Impulse Response Measurements in the Presence of Clock Drift

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Introduction + Motivation

- Characterize the acoustic properties of a room in the form of an impulse response (IR)
- Occasionally difficult to record and playback on a single device as found in archeological acoustics (Chavín de Huántar—Miriam Kolar, et al.)
IR Measurements w/Clock Drift

• Different devices result in different digital clocks causing small timing differences in playback and record signals

• The misalignment accumulates over time for robust impulse response measurement techniques using convolution/correlation

Recorded impulse train with clock drift over two minutes, drift 1-2 ms / minute

Measurement with and without clock drift
Impulse Response Measurements

- Measurement model \( r(t) = s(t) \ast h(t) + n(t) \)

Measurement system setup

Measurement model
Convolution IR Measurement Methods

Cyclic convolution
• Sinusoidal sweeps
  – linear, exponential, etc.
• Maximum length sequences
• Allpass chirps

Acyclic convolution
• Sinusoidal sweeps
  – linear, exponential, etc.
• Golay codes
  – binary, ternary, etc.
Sinusoidal Sweeps

• Increasingly popular, straightforward implementation
• Robust measurements against weak non-linearities
• Offer thorough theoretical analysis and alternative methods in the presence of clock drift
Acyclic IR Measurements

• Given the measurement model \( r(t) = s(t) \ast h(t) + n(t) \) we assume, there exist an inverse filter \( f(t) \) such that \( s(t) \ast f(t) \approx \delta(t) \)

• The from the measurement model we can recover the impulse response via

\[
\begin{align*}
  h(t) &= r(t) \ast f(t) \\
  &= h(t) \ast s(t) \ast f(t) + n(t) \ast f(t) \\
  &\approx h(t)
\end{align*}
\]
Sinusoidal Sweeps

\[ s(t) = \sin \phi(t) \quad \phi(t) = \int_0^t \omega(\nu) d\nu \]

\[ \omega_{\text{lin}}(t) = \left( \frac{\omega_1 - \omega_0}{\tau} \right) t + \omega_0 \]

\[ \omega_{\text{exp}}(t) = \omega_0 \exp\left\{ \frac{t}{\tau} \ln \frac{\omega_1}{\omega_0} \right\} \]
Sine Sweep Group Delay

- For smooth phase, the group delay is the time delay of the amplitude envelope of a sinusoid
- Functional inverse of frequency trajectory
**Sinusoidal Sweep Transforms**

- The frequency response $S(\omega)$ can be formulated via a magnitude and phase decomposition $|S(\omega)|e^{j\phi(\omega)}$.

$$|S(\omega)| \approx 1 \sqrt{\frac{1}{2} \left| \frac{d\gamma(\omega)}{d\omega} \right|} \quad \phi_s(\omega) = -\int_0^\omega \gamma(\nu) d\nu,$$

where the group delay $\gamma(\omega)$ is the functional inverse of $\omega(t)$ (Abel 2004).
Sinusoidal Sweep Inverse Filter

- $f(t)$ can be constructed using various methods
  - Time-reversal plus whitening filter
  - Numerical inversion via $\text{IFFT} \left( \frac{1}{\text{FFT}(s(t))} \right)$
  - Closed form approximation

$$f(t) \approx \left| \frac{d\omega(-t)}{dt} \right| s(-t)$$

$$|F(\omega)| \approx \sqrt{\frac{1}{2} \left| \frac{d\gamma(\omega)}{d\omega} \right|}$$

$$\phi_f(\omega) = \int_0^\omega \gamma(\nu)d\nu$$
Clock Drift Analysis

• For convenience, a single clock can be chosen as reference with the other as drifting resulting in two scenarios
  – drifting playback clock
  – drifting record clock

\[
h(t) = r(t) \ast f(t) = h(t) \ast s(t) \ast f(t)
\]

