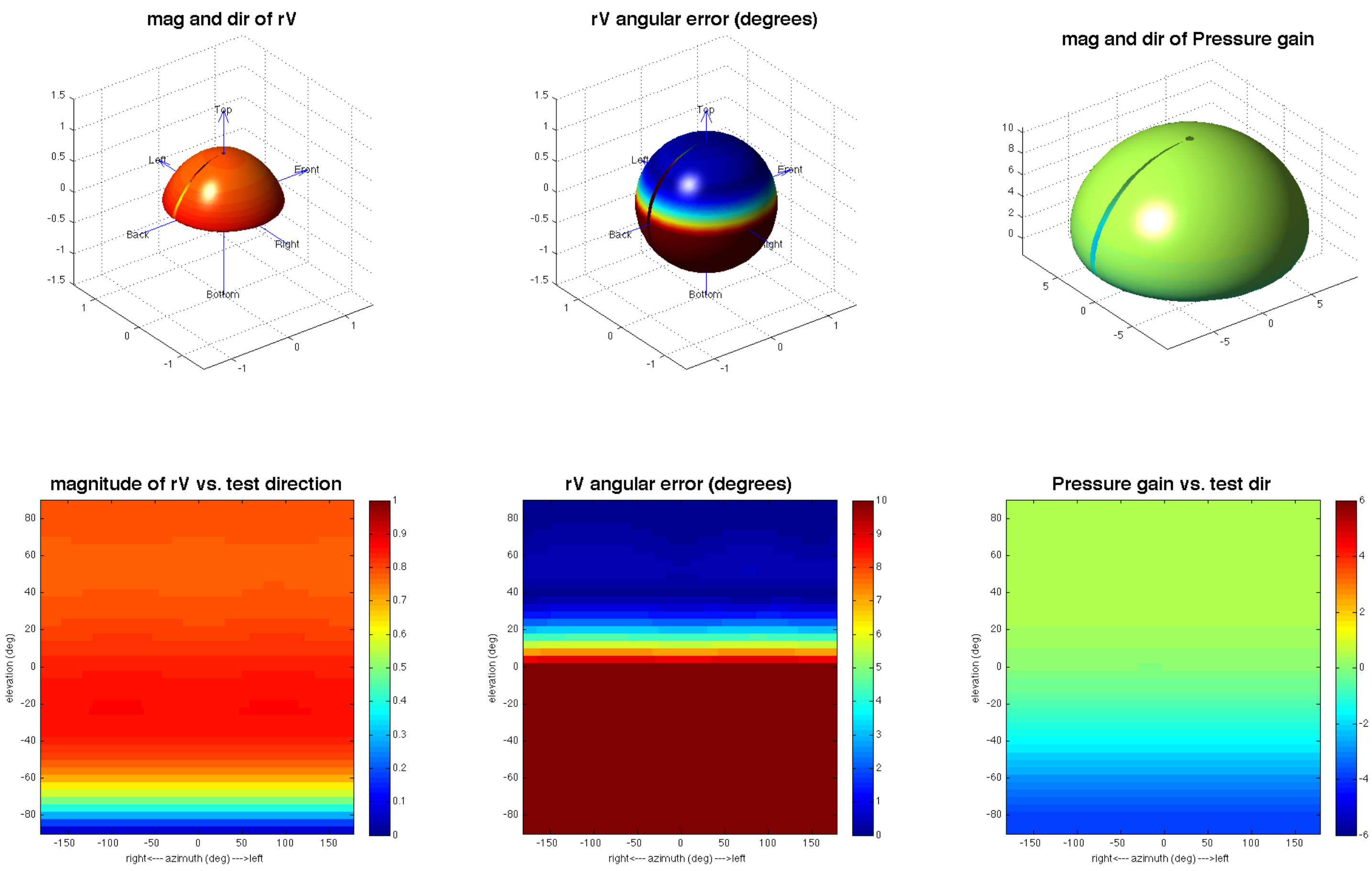
(a) r_E vs. Test Direction (b) r_E Direction Error (degrees) (c) Energy Gain (dB)

AllRAD – Hybrid Ambi-VBAP

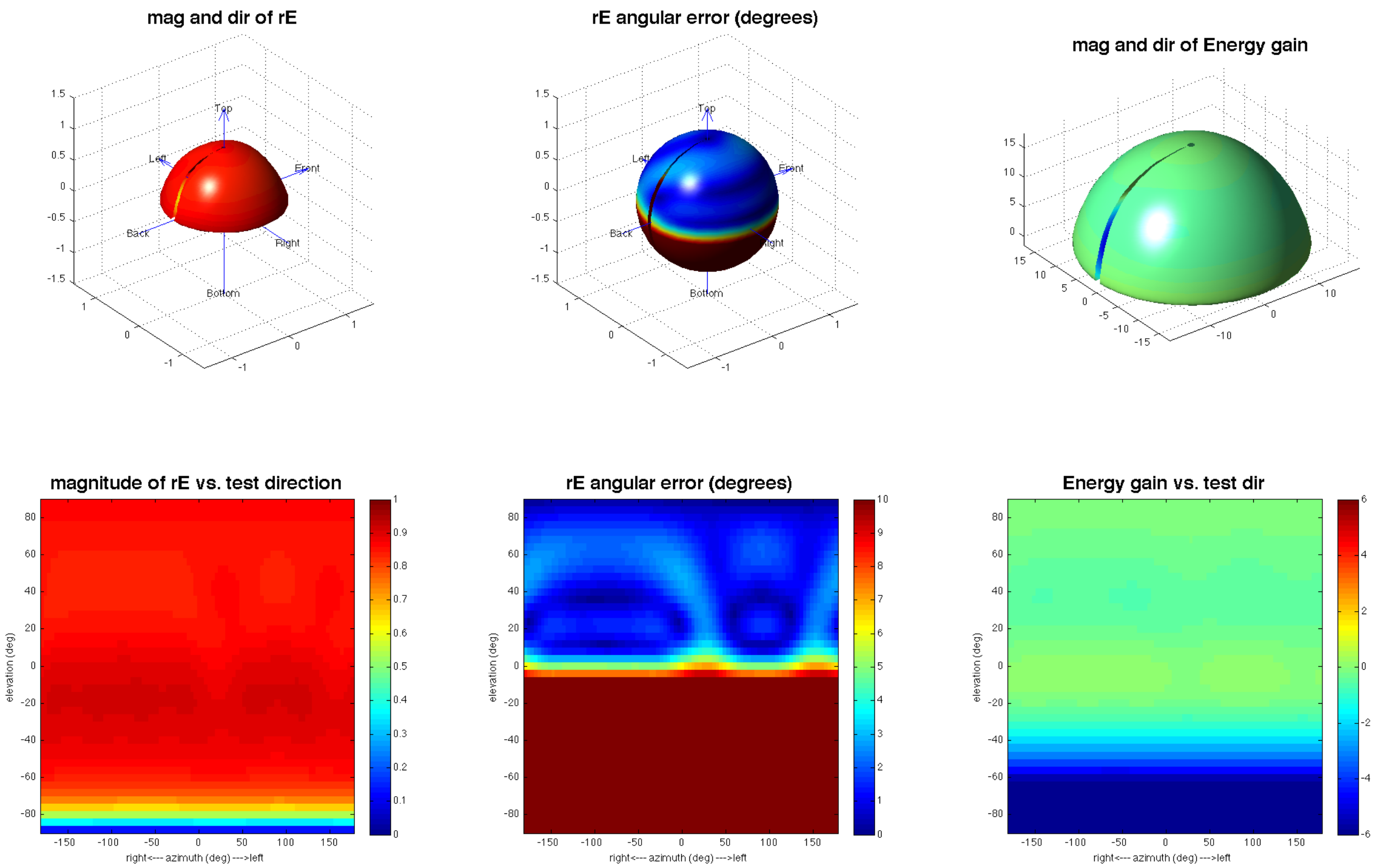


- 240-point spherical design for virtual speaker array
- Dome of upper 15 loudspeakers of CCRMA Listening Room, 8-6-1
- Imaginary speaker at bottom
- Design procedure detailed in LAC 2014 paper

