A SYSTEM FOR ACOUSTIC CHORD TRANSCRIPTION AND KEY EXTRACTION FROM AUDIO USING HIDDEN MARKOV MODELS TRAINED ON SYNTHESIZED AUDIO

A DISSERTATION
SUBMITTED TO THE DEPARTMENT OF MUSIC
AND THE COMMITTEE ON GRADUATE STUDIES
OF STANFORD UNIVERSITY
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Kyogu Lee
March 2008

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

(Julius Smith, III) Principal Adviser

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

(Jonathan Berger)

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

(Chris Chafe)

I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

(Malcolm Slaney)

Approved for the University Committee on Graduate Studies.

Abstract

Extracting high-level information of musical attributes such as melody, harmony, key, or rhythm from the raw waveform is a critical process in Music Information Retrieval (MIR) systems. Using one or more of such features in a front end, one can efficiently and effectively search, retrieve, and navigate through a large collection of musical audio. Among those musical attributes, harmony is a key element in Western tonal music. Harmony can be characterized by a set of rules stating how simultaneously sounding (or inferred) tones create a single entity (commonly known as a chord), how the elements of adjacent chords interact melodically, and how sequences of chords relate to one another in a functional hierarchy. Patterns of chord changes over time allow for the delineation of structural features such as phrases, sections and movements. In addition to structural segmentation, harmony often plays a crucial role in projecting emotion and mood. This dissertation focuses on two aspects of harmony, chord labeling and chord progressions in diatonic functional tonal music.

Recognizing the musical chords from the raw audio is a challenging task. In this dissertation, a system that accomplishes this goal using hidden Markov models is described. In order to avoid the enormously time-consuming and laborious process of manual annotation, which must be done in advance to provide the ground-truth to the supervised learning models, symbolic data like MIDI files are used to obtain a large amount of labeled training data. To this end, harmonic analysis is first performed on noise-free symbolic data to obtain chord labels with precise time boundaries. In parallel, a sample-based synthesizer is used to create audio files from the same symbolic files. The feature vectors extracted from synthesized audio are in perfect alignment with the chord labels, and are used to train the models.

Sufficient training data allows for key- or genre-specific models, where each model is trained on music of specific key or genre to estimate key- or genre-dependent model parameters. In other words, music of a certain key or genre reveals its own characteristics reflected by chord progression, which result in the unique model parameters represented by the transition probability matrix. In order to extract key or identify genre, when the observation input sequence is given, the forward-backward or Baum-Welch algorithm is used to efficiently compute the likelihood of the models, and the model with the maximum likelihood gives key or genre information. Then the Viterbi decoder is applied to the corresponding model to extract the optimal state path in a maximum likelihood sense, which is identical to the frame-level chord sequence.

The experimental results show that the proposed system not only yields chord recognition performance comparable to or better than other previously published systems, but also provides additional information of key and/or genre without using any other algorithms or feature sets for such tasks. It is also demonstrated that the chord sequence with precise timing information can be successfully used to find cover songs from audio and to detect musical phrase boundaries by recognizing the cadences or harmonic closures.

This dissertation makes a substantial contribution to the music information retrieval community in many aspects. First, it presents a probabilistic framework that combines two closely related musical tasks — chord recognition and key extraction from audio — and achieves state-of-the-art performance in both applications. Second, it suggests a solution to a bottleneck problem in machine learning approaches by demonstrating the method of automatically generating a large amount of labeled training data from symbolic music documents. This will help free researchers of laborious task of manual annotation. Third, it makes use of more efficient and robust feature vector called tonal centroid and proves, via a thorough quantitative evaluation, that it consistently outperforms the conventional chroma feature, which was almost exclusively used by other algorithms. Fourth, it demonstrates that the basic model can easily be extended to key- or genre-specific models, not only to improve chord recognition but also to estimate key or genre. Lastly, it demonstrates the usefulness of recognized chord sequence in several practical applications such as cover

song finding and structural music segmentation.

Acknowledgments

I would like to begin by thanking Julius O. Smith, my principal adviser, who gave me constant support and all kinds of research ideas throughout the development of this dissertation work. The enthusiasm and encouragement he has shown through his courses, seminars, individual studies and a number of meetings inspired me to accomplish my academic goals. I would also like to thank the other two professors at CCRMA — Jonathan Berger and Chris Chafe — for their invaluable help and friendship for these many years through their courses, seminars and research projects.

I would like to thank Malcolm Slaney at Yahoo! Research for his support, supply of ideas and friendship, which were indispensable in finishing this dissertation work. My research benefited greatly from a number of project meetings we had together and from the Hearing Seminar where he invited many expert speakers from all around the world.

The CCRMA community has always provided me with an excellent research environment. Especially, I would like to thank my research colleagues and friends in the DSP group: Ed Berdahl, Ryan Cassidy, Pamornpol Jinachitra (Tak), Nelson Lee, Yiwen Liu, Aaron Master, Gautham Mysore, Greg Sell and David Yeh for their support and friendship. Not only their friendship but also research ideas we exchanged at the regular meetings were invaluable. I am also grateful to my other friends at the CCRMA, including Juan Pablo Caceres, Michael Gurevich, Patty Huang, Randal Leistikow, Craig Sapp, Hiroko Terasawa and Matthew Wright.

My thank also extends to the Korean community at the CCRMA: Song Hui Chon, Changhyun Kim, Mingjong Kim, Moonseok Kim, Dongin Lee, Jungsuk Lee, Zune Lee, Juhan Nam, and Sookyoung Won. My special thanks go to Woonseung Yeo,

who shared the past five years at the CCRMA as a research colleague, a classmate, and most importantly, as a true friend.

I would like to thank Lawrence Rabiner at Rutgers University and Juan Pablo Bello at New York University for their fruitful comments through which my work could make an improvement. I would also like to thank Markus Cremer at Gracenote and Veronique Larcher at Sennheiser for giving me opportunities to have real-world experience via their internships. The internships I did at these places helped me understand how academic research is actually applied in the industries.

My special thanks go to my parents and my wife's parents. Their endless love and support made it possible to complete my dissertation work. My thanks extend to my two brothers who have been very supportive and encouraging all the time. My adorable two sons, although they were not always helpful, deserve my gratitude for just being there with me and for calling me "dad". Last, but most of all with no doubt, I would like to thank my lovely wife, Jungah, for her endless support, encouragement, patience, and love she has never failed to show me during the completion of my work. It would have been impossible without her. I dedicate this dissertation to her.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

This dissertation discusses automatically extracting harmonic content — musical key and chords — from the raw audio waveform. This work proposes a novel approach that performs simultaneously two tasks — *i.e.*, chord recognition and key estimation — from real recordings using a machine learning model trained on synthesized audio. This work also demonstrates several potential applications that use as a front end harmonic descriptions derived from the system. This chapter serves as an introduction presenting the fundamental ideas on which this thesis was based and written, which include the current research efforts in the field of music information retrieval, the motivation and goals of this work, potential applications, and how this thesis is organized.

1.1 Music Information Retrieval

Information retrieval (IR) is a field of science that deals with the representation, storage, organization of, and access to information items [2]. Likewise, music information retrieval (MIR) deals with the representation, storage, organization of, and access to *music* information items. This is best described with an example by Downie [21]:

> *Imagine a world where you walk up to a computer and sing the song fragment that has been plaguing you since breakfast. The*

> *computer accepts your off-key singing, corrects your request, and promptly suggests to you that "Camptown Races" is the cause of your irritation. You confirm the computer's suggestion by listening to one of the many MP3 files it has found. Satisfied, you kindly decline the offer to retrieve all extant versions of the song, including a recently released Italian rap rendition and an orchestral score featuring a bagpipe duet.*

Music information retrieval is an interdisciplinary science, whose disciplines include information science, library science, computer science, electrical and computer engineering, musicology, music theory, psychology, cognitive science, to name a few. The problems in MIR are also multifaceted, due to the intricacies inherent in the representation of music information. Downie defines seven facets which play a variety of roles in defining the MIR domain — pitch, temporal, harmonic, timbral, editorial, textual, and bibliographic facets [21]. These facets are not mutually exclusive but interact with each other, making the problems more perplexing.

Finding more efficient and effective ways to search and retrieve music is attracting increasing number of researchers both from academia and industries as it becomes possible to have thousands of audio files in a hand-held device such as portable music players or even in cellular phones. Furthermore, the distribution of music is also changing from offline to online: more and more people are now buying music from the web[1], where more than tens of millions of songs are available. Therefore, users need to manage and organize their large collection of music in a more sophisticated way, and the content providers need to provide the users with an efficient way to find music they want.

1.2 Motivation

People have been using the meta-data, almost exclusively, as a front end to search, retrieve and organize through a large collection of music in their computer or on the

[1]As of 2/23/2006, more than 1,000,000,000 songs were downloaded from iTunes only, which is an online music store by Apple Computer [67].

Internet. Meta-data, sometimes called *tags*, are textual representations that describe data, to facilitate the understanding, organization and management. In the music domain, meta-data may include the title of a piece, the name of an artist or a composer, the name of a performer, or the musical genre. An example is "Eight Days A Week (song title)" by "Beatles (artist, composer)", which belongs to "rock (genre)" music. Another example is "Piano Sonata No. 14 in C♯ minor, Op. 27 (title)" by "Beethoven (composer)", performed by "Glenn Gould (performer)," which is "classical (genre)" music.

These textual descriptions of music are very compact and therefore extremely efficient for search and retrieval, especially when there are a great number of items to be searched. However, the textual meta-data is far from being ideal in many situations. First of all, their usage is limited in that all the required information must be previously prepared — *i.e.*, someone has to manually enter the corresponding information — or the search is bound to fail. This is likely to happen especially when new songs are released.[2] Second, the user must know exactly what the query is, which might not be always the case: he/she might not be able to recall the name of a song or an artist. Third, they might not be as effective in particular tasks. For instance, imagine a situation where a user wants to find *musically* similar songs to his/her favorite ones using the meta-data only. Chances are the returned items may not be similar at all because there is very little correspondence between the distance in a meta-data space and the distance in a musical space.

It is not too difficult to infer the cause of the problems described above: a piece of music is a whole entity, and music listening is an experience that requires a set of hierarchical processes from low-level, auditory perception to high-level cognition, often including emotional processes. Therefore, it is almost impossible to fully describe music with just a few words, although they may convey important and/or relevant information about music. Meta-data may help us guess what it's like but we can truly comprehend the actual content only through listening experiences. This is why content-based approaches in MIR attract researchers from diverse disciplines.

[2] 10,000 new albums are released and 100,000 works registered for copyright every year as of 1999 [97].

1.2.1 Content-based Approach

Recently, due to limitations and problems of the meta-data-driven approaches in music information retrieval, a number of researchers are actively working on developing systems based on *content* description of musical audio. Cambridge Online Dictionary defines the word *content* as: *the ideas that are contained in a piece of writing, a speech or a film.*[3] Therefore, instead of describing music with only a few words such as its title, composer, or artist, content-based methods try to find more meaningful information implicitly conveyed in a piece of music by analyzing the audio itself.

It is nearly impossible to figure out from the raw audio the high-level ideas that the composers tried to express via musical pieces. However, music is not written based on random processes. There are rules and attributes in music to which most composers conform when writing their music. This is especially true in Western tonal music. Therefore, if we can extract these musical rules and/or attributes from audio that the composers used in writing music, it is possible to correlate them with higher-level semantic description of music — *i.e.*, the ideas, thoughts or even emotional states which the composers had in mind and tried to reflect through music they wrote. We are not trying to say that the conventional meta-data like song title or genre are useless; they are very useful and efficient in some tasks. However, these high-level semantic descriptions obtained through content analysis give information far more relevant to music itself, which can be obtained only through listening experience.

There are several attributes in Western tonal music, including melody, harmony, key, rhythm or tempo. Each of these attributes has its unique role, and yet still interacts with the others in a very controlled and sophisticated way. In the next section, we explain why we believe harmony and key, among many attributes, are particularly important in representing musical audio signals of tonal music where the tonal organization is primarily diatonic.

[3] `http://dictionary.cambridge.org`

1.2.2 Harmonic Description of Tonal Music

Harmony is one of the most important attributes of Western tonal music. The importance of harmony in tonal music is emphasized by a number of treatises written over the centuries. Ratner explains harmony in three different and yet closely related contexts: *harmonic sonorities, harmonic action* and *harmonic color* [88].

Harmonic sonorities are about the stability/instability embodied in a set of simultaneously sounding notes (intervals, chords) and their contributions to the sense of a key and to musical movement and arrival. Mutual relationships of tones and their qualities and effects expressed by various intervals and chords constitute the basis of harmony in Western tonal music.

Changes over time in these harmonic relationships — intervals, the sense of key, stability/instability, chords — ensure the power, or harmonic action, to carry musical movement forward. Like all action, harmonic action proceeds through a cycle of departure, movement and arrival. Among a number of characteristic patterns and formulas that realize this cycle, including melodic elaboration, rhythmic patterns of chord change, voice leading and modulation, the cadential formula is by far the most important. Two basic aspects of harmony embodied in the cadential formula — *i.e.*, the movement-instability and the arrival-stability represented by the tritone[4] and the tonal center, respectively — create a clear sense of key, from which a small segment of musical structure has taken.

While it was the most important role of harmony to create the organic, structurally firm embodiment of key in the eighteenth century and in the early nineteenth century, the element of color in harmony was exploited and became increasingly important as composers sought to develop and enrich their sounds and textures, especially in the late nineteenth century and in the twentieth century.

Among the three aspects of harmony described above, the description of harmonic action in particular clearly suggests that we can infer a musical structure from harmony by detecting changes in local key or by finding the cadential formula. Furthermore, because a strong sense of key is created by some specific harmonic movement

[4]A tritone is also called an augmented fourth and is an interval that is composed of three whole tones, *e.g.*, C-F♯.

or the cadential formula, it shouldn't be difficult to infer a key once we figure out how harmony progresses over time.

Other than analyzing the structure of music, harmonic description of tonal music can be used as an efficient, robust front end in practical applications. For example, when finding different versions of the same song (or so-called *cover* songs), harmonic content remains largely preserved under severe acoustical variations due to changes in tempo, dynamics and/or instrumentation, and therefore it can be a robust front end in such application. Another potential application is *recommendation* or *playlist generation*; *i.e.*, use harmonic content to compute (dis)similarity between songs and recommend only those that are *harmonically* similar to the user's existing collection of music.

1.2.3 Machine Learning in Music Applications

Machine learning is the study of computer algorithms that improve automatically through experience [68]. There are a number of real-world applications that benefit greatly from the machine learning techniques, including data mining, speech recognition, hand-writing recognition, and computer vision, to name a few. In MIR, many systems also use machine learning algorithms to solve problems such as genre classification [96, 4, 5], instrument identification [99, 24] and key estimation [14, 73].

Speech recognition is one of many application areas for which machine learning methods have made significant contributions. Research for the last 20 years shows that among many machine learning models, HMMs are very successful for speech recognition. A hidden Markov model [86] is an extension of a discrete Markov model, in which the states are *hidden* in the sense that we can not directly observe the underlying stochastic process, but can only observe it through another set of stochastic processes. The more details about the HMMs are found in Appendix A.

Much progress in speech recognition has been made with gigantic databases of labeled speech. Such a huge database not only enables researchers to build richer models, but also allows them to estimate the model parameters precisely, resulting in improved performance. However, there are very few such databases available for

music. Furthermore, the acoustical variance in music is far greater than that in speech, in terms of its frequency range, timbre due to different instrumentations, dynamics and/or duration. Consider the huge acoustical differences among: a C major chord in root position played by a piano, the same chord in first inversion played by a rock band, and the same chord in second inversion played by a full orchestra. All of these sounds must be transcribed as the same C major chord; this in turn means even more data are needed to train the models so they *generalize*.[5]

However, it is very difficult to obtain a large set of training data for music, particularly for chord recognition. First of all, acquiring a large collection of music is expensive. In addition, in order to obtain the ground-truth annotation, we need a certain level of expertise in music theory or musicology to perform harmony analysis. Furthermore, hand-labeling the chord boundaries in a number of recordings is not only an extremely time consuming and tedious task, but also is subject to errors made by humans. In the next section, we describe how we solve this bottleneck problem of acquiring a large amount of training data with minimal human labor by using symbolic music documents.

1.2.4 Symbolic Music Documents

In this dissertation, we propose a method of automating the daunting task of providing the machine-learning models with labeled training data. To this end, we use symbolic music documents, such as MIDI files, to generate chord names and precise corresponding boundaries, as well as to create audio. Instead of a digitized audio signal like a PCM waveform, MIDI files contain a set of event messages such as pitch, velocity and note duration, along with clock signals from which we can synthesize audio. Audio and chord-boundary information generated this way are in perfect alignment, and we can use them to directly estimate the model parameters. The overall process of training is illustrated in Figure 3.9.

The rationale behind the idea of using symbolic music files for automatic harmony

[5] A model is called *generalized* when it performs equally well on various kinds of inputs. On the other hand, it is called *overfitted* if it performs well only on a particular input.

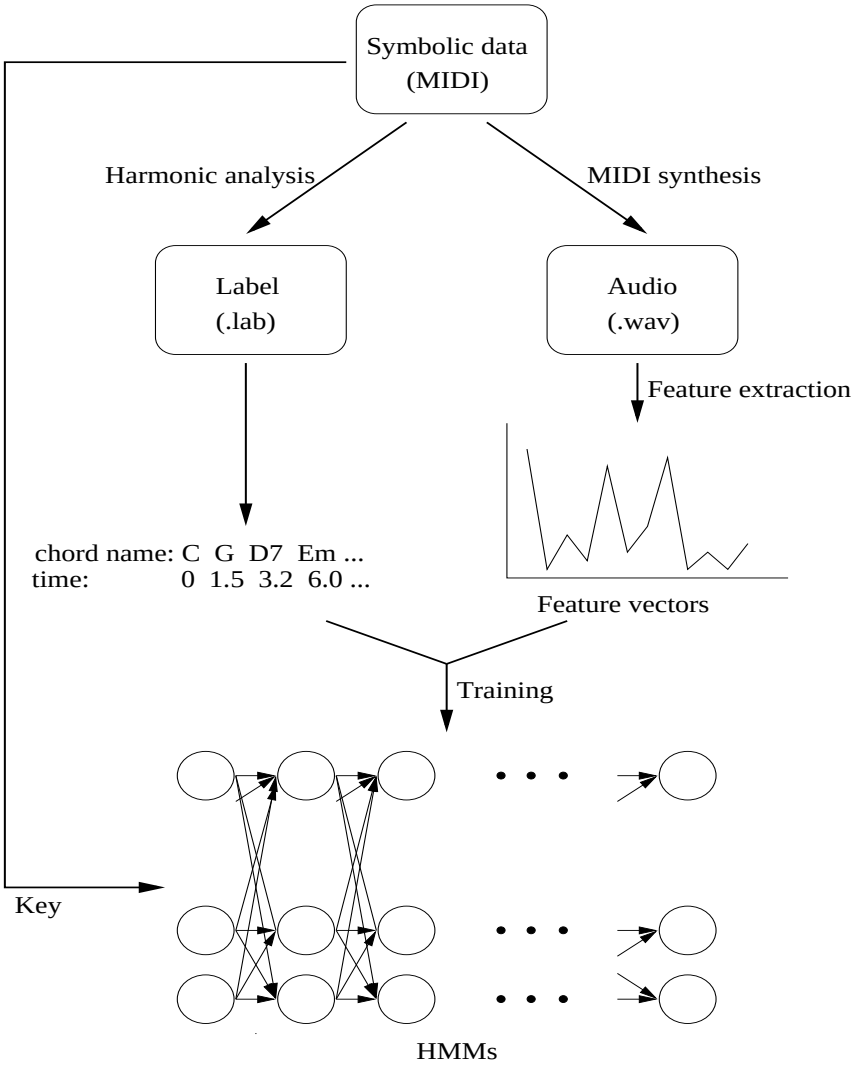


Figure 1.1: Training the chord transcription system. Labels obtained through harmony analysis on symbolic music files and feature vectors extracted from audio synthesized from the same symbolic data are used to train HMMs.

analysis is that they contain noise-free pitch and on/offset time information of every single note, from which musical chords are transcribed with more accuracy and easiness than from real acoustic recordings. In addition, other information included in symbolic music files, such as key, allow us to build richer models, like key-specific models.