IR deconvolution process is corrupted
Drifting Playback Clock

- Recorded signal \( \tilde{r}(t) = h(t) \ast s(\alpha_p t) \)
- Propagates through to the impulse response convolution process via
  \[
  \tilde{h}(t) = \tilde{r}(t) \ast f(t) \\
  = h(t) \ast (s(\alpha_p t) \ast f(t)) \\
  = h(t) \ast d_p(t)
  \]
- IR is filtered by a drift filter
  \[
  d_p(t) = s(\alpha_p t) \ast f(t)
  \]
Drifting Record Clock

- Recorded signal \( \tilde{r}(t) = h(\alpha_r t) \ast s(\alpha_r t) \)
- Propagates through to the impulse response convolution process via

\[
\tilde{h}(t) = \tilde{r}(t) \ast f(t) = h(\alpha_r t) \ast (s(\alpha_r t) \ast f(t))
\]

- IR is filtered by a drift filter \( d_r(t) = s(\alpha_r t) \ast f(t) \) and resampled
- Resampling is typically negligible
Sine Sweep Clock Drift

• A stretch $\alpha$ in the time scale is equivalent to a stretch in the frequency trajectory and hence the group delay

\[ \tilde{\gamma}(\omega) \approx \alpha \gamma(\omega) \approx (1 + \epsilon) \gamma(\omega) \]

• As before, the magnitude and phase of a sweep can be solely expressed as a function of the group delay
Sine Sweep Transform w/Drift

• Modify the magnitude and phase of a sine sweep with drifting clocks resulting

\[ |\tilde{S}(\omega)| \approx \sqrt{1 + \epsilon} \ |S(\omega)| \quad \tilde{\phi}_s(\omega) = -(1 + \epsilon) \phi(\omega) \]

• The drift filter is then

\[ D(\omega) \approx \sqrt{1 + \epsilon} \exp\{-j \epsilon \phi_s(\omega)\} \approx \exp\{-j \epsilon \phi_s(\omega)\} \]
Sine Sweep Transform w/Drift

• The drift filter is an allpass filter for sine sweeps
  – Frequency trajectory of the same type as input sweep
  – Function of drift rate and sine sweep length

\[ D(\omega) \approx \sqrt{1 + \epsilon \exp\{-j \epsilon \phi_s(\omega)\}} \approx \exp\{-j \epsilon \phi_s(\omega)\} \]
Dependent on Sine Sweep Length

Sweep length in samples = $2^N$
Dependent on Frequency Trajectory

- No drift
- Exponential trajectory w/drift
- Linear trajectory w/drift
Clock Drift Compensation

- Desirable to remove drift effects via post-processing
  - Resampling
  - Compensation Filtering
Resampling Compensation

- Resample the recorded test signal or inverse filter prior to convolution
- Applicable to all convolution-based IR methods
Compensation Filtering

• Apply a compensation filter after convolution
• Applicable to sinusoidal sweeps
• Applied drift filter is allpass, so the compensation filter is the time-flip

Warped IR * Compensation Filter → Corrected IR
Clock Drift Estimation

• Both compensation methods need an estimate of the clock drift

• Explicit Estimation
  – Direct electronic loopback recording of an periodic impulse train, noting the recorded time indices

• Implicit Estimation
  – Direct electronic loopback of sine sweep or inverse
Explicit Estimation

- Record high frequency impulse train

- Note time indices of peaks and compute least-squares estimate of the drift rate $\alpha$ via

  $$\min_{\alpha} \| t - t_{meas}\alpha \|$$

- Allpass chirps can be used for a more robust measure
Implicit Estimation

- Record direct loopback of sine sweep or inverse
- Simultaneously estimate drift and resample
- Drift rate *perfectly* estimated, but *unknown*

![Diagram of Devices]

Device 1

Device 2
Compensation Results

IR direct path with clock drift

IR compensated for drift vs. reference IR with no drift
Compensation Results

Compensated IR direct path and early reflections
Conclusions

- Clock drift can influence impulse response measurements in various ways
- Unwanted drift imposes allpass filtering on the resulting IR for sinusoidal sweep measurements
- Two methods of compensation are proposed and achieve near perfect compensation
- Post-processing solution for measuring room impulse responses in the presence of clock drift
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  https://ccrma.stanford.edu/groups/chavin/