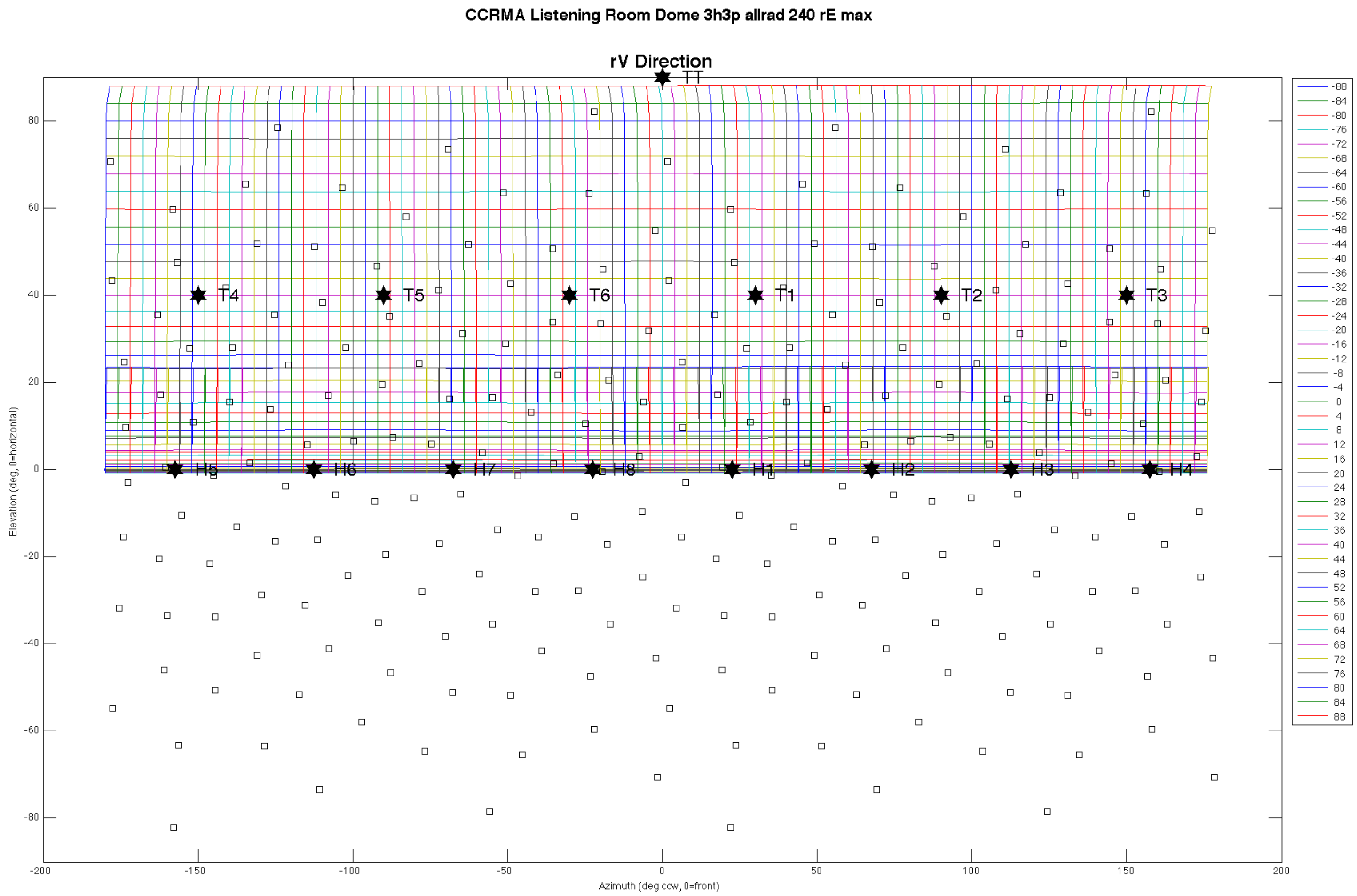
AllRAD performance r_v (3rd order)



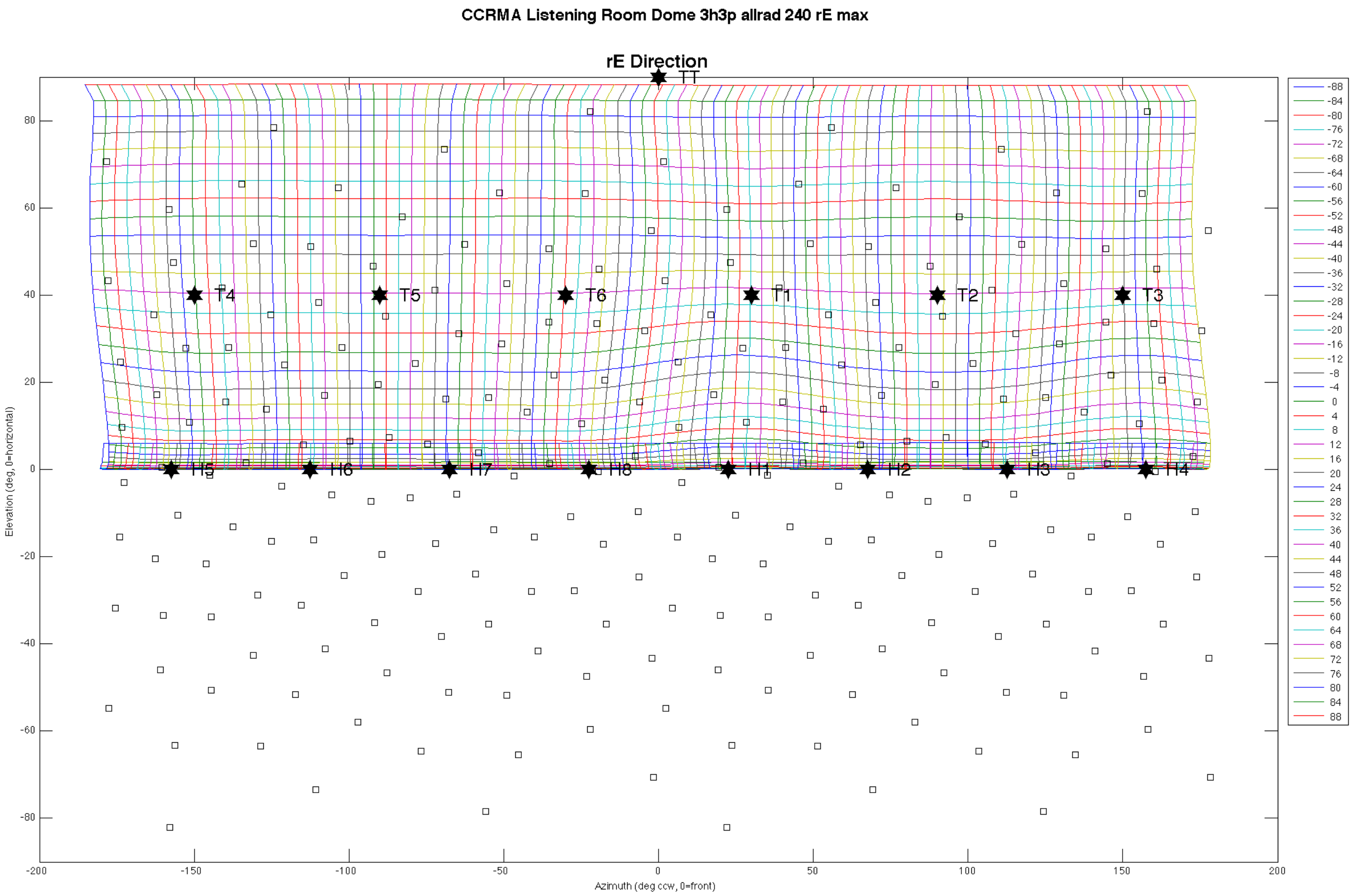
AllRAD performance r_E (3^{rd} order)