There are several advantages to this approach. First, a great number of symbolic music files are freely available. Second, we do not need to manually annotate chord boundaries with chord names to obtain training data. Third, we can generate as much data as needed with the same symbolic files but with different musical attributes by changing instrumentation, tempo or dynamics when synthesizing audio. This helps avoid overfitting the models to a specific type of sound. Fourth, sufficient training data enables us to build richer models so that we can include more chord types such as a 7th, augmented or diminished. Lastly, by using a sample-based synthesis technique, we can generate harmonically rich audio as in real acoustic recordings. Although there may be noticeable differences in sonic quality between real acoustic recording and synthesized audio, we do not believe that the lack of human touch, which makes a typical MIDI performance dry, affects our training program. The fact that our models trained on synthesized audio perform very well on real recordings supports this hypothesis. In the next section we present the goals of the current work.

1.3 Goals

We present here the goals of this dissertation work.

1. Review and analyze related work on chord transcription and key finding. Present not only technical perspectives such as audio signal processing and machine learning algorithms, but also music theoretical/cognitive studies, which provide fundamental background to current work.

2. Describe the system in complete detail, which includes: 1) feature extraction from the raw audio; 2) building and training of the machine learning-models;

3) recognition of chords and key from an unknown input; 4) and the further extensions of the models.

3. Provide quantitative evaluation methods to measure the performance of the system.

4. Perform thorough analysis of the experimental results and prove the state-of-the-art performance, including robustness of our system using various types of test audio.

5. Validate our main hypothesis — *i.e.*, harmony is a very compact and robust mid-level representation of musical audio — by using it as a front end to practical applications.

6. Discuss the strength and weakness of the system and suggest the directions for improvement.

In the next section, we describe several potential applications in which we use harmonic content automatically extracted from audio.

1.4 Potential Applications

As aforementioned in Section 1.2.2, in Western tonal music, once we know musical key and chord progression of a piece over time, it is possible to perform structural analysis from which we can define themes, phrases or forms. Although other musical properties like melody or rhythm are not irrelevant, harmonic progression has the closest relationships with the musical structure. Finding structural boundaries in musical audio is often referred to as **structural music segmentation**.

Once we find the structural boundaries, we can also group similar segments into a **cluster**, such as a verse or chorus in popular music, by computing the (dis)similarity between the segments. Then it is also possible to do **music summarization** by finding the most repetitive segment, which in general is the most representative part in most popular music.

A sequence of chords and the timing of chord boundaries are also a compact and robust mid-level representation of musical signals that can be used to find the cover version(s) of the original recording, or **cover song identification**, as shown by Lee [56] and by Bello [7]. Another similar application where chord sequence might be useful is **music similarity finding** because a original-cover pair is an extreme case of similar music.

We describe these applications in more detail and demonstrate the possibilities of our approach with real-world examples in Chapter 6.

1.5 Organization

This dissertation continues with a scientific background in Chapter 2, where related works are reviewed. In Chapter 3, we fully describe our chord recognition system, including the feature set we use as a front end to the system, and the method of obtaining the labeled training data. We also describe the procedure of building the model, and present the experimental results. In Chapter 4, we propose the method of extending our chord recognition model to key or genre-specific models to estimate key or genre as well as to improve the chord recognition performance. In Chapter 5, we present a more advanced model — discriminative HMMs — that uses a powerful discriminative algorithm to compute the posterior distributions. In Chapter 6, we demonstrate that we can use the chord sequence as a front end in applications such as audio cover song identification, structural segmentation/clustering of musical audio, and music summarization. Finally, in Chapter 7, we summarize our work and draw conclusions, followed by directions for future work.

Chapter 2

Background

2.1 Introduction

In this chapter, we review previous research efforts on key extraction and chord recognition, and on tonal description of musical audio, which are very closely related with each other. In doing so, we first present related studies from the perspective of music theory and cognition that serve as fundamental background to most computational algorithms, including ours.

2.2 Tonality

Hyer explains the term *tonality* as:

> *A term first used by Choron in 1810 to describe the arrangement of the dominant and subdominant above and below the tonic and thus to differentiate the harmonic organization of modern music (tonalité moderne) from that of earlier music (tonalité antique). One of the main conceptual categories in Western musical thought, the term most often refers to the orientation of melodies and harmonies towards a referential (or tonic) pitch class. In the*

> *broadest possible sense, however, it refers to systematic arrangements of pitch phenomena and relations between them [40].*

According to Hyer, there are two main historical traditions of theoretical conceptualization about tonal music: the function theories of Rameau and Riemann on the one hand and the scale-degree theories of Gottfried Weber and Schenker on the other. The first tradition corresponds to the definition of tonality by Choron and the second corresponds to that by Fétis, who places more emphasis on the order and the position of pitches within a scale. However, in both traditions, tonal theories tend to concentrate on harmonic matter, virtually excluding all other musical phenomena such as register, texture, instrumentation, dynamics etc. These features are considered only to the extent that they articulate or bring out relations between harmonies.

We can describe tonality using two musical properties — *key* and *harmony*. A key is a tonal center or a referential point upon which other musical phenomena such as melody or harmony are arranged. Ratner states that the key is a comprehensive and logical plan coordinated from the interplay of harmonic stability and instability of intervals, which leads to a practical working relationship between tones [88]. There are two basic *modes* in the key: major and minor mode. Therefore, key and mode together define the relative spacing between pitch classes, centered on a specific pitch class (tonic). When the two keys share the same tonal center (such as in C major and C minor key), they have *parallel* major-minor relationship. On the other hand, if a sequence of notes in the two keys have the same relative spacing but have different tonal centers (such as in C major and A minor key), we refer to them as *relative* keys.

If a key is a tonal center that functions as a point of departure, reference en route and arrival, it is harmony that carries musical movement forward and at the same time control and focus movement [88]. A phase of harmonic movement or *action* is created through a cycle of departure, movement and arrival, and each point of arrival becomes in turn a new point of departure. This series of phases of harmonic movement shape musical structure. Among many rules and patterns in realizing a phase of harmonic movement, the *cadential formula* is the most important one. The cadence or cadential formula is defined as:

> *The conclusion to a phrase, movement or piece based on a recognizable melodic formula, harmonic progression or dissonance resolution; the formula on which such a conclusion is based. The cadence is the most effective way of establishing or affirming the tonality — or, in its broadest sense, modality — of an entire work or the smallest section thereof; it may be said to contain the essence of the melodic (including rhythmic) and harmonic movement, hence of the musical language, that characterizes the style to which it belongs [89].*

As is defined above, a cadence concludes a phrase or movement and is recognized by a specific harmonic progression or melodic formula. Therefore, if we know key and harmonic progression in tonal music, then we can identify cadences which indicate phrase boundaries.

2.2.1 Perception/Cognition of Tonality

The scope of the current dissertation is bounded by recognition of chords/key of simple diatonic music like pop or rock music. Furthermore, it does not include any sort of music cognitive or music theoretical experiments/studies, which cover more general and a broader range of Western tonal music. However, many computer algorithms make use of findings by such music cognitive/theoretical studies to infer tonality in Western tonal music, and we believe that presenting some landmark studies in perception/cognition of tonal music will help better understand the computer models as well.

There are many approaches to understand and model how humans perceive tonal music from the perspectives of cognitive science and psychology. We describe some of them here, but interested readers are advised to see Gómez [32] for a more complete summary.

Krumhansl and her colleagues did extensive research to investigate and model the relationship between musical attributes that are perceived and psychological experiences corresponding musical attributes through empirical studies [49, 50, 52].

Krumhansl and Shepard introduced the probe tone method to quantify the hierarchy of stability within a major key context [52]. In this study, they observed that when an "incomplete" scale is sounded, such as the successive tones C, D, E, F, G, A, B, this creates strong expectations about the tone that is to follow. In their experiments, they presented to the subjects two sequence of notes in a C major scale — the ascending scale of C, D, E, F, G, A, B and the descending scale of C, B, A, G, F, E, D — followed by the probe tone. The task of the subjects was to rate the degree of how well the given probe tone completes a sequence of notes from 1 (very bad) to 7 (very good). The probe tones were the equal-tempered semitones in the chromatic scale ranging from middle C to the C an octave above (thus the 13 probe tones total). From the results of the experiments, they found three distinct patterns depending on the level of prior musical experience.

Based on the initial study, Krumhansl and Kessler extended their experiments to minor-key contexts, resulting in two probe tone ratings, one for each key context [50]. Figure 2.1 shows the key profiles for a major and a minor key obtained from these probetone ratings.

Using these key-profiles obtained from the probe-tone experiments, they calculated the key-distances between the two arbitrary keys following the steps: 1) perform circulative permutation on the two key-profiles (major and minor) to obtain 12 major and 12 minor key-profiles, one for each pitch class; 2) compute correlations between a pair of key-profiles for every possible pair. Figure 2.2 shows correlations between a C major key-profile and all other major and minor key-profiles, and correlations between a C minor key-profile and all other major and minor key-profiles.

They further extended their studies to investigate how the relation between a chord and its abstract tonal center, or key, may be represented in a two-dimensional map of the multidimensional scaling solution of the 24 major and minor keys. Figure 2.3 illustrates how the C major, A minor, and B diminished chords appear in the spatial map of musical keys.

Krumhansl proposed a key-finding algorithm using the key-profiles and the maximum correlation analysis. She first computed the input vector $I = d_1, d_2, \cdots, d_{12}$,

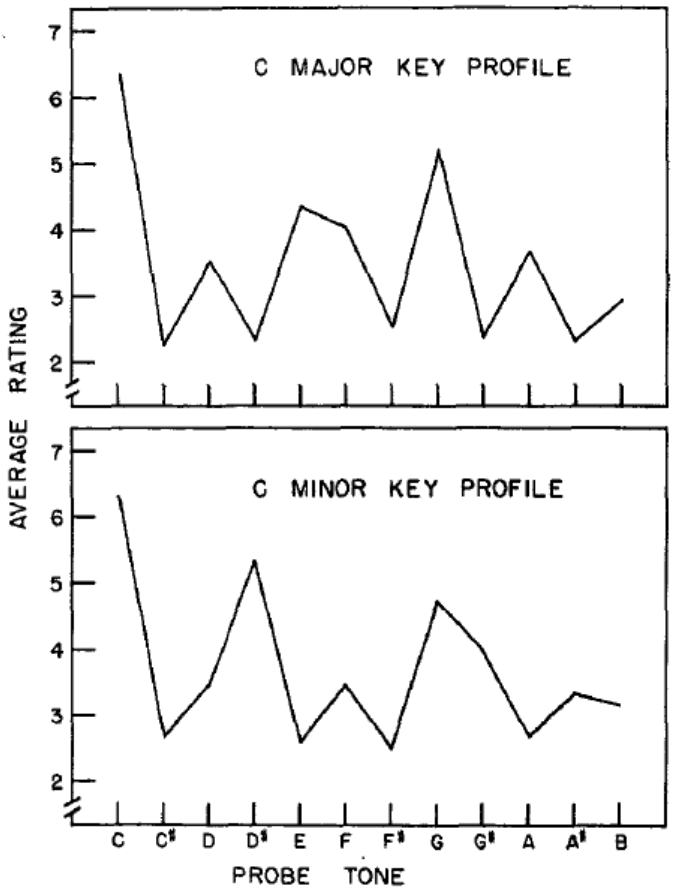


Figure 2.1: C major and C minor key-profiles obtained from the probe tone rating experiments (from Krumhansl and Kessler [50]).

each dimension representing the tone duration in musical selection. She then computed correlations between the input vector I and the probe tone profiles $K_i, k = 1, 2, \cdots, 24$ for 24 keys. The output is a 24-dimensional vector, where each element represents correlation between the input vector and each key. Using the 48 fugues of Bach's, Krumhansl and Schmuckler showed that their algorithm needs fewer number of tones to find the correct key [51].

A series of studies and experiments done by Krumhansl and her collaborators made significant contributions to understanding how listeners encode, organize, and

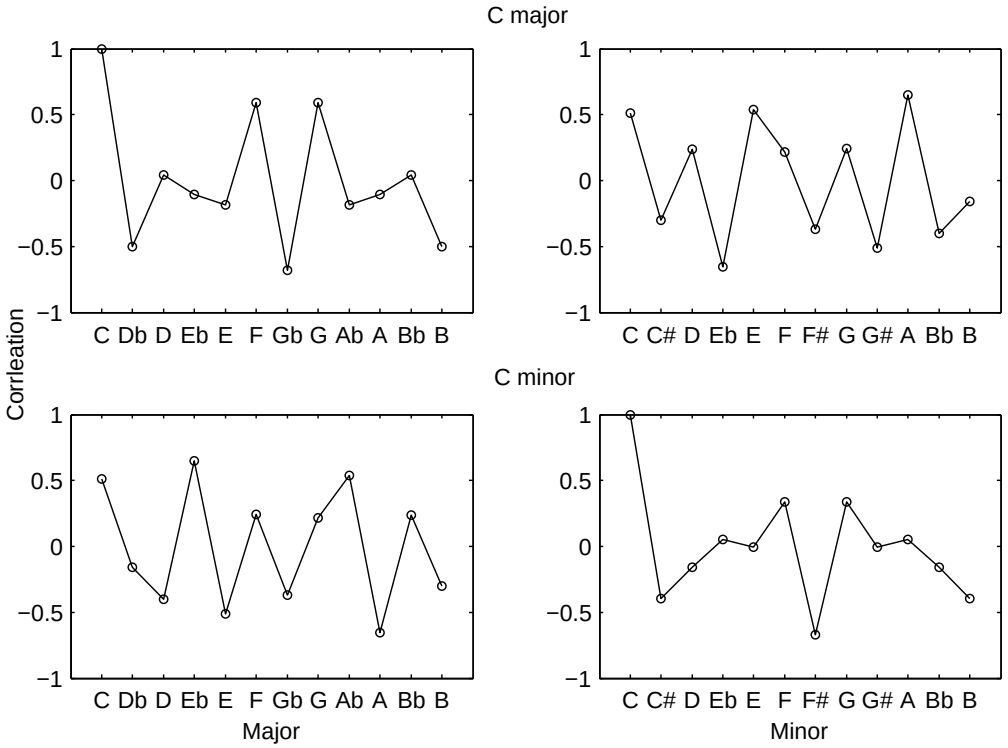


Figure 2.2: Correlations between C major and all other key-profiles (top) and correlations between C minor and all other key-profiles (bottom) calculated from the probe tone ratings by Krumhansl and Kessler [50].

remember pitch patterns in music. Furthermore, although they approached the problems from the perspectives of cognitive science and psychology, they had a giant impact on a number of computer algorithms that try to infer tonality from music, both from symbolic music and from musical audio.

In the next section, we review computer systems that automatically identify musical key and/or chords from the real recordings.

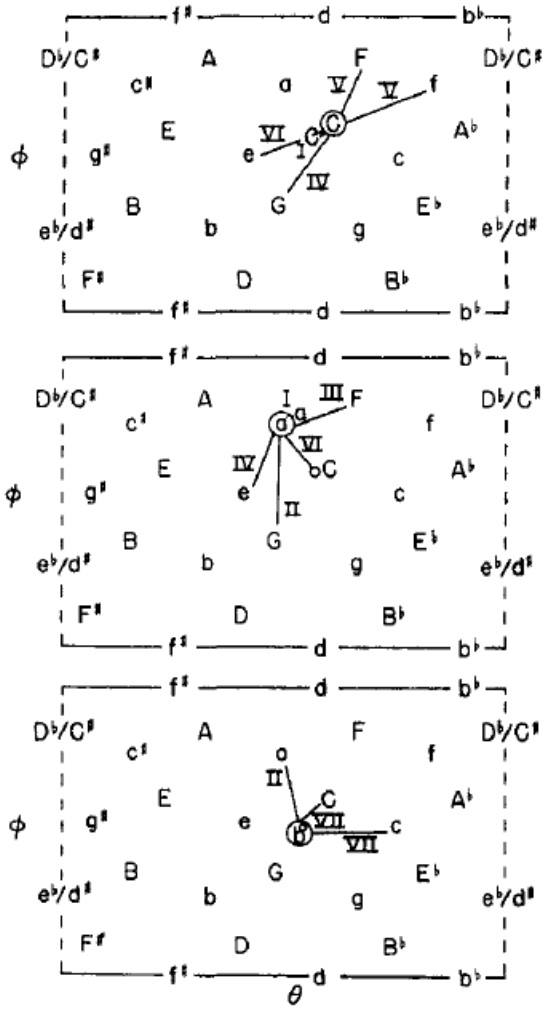


Figure 2.3: The relations between the C major, A minor and B diminished chords and the 24 keys displayed in a 2-dimensional map of keys (from Krumhansl and Kessler [50]).

2.3 Computer Systems

In Western tonal music, key and chord are two closely-related attributes, as mentioned above. Musical key defines a set of rules that harmony, as well as melody, abide by in the course of musical development. Therefore, if a piece of Western tonal music is in a certain key, we can predict not only the kind of chords that are most likely to appear but also how they are going to progress over time. Conversely, analyzing the chords and their pattern of progression also suggests key information.

