Complex Waves

Complex tones have more than one sinusoidal component. In fact, simple pure sinusoidal tones are rare in nature possibly with the exception of tuning forks, coke bottle resonances, and some whistles and flutes. Much effort is spent recreating the exact complexes that make up the majority of sounds. The following presents three ways of building up complex tones, by addition of sine oscillators and by two types of modulation of one oscillator by another. A waveform with any kind of non-sinusoidal “kink” or distortion will have a spectrum with more than one component. Techniques using fast frequency, phase or amplitude modulation of a sinusoidal oscillator will distort its waveshape and produce extra components.

The components of a complex tone are also called partials. Clearly pitched sounds are formed by a fundamental frequency and other components in a harmonic relationship to it. Harmonic partials, also called overtones, are defined by a simple equation: \( \text{frequency} = \text{fundamental} \times \text{rank} \) where frequency and fundamental are expressed in Hz and rank is an integer defining the harmonic number. For instance if the fundamental (or first harmonic) of a sound is 440 Hz then the second harmonic is \( 2 \times 440 = 880 \) Hz, the third harmonic is \( 3 \times 440 = 1320 \) Hz, and so on.
Complex tones in nature can have vast arrays ofpartials, with varying degrees of harmonicity. Some tones support more than one pitch, especially metalophones like carillon bells or cymbals. The different amplitudes of the partials of a sound and their evolution in the time domain will determine the characteristic timbre or color of a sound. When an instrument is vibrating, its physical structure emphasizes or attenuates the different partials of the sound and lead to categorizations based on type. For instance the clarinet working like a closed tube, has largely only odd harmonics therefore creating its distinct spectrum.

**Additive Synthesis**

Additive synthesis is the technique of building up complex sounds by the combination of a large number of sine waves with their different frequencies, amplitudes and durations. Sounds resulting from simple additive synthesis are often too static and predictable. To generate a rich sound, a sufficient number of oscillators and envelopes are required. In patch `jcrSynth` you can change the pitch, duration and internal structure of the sound.

`jcrSynth`
Modulations: frequency and ring types.

A complex sound can be created when one sine wave distorts a second one. In the case of Frequency Modulation (FM) Synthesis, the first “carrier” oscillator's frequency is modulated by a second “modulation” oscillator's frequency. This creates multiple sidebands (or partials with frequencies symmetrically above and below the carrier). Therefore, we need only two oscillators to create a partial-rich complex sound.

This technique was discovered in the late sixties by John Chowning here at CCRMA and used in the very successful Yamaha DX-7 synthesizer. In fmSynth you can see how
simple FM synthesis works. If the modulation oscillator is tuned around 6 Hz the carrier continues being a sine wave with vibrato. But if the modulator is tuned higher than 20 Hz a complex tone results. The internal structure is determined by the FM parameters: carrier frequency, modulator frequency, and modulation index (aka, amplitude of the modulating oscillator).

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fmSynth.pd
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Hearing is believing, but seeing is useful, too. Here's how to launch a real-time spectrum analyzer program and send the Pd output to it. Find the menu item for launching the
Freqtweak application, or in the terminal window, type `freqtweak &`
Hit the “Input 1” button and select: `pure_data_0:output0`
Set the FM parameters in the patch above to $cf = 8000$, $mf = 1500$, $mi = 355$ and the following spectral lines should appear in the waterfall plot:

With the analyzer running, watch the effect of changing $mf$ back down to vibrato rate, and then $mi$ to 0. For more information and references about FM synthesis, see Bill Schottstaedt's *An Introduction to FM*.

Ring Modulation is the multiplicative product of the signals of two sounds. The output waveform contains their sum and difference frequencies, e.g. 1000 Hz & 100 Hz inputs will output 900 Hz and 1100 Hz. Check it in Freqtweak...

`ringSynth.pd`