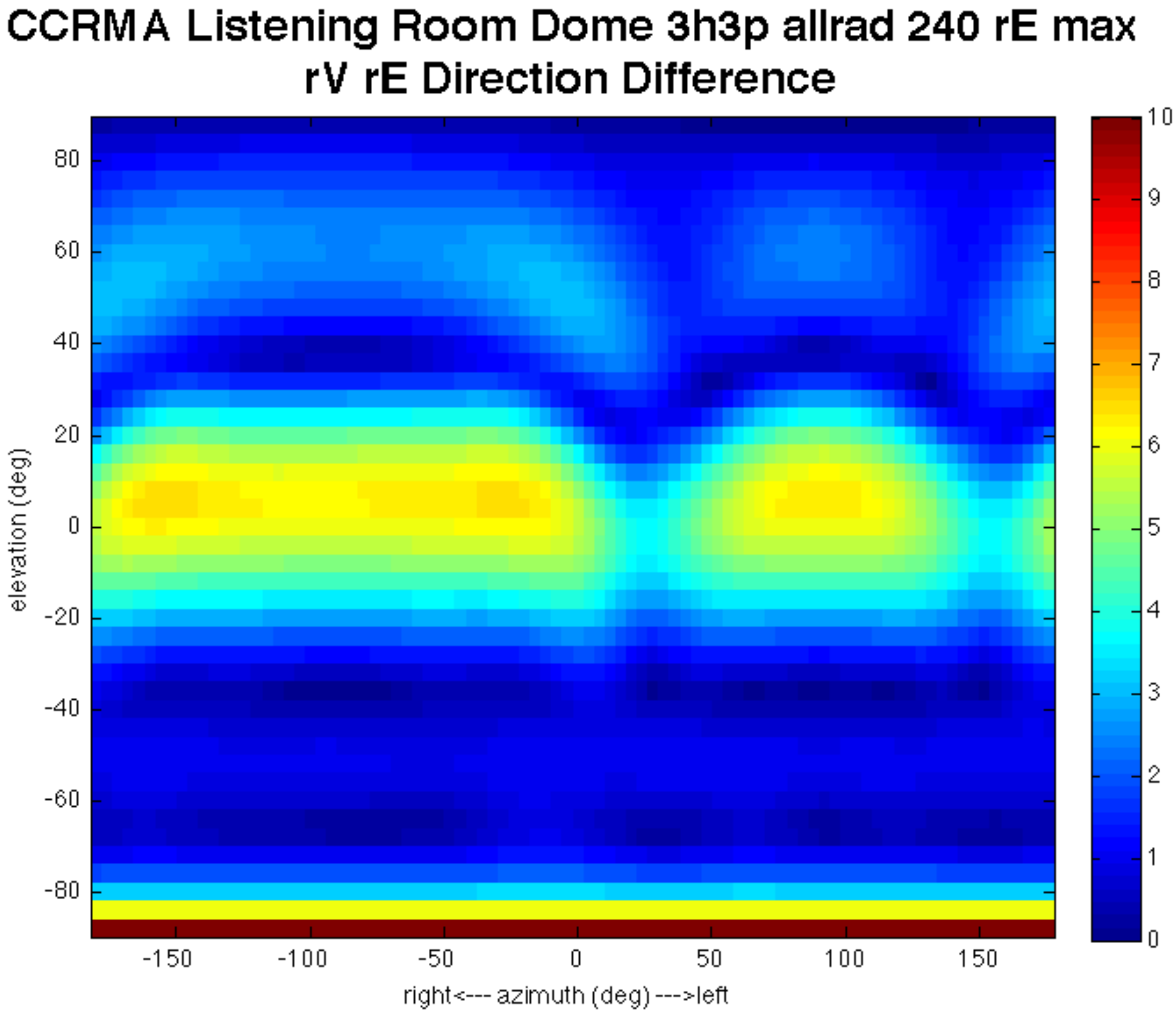


AllRAD r_V direction grid



AllRAD r_E direction grid



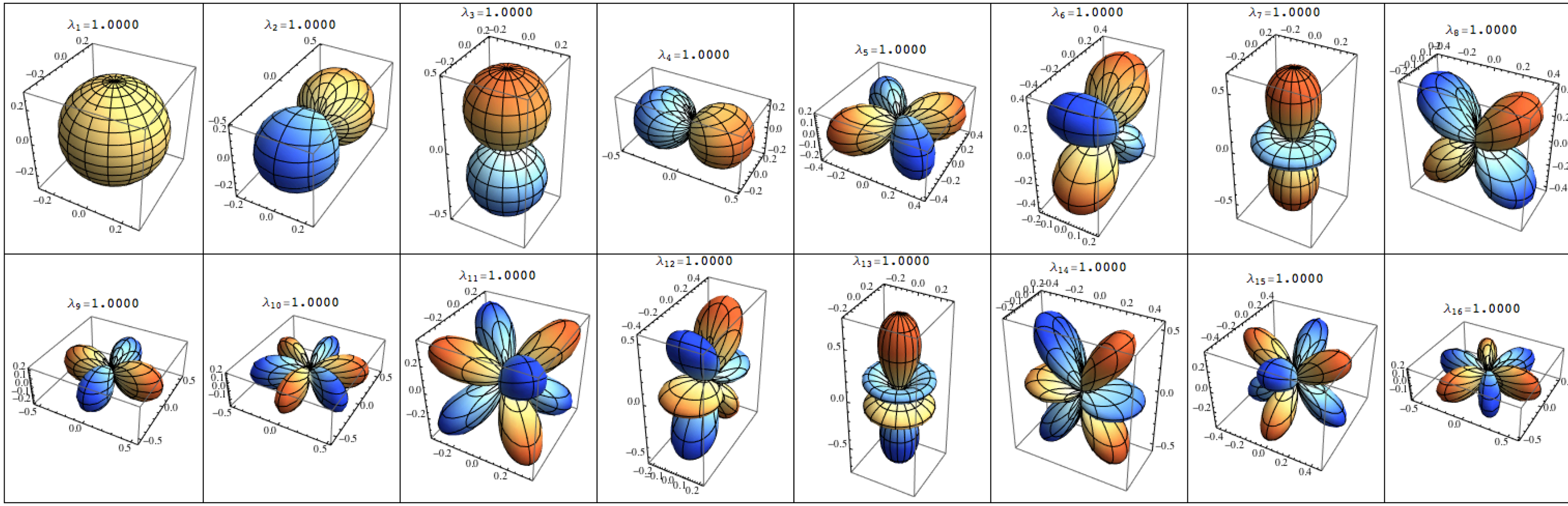

CCRMA Listening Room Dome 3h3p allrad 240 rE max
rV rE Direction Difference
elevation (deg)
right<--- azimuth (deg) --->left
80
60
40
20
0
-20
-40
-60
-80
-150
-100
-50
0
50
100
150
10
9
8
7
6
5
4
3
2
1
0