Because musical key and chords have such close relatioships, computer systems for key estimation and those for chord recognition are also alike. The biggest difference lies in the scope of temporal dynamics — key is more a global attribute while chords are much more local. That is, while key seldom changes (or never for many popular music), chords changes even more frequently. This difference in temporal dynamics has a great impact on building systems, particularly in machine learning approaches, because it is much more difficult to obtain the *training data* for chord recognition systems.

Most algorithms for key-finding and chord recognition from audio are divided into two main stages: feature extraction and classification, as illustrated in Figure 2.4. In the feature extraction stage, the computer system extracts appropriate feature vectors from the raw audio because raw audio samples are redundant and not robust. The feature vectors must satisfy the following conditions. First, the feature must be low in dimension or the system will be computationally expensive. This is unacceptable for real-time applications. Furthermore, high dimensionality is likely to cause a data sparsity problem in statistical classifiers.

Second, the feature must be robust to the kinds of acoustical variations present in music. That is to say, it must remain largely invariant no matter how the musical acoustics vary unless the key or chord should change. For example, the features from *Bach's Prelude in C major* and *The Beatles' No Reply*, which is also in the key of C major, should be closer to each other in distance than to other keys. A similar argument holds for chord recognition as well. Once suitable features are extracted from the raw audio, the classification algorithms classify the given feature to the

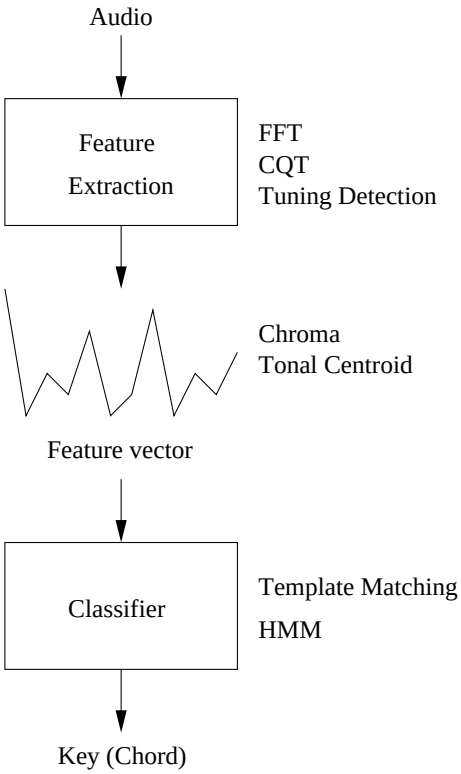


Figure 2.4: General overview of key estimation and chord recognition system.

correct chord or key.

While there is an almost unanimous agreement in the choice of the feature for both key estimation and chord recognition, the classifiers are mainly divided into two groups: some use a simple pattern matching algorithm based on pre-defined key or chord templates, and others depend on more complex, sophisticated statistical learning algorithms. Therefore, in the following two sections, we sub-divide systems for each task into two categories — template matching algorithms and machine learning algorithms — and describe each algorithm in more detail.

2.3.1 Key Estimation

Template Matching Approaches

Template matching techniques for key estimation use pre-defined key templates (*e.g.*, Krumhansl's key-profiles) for each key, and correlate them with the feature vector computed from audio to find the key template that yields the maximum correlation coefficient.

Pauws proposed a key extraction algorithm from audio based on human auditory perception and music cognition, and evaluated his algorithm using 237 classical piano sonatas [79]. He first computed a non-overlapping chromagram[1] over six octaves from A0 to A6 (27.5 Hz to 1760 Hz), and used it as input to the maximum-key profile correlation (MKC) algorithm to estimate musical key. Chroma vector is a 12-dimensional vector, each dimension representing the spectral energy in a chromatic scale. Because it is not only based on the actual musical scale (*i.e.*, equal-tempered scale[2]), but also because of its computational efficiency and low dimensionality, chroma feature is a common choice for key extraction and/or chord recognition.

In computing a chromagram, however, he went through several pre-processing stages to obtain the harmonically compressed spectrum, including low-pass filtering, enhancing spectral components to cancel out spurious peaks, logarithmic frequency scaling, and the use of weighting function to model the human auditory sensitivity. After adding and normalizing the chromagrams over all frames, he applied the maximum-key profile correlation (MKC) algorithm based on key profiles derived by Krumhansl and Kessler [50]. The key profile yielding the maximum correlation value out of 24 possible keys was recognized as the key of input musical audio. Using 237 classical piano sonatas as test data, his algorithm gave an accuracy of 75.1% by counting only exact matches as correct. Including related keys — relative, dominant, sub-dominant and parallel — as correct matches, the accuracy went up to 94.1%.

Based on the observation that the scale root note plays the foremost role in tonal

[1] A chromagram is a two-dimensional representation of chroma vectors, similar to a spectrogram.

[2] In the 12-tone equal-tempered scale, an octave is divided into 12 logarithmically equal steps.

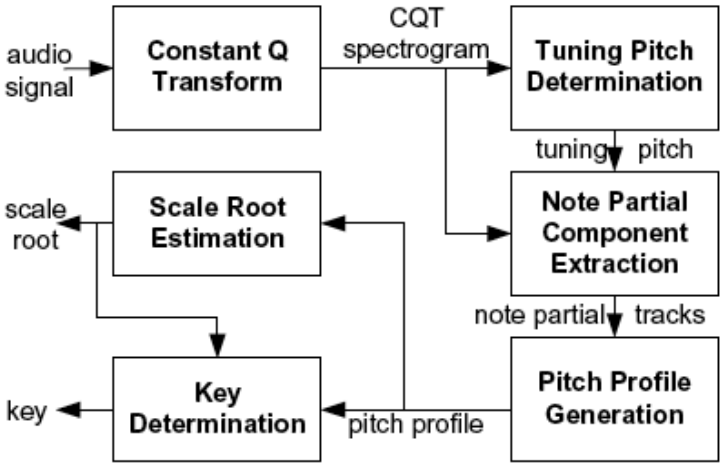


Figure 2.5: Diagram of key detection method (from Zhu *et al.* [102]).

music, Zhu *et al.* proposed an approach to detect the scale root and the key from audio [102]. They first extracted the pitch profile feature, similar to the chroma feature, which characterizes the probability of the presence of particular tones in the audio signal. In doing so, they performed four steps: (1) constant-Q transform of the signal (CQT) to represent the signal in frequency domain; (2) the tuning pitch is determined to correct mis-tuning of the music signal; (3) note partials are extracted based on the energy of the CQT spectrum; (4) with the extracted note partial tracks, a pitch profile feature is generated by a consonance filtering and pitch profiling process. An overview of the system is shown in Figure 2.5.

A novel part of their algorithm is the key detection stage. From the pitch profile feature generated using the processes described above, they first estimated the root of the diatonic scale, whose structure they argued is much more salient and robust than the structure of the key. They then conducted the key detection using both the pitch profile feature and the estimated diatonic scale root. Using 60 pieces of pop music and 12 pieces of classical music, the accuracy they obtained was 90% and 83.3%, respectively, arguing most errors were due to key modulation.

Most errors in these key-finding algorithms come from the confusion between closely-related keys such as relative, dominant, sub-dominant or parallel. After careful

examination of the source of most errors, Chuan and Chew proposed a fuzzy analysis technique to extract more correct pitch classes, which improves the accuracy of key finding [16, 17]. They also found that the source of this confusion was caused by noisy detection of lower pitches, and by the biased raw frequency data.

Chuan and Chew's fuzzy analysis technique to clarify pitch information from the FFT has three steps: 1) clarify membership in the lower frequency range using knowledge of the overtone series; 2) scale the FFT in pre-defined range to acknowledge the presence of important pitches, or so called *adaptive level weighting*; 3) refine pitch class distribution by setting all normalized pitch class values 0.2 and below to zero, and 0.8 and above to one. They then applied Spiral Array Center of Effect Generator (CEG) algorithm to determine the key. In the CEG algorithm, the key selection is done by a nearest neighbor search in the Spiral Array space, which is a three-dimensional helical representation of pitch relations [15].

Their evaluation with 410 classical music pieces shows an increase in accuracy with their fuzzy analysis technique compared to a simple peak detection technique, achieving a maximum correct rate of 75.25%. The test audio was all synthesized from the MIDI files, and thus they failed to provide the results with real recordings.

While the previous algorithms used pre-defined key-profiles against which the extracted feature was correlated, Van de Par *et al.* [98] used chromagrams extracted from a collection of musical audio to train three key profiles for major and another three for minor keys, respectively. The three trained key profiles are different in their temporal weighting of information across the duration of each song. In other words, one profile uses uniform weighing, another weighs more on the beginning, and the third puts emphasis on the end of the song, respectively.

Once the key profiles have been trained, three chromagrams are extracted from the test audio as well using the three temporal weighting functions. Each of these extracted chromagrams is correlated with their respective key profiles, yielding three correlation values that relate to the beginning, ending, and totality of the song.

Although an evaluation with the same database used by Pauws [79] (237 classical piano pieces by Bach, Shostakovich, Brahms and Chopin) yields a very high rate of 98% correct classification, the test set was very small (2% or 5 pieces) compared with

a large training data set (98% or 232 pieces). For a more reliable and complete test, a k-fold cross validation with $k = 10$ or so would be more desirable. Furthermore, the method of combining the results from the three different key profiles was empirical, failing to provide logical grounds.

All key-finding algorithms described above find a single, global key from an entire song, which is not true when there is key modulation. Furthermore, key modulation is often the source of errors. Izmirli proposed a model for localized key finding from audio using non-negative factorization for segmentation [76, 77]. He used the conventional chroma vector as a feature. To compensate for possible mis-tuning present in recordings, however, he applied an adaptive tuning algorithm before computing a chromagram. He then performed non-negative factorization on the tuned chromagram for segmentation based on tonal features to reveal the additive contributions of tonal elements in a piece.

After segmenting the whole chromagram into sections with localized keys using the NMF algorithm, he used a correlation method over the first segment to obtain the global key of a piece. Using three different sets of test data (17 pop, 152 classical from the Naxos set[3] and 17 classical excerpts from Kostka and Payne [46]), he obtained a performance of 87.0%, 84.1% and 97.1%, respectively for each test data set. His algorithm also ranked first in the Audio Key Finding task in the MIREX 2005.[4]

Machine Learning Approaches

The key finding systems described so far use pre-defined key templates with which the extracted feature from audio is correlated to find the closest key template. Therefore, their performance is dependent on the choice of the templates, which are either heuristically defined or based on the experimental data.

While these algorithms work fairly well, the pre-defined templates fail to represent the actual organization of pitch classes in real music. Furthermore, it is very difficult to directly relate the low-level acoustical features such as chroma vector to the

[3] `http://www.naxos.com`

[4] Music Information Retrieval Evaluation eXchange (`http://www.music-ir.org/mirexwiki/index.php/Main_Page`).

high-level feature obtained from cognitive studies. Therefore, researchers turn their attentions to alternative approaches, where they *learn* the rules that may define the key from the actual musical audio.

Chai and Vercoe presented a hidden Markov model-based approach to detect the segmental boundaries determined by key changes in musical audio, and identified the key in each segment [14]. They used the 24-dimensional chromagram as a front end instead of the conventional 12-dimensional one. Key detection is composed of two sequential stages. First, they estimate the key without differentiating by *mode*. For instance, C major and A minor key are regarded as the same key without considering its mode. This results in only 12 different keys for each pitch class. Second, after the key is identified, they determine the mode. This two-stage approach is based on the assumptions that they use diatonic scales, and relative modes share the same diatonic scale.

Chai and Vercoe then built hidden Markov models (HMMs) for each task: 12-state HMMs for key detection and 2-state HMMs for mode detection. In estimating the model parameters, *i.e.*, the initial state distribution, the state transition probability distribution and the observation probability distribution, they used empirical values based on music-theoretical knowledge, instead of learning from the training data.

Using 10 pieces of classical piano music, Chai and Vercoe evaluated their system with three different measures, including recall, precision and label accuracy, with promising results. Instead of using a simple pattern-matching algorithm, they chose to use a more sophisticated, statistical approach to detect the key modulation as well as extract the key. However, their estimation of model parameters, which is the most important step in all statistical learning models, was 100% heuristic and therefore fails to make a full use of powerful machine-learning algorithms. This is probably due to lack of sufficient training data, which is usually obtained through a extremely laborious process of manual annotation, and again supports our method of using symbolic music files to generate a large amount of training data *for free*.

While the abovementioned model is in part knowledge-based, *i.e.*, the model parameters are initialized based on music-theoretical knowledge, Peeters proposed a key-finding system that is purely data-based [81, 80]: that is, no prior knowledge is

encoded and the model parameters are learned from the data only. The advantages of a machine-learning approach are, he argues: 1) it does not make assumptions about the presence of harmonics of the pitch notes in chroma vectors; 2) it does not require the choice of a specific key-profile such as Krumhansl, Temperley or Diatonic, etc.; 3) it allows possible key modulation over time through the transition probabilities. He also used HMMs with chroma features as a front end, and compared his model with the ones based on cognitive models

Based on the chroma feature vectors and corresponding key labels, Peeters built 24 HMMs, one for each key. Due to different number of instances for each key, he first trained only two models, a major and a minor *mode* model, and mapped the two trained models to the various other *key-note* models, resulting in 24 major/minor key models. This mapping is performed by circular permutation of observation distribution parameters, *i.e.*, mean vectors and covariance matrices. The final key estimation is done by computing the likelihood of each key model given an unknown chroma sequence, and selecting a model with the maximum likelihood. Figure 2.6 shows an overview of his key estimation system.

A comparison with the cognitive models based on Krumhansl's key-profiles using 302 classical music excerpts shows that his model is outperformed in several different measures, although the difference in performance is not significant. Particularly, the confusion with closely-related keys was larger than the one in the cognitive model-based approach. Counting the related keys as correct matches, the accuracy between the two models is similar (94.9% for HMMs and 95.0% for cognitive models).

Although Peeters' data-driven approach achieves lower scores than the template-matching methods based on cognitive models, it is noteworthy that no prior musical knowledge was used in his model, proving that the system could successfully learn the characteristics of keys from the data. In his models, he used three to nine hidden states and the meaning of the state remains unspecified. This is because he is not interested in decoding the state path, but only interested in finding the key model giving the maximum likelihood. If the states had musical meanings, such as *chords*, however, the results might have been better because musical keys and chords are very closely related. In addition, the optimal state path, which comes free when computing

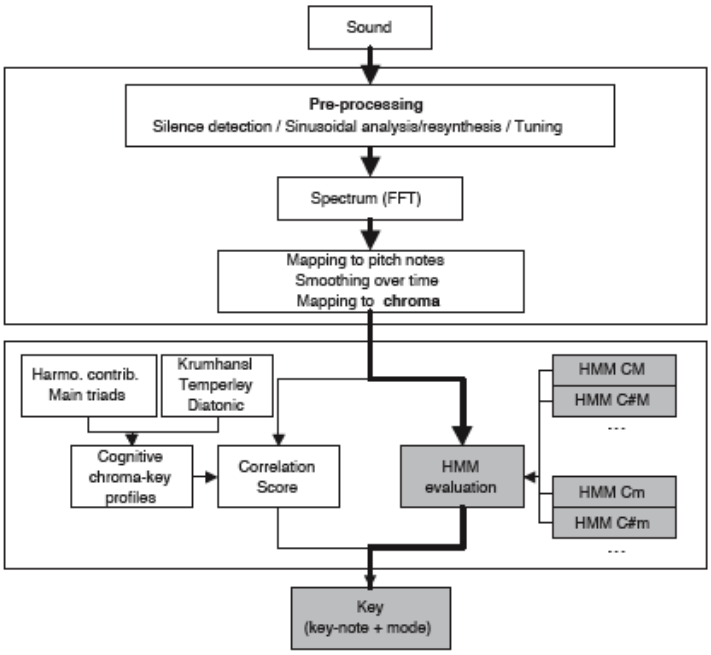


Figure 2.6: Overview of key estimation systems: based on cognitive models (thin arrows) and on HMMs (thick arrows) (from Peeters [80, 81]).

the likelihood with Viterbi search, is identical to chord sequence. This is exactly how our models accomplished *chord recognition* as well as *key estimation* at the same time, as described in Chapter 4.

2.3.2 Chord Transcription

Identifying musical chords from raw audio is a challenging task and has recently attracted the interest of many researchers in a music information retrieval society. Key and chord are very closely related attributes of tonal music. Thus, recognizing the chords and estimating the key from audio share some processes in common, from feature extraction to classification as has already been shown in Figure 2.4.

Because musical chords have much shorter time span than a key has, as explained in Section 2.3, the most prominent difference in a chord recognition task lies in its *evaluation*. It is not too difficult nor laborious to have the ground truth data or labels for key estimation, both for training and test, because one piece of music in general,

especially in popular music, has only one key or a few at best, considering modulation. On the other hand, chord progression, as the name suggests, is a very dynamic property, and thus chord usually keeps changing over time. Therefore, in order to evaluate the chord-recognition systems, we need ground truth which must have not only the correct chord labels, but also have precise, corresponding boundaries. This is why very few systems were systematically evaluated using a fairly large amount of real recordings as test data.

Similar to key estimation systems, the chroma vector has been almost exclusively used as a front end feature for chord recognition systems as well. What makes each system unique is mostly dependent on the decision mechanism, or classifier, which is divided into two main groups, similar to key estimation: template matching techniques and machine learning techniques. In the following sections, both techniques for chord recognition are reviewed in detail.

Template Matching Approaches

In template matching or pattern matching techniques, as in key estimation, chord templates are first defined, either heuristically or based on cognitive studies. For example, some systems use binary chord templates in which only pitch classes corresponding to chord tones are set to ones and the others are set to zeros. Therefore, if we denote 12 pitch classes from C, C♯, $\cdots$, to B, the template for C major triad is [1 0 0 0 1 0 0 1 0 0 0 0]. Other systems use the chord templates derived from the cognitive studies.

Chord classification also follows the similar steps as in key estimation. That is, the feature vector is first computed from audio, and then is correlated with the predefined chord templates. Finally, the template yielding the maximum correlation is recognized as the chord of an given input vector. However, because chords are more locally defined than a key, the analysis must be more granular; *i.e.*, chords must be recognized every frame or every beat in order to detect *when* they change, which is important in the course of musical development.

Fujishima proposed a real-time chord recognition system using a pitch class profile (PCP) vector, which is almost identical to a chroma vector, as a front end [31].

Fujishima described the technique of computing a 12-dimensional chroma vector from the discrete Fourier transform (DFT) of the audio signal. He then used a correlation method with pre-defined binary chord templates to identify a chord type. For example, C major triad template is [1 0 0 0 1 0 0 1 0 0 0 0] and A minor triad template is [1 0 0 0 1 0 0 0 0 1 0 0], where the first bin corresponds to C, the second bin corresponds to C♯, and so on. His system performed well on musical audio containing a single instrument.