Spherical Slepian Functions

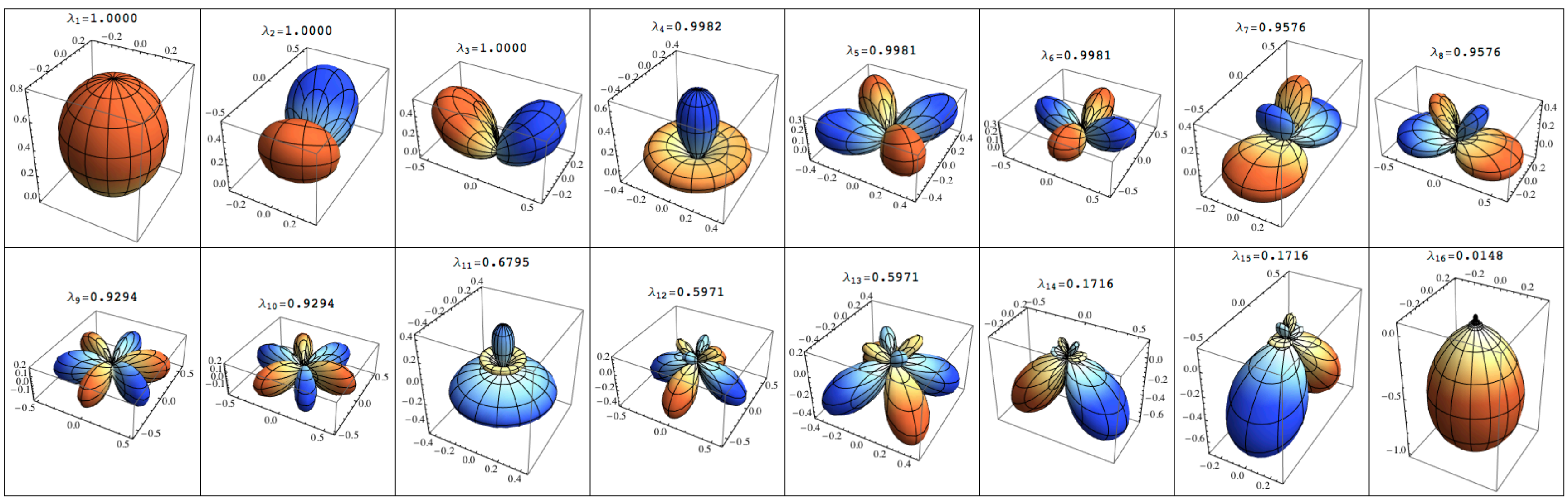
- Linear combinations of spherical harmonics
- Produce a new set of basis functions that are zero outside the region of interest on the sphere
- Remain orthogonal within the region
- Used in satellite geodesy to model earth's gravitational and magnetic fields from incomplete data

- In Ambisonic decoding, we can specify a region of the sphere, a dome or a ring, and derive a well behaved set of basis functions for that region.
- Design procedure detailed in LAC 2014 paper

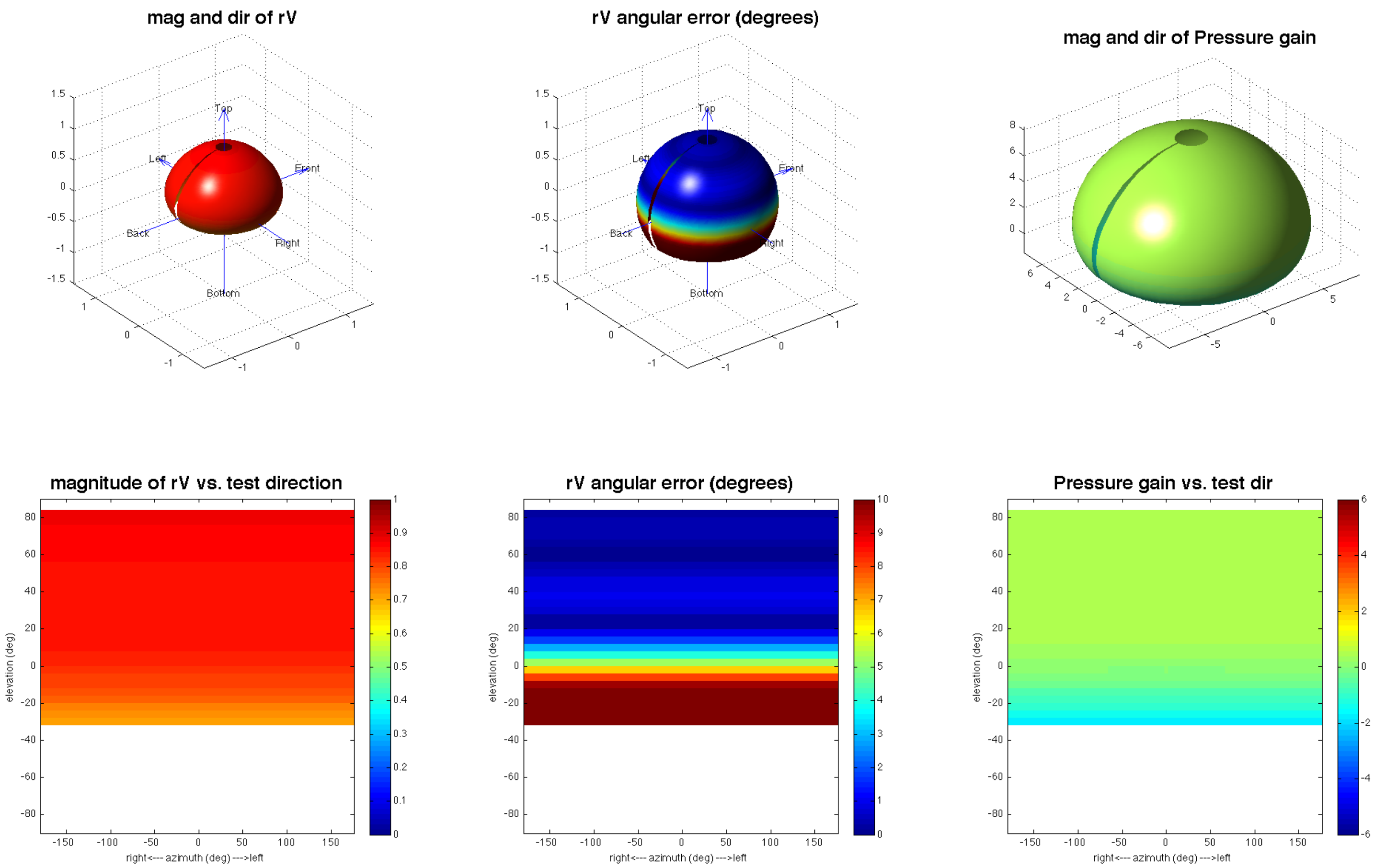
3rd order spherical harmonics (blue = inverted polarity)



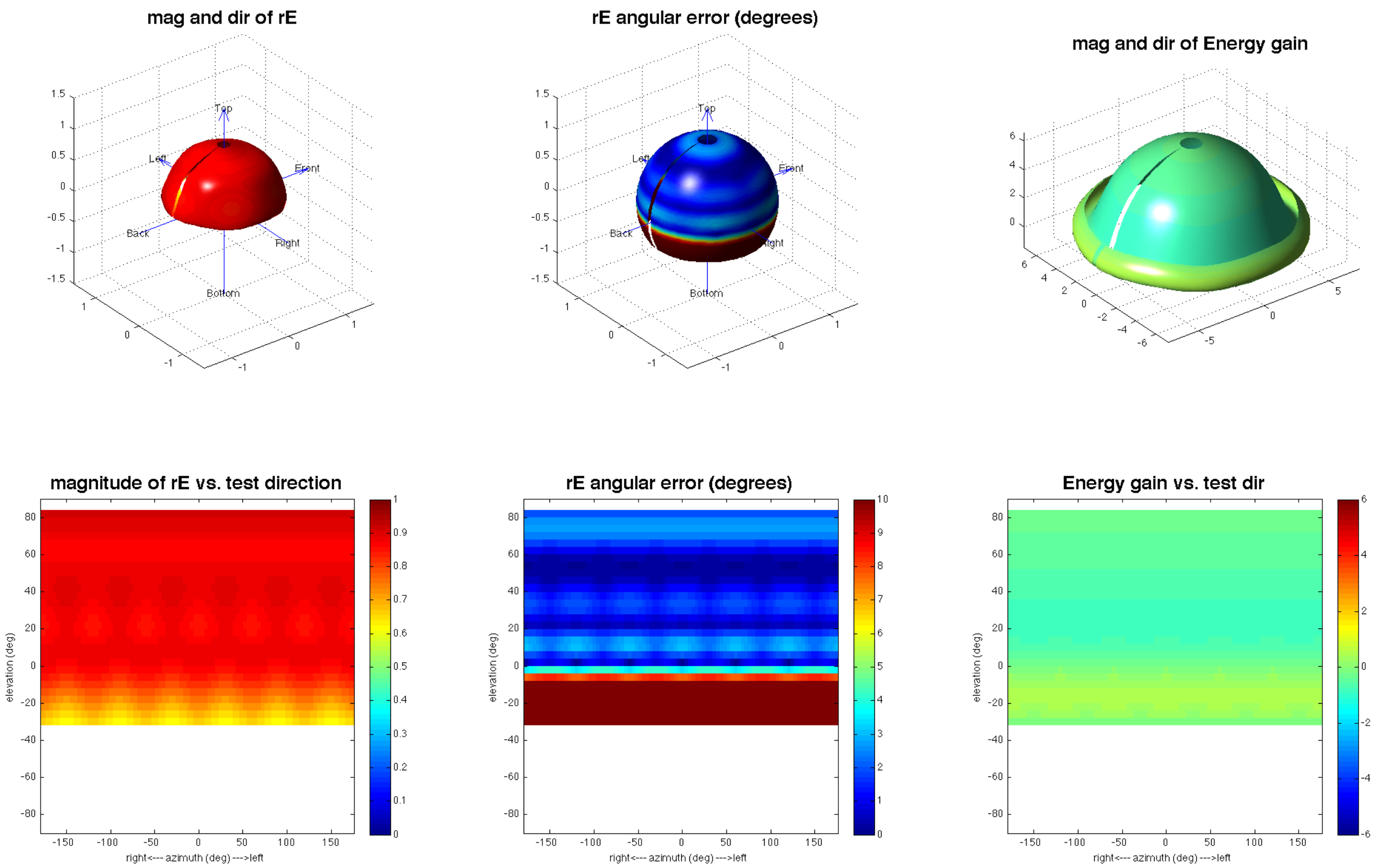
3rd order spherical Slepian functions for +90° to -30° dome (first 13 used for decoder)



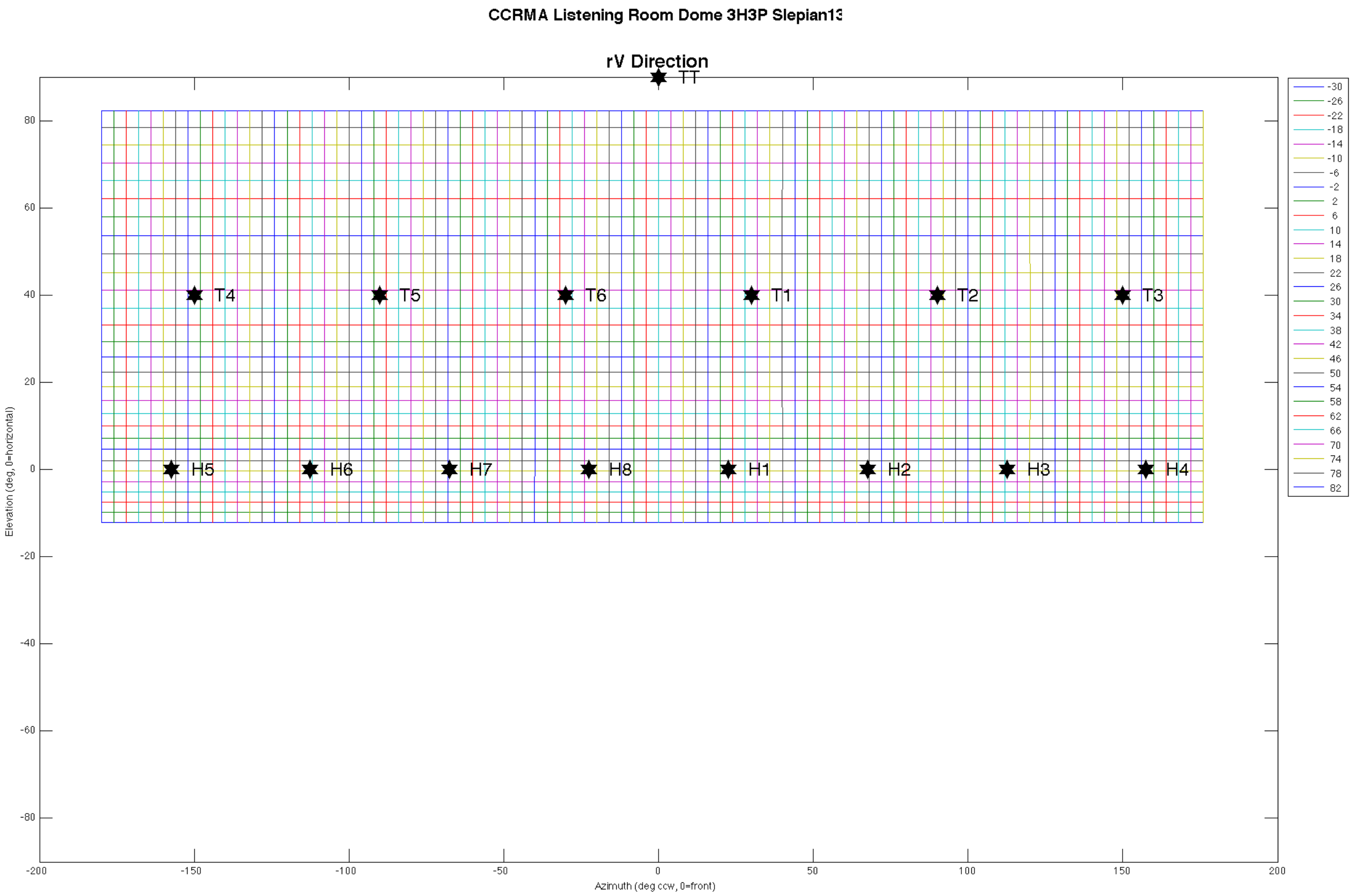
Spherical Slepian performance r_V



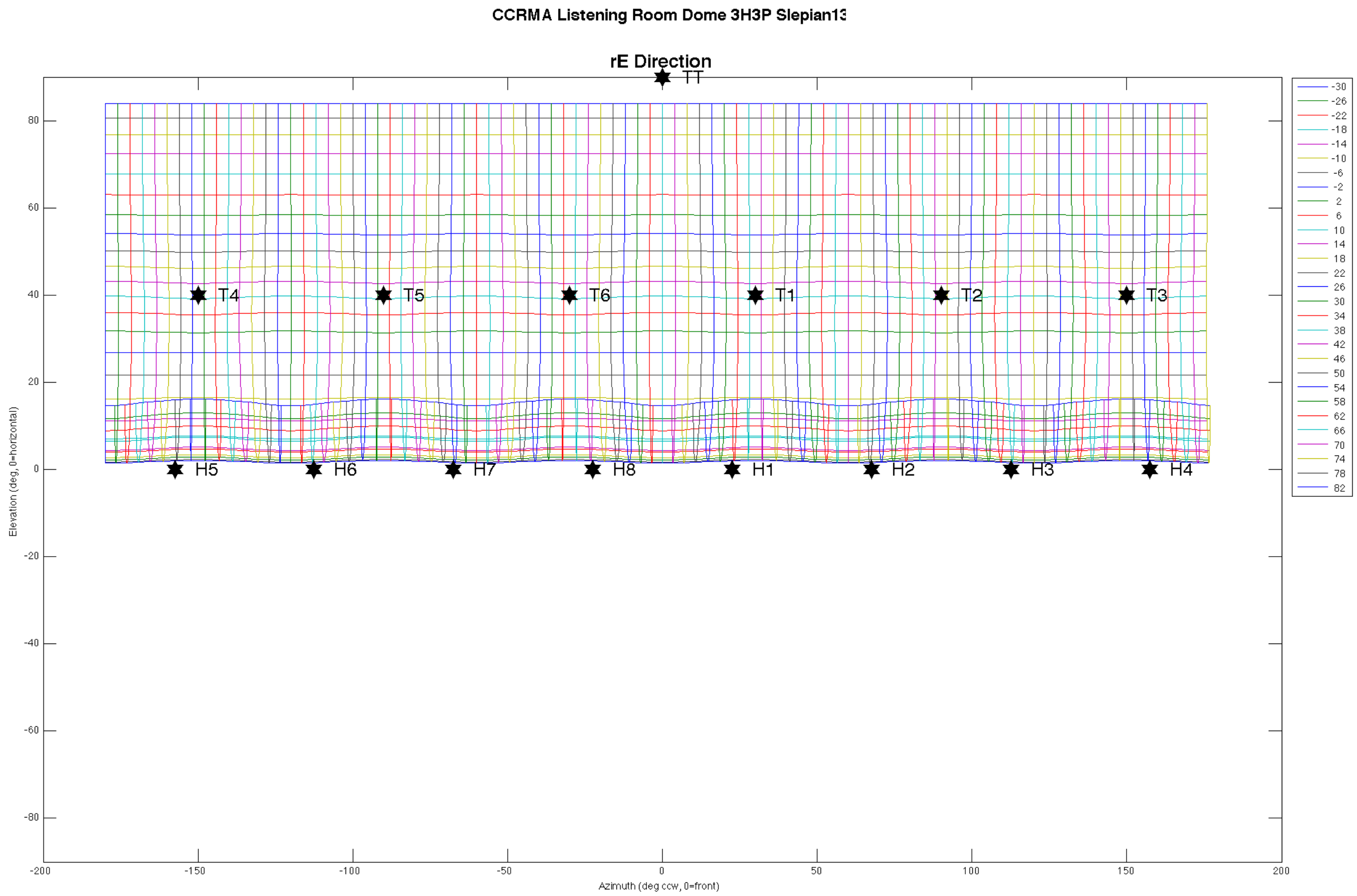
Spherical Slepian performance r_E



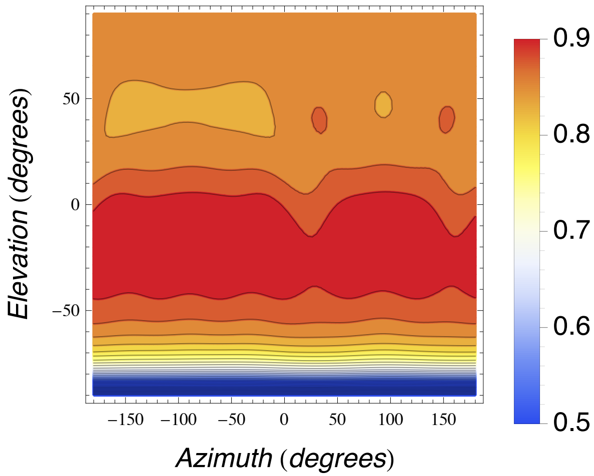
Spherical Slepian r_V direction grid



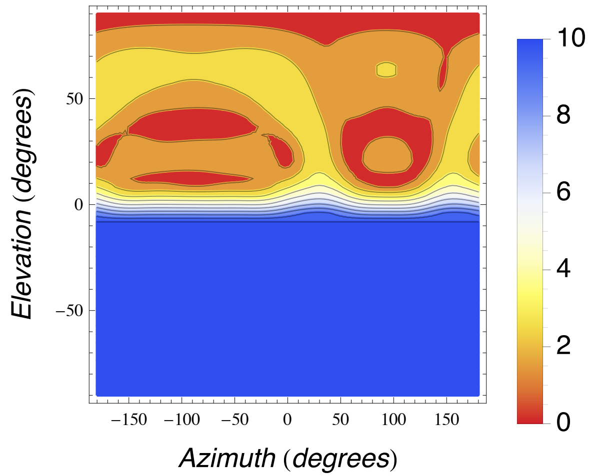
Spherical Slepian r_E direction grid



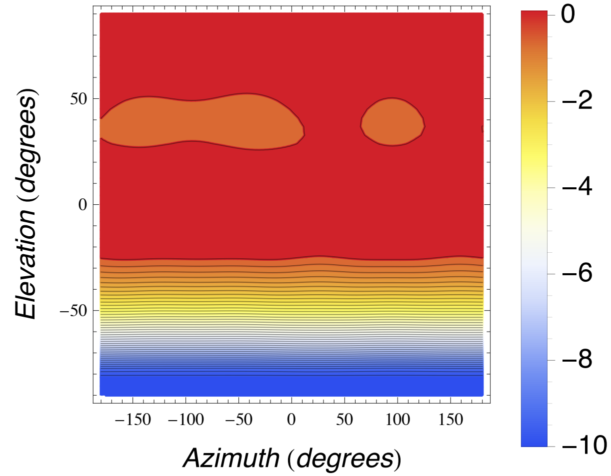
3rd order Hybrid Ambi-VBAP (AllRAD)



(a) r_E vs. Test Direction

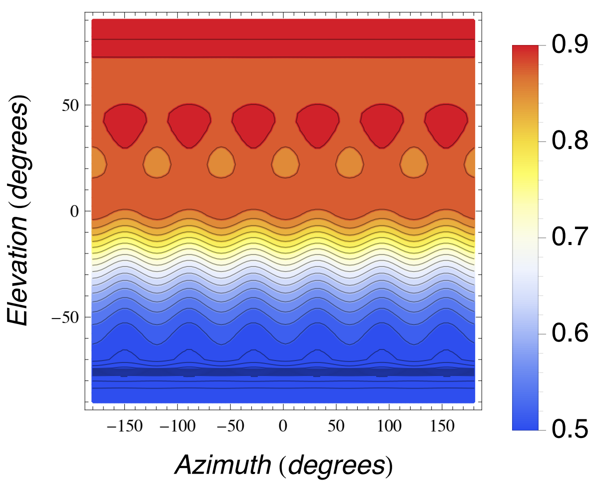


(b) r_E Direction Error (degrees)

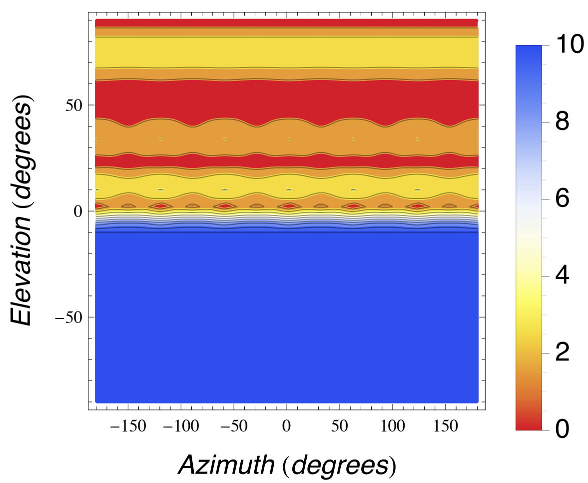


(c) Energy Gain (dB)

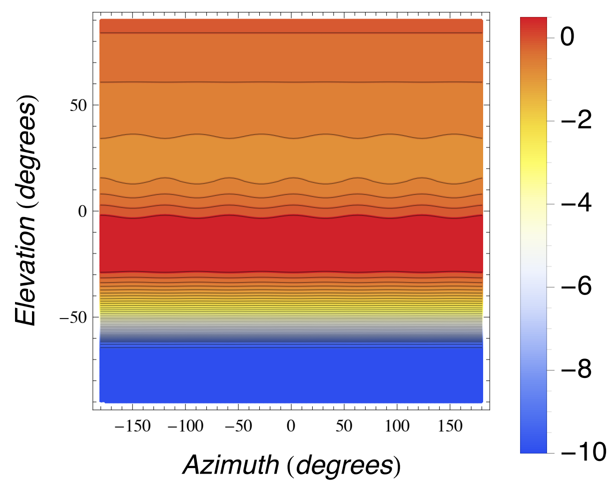
3rd order spherical Slepian function (EPAD)



(a) r_E vs. Test Direction



(b) r_E Direction Error (degrees)



(c) Energy Gain (dB)

Decoding Engine

- Decoding engine written in FAUST
  - No inherent limit on order
  - Dual band, NFC filters, distance compensation, …
- Toolbox writes out configuration section, appends implementation
- Compiles to LADSPA, LV2, Pd, Supercollider, VST, AU …
- Can be used independently of toolbox
- Drawback: Configuration “baked into” plugin

- Toolbox also writes out configuration files for
  - Kronlachner’s ambiX plugin suite
  - Adriaensen’s Ambdec
  - Bare matricies