It is possible that musical instruments are slightly mis-tuned when they are recorded. This mis-tuning may cause mis-recognition of chords. Harte and Sandler used a quantized chromagram for automatic chord recognition to compensate for possible mis-tuning which may be present in musical audio [36]. To this end, they first computed a 36-bin chromagram, instead of a conventional 12-bin chromagram, which has a resolution higher than a quarter-tone. They then applied the peak-picking algorithm with interpolation to a 36-bin chromagram, and computed a histogram of chroma peaks over the entire piece of music. From the peak histogram, they selected a global tuning center, and used it to convert a 36-bin chromagram to a 12-bin chromagram by merging three chroma bins into one. For the task of chord identification, each frame of the quantized chromagram is compared with a set of chord templates and the closest match is then recorded as the chord estimate for that frame. The flow diagram of the system is shown in Figure 2.7.

They defined 48 chord templates — triads of major, minor, diminished and augmented for each pitch class — and used two full albums of Beatles (28 songs total) for evaluation. The average frame-level accuracy they achieved was 62.4%.

Machine Learning Approaches

As in key estimation using the templates, chord recognition systems using the predefined chord templates also are limited because the templates — either binary or cognitive-based — don't reflect appropriately the distributions of chroma vectors in *real* music. For instance, the binary templates are far from the actual distribution of pitch classes because almost all musical instruments produce *harmonics* of the fundamental notes. Binary templates are only possible when pure sinusoids are used.

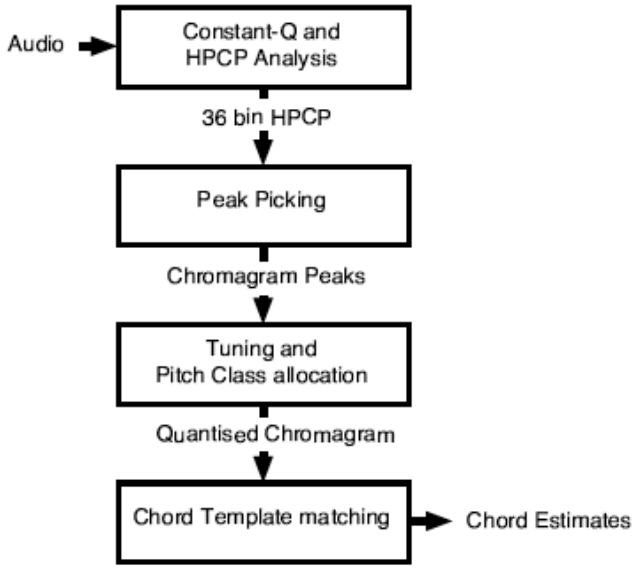


Figure 2.7: Flow diagram of the chord recognition system using a quantized chromagram (from Harte and Sandler [36]).

Using machine learning methods, however, we can understand the real-world characteristics of chords — their progression and observation distribution — and therefore can build the appropriate model from which we can infer chords for an unknown input.

Sheh and Ellis proposed a statistical learning method for chord segmentation and recognition using the 24-bin pitch-class profile features as a front end [93]. They used HMMs trained by the Expectation-Maximization (EM) algorithm, and treated the chord labels as hidden values within the EM framework. This idea was a major breakthrough in HMM-based chord recognition systems because by treating the chord names as hidden states in HMMs, state transitions are identical to chord transitions or *chord progression*. Furthermore, the optimal state path found by the Viterbi decoder is the same as chord sequence.

When training the models, they used only the sequence of chord names, without chord boundaries, as an input to the models, and applied the forward-backward algorithm to estimate the model parameters. After estimating the model parameters, Sheh and Ellis applied the Viterbi algorithm to either forcibly align or recognize

these labels. In forced alignment observations are aligned to a composed HMM whose transitions are dictated by a specific chord sequence, as in training *i.e.*; only the chord-change times are being recovered, since the chord sequence is known. In recognition, the HMM is unconstrained, in that any chord may follow any other, subject only to the Markov constraints in the trained transition matrix.

Using 18 songs and 2 songs of Beatles' as training and test data, respectively, the frame-level accuracy they obtained is about 76% for segmentation and about 22% for recognition. The poor performance for recognition may be due to insufficient training data for a large set of classes (20 songs for 147 chord types). It is also possible that the flat-start initialization of training data yields incorrect chord boundaries resulting in poor parameter estimates.

Although the performance is no longer state-of-the-art, their work made significant contributions in several aspects. First, no music theoretical or music cognitive knowledge such as key-profiles is encoded in their system; it is purely data-based. Second, they applied much of the speech recognition framework with minimal modification, by making a direct analogy between the sequence of discrete, non-overlapping chord symbols used to describe a piece of music, and the word sequence used to describe recorded speech. Third, they avoided the extremely time-consuming processes of labeling the times of chord changes. Lastly, they showed quantitatively that the chroma feature is superior to Mel-frequency cepstral coefficients (MFCCs), supporting the hypothesis that chroma representation is ideal in chord recognition.

Paiement *et al.* proposed a probabilistic model for chord progression based on graphical models [78]. In their model they designed a distributed representation for chords such that Euclidean distances roughly correspond to psychoacoustic dissimilarities. They then derived estimated probabilities of chord substitutions from this representation and used them to introduce smoothing in graphical models observing chord progressions. Parameters in the graphical models are learned with the EM algorithm and the classical Junction Tree algorithm is used for inference.

They used excerpts of 52 jazz standard to train the model and evaluated the model on the same dataset to show that the tree models are slightly better than the HMM, which was used as a benchmark. Although their model is not intended to recognize

chords but to generate them, it is believed that the same model can be used as an analysis tool for chord recognition as well.

Based on the observations that the initialization is critical when estimating the model parameters, Bello and Pickens proposed a chord recognition system in which some music theoretical knowledge is encoded [8]. They also use an HMM with chroma features as a front end, which in essence is similar to what Sheh and Ellis presented [93]. What is interesting in their approach is that they don't use any training database to train their model. Instead, their approach is *knowledge-based*; that is, they incorporate musical knowledge into the models by defining a state transition matrix based on the key distance in a circle of fifths, and avoid random initialization of a mean vector and a covariance matrix of observation distribution. In addition, in training the model's parameters, they selectively update the parameters of interest on the assumption that a chord template or distribution is almost universal regardless of the type of music, thus disallowing adjustment of distribution parameters. The only parameter that is updated and estimated is a transition probability matrix — they use EM algorithm to find the crude transition probability matrix for each input, and use adaptation for final recognition. For example, Figure 2.8 shows the initial state-transition distribution and those trained on specific inputs.

Bello and Pickens tested their system on Beatles' two full albums that Harte and Sandler used [36]. The frame-level accuracy is about 67%, and it increases up to about 75% with beat-synchronous segmentation, which we believe is the state-of-the-art performance. They focus on extracting from the raw waveform a robust mid-level representation, however, and thus use a much smaller set of chord types (24 major/minor triads only) compared with 147 chord types defined by Sheh and Ellis [93] or 48 chord types used by Harte and Sandler [36].

Their main contribution, they argue, is the creation of an effective mid-level representation of musical audio, which in fact is the sequence of major/minor chords on the beat level. They have shown that by incorporating the inherent musical knowledge into machine learning models, the performance increases significantly.

Cabral *et al.* presented a chord recognition system using an *extractor discovery system* (EDS) [12, 13]. EDS is a heuristic-based generic approach for automatically

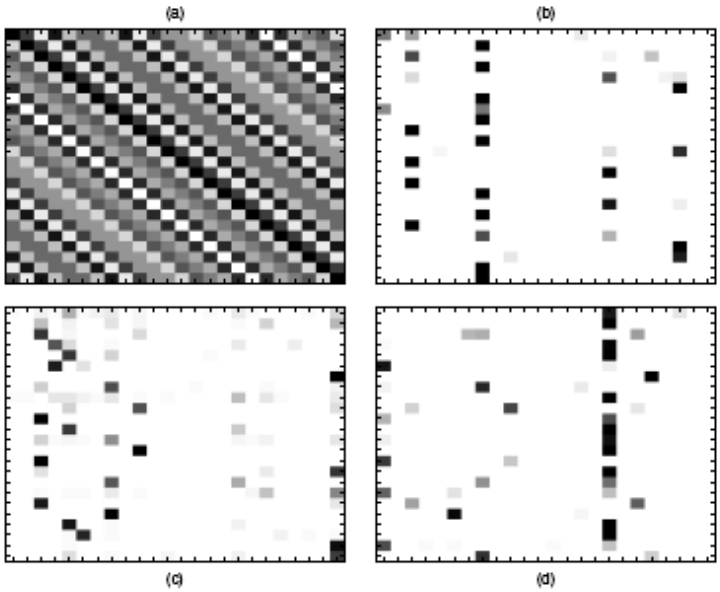


Figure 2.8: State-transition distribution A: (a) initialization of A using the circle of fifths, (b) trained on *Another Crossroads* (M. Chapman), (c) trained on *Eight days a week* (The Beatles), and (d) trained on *Love me do* (The Beatles). All axes represent the 24 lexical chords (C→B then c→b). Adapted from Bello and Pickens [8].

extracting high-level musical descriptors from acoustic signals, based on genetic programming. Given a database of audio signals with their associated perceptive values, EDS is capable to generalize a descriptor. Such descriptor is built by running a genetic search to find relevant signal processing features to match the description problem, and then machine learning algorithms to combine those features into a general descriptor model.

Although they avoid using the conventional chroma feature and try instead to select the *optimal* feature set for chord recognition using genetic algorithm and machine learning techniques, the experiments with 1885 samples and 60 chord types shows the EDS algorithm is outperformed by traditional approaches such as a PCP template matching method or kNN (k-nearest neighbors) classifier.

In a musical chord detection system Maddage *et al.* presented a hierarchical approach based on the analysis of tonal characteristics of musical audio [64]. They

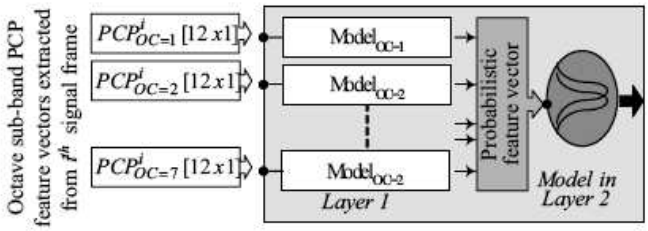


Figure 2.9: Two-layer hierarchical representation of a musical chord (from Maddage *et al.* [64]).

first examined two different methods of representing tonal characteristics of musical audio, *i.e.*, pitch class profile (PCP) method and psycho-acoustical method, to find out not only the fundamental frequency of music note but also its harmonics and sub-harmonics contribute for detecting related chords. Then they modeled each chord using a hierarchical approach, which is composed of two layers. In the first layer, they build individual tonal characteristic model for each octave. The responses of the individual models in the first layer are fed to the model in the second layer to detect the chord. The models in the 1st layer are trained using 12-dimensional PCP feature vectors which are constructed from individual octaves. The 2nd layer model is trained with the vector representation of the 1st layer model responses. The hierarchical model is illustrated in Figure 2.9.

For the dataset for training and test, they used more than 2000 synthesized samples for each chord as well as real-world samples extracted from 40 pieces of pop music, and compared three different chord models — pitch class profile (PCP), psycho-acoustical profile (PAP) and two-layer hierarchical model. The experimental results show that the proposed two-layer hierarchical model consistently outperforms the other two in both datasets — synthesized and real. The contribution of their work is the demonstration that a hierarchical model based on the PCP from *each octave* increases the performance in chord recognition.

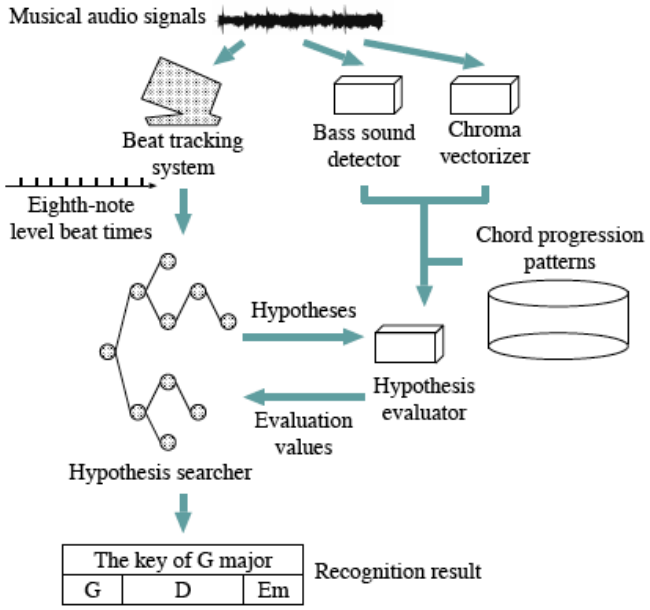


Figure 2.10: Overview of the automatic chord transcription system based on hypothesis search (from Yoshioka *et al.* [101]).

Other Approaches

Yoshioka *et al.* presented an automatic chord-transcription system by first generating hypotheses about tuples of chord symbols and chord boundaries, and evaluating the hypotheses based on three cues: acoustic features, chord progression patterns and bass sounds [101]. To this end, they first performed beat-analysis on raw audio to detect the eighth-note level beat times of an input musical piece. Then, the hypothesis searcher searches the most plausible hypothesis about a chord sequence and a key. The search progresses every eighth-note level beat time from the beginning of the input. Finally, the searcher outputs the obtained most plausible hypothesis. Figure 2.10 shows an overview of the system.

A conventional 12-dimensional chroma feature is used as a front end to the system. Pre-defined chord progression patterns reduce the ambiguities of chord symbol-identification results. Finally, using the bass sounds, they argue, improves the performance of automatic chord transcription, because bass sounds are closely related to

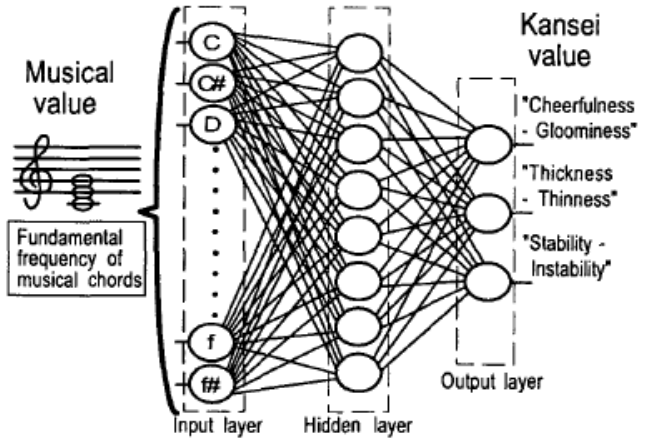


Figure 2.11: Network structure of Kansei model (from Onishi *et al.* [75]).

musical chords, especially in popular music. They tested their system on one-minute excerpts from seven popular songs, and achieved 77% average accuracy.

Onishi *et al.* presented an interesting approach to map different chords to different *"Kansei"* or emotion values using a neural network model that simulates a human auditory system [75]. They used 19 frequency units in the input layer of the neural network, each frequency corresponding to a musical tone between C and f♯, put eight units in the hidden layer, and defined three Kansei values — "cheerfulness-gloominess", "thickness-thinness" and "stability-instability" — in the output layer as illustrated in Figure 2.11.

Using 326 chords for training and 40 for testing, all synthesized using pure sinusoids, they could successfully map the chords to the corresponding Kansei values or emotions. Although the purpose of the algorithm is not recognizing the chords, and their experiment is limited in a sense they didn't analyze the real recording but used synthesized chords as an input, it is noteworthy in that they tried to extract high-level emotional attributes from the audio.

2.4 Summary

In this chapter, we have presented the scientific background on which this dissertation work is largely based. We first presented music theoretical/cognitive studies that address the relation between the perceived musical attributes and the corresponding responses from the subjects in the context of tonal music. The findings from these studies provide a fundamental background to a number of computational algorithms for both key estimation and chord recognition. We then reviewed many different approaches to tackle these challenging tasks, concentrating on those that deal with the raw audio. With all the differences in background and implementation, we have found that most algorithms are largely divided into two groups for both key estimation and chord recognition — template matching algorithms and machine learning algorithms.

In template matching algorithms, the key/chord templates are first defined based on music cognitive studies (or simple binary-types are used), and the features extracted from audio are compared against these templates to find the closest one in distance. These methods are very efficient and don't require the training data that machine learning algorithms do. However, their performance is very much dependent on the pre-defined templates or profiles, which may not be universal; they may vary from genre to genre, for example. Furthermore, they don't take into account one of very important characteristics inherent in music — its *temporal dynamics* — which makes music distinct from other forms of art like visual art. A template matching approach doesn't care about the *progression* of music over time. For example, in case of chord recognition, when classifying the current feature frame, they make a decision based on that frame only, not considering what preceded or will follow. Or in key-finding algorithms, the chroma features are usually averaged to give a global chroma vector, which is matched against the key profiles. This in turn means that these algorithms will perform exactly the same even if a musical piece is in reverse order or even randomized, which will probably not be defined as music any more. We can take great advantage of this time-dependent attribute of music in analyzing music.

On the other hand, machine-learning algorithms such as hidden Markov models

depend on what happened previously in determining what is happening now. This is crucial, in chord recognition in particular, because in Western tonal music, chords progress based on a set of rules, and thus knowing the current chord significantly helps decide what will follow. In addition, using machine-learning approaches, we don't need any prior knowledge such as key profiles. The model learns all the rules from the data, if we build the model appropriately.

However, the biggest bottleneck problem in machine learning models is the preparation of the training data, on which the performance is heavily dependent. This is even more problematic in supervised learning, where the training data must have *ground truth*, which is provided by humans, involving a great amount of time and labor. For instance, in order to train a chord-recognition model, we must hand-label all the chords for a number of songs. This implies that we should perform harmony analysis first to obtain chord names and find precise timing boundaries. This manual annotation process can be extremely difficult and time-consuming even for experienced musicians.

In the next chapter, we propose a novel and almost labor-free method of obtaining a large amount of labeled training data to train hidden Markov models using symbolic music files.

Chapter 3

System Description

3.1 Introduction

A system for automatically recognizing musical chords from raw waveforms involves two stages, as shown in Figure 3.1. It must first extract the appropriate feature vector from the raw audio samples. The extracted feature for the chord-recognition task must satisfy these requirements: first, it must be low in dimension, particularly when used in real-time applications or when there is not enough training data in machine learning approaches; second, it must be robust to noise-like signals such as percussive sounds or distorted guitar sounds because they don't constitute chord tones; finally, it must remain largely invariant under acoustical differences caused by changes in tempo, melody, instrumentation, dynamics, and so on. This is because, for example, a C major chord in root position played by a piano, the same chord in first inversion played by a rock band, and the same chord in second inversion played by a full orchestra, however their acoustical properties may vary, must all be transcribed as the same C major chord.