Alternatives

- Commercial
  - Blue Ripple -- http://www.blueripplesound.com
    - max rE only
  - Harpex -- https://harpex.net
    - Parametric decoding
- Free
  - ATK -- http://www.ambisonictoolkit.net
    - First-order only, Reaper and Supercollider, kernels can be used in other systems
  - ambiX – Ambisonic plug-in suite
    - max rE only, no distance, level, or NFC compensation
    - Uses ADT for new configurations
    - Can do binaural rendering
  - IEM Plug-in Suite -- https://plugins.iem.at
    - AllRAD, max rE only, no NFC
  - Spatial Audio Real-time Applications (SPARTA)
    - http://research.spa.aalto.fi/projects/sparta_vsts/plugins.html
    - Looks very promising, haven't tried it yet
  - Other Ambisonics libraries and plugins found on the internet
    - Beware! Test carefully! (feel free to contact me for an opinion)

ambi2youtube

- A python script to convert “amb” files to YouTube spatial media format
  - https://bitbucket.org/ambidecodertoolbox/amb2yt

- Dependencies
  - sox, ffmpeg, Google spatial media injector

- Examples
  - Eight directions
    - https://youtu.be/eY9DMn8pgGA
  - Dvořák: Carnival Overture, Op92
    - https://youtu.be/mDVVyNKkhSA

Recent Overviews

1. S. Spors, H. Wierstorf, A. Raake, F. Melchior, M. Frank, and F. Zotter, "Spatial Sound With Loudspeakers and Its Perception: A Review of the Current State," *Proceedings of the IEEE*, vol. 101, no. 9, pp. 1920–1938, Sept. 2013.

2. R. L. Bleidt, D. Sen, A. Niedermeier, B. Czelhan, S. Fug, S. Disch, J. Herre, J. Hilpert, M. Neuendorf, H. Fuchs, J. Issing, A. Murtaza, A. Kuntz, M. Kratschmer, F. Kuch, R. Fug, B. Schubert, S. Dick, G. Fuchs, F. Schuh, E. Burdiel, N. Peters, and M.-Y. Kim, "Development of the MPEG-H TV Audio System for ATSC 3.0," *IEEE Trans. on Broadcast.*, vol. 63, no. 1, pp. 202–236, Mar 2017.

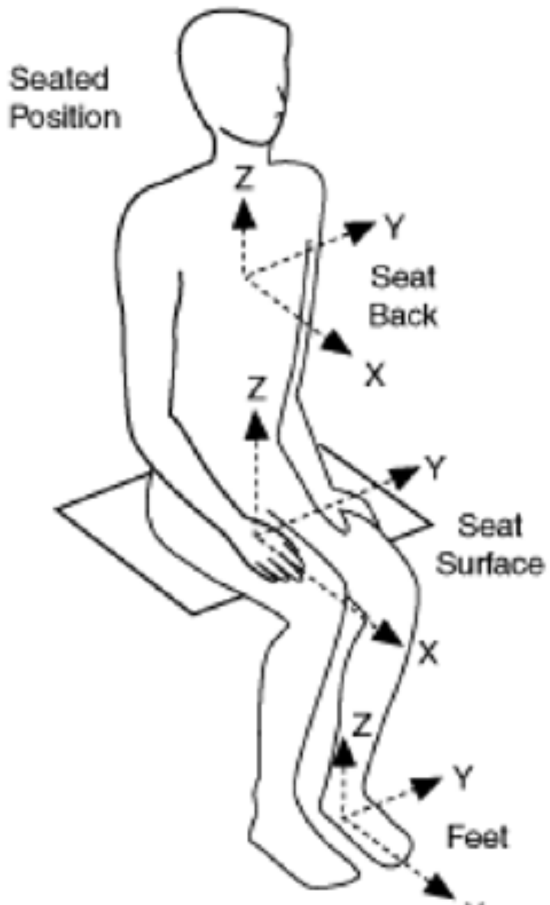
INTERCHANGE

Interchange, the bad news

- Many different conventions in use
- Many bad, poorly tested, or untested software packages
- Very difficult to diagnose by listening

Cartesian Coordinate Systems

Ambisonics

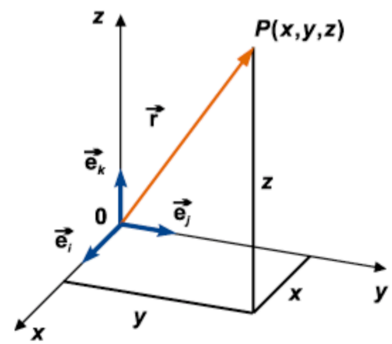


ISO 2631-1:1997(E)

Other ones you will encounter

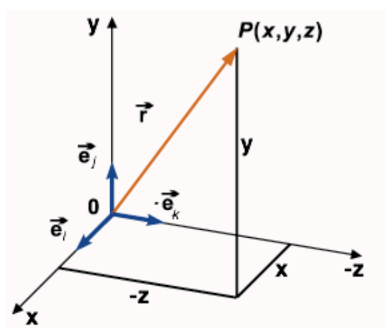
Mathematical System: (Navigation)

- right-handed system, y to the front, x to the right, z to the top



OpenGL, OpenAL System (Computer Graphics)

- right-handend system, x-axis to the right, y-axis to the top, z-axis to the front



Spherical Coordinate Systems

```
# physics
#   theta is zenith angle, angle from positive z axis
#   phi is counter-clockwise horizontal angle around the z-axis, 0 is x-axis
#   Mathematica, Boost, and Wikipedia (on 5/16/17) use this.
spherical_coordinates_physics = {
    x: cos(phi) * sin(theta),
    y: sin(phi) * sin(theta),
    z: cos(theta)}

# mathematics
#   theta is counter-clockwise horizontal angle around the z-axis, 0 = x-axis
#   phi is zenith angle, angle from positive z-axis
#   Wolfram MathWorld and most Calculus textbooks use this
spherical_coordinates_mathematics = {
    x: cos(theta) * sin(phi),
    y: sin(theta) * sin(phi),
    z: cos(phi)}

# geodesy and ambisonics
#   theta is counter-clockwise horizontal angle around the z-axis, 0 = x-axis
#   phi is elevation, angle from x-y plane.
#   MATLAB cart2sph and sph2cart use this
spherical_coordinates_azimuth_elevation = {
    x: cos(theta) * cos(phi),
    y: sin(theta) * cos(phi),
    z: sin(phi)}
```

Real Spherical Harmonics

C-S phase $(-1)^{|m|}$

https://en.wikipedia.org/wiki/Spherical_harmonics#Real_form

$$Y_{\ell m} = \begin{cases} \sqrt{2}\sqrt{\dfrac{(2\ell+1)}{4\pi}\dfrac{(\ell-|m|)!}{(\ell+|m|)!}}\,P_\ell^{|m|}(\cos\theta)\sin|m|\varphi & \text{if } m<0 \\ \sqrt{\dfrac{(2\ell+1)}{4\pi}}\,P_\ell^{m}(\cos\theta) & \text{if } m=0 \\ \sqrt{2}\sqrt{\dfrac{(2\ell+1)}{4\pi}\dfrac{(\ell-m)!}{(\ell+m)!}}\,P_\ell^{m}(\cos\theta)\cos m\varphi & \text{if } m>0 \end{cases}$$

orthonormal unit Zenith angle

l: degree; m: order. Also see m, n and n, m for degree and order

Note: The term "order" has been used with two distinct meanings in Ambisonic literature. Firstly, to refer to the total collection of channels. Secondly to refer to the components in that total collection of channels that are of that degree.

Modern literature deprecates the latter and uses the term *degree* (which is common across the rest of the literature on SHs). It continues the usage for the totality of channels: for example, "a third order file", "a second order microphone", "a first order recording".

Sampling the on the Unit Sphere

- Sampling schemes for integration
  - Platonic solids
  - Geodetic
  - Lebedev-Laikov
  - Spherical designs
- Inner product on the unit sphere

$$
\begin{aligned}
g_{lm,l'm'} &= \langle Y_{lm}, Y_{l'm'} \rangle_{\mathcal{R}} \\
&= \int_{\mathcal{R}} Y_{lm}(\hat{\boldsymbol{\theta}})\; Y_{l'm'}(\hat{\boldsymbol{\theta}})\; d\boldsymbol{\theta}
\end{aligned}
$$

where lm is a single-index designator for the real spherical harmonic of degree l and order m, $\hat{\boldsymbol{\theta}} = \begin{bmatrix} \cos\theta\cos\phi & \sin\theta\cos\phi & \sin\phi \end{bmatrix}^\top$, and θ and ϕ are azimuth and elevation.

SymYlm package

```
SymYlm command line
$ python SymYlm.py --help
usage: SymYlm.py [-h] [-v] [-s] [-c] [--four_pi | --unity]
                 [--orthonormal | --seminormalized | --unnormalized | --max | --FuMa]
                 [--condon_shortley_phase]
                 [-t {c,faust,fortran,javascript,mathematica,mathml,matlab,none,octave,pretty,tree}]
                 degree order

returns a polynomial expression for the real spherical harmonic of the
specified degree (l) and order (m)

positional arguments:
  degree                l, degree of the spherical harmonic. l>=0
  order                 m, order of the spherical harmonic. |m|<=l

optional arguments:
  -h, --help            show this help message and exit
  -v, --verbose         increase output verbosity
  -s, --spherical       use spherical coordinates.
  -c, --cartesian       use Cartesian coordinates. (default)
  --four_pi             inner product is 4pi
  --unity               inner product is 1 (default)
  --orthonormal         fully orthonormalized (default)
  --seminormalized      Schmidt seminormalized
  --unnormalized        no normalization
  --max                 max gain is 1 (note: may take a while to compute and
                        result may not exact for l>3
  --FuMa                Furse-Malham normalization
  --condon_shortley_phase
                        Use Condon-Shortley Phase (default is off)
  -t {c,faust,fortran,javascript,mathematica,mathml,matlab,none,octave,pretty,tree},
  --translation {c,faust,fortran,javascript,mathematica,mathml,matlab,none,octave,pretty,tree}
```

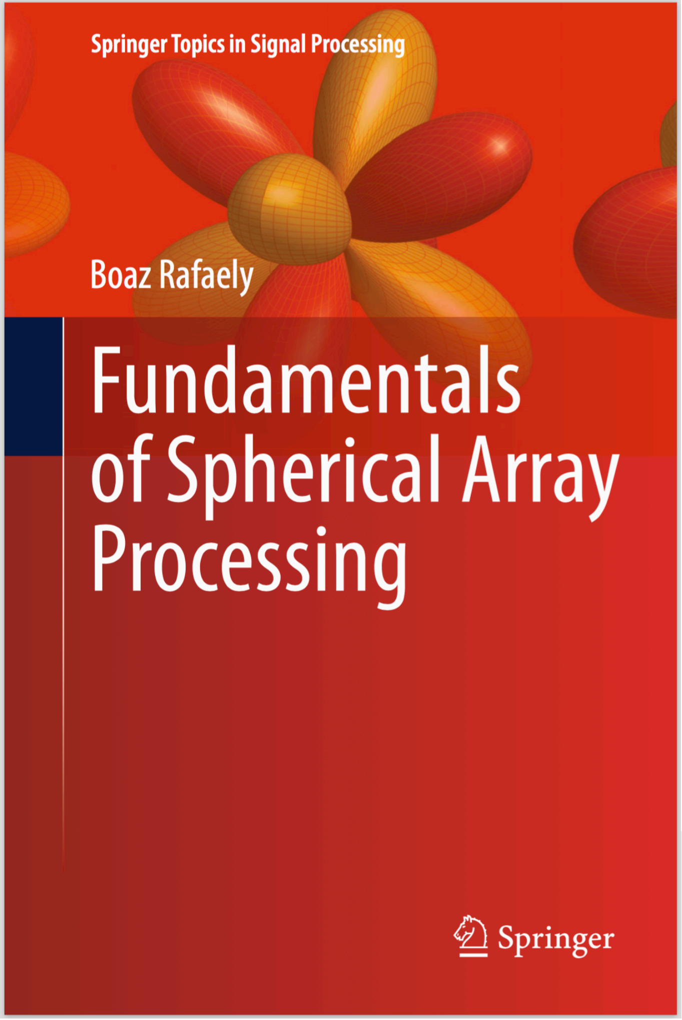
https://bitbucket.org/ajheller/symbolic_spherical_harmonics

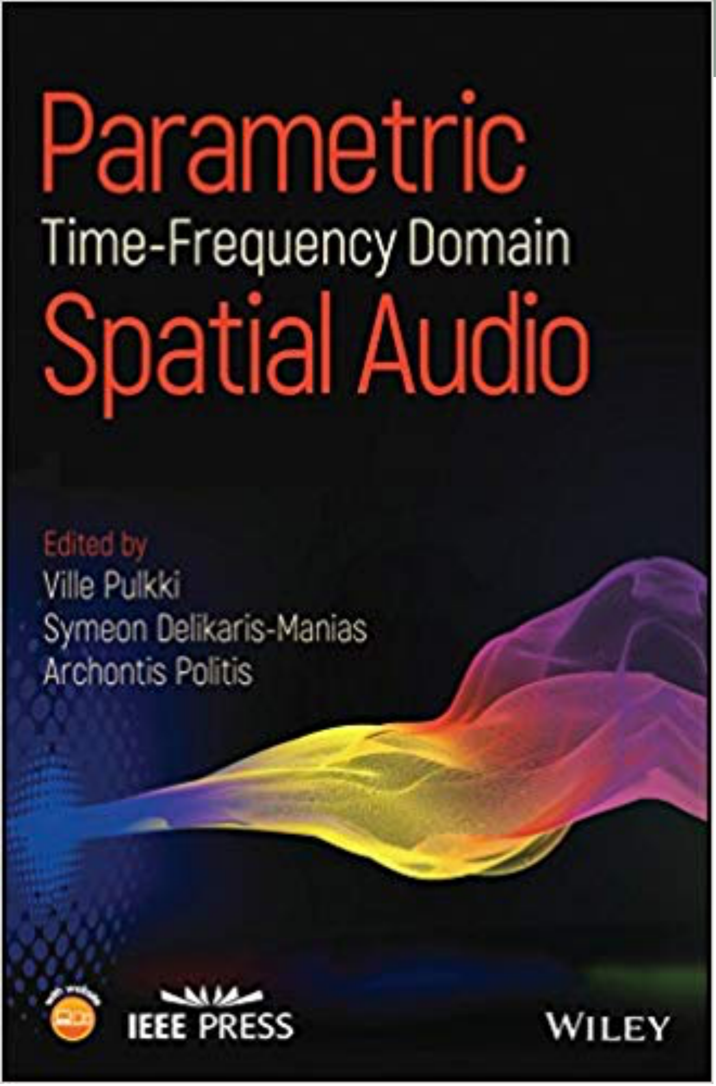
Channel Order

- FOA: XWYZ, WXYZ, WYXZ
- Furse-Malham: W XYZ RSTUV KLMNOPQ (Ambisonia)
- Ambisonic Channel Number (AmbiX, MPEG-H)
  - W YZX VTRSU QOMKLNP
  - 44S 43S 42S 41S 40C 41C 42C 43C 44C
- SID (Daniel, MPEG-4)
  - W XYZ UVSTR PQNOLMK
  - 44C 44S 43C 43S 42C 42S 41C 41S 40C

- ambi_channel_definitions_convention()

FURTHER READING


Springer Topics in Signal Processing
Boaz Rafaely
Fundamentals of Spherical Array Processing
Springer



1. Analysis and Synthesis of Spatial Sound
2. Reproduction of Spatial Sound
3. Signal-Dependent Spatial Filtering
4. Applications

Springer Topics in Signal Processing

Franz Zotter
Matthias Frank

Ambisonics

A Practical 3D Audio Theory for Recording, Studio Production, Sound Reinforcement, and Virtual Reality

Springer Open

Links

- Papers
  - http://www.ai.sri.com/ajh/
  - Everything BLaH http://www.ai.sri.com/ajh/ambisonics/Everything_BLaH.pdf
- ADT
  - https://bitbucket.org/ambidecodertoolbox/adt
- PyADT
  - https://bitbucket.org/ambidecodertoolbox/adt_evaluation
- SymYlm
  - https://bitbucket.org/ajheller/symbolic_spherical_harmonics/