Once the appropriate feature vector is extracted from the raw audio, it becomes an input to the recognition system, where each feature vector is classified as a certain chord. Some systems use a simple pattern matching algorithm for classification [31, 36, 55] while others use more sophisticated machine learning approaches [93, 8, 69, 59, 58, 60, 61, 57]. We use HMMs for chord classification because they represent a

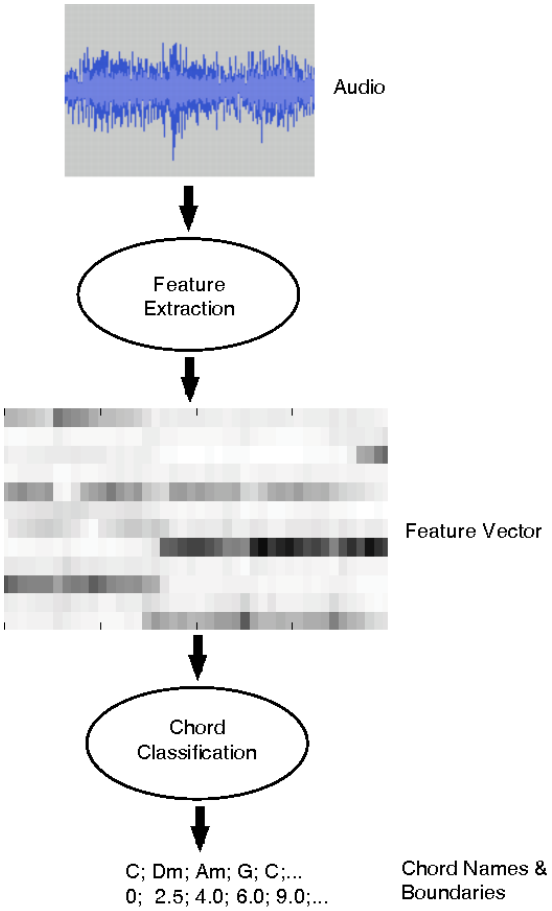


Figure 3.1: Two main stages in chord recognition systems.

probabilistic framework that not only provides the observation distribution of chords for the given input vector, but also explains the progression of chords over time through a Markov process.

A hidden Markov model [86] is an extension of a discrete Markov model, in which the states are *hidden* in the sense that we can not directly observe the underlying stochastic process, but can only observe it through another set of stochastic processes. The three parameters that define an HMM are the observation probability distribution, the state transition probability distribution and the initial state distribution; we can accurately estimate these parameters from the labeled training data. More

details about an HMM are found in Appendix A.

Our system is based on the work of Sheh and Ellis and Bello and Pickens, in that the states in the HMM represent chord types, and the optimal path, *i.e.*, the most probable chord sequence, is found in a maximum-likelihood sense. The most prominent difference in our approach is, however, that we use labeled training data from which model parameters can be directly estimated without using an EM algorithm because each state in the HMM is treated as a single chord and therefore we can learn the observation distribution for each chord and the chord-to-chord or state-transition probabilities. In addition, we propose a method to automatically obtain a large set of labeled training data, removing the problematic and time-consuming task of manual annotation of precise chord boundaries with chord names. Finally, we train our models on the data sets of different musical genres and investigate the effect of each parameter set when various types of input are given. We also demonstrate the robustness of the tonal centroid feature because it yields better performance than the chroma feature when tested on different kinds of input.

In the following section, the procedure for obtaining the feature set from the raw waveform will be described in more detail.

3.2 Feature Vector

As has been mentioned above, the raw audio samples are not appropriate to be used as an input to the chord recognition system, which must make a decision as to which chord type a given feature vector should be classified as. The feature should not only be able to represent musical chords properly but also have low dimensionality. The requirement for low dimensionality and computational efficiency is especially important when a real-time implementation is needed, or when the number of training samples is small in statistical learning methods. Furthermore, it must be robust to non-harmonic noisy signals caused by sharp attacks or percussive sounds since they don't tell us about the chord tones.

We use two different types of feature vectors in our chord recognition system. The first we use is the conventional 12-dimensional chroma feature. We use chroma

feature not only because it is widely accepted for chord recognition, but also because we try to compare our system with others just based on the classification algorithm. As a second feature vector, we use *tonal centroid* which is a 6-dimensional vector derived from a 12-dimensional chroma vector. In the following sections, we describe each feature in more detail.

3.2.1 Chroma Vector

A chroma vector or a Pitch Class Profile (PCP) is the feature of choice in automatic chord recognition or key extraction ever since it was introduced by Fujishima [31]. Perception of musical pitch involves two dimensions — *height* and *chroma* — often described as the *Pitch Helix*. This is illustrated in Figure 3.2. Pitch height moves vertically in octaves telling which octave a note belongs to. On the other hand, chroma tells where a note stands in relation to others within an octave. A chroma or a PCP feature is a 12-dimensional vector representation of a chroma, which represents the relative intensity in each of twelve semitones in a chromatic scale. Since a chord is composed of a set of tones, and its label is only determined by the position of those tones in a chroma, regardless of their heights, chroma vectors appear to be an ideal feature to represent a musical chord or a musical key. Fujishima developed a real-time chord-recognition system, where he derived a 12-dimensional pitch class profile from the discrete Fourier transform (DFT) of the audio signal, and performed pattern matching using binary chord type templates [31].

There are some variations when computing a 12-bin chroma feature, but it usually follows these steps. First, the DFT of the input signal $X(k)$ is computed, and the constant-Q transform X_{CQ} is calculated from $X(k)$, using a logarithmically spaced frequencies to reflect the way a human perceives sound [11]. The frequency resolution of the constant-Q transform follows that of the equal-tempered scale, which is also logarithmically based, and the kth spectral component is defined as

$$f_k = (2^{1/B})^k f_{min}, \tag{3.1}$$

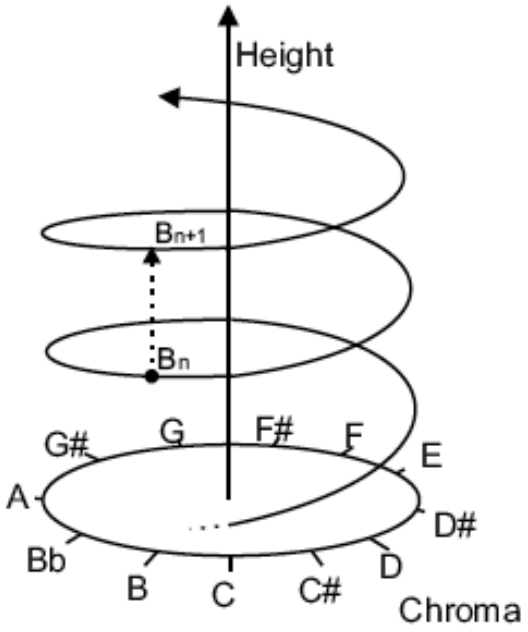


Figure 3.2: The pitch helix and chroma representation. Note B_{n+1} is an octave above note B_n (from Harte and Sandler [36]).

where f_k varies from f_{min} to an upper frequency, both of which are set by the user, and B is the number of bins in an octave in the constant-Q transform.

Once $X_{CQ}(k)$ is computed, a chroma vector CH can be easily obtained as:

$$CH(b) = \sum_{m=0}^{M-1} \big|X_{CQ}(b + mB)\big|, \tag{3.2}$$

where $b = 1, 2, \cdots, B$ is the chroma bin index, and M is the number of octaves spanned in the constant-Q spectrum. That is, all DFT bins are collapsed to create a 12-dimensional vector without absolute frequency height, each bin representing the spectral energy in each pitch class in a chromatic scale. Although a chromatic scale has only 12 pitch classes ($B = 12$), $B = 24$ or $B = 36$ can also be used to deal with finer tuning than a semitone. This fine tuning is especially useful when compensating for possible mistuning present in audio. Figure 3.3 shows a chromagram

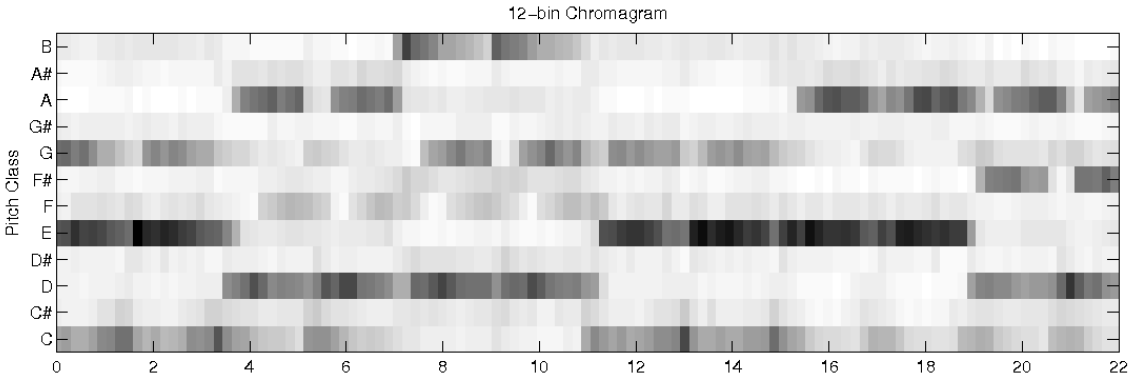


Figure 3.3: 12-bin chromagram of an excerpt from *Bach's Prelude in C major* performed by Glenn Gould. Each chroma vector is computed every 185 millisecond.

example, where each chroma vector is horizontally concatenated over time to form a 2-dimensional representation like a spectrogram.

While the chroma feature not only appears to be ideal for both chord recognition and key extraction but also is efficient because of its low dimensionality, the semitone resolution might not be sufficient to fully describe the tonality present in musical audio. Furthermore, a constant-Q transform assumes a fixed tuning frequency (*e.g.* A4 = 440Hz), which is not always the case for all recordings.

Researchers therefore proposed various algorithms to compute different features as an alternative to the conventional chroma feature. Gómez proposed a Harmonic Pitch Class Profile (HPCP) with a few modifications on the pitch class profile (PCP) [32]. First, she introduces a weight function in the feature computation. Second, she considers the harmonics. Third, she uses a higher resolution in the HPCP bins, resulting in 36-dimensional feature vectors. She and her colleagues applied the proposed HPCP vector successfully in applications such as audio cover song finding [33, 92] and structural segmentation [74]. Lee used a summary autocorrelation function (SACF) based on the human auditory model as a front end to template matching classifier to recognize chords from audio [54]. Lee also proposed an enhanced pitch class profile or EPCP vector for chord recognition [55], where he used a harmonic product spectrum (HPS) computed from the STFT, before collapsing it down to a 12-dimensional vector, in order to suppress the effect of the harmonics of non-chord tones, which might

cause confusion to the classifier.

3.2.2 Quantized Chromagram

The conventional chroma feature explained above can be problematic when a possible mistuning is present in acoustic recordings. For instance, instruments may have been slightly mis-tuned (*e.g.* A4 = 445 Hz) when recording a rock band. In such cases, the resulting chroma feature obtained using the above formulas will not be precise, and may cause mis-classification in the recognition system.

In order to avoid the mis-tuning problem, Harte and Sandler proposed a 12-bin Quantized chromagram [36]. They first derived a 36-bin chromagram instead of a 12-bin chromagram, which can detect a quartertone difference in the signal. They then obtained the sinusoidal peak distribution from the Short-Time Fourier Transform (STFT) frames for the whole piece of music, and determined how to group three contiguous bins into one to generate a 12-bin quantized chromagram based on the peak distribution. The overall flow of these processes is shown in Figure 3.4.

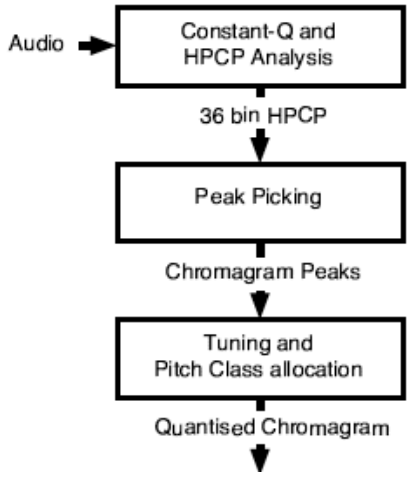
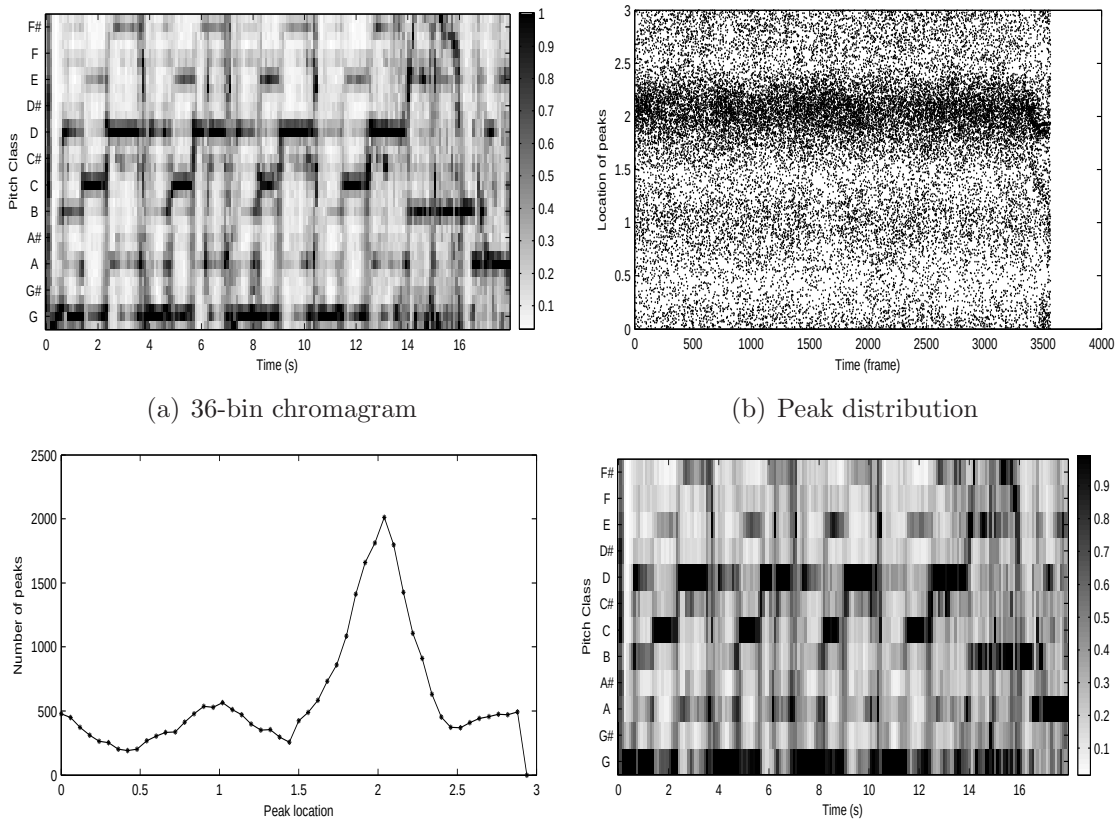


Figure 3.4: Computing the quantized chromagram (from Harte and Sandler [36]).

Figure 3.5 illustrates the tuning algorithm with an actual example. Figure 3.5(a) is an original 36-bin chromagram. Figure 3.5(b) and Figure 3.5(c) show the distribution and the histogram of peaks obtained from the 36-bin chromagram. Finally, Figure



(a) 36-bin chromagram

(b) Peak distribution

(c) Peak histogram

(d) 12-bin tuned chromagram

Figure 3.5: Tuning of *Another Crossroads* by Michael Chapman. (a) 36-bin chromagram (b) Peak distribution (c) Peak histogram and (d) 12-bin tuned chromagram

3.5(d) shows a tuned, semitone-quantized chromagram.

3.2.3 Tonal Centroid

Based on the observation that close harmonic relations such as fifths and thirds appear as small Euclidean distances in an interior space mapped from the 12-bin chroma space, Harte *et al.* proposed a 6-dimensional feature vector called *Tonal Centroid*, and used it to detect harmonic changes in musical audio [37]. It is based on the Harmonic Network or *Tonnetz*, which is a planar representation of pitch relations where pitch classes having close harmonic relations such as fifths, major/minor thirds have smaller Euclidean distances on the plane as shown in Figure 3.6. The Harmonic Network is

a theoretically infinite plane, but is wrapped to create a 3-D Hypertorus assuming enharmonic and octave equivalence, and therefore there are just 12 chromatic pitch classes. The Hypertorus is illustrated in Figure 3.7.

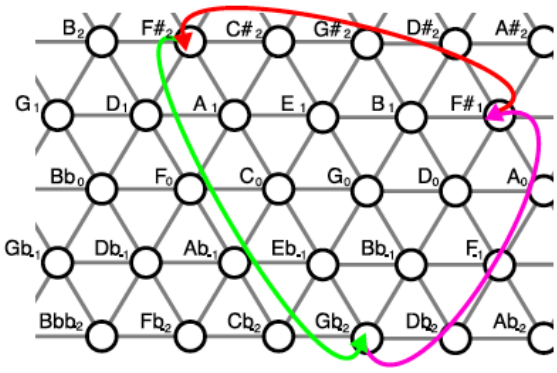


Figure 3.6: The Harmonic Network or *Tonnetz*. Arrows show the three circularities inherent in the network assuming enharmonic and octave equivalence (from Harte *et. al* [37]).

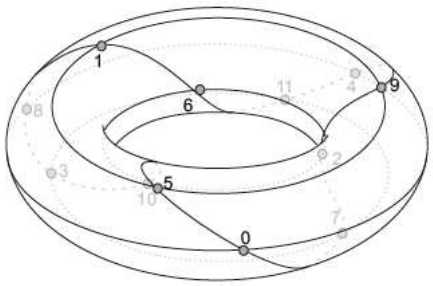


Figure 3.7: A projection showing how the 2-D Tonnetz wraps around the surface of a 3-D Hypertorus. Spiral of fifths with pitch classes is shown as a helical line (from Harte *et. al* [37]).

If we reference C as a pitch class 0, then we have 12 distinct points on the circle of fifths from 0-7-2-9-$\cdots$-10-5, and it wraps back to 0 or C (shown as a spiral line in Figure 3.7). If we travel on the circle of minor thirds, however, we come back to a referential point only after three steps as in 0-3-6-9-0. The circle of major thirds is defined in a similar way. This is visualized in Figure 3.8. As shown in Figure 3.8, the six dimensions are viewed as three coordinate pairs $(x1, y1), (x2, y2)$, and $(x3, y3)$.

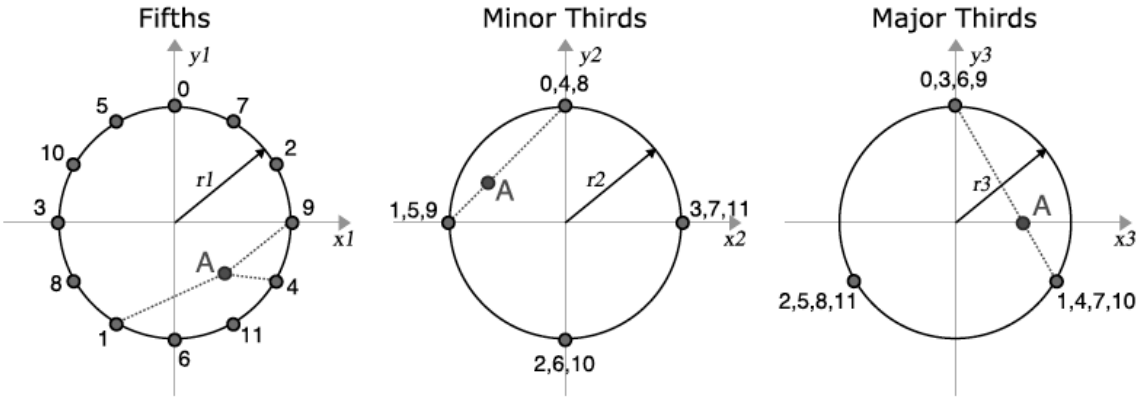


Figure 3.8: Visualizing the 6-D Tonal Space as three circles: fifths, minor thirds, and major thirds from left to right. Numbers on the circles correspond to pitch classes and represent nearest neighbors in each circle. Tonal Centroid for A major triad (pitch class 9,1, and 4) is shown at point A (from Harte *et. al* [37]).

Using the aforementioned representation, a collection of pitches like chords is described as a single point in the 6-D space. Harte *et. al* obtained a 6-D Tonal Centroid vector by projecting a 12-bin tuned chroma vector onto the three circles in the equal tempered Tonnetz described above. Mathematically, this is equivalent to multiplying a a 12×1 chroma vector by a 6×12 linear transformation matrix Φ to obtain a 6×1 tonal centroid vector. The 6×12 transformation matrix Φ represents the basis in the 6-D tonal centroid space and is given as:

$$\Phi = [\phi_0, \phi_1, \cdots, \phi_{11}], \tag{3.3}$$

where

$$\phi_b = \begin{bmatrix} \Phi(0,b) \\ \Phi(1,b) \\ \Phi(2,b) \\ \Phi(3,b) \\ \Phi(4,b) \\ \Phi(5,b) \end{bmatrix} = \begin{bmatrix} r_1 \sin b\frac{7\pi}{6} \\ r_1 \cos b\frac{7\pi}{6} \\ r_2 \sin b\frac{3\pi}{2} \\ r_2 \cos b\frac{3\pi}{2} \\ r_3 \sin b\frac{2\pi}{3} \\ r_3 \cos b\frac{2\pi}{3} \end{bmatrix}, b = 0, 1, \cdots, 11. \tag{3.4}$$

The values of $r1, r2$ and $r3$ are the radii of the three circles in Figure 3.8. Harte *et. al* set these values to 1, 1 and 0.5, respectively to ensure that the distances between pitch classes in the 6-D space correspond to our perception of harmonic relations between pitches (*i.e.* that the fifth is the closest relation followed by the major third then the minor third and so on)

By calculating the Euclidean distance between successive analysis frames of tonal centroid vectors, they successfully detect harmonic changes such as chord boundaries from musical audio.

We used the tonal centroid feature as well as the conventional 12-dimensional chroma vector, and compared their performance. We hypothesize the tonal centroid vector is more efficient because it has only 6 dimensions, and more robust because it puts emphasis on the interval relations such as fifths, major/minor thirds, which are key intervals that comprise most of musical chords in Western tonal music.

In the next section, we present a chord-recognition system using hidden Markov models trained on synthesized audio.

3.3 Chord Recognition System

Our chord transcription system performs both symbolic and acoustic analysis of the symbolic music files. We perform harmony analysis on symbolic data to automatically obtain label files with chord names and precise time boundaries. In parallel, we synthesize audio files from the same symbolic files using a sample-based synthesizer. The audio files and the label files are in perfect synchronization, and we can use them

to train our models. The overview of the training process is shown in Figure 3.9.

3.3.1 Obtaining Labeled Training Data

In order to estimate the model parameters in supervised learning, we need training data; namely, for chord recognition, audio files with corresponding labels that annotate chord boundaries and chord names. As the amount of the training data increases, the model can better generalize to various types of unseen input, resulting in better performance. The high performance in automatic speech recognition is due to a gigantic amount of speech corpora accumulated over decades. However, manual annotation of chord names and boundaries is very difficult and extremely time-consuming. Previous work by Sheh and Ellis has just 0.72 hours of audio for training their model, and the poor performance is probably explained by insufficient training data.

We tackle this bottleneck problem using symbolic music documents like MIDI files. That is, in order to automate the laborious process of manual annotation, we use symbolic data to generate label files as well as to create audio data. We hypothesize that this will produce adequate training data. Although there may be noticeable differences in sonic quality between real acoustic recording and synthesized audio, we do not believe that the lack of human touch, which makes a typical MIDI performance dry, affects our training program.

To this end, we first convert a symbolic file to a format which can be used as an input to a chord-analysis tool. The chord analyzer then performs harmony analysis and outputs a file with root note information and note names from which we obtain complete chord information (*i.e.*, root and its sonority–major, minor or diminished triad/seventh). We use sequence of chords as ground truth, or labels, when training the HMMs.

To examine the model's dependency on the training data, we choose two different training data sets with different types of music. For the first parameter set, we use 765 classical symbolic music files as a training data set, which comprise 406 pieces of solo keyboard music and 359 string quartets by J. S. Bach, Beethoven, Haydn,

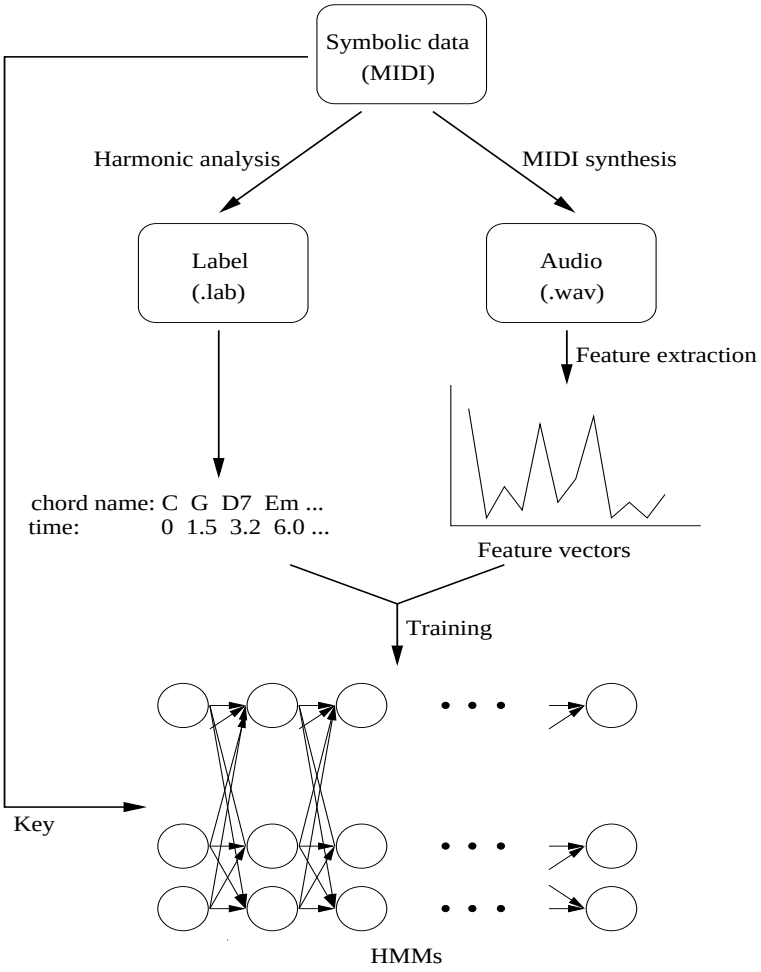


Figure 3.9: Training the chord transcription system. Labels obtained through harmony analysis on symbolic music files and feature vectors extracted from audio synthesized from the same symbolic data are used to train HMMs.

Mozart and other composers. All classical symbolic music files are in a Humdrum data format from the Center for Computer Assisted Research in the Humanities at Stanford University. Humdrum is a general-purpose software system intended to help music researchers encode, manipulate and output a wide variety of musically-pertinent representations [39]. These files are converted to a format that can be used in the Melisma Music Analyzer, as well as to a MIDI format using the tools developed by Craig Sapp.[1]

For the second training set, we use 158 MIDI files of Beatles available from `http://www.mididb.com`.

The audio data synthesized from these symbolic music files of the classical and Beatles data set are 26.73 hours long or 517,945 feature frames, and 5.73 hours long or 111,108 feature frames, respectively.

We perform harmony analysis to obtain chord labels using the Melisma Music Analyzer developed by Sleator and Temperley [94]. Melisma performs harmony analysis on a piece of music represented by an event list and extracts information about meter, harmony and key, and so on. We configure Melisma so that it outputs a chord name every beat and use its output as ground truth. When building key-dependent models, we take the beginning key as a home key for an entire piece.

Temperley tests the symbolic harmony-analysis program on a corpus of excerpts and the 48 fugue subjects from the *Well-Tempered Clavier*; Melisma harmony analysis and key extraction yields an accuracy of 83.7% and 87.4%, respectively [95].

Figure 3.10 shows the normalized distributions of chords and keys extracted from Melisma for each training set.

We synthesize the audio files using Timidity++ (Timidity++ is a free software synthesizer and converts MIDI files into audio files in a WAVE format.[2] It uses a sample-based synthesis technique to create harmonically rich audio as in real recordings.) We use the GUS (Gravis Ultra Sound) sound font, which is used by Timidity, to synthesize the MIDI files.[3] The set of instruments we use to synthesize classical music are piano, violin, viola and cello. When rendering Beatles' MIDI files we use

[1] `http://extras.humdrum.net`
[2] `http://timidity.sourceforge.net/`
[3] `http://www.gravis.com`

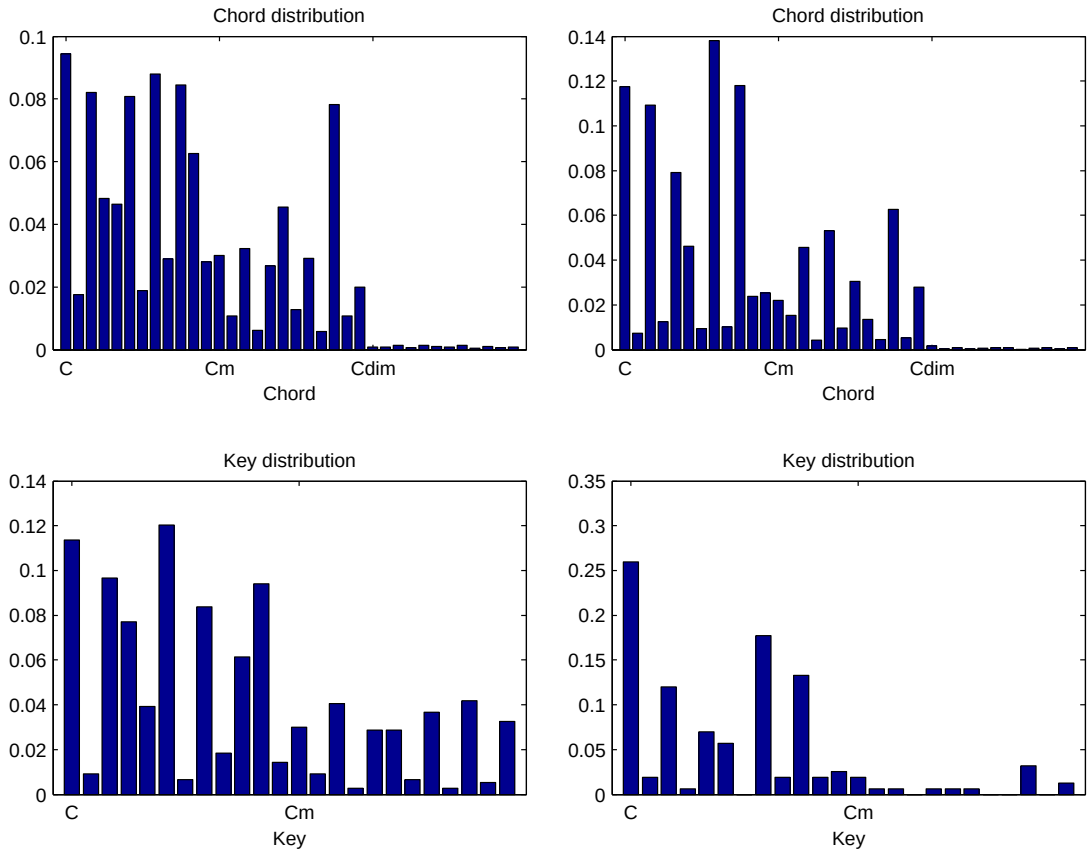


Figure 3.10: Chord and key distribution of classical (left) and Beatles (right) training data.

electric piano, electric guitar, steal string guitar, electric bass and orchestral strings.

Our feature analysis from audio follows these steps. The raw audio is first down-sampled to 11025 Hz; then 12-bin chroma features and 6-dimensional tonal centroid features are extracted from it with the frame size of 8192 samples and the hop size of 2048 samples, which gives the frame rate of approximately 5.4 frames/second. The frame-level chroma vectors or tonal centroid vectors are then used as input to the HMMs along with the label files obtained above.

3.3.2 Hidden Markov Model

We recognize chords using either 24-state (trained on Beatles music) or 36-state HMMs (trained on classical music). Each state in our model represents a single chord. The observation distribution is modeled by a single multivariate Gaussian—in 12 dimensions for the chroma feature or in six dimensions for the tonal centroid feature—defined by its mean vector μ_i and covariance matrix Σ_i, where i denotes ith state. We assume the dimensions of the features are uncorrelated with each other, and thus use a diagonal-covariance matrix.[4] State transitions obey a first-order Markov property; *i.e.*, the future is independent of the past given the present state. In addition, we use an ergodic model since we allow every possible transition from chord to chord, and yet the transition probabilities are learned.

In our model, we define 36 classes or chord types according to their sonorities only—major, minor and diminished chords for each pitch class. We ignore the augmented chords since they rarely appear in Western tonal music. We group triads and seventh chords with the same sonority into the same category. For instance, we treat E minor triad and E minor seventh chord as just E minor chord without differentiating the triad and the seventh. Most pop or rock music, as in the Beatles, makes use of only 24 major/minor chords, so for our experiments with popular music we recognized only 24 chords, as done by Bello and Pickens [8].

With the labeled training data we obtain from the symbolic files, we first train our models to estimate the model parameters. Once we learn the model parameters—initial state probabilities, state transition probability matrix, and mean vector and covariance matrix for each state—we recognize chords for an unknown input by extracting the feature vectors from the raw audio and applying the Viterbi [100, 30] algorithm to the appropriate model to find the optimal path, *i.e.*, chord sequence, in a maximum likelihood sense.

[4]We tried full-covariance observation matrices, but our recognition was lower, suggesting that we don't have enough data.

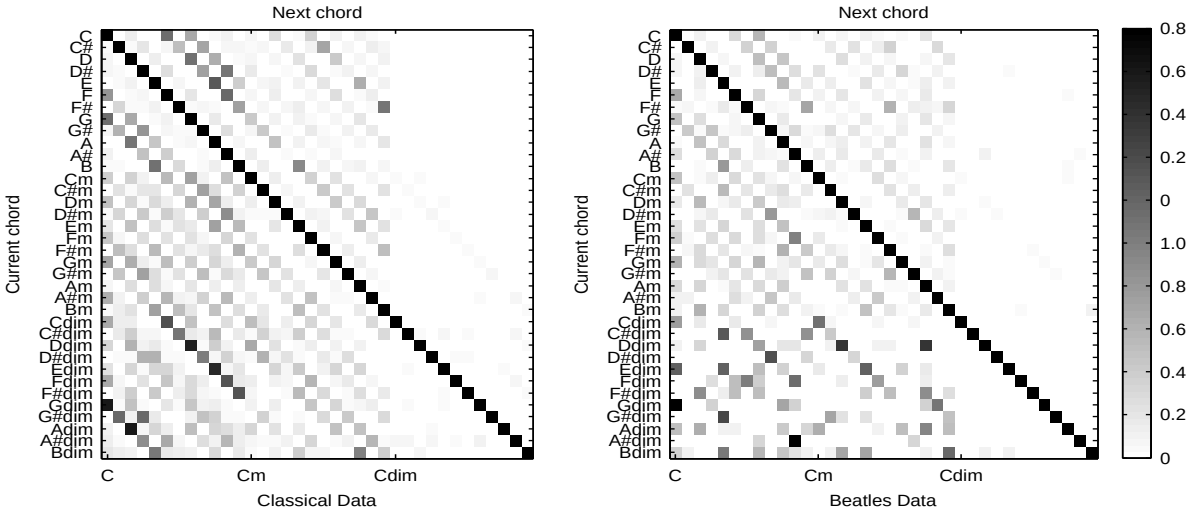


Figure 3.11: 36x36 transition probability matrices obtained from 765 pieces of classical music and from 158 pieces of Beatles' music. For viewing purpose, logarithm is taken of the original matrices. Axes are labeled in the order of major, minor and diminished chords. The right third of these matrices are mostly zero because these musical pieces are unlikely to transition from a major or minor chord to a diminished chord, and once in a diminished chord, the music is likely to transition to a major or minor chord again.

3.3.3 Parameter Estimates

Figure 3.11 shows chord-to-chord transition probability matrices estimated from each training data set. The transition matrices are strongly diagonal since a chord's duration is usually longer than the frame rate, and thus the state does not change for several frames, which makes a transition probability to itself highest.

As further illustrated in Figure 3.12, however, the chord progressions observed in the transition probabilities are rooted in music theory. The C major chord has the largest probability of staying within the same state, *i.e.*, within a C major chord, because of faster frame rate than the rate of chord changes. But it has comparably higher probabilities for making a transition to specific chords such as an F major, G major, F minor or A minor chord than to others. F major and G major have subdominant-tonic and dominant-tonic relationships with C major, respectively, and transitions between them happen very often in Western tonal music. A C major

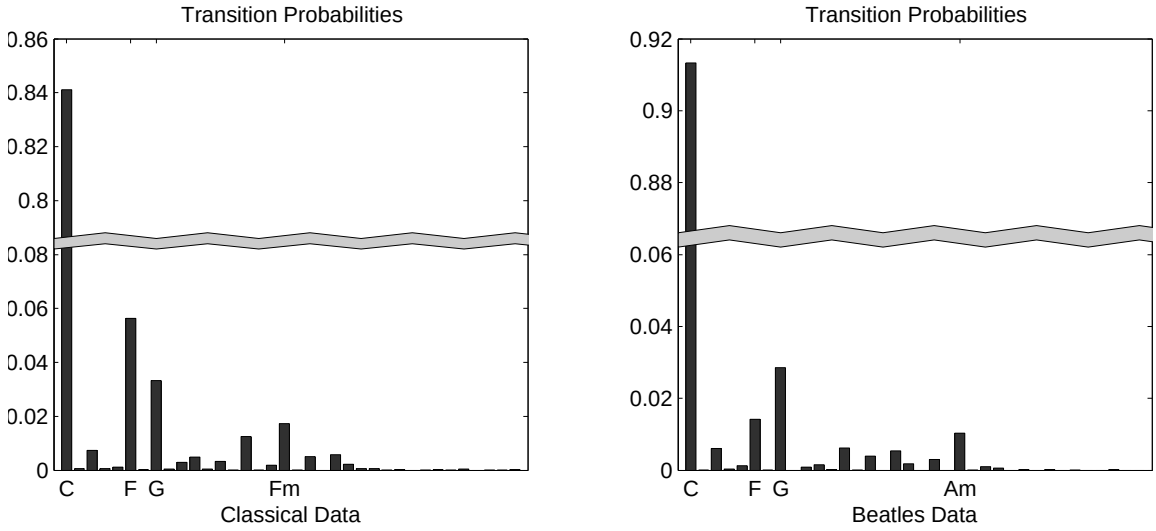


Figure 3.12: Transition probabilities from C major chord estimated from classical and from Beatles' data. The X axis is labeled in the order of major, minor and diminished chords.

chord is also a dominant chord of an F minor, and therefore a C major to F minor transition is frequent as well. Finally, an A minor chord is a relative minor of the C major chord, and a C-to-Am transition also occurs quite often. This tonic-dominant-subdominant relationship is also shown in Figure 3.11 as off-diagonal lines with five and seven semitone offsets with respect to their tonics.

Figure 3.13 shows the observation distribution parameters for chroma feature estimated from each training data set for a C major chord. On the left are the mean chroma vector and diagonal covariance vector for an HMM trained on classical music, and those for Beatles' music are on the right. It is obvious, as expected, that they both have three large peaks at chord tones or at C, E and G. In addition, we can also see relatively large peaks at D and B, which come from the third harmonics of chord tones G and E. Mean vectors and covariance matrices of tonal centroid feature are also shown in Figure 3.14 for each data set.

Although we can estimate the model parameters for observation distribution for each chord, the number of feature samples in training data not only varies to a great degree from chord to chord but also is limited for some chords, as shown in

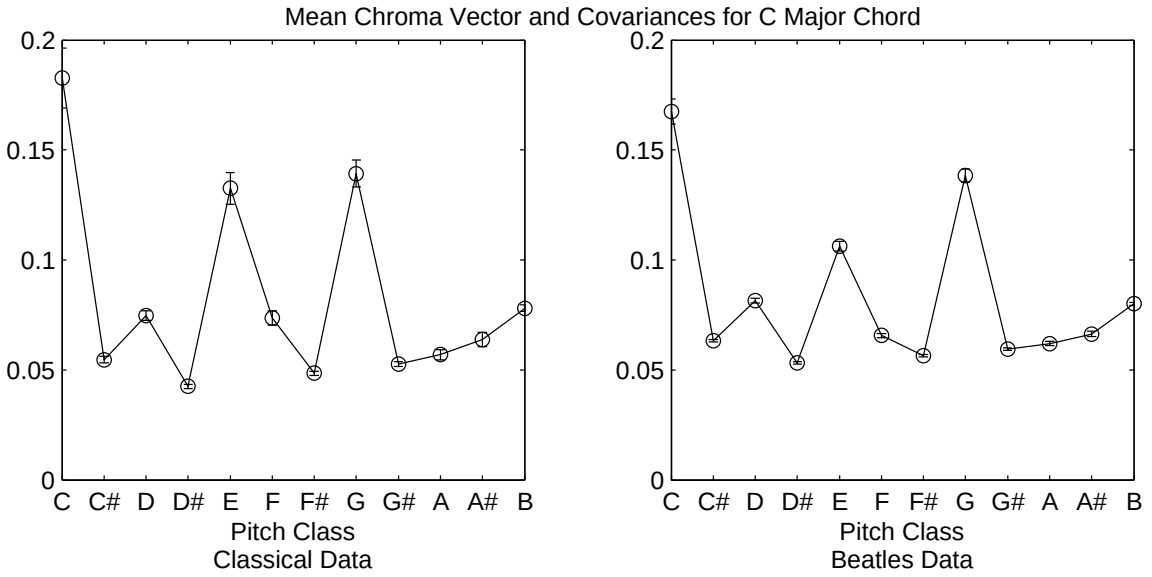


Figure 3.13: Mean chroma vector and covariances for C major chord estimated from classical and from Beatles' data. Because we use diagonal covariance, the variances are shown with "error" bars on each dimension.

the chord distributions in Figure 3.10. This is likely to cause class statistical errors when estimating the mean vectors and covariance matrices from the available training samples, which may lead to overfitting. We therefore transpose all the major chords to a single major chord no tonal center or root information, and then estimate its probability distribution.

For example, if we wish to estimate the parameters for the C major chord, we downshift the chroma vectors of C♯ major chord by 1, those of D major chord by 2, $\cdots$ and those of B major chord by 11, respectively. We now have more feature samples than we had for the original C major chord. Such a transposition method is valid because in a 12-dimensional chroma representation, only the relative spacing between pitch classes is important, not the absolute location. Similarly, we estimate the distribution parameters for 12 minor and 12 diminished chords. This simple transposition method increases the number of feature samples per class to give more accurate parameter estimates. We use this method to obtain the distribution parameter estimates shown in Figure 3.13 and in Figure 3.14.

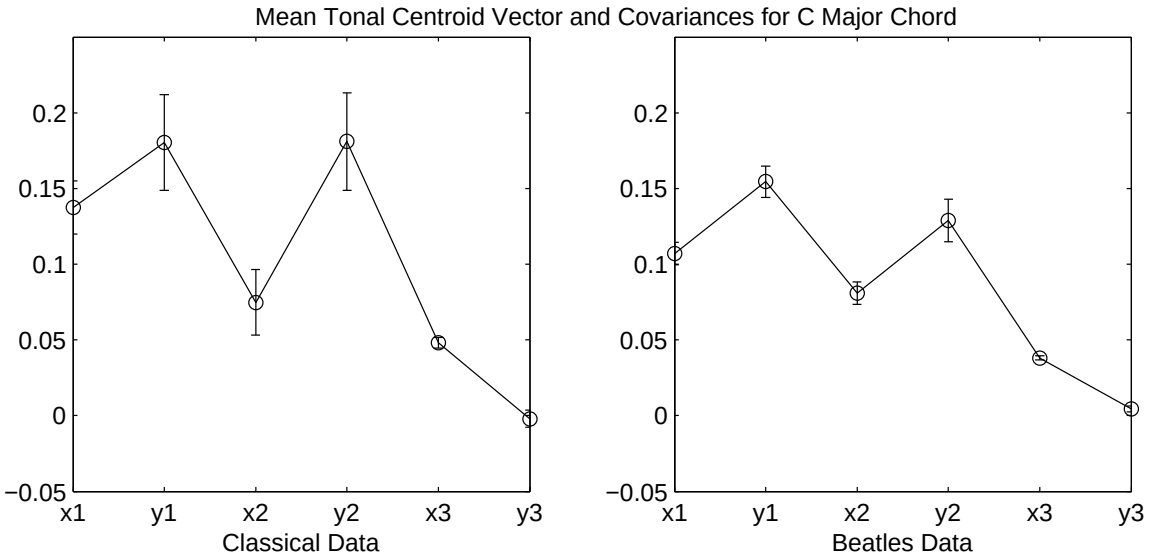


Figure 3.14: Mean tonal centroid vector and covariances for C major chord estimated from classical and from Beatles' data. Because we use diagonal covariance, the variances are shown with "error" bars on each dimension.

3.4 Experimental Results and Analysis

3.4.1 Evaluation

We test our models' performance on two types of musical audio. First, we used Bach's keyboard piece (*Prelude in C Major*) and Haydn's string quartet (*Op.3, No.5: Andante*, mm.1–46) as a test set of classical music. For these test data, the authors perform harmony analysis to obtain the ground-truth annotation. For a more complete test, we then test our models on the two whole albums of Beatles (CD1: *Please Please Me*, CD2: *Beatles For Sale*) as done by Bello and Pickens [8]. Ground-truth annotations are provided by Harte and Sandler at the Digital Music Center at the University of London in Queen Mary.[5] We reduce the class size from 36 to 24 by discarding the 12 diminished chords for the Beatles' test set since they rarely appear in rock music. In computing frame-level scores, we only count exact matches as correct recognition.

[5] `http://www.elec.qmul.ac.uk/digitalmusic/`

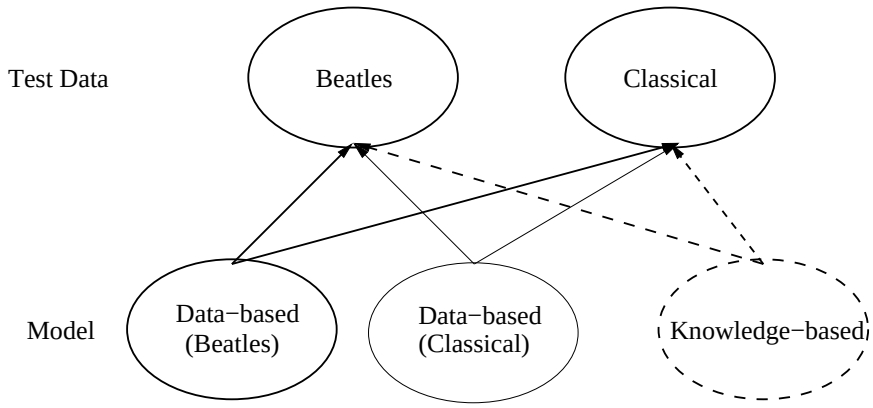


Figure 3.15: Cross-evaluation between data-based and knowledge-based model.

We measure the performance of the models in several configurations. First, because we have two separate parameter sets trained on two different training data sets (classical and Beatles), we perform tests for each parameter set to measure how each model's performance changes with training data. None of the symbolic files corresponding to the test audio is included in the training data sets. Second, we compare two feature sets—chroma feature and tonal centroid feature. Finally, we perform cross-evaluation in order to fully investigate how our data-based model performs compared with the knowledge-based model. Figure 3.15 illustrates how these models are cross-evaluated.

Finally, we also compare two different features, namely the conventional 12-D chroma feature and the 6-D tonal centroid feature.

3.4.2 Results and Discussion

Figure 3.16 shows the first 22 seconds of a 12-bin chromagram of Bach's *Prelude in C Major* as performed by Glenn Gould. Below the chromagram are the ground truth and the chord recognition results using a C major key HMM trained on classical symbolic music. As shown, chord boundaries, as well as chord names, are almost identical to those of ground truth except that our system classifies the dominant seventh chords as major chords with the same root, which are equivalent because of our definition of chord classes (see Section 3.3.2).

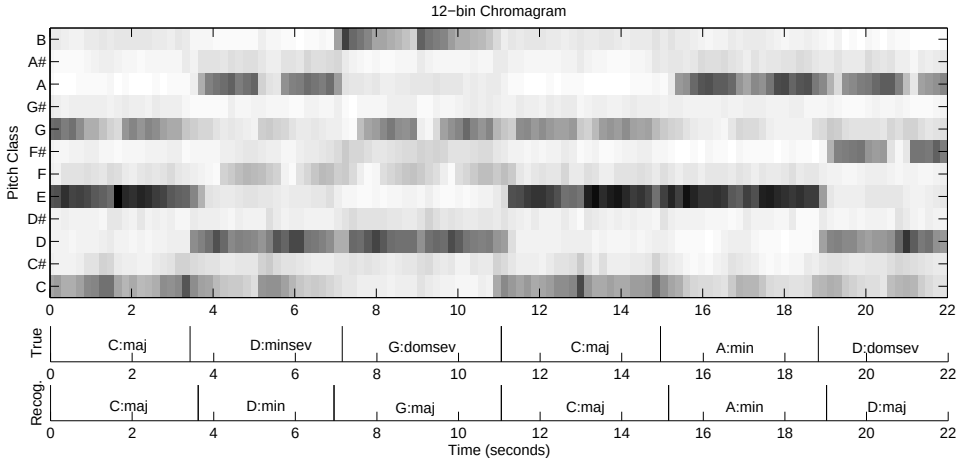


Figure 3.16: Frame-rate recognition results for Bach's Prelude in C Major performed by Glenn Gould. Below 12-bin chromagram are the ground truth and the recognition result using a C major key HMM trained on classical symbolic music.

Table 3.1 shows the frame-level accuracy in percentage for all the test data for various model parameters. In order to show that the data-based model performs better than, or comparably to, the knowledge-based model, we include the frame-rate results[6] on the same Beatles' test set by Bello and Pickens [8]. Because the results of the knowledge-based model on the classical test data are not available, however, we simulate them by combining the knowledge-based output distribution parameters with the transition probabilities learned from our training data without adaptation. We test this method on the Beatles' test data; the difference in results is less than 1% compared with the results of the original model [8], which uses fixed output distribution parameters and adapts the transition probabilities for each input. We therefore believe that the results on the classical data too are not significantly different from what would be obtained with the original knowledge-based model. The best result for each test material is in boldface. All the best results use a tonal centroid vector and the model trained on the same kind of music. This is encouraging in that the results are consistent with our expectations. If we take a closer look at the numbers, however, we find a few items worthy of further discussions.

[6]Although they show higher accuracy with beat-synchronous analysis, we present here frame-level accuracy for apple-to-apple comparison.

Table 3.1: Test results for various model parameters (% correct)

Model	Training Set	Feature	Test Set			
			Beatles		Classical	
			CD1	CD2	Bach	Haydn
Data-based	Beatles	Chroma	58.56	78.21	70.92	56.20
		Tonal Centroid	**66.89**	**83.87**	75.03	69.07
	Classical	Chroma	51.82	78.77	88.31	67.69
		Tonal Centroid	63.81	81.73	**94.56**	**71.98**
Knowledge-based	N/A	Chroma	58.96	74.78	83.40	69.37

First of all, we observe a strong dependence on the training set, especially with classical test data. This is because the model parameters, *i.e.*, observation distribution and transition characteristics are different for the two distinct musical styles. We noticed such a genre dependency in our earlier work [57]. We also find that the model trained on classical data is more robust to the change in musical genre of the test input. That is, the classical model performs equally well on both test sets while the performance of the Beatles' model drops sharply when a different style of music is used as an input. We believe this is because the model trained only on Beatles' music fails to generalize; it fails because it is only trained on music by one specific artist and the training data set is small. On the other hand, the classical model is trained on a larger training data set by more than eight composers, and thus performs equally well on both test data sets.

Second, we can see that the tonal centroid feature performs better than the chroma feature. As we mentioned earlier, a possible explanation for this is because the tonal centroid vector is obtained by projecting the 12-bin chroma vector only on specific interval relations, like fifths and major/minor thirds; thus, it is more suitable and robust for identifying musical chords since these interval relations define most chords in Western tonal music.

Our results compare favorably with other state-of-the-art systems by Harte and Sandler [36] and by Bello and Pickens [8]. Using the same Beatles' test data set, Harte and Sandler obtain frame-level accuracy of 53.9% and 70.8% for CD1 and CD2, respectively. They define 48 different triads including augmented triads, and

use a pattern matching algorithm for chord identification, followed by median filtering for smoothing. Using the HMMs with 24 states for just major/minor chords, Bello and Pickens' knowledge-based system yields the performance of 68.55% and 81.54% for CD1 and CD2, respectively, after they go through a pre-processing stage of beat detection to perform a tactus-based analysis. Without a beat-synchronous analysis, their accuracy drops down to 58.96% and 74.78% for each CD, as is shown in Table 3.1.

In computing the frame-level accuracy shown in Table 3.1, we count only exact matches as correct. However, we believe it is more accurate to measure performance with a tolerance of one frame. In other words, if a detected frame boundary or its neighbor is equal to the ground truth, we classify it as a correct match. This assumption is fair since the segment boundaries are generated by humans listening to audio, and thus they are not razor sharp. Using this error metric, the accuracy of our key-dependent tonal centroid models rises to 69.15% and 86.66% for Beatles' CD1 and CD2, respectively.

3.5 Summary

In this chapter, we have described a HMM-based chord recognition system, and we have demonstrated that symbolic music data, such as MIDI files, can be used to train machine-learning models like HMMs, with a performance that matches the best knowledge-based approach. The key idea behind our data-based approach was to use automatic generation of labeled training data to free researchers from the laborious task of manual annotation.

In order to accomplish this goal, we used symbolic data to generate label files, as well as to synthesize audio files. The rationale behind this idea was that it is far easier and more robust to perform harmony analysis on the symbolic data than on the raw audio data since symbolic music files, such as MIDI, contain noise-free pitch information. In addition, by using a sample-based synthesizer, we created audio files that have harmonically rich spectra as in real acoustic recordings. This nearly labor-free procedure to obtain labeled training data enabled us to build a generalized

model, resulting in improved performance.

As feature vectors, we first used conventional 12-bin chroma vectors, which have been successfully used by others in chord recognition. In addition, we tried another feature vector called tonal centroid which yielded better performance.

Each state in our HMM was modeled by a multi-variate, single Gaussian completely represented by its mean vector and a diagonal covariance matrix. We defined 36 classes or chord types in our models, which include for each pitch class three distinct sonorities—major, minor and diminished. We treated seventh chords as their corresponding root triads and disregarded augmented chords since they very rarely appear in Western tonal music.

In order to examine the generality of our approach, we obtained two different model parameters trained on two musically distinct data sets. Experiments with various kinds of unseen test input, all real acoustic recordings, showed that there is positive correlation between the test and the training data. In other words, the results were better when the test and training data are of the same kind. This dependency on the training set, however, was less significant when the size of the training set was larger. This in turn suggests that we can generalize our model with even larger amount of training data.

Bello and Pickens showed approximately an 8% performance increase using beat-synchronous analysis. While there is some chance for increased errors if beat-tracking is done incorrectly, we believe that this result is orthogonal to the arguments presented in this paper. Thus a state-of-the-art system for chord recognition should combine the data-driven approach described here with tonal centroid features and beat-synchronous analysis.

The one and only goal of the system described so far was chord recognition. However, musical chord is closely related to a key in tonal music. Therefore, if we know the key of a musical piece, we expect that certain chords will more likely appear than the others, or we can infer a musical key by observing the progression of chords over time. Furthermore, musical genre is also dependent on the use of chords and their progression. For instance, rock music makes extensive use of I, IV and V chords, and the most typical chord progression found in blues music is I-VI-I-V-I.

In the next chapter, we present how we can extend the chord recognition system described above to identify or provide information as to other musical attributes such as key or genre and to improve chord recognition

Chapter 4

Extension

4.1 Introduction

In this chapter, we present two extensions of the HMM-based chord recognition system described in Chapter 3 that increase chord recognition performance and provide other useful information such as key or genre. The rationale behind these extensions — key-dependent and genre-specific HMMs — is that musical chords are closely related to key or genre, and therefore a priori information as to key or genre makes chord recognition easier, especially when the feature vectors appear similar as in the related chords such as a major-minor pair sharing the root note (*C major and C minor*) or relative major-minor chords (*C major and A minor*). However, we don't need to have such prior information beforehand. In fact, by building key- or genre-specific models and computing the likelihood of each model, we can estimate the key or identify the genre of the input audio. Furthermore, we show that the selected key or genre model outperforms the generic model described in the previous chapter.

4.2 Key-dependent HMMs

4.2.1 System

In Western tonal music, a song's key and chords are closely related; thus, knowing the key of a piece provides valuable information about the chords as well. For instance, if a musical piece is in the key of C major, then we can expect frequent appearances of chords such as C major, F major and G major, which correspond to the tonic, subdominant and dominant chord, respectively. On the other hand, F♯ minor or A♭ major chord will seldom appear, since neither has any harmonic function in a C major key. Based on this close relationship between musical key and chords inherent in Western tonal music, this thesis proposes key-dependent HMMs trained on synthesized audio for chord recognition and key estimation [60, 61].

An important advantage of using symbolic music files is that other information, such as the key or the tempo, comes for free. We therefore build key-dependent models using the key information already contained in the symbolic data. We define major/minor keys for each pitch class, resulting in 24 different HMMs. After building 24 key models, $\lambda_k, 1 \leq k \leq 24$, we simultaneously perform key estimation and chord recognition of an unknown input as follows. First, given an acoustic input, we extract the observation sequence $O = \{O_1 O_2 \cdots O_T\}$ of appropriate feature. Then, we calculate the model likelihoods for all 24 key-dependent models, $P(O, Q|\lambda_k), 1 \leq k \leq 24$ [1]. We then estimate the key by selecting the key model whose likelihood is highest, *i.e.*,

$$k^* = \underset{1 \leq k \leq 24}{\operatorname{argmax}} P(O, Q|\lambda_k). \tag{4.1}$$

By using the Viterbi algorithm in Equation 4.1, however, we not only estimate the key k^*, but we also obtain the optimal state path $Q^* = \{Q_1 Q_2 \cdots Q_T\}$, which is an estimate of the frame-level chord sequence. This process is illustrated in Figure

[1] An algorithm for efficient computation of $P(O, Q|\lambda_k)$, also known as *Viterbi* decoding, is described in Appendix A.

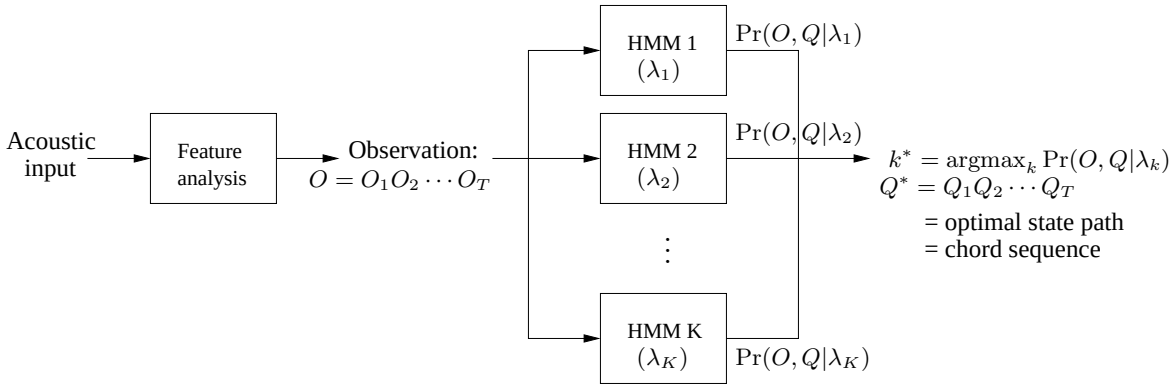


Figure 4.1: System for key estimation and chord recognition using key-dependent models ($K = 24$).

4.1.

Although similar systems for key estimation were previously proposed by others using the HMMs with chroma features as observation [81, 80], the one and only goal of these systems was key estimation; the states in the HMMs had no *musical* meanings. However, in our system, we treat the hidden states as *chords*, and therefore we also identify the chord sequence by finding the optimal state path using a Viterbi decoder when computing the likelihood of the key model.

4.2.2 Parameter Estimates

We use the same sets of training data we used in Chapter 3 (classical and the Beatles). Figure 4.2 shows the distribution of 24 keys of the whole training data set. Because we assume there is no key modulation within a piece, each musical piece is assigned a single key. Therefore, it is not impossible that there are keys without any instances or very few, if any, which might result in poor models. For example, there is only one instance of A♭ minor key, while there are 174 pieces of music in the key of C major. Such a discrepancy in the class size will result in statistical errors when estimating the model parameters.

Therefore, in the same manner we estimate observation distribution parameters for each chord, by transposing all the major[minor] chords to C major[minor] chord and transposing back to all the other major (minor) chords, we also perform circular

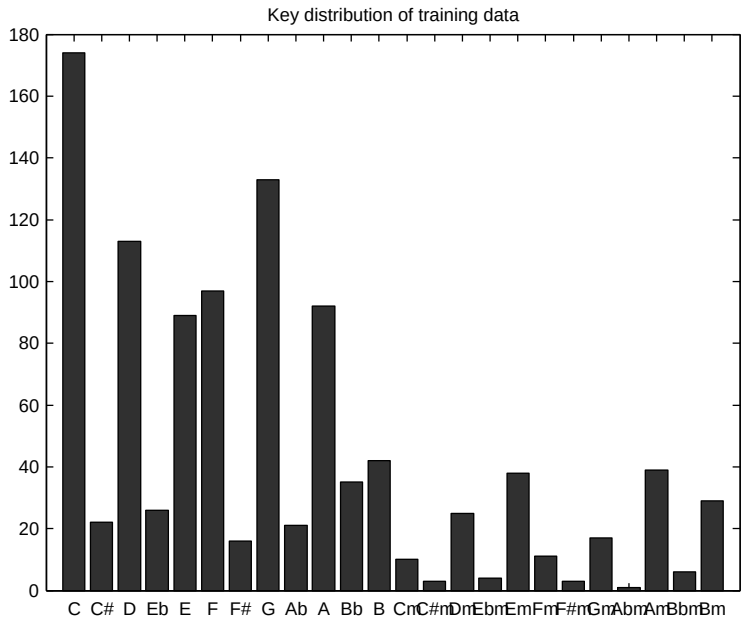


Figure 4.2: Key distribution of the training data (classical and the Beatles).

permutation for keys as well. That is, we transpose all major[minor] keys to a C major[minor] key, estimate the parameters of a C major[minor] key model, and then rotate them to obtain the parameters of other major[minor] keys. This helps us fit more accurate models because we have more data. This circular permutation approach was also used in the HMM-based key estimation algorithms by Peeters [81, 80].

Figure 4.3 contrasts the transition probabilities from a C major chord for HMMs based on C major and C minor keys for classical data. They share some general properties like the high transition probability to the same chord, and the relatively high probability to the harmonically close chords such as dominant (G) or subdominant (F) chord. The most prominent distinction is found in the higher transition probability to F minor chord in the C minor key HMM than in the C major key HMM. This is because the fourth degree (F) or the subdominant degree is defined as a minor chord in the C minor key context and therefore it is much more likely to occur than in the C major key context where the fourth degree is defined as a major chord. Similar distinctions are found in the Beatles' training data set.

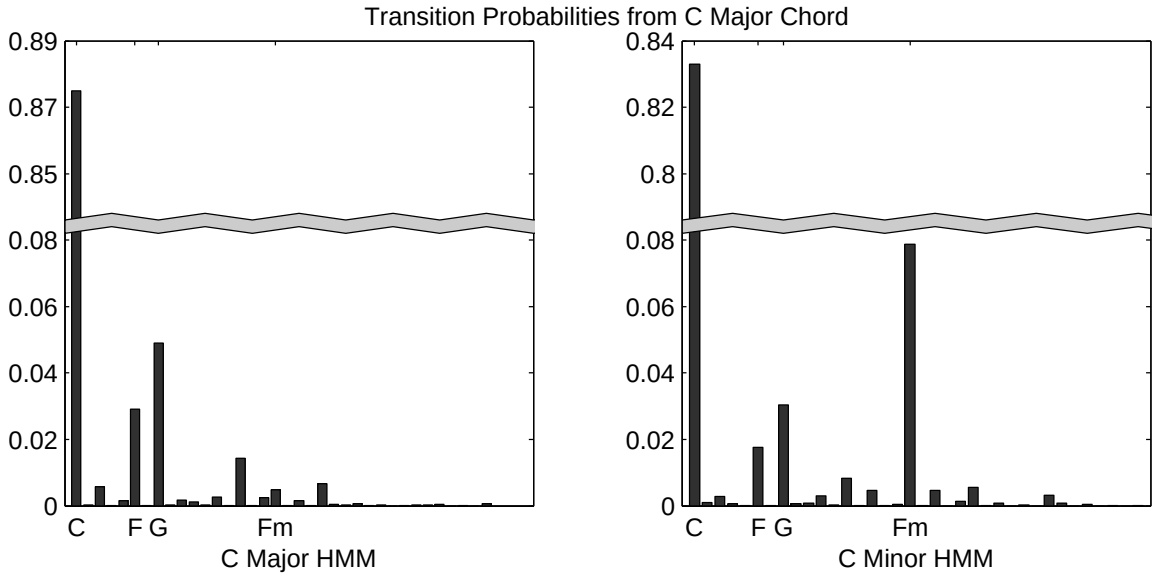


Figure 4.3: Transition probabilities from a C major chord in C major key and in C minor key HMM from classical data. The X axis is labeled in the order of major, minor and diminished chords. Compare this to the generic model shown in Figure 3.12.

Such key-specific transition probability matrices help us make a correct decision, particularly in situations where the observation feature vectors are ambiguous. For example, those chords in relations such as relative major/minor pairs share two notes in common, and thus their observation vectors look quite similar, which may cause great confusion. Discriminating the transition probabilities even from the same chord by using key-specific HMMs helps avoid mis-recognition caused by the confusion described above.

Even with key-dependent HMMs, however, we use mean feature vectors and covariance matrices that are obtained from the universal, key-independent HMM because we believe the chord quality remains the same, independent of key context. For instance, the sonic quality of a C major chord in C major key will be the same as that of a C major chord in A minor key. What differs in each key is the distribution of chords, as well as their transition probabilities.

Although changes in key within a piece of music, or modulations, are not rare

in Western tonal music, we did not take them into account in building the models because modulations occur mostly between harmonically closely-related keys such as parallel, relative major/minor keys or those in fifth relation, and therefore don't cause significant alterations in chord distribution or progression characteristics.

4.2.3 Results and Discussion

We used the same evaluation setup as we did in Chapter 3, and made comparison with the knowledge-based model. Table 4.1 shows the frame-level accuracy in percentage for the Beatles test data for various model parameters. Results in parenthesis are obtained using a key-dependent model; the best result for each test material is in boldface. Comparison between a genric and a key-dependent model is also illustrated in Figure 4.4.

Table 4.1: Test results for various model parameters (% correct). In parenthesis are accuracies of key-dependent models.

Model	Training Set	Feature	Test Set	
			Beatles	
			CD1	CD2
Data-based	Beatles	Chroma	58.56 (57.89)	78.21 (78.38)
		Tonal Centroid	66.89 (**67.50**)	83.87 (**85.24**)
	Classical	Chroma	51.82 (52.46)	78.77 (79.35)
		Tonal Centroid	63.81 (64.79)	81.73 (83.19)
Knowledge-based	N/A	Chroma	58.96	74.78

We observe the overall effect that a key-dependent model has on the performance. Except for one case (chroma feature; Beatles CD1), we find that a key-dependent model always increases performance. As mentioned in Section 4.2.1, chord progression is based on the musical key; therefore, knowing the key helps determine which chord is more likely to follow. Such an example is illustrated in Figure 4.5.

This Figure shows an excerpt of frame-level recognition results of the Beatles' song *Eight Days A Week*, which is in the key of D major. The results of the D major key model are shown with circles, and those of a key-independent model are indicated with x's. We observe that a key-independent model makes a wrong transition from

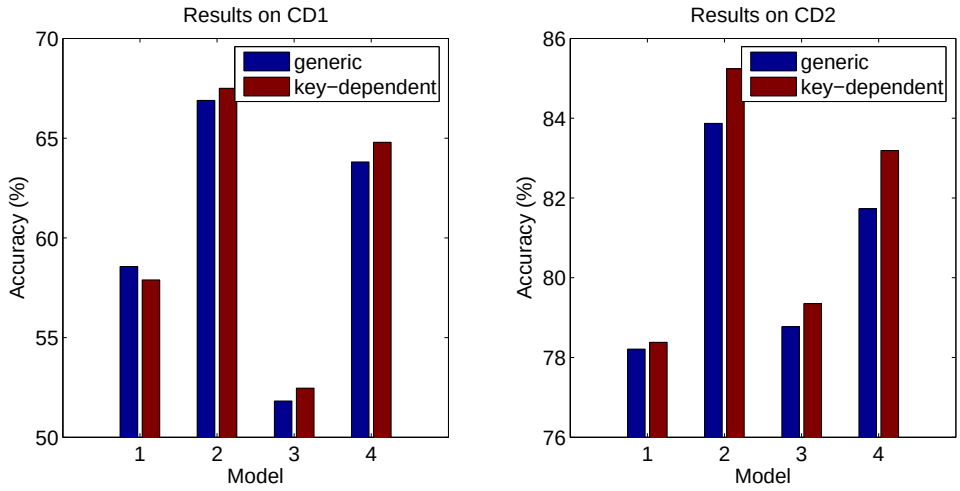


Figure 4.4: Comparison of a generic model with a key-dependent model. Model numbers on the x-axis denote: 1) chroma, trained on Beatles; 2) tonal centroid, trained on Beatles; 3) chroma, trained on classical; 4) tonal centroid, trained on classical.

G major to D minor chord near 158 seconds while the D major key model correctly switches to D major chord. As mentioned, D major and D minor chord have a parallel major/minor relation. They share two chord tones—tonic and dominant or D and A—which make the observation vector of those chords look similar, and thus causes similar output probabilities. In the D major key, however, scale degree 1 or D is a tonic center and is defined as a major chord, although it is possible to use a D minor chord. Therefore, a D major chord occurs more often in D major key than for example, in the C major or in the D minor key, resulting in higher transition probability to it than to other chords. For the same reason, since a D minor chord is rarely used in the D major key, it is less probable. This is clearly indicated in Table 4.2, which shows transition probabilities learned from the data. As shown, a transition from G major to D major chord is almost three times more likely in the key-specific model than in the key-independent model, while a G major to D minor chord transition is three times less likely.

We performed a quantitative evaluation on the key estimation algorithm. We compared our results to manually-labeled ground truth for the Beatles' test set [84].

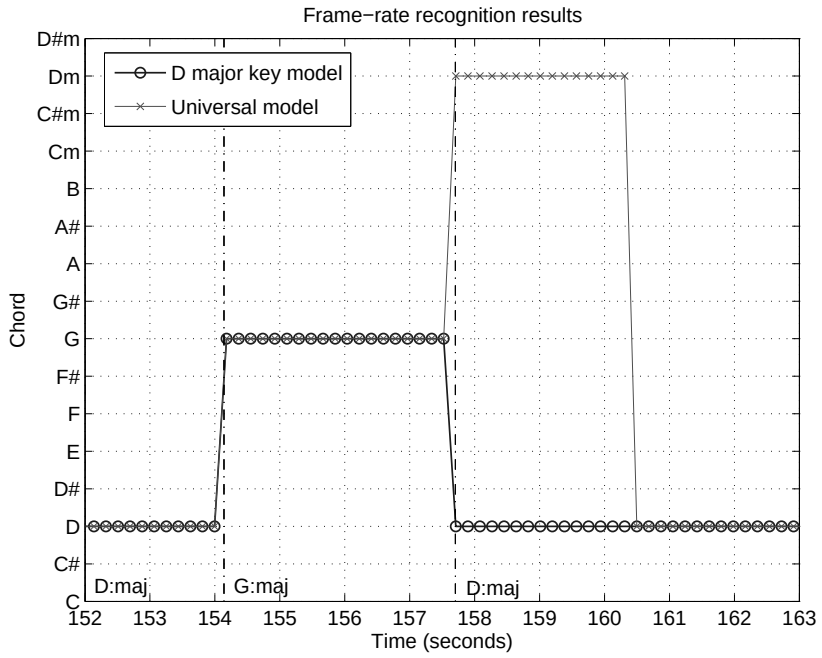


Figure 4.5: Frame-rate recognition results from Beatles' *Eight Days A Week*. In circles are the results of D major key model and in x's are those of universal, key-independent model. Ground-truth labels and boundaries are also shown.

Of all the 30 pieces (28 Beatles and 2 classical) in the test set, our system correctly estimated 29 of them, achieving an accuracy of 97%. The only song our system mis-recognized was *A Taste of Honey*, which is in the key of F♯ minor. Our algorithm recognized it as a E major key instead, which is not a related key. One possible explanation is that the extensive use of E, A and B major chords strongly implies the key of E major because those chords form the most important functions in tonal harmony, namely tonic, subdominant and dominant of E major key. The 97% accuracy for finding key from audio is very encouraging, although the test set was small.

We also tested our key estimation system on Bach's *Well Tempered Clavier I*, which consists of 24 preludes and fugues in each key. Using our algorithm, 41 out of total 48 pieces were correctly identified, corresponding to 85.42% of accuracy. Six of seven mis-recognized keys were related keys (five relative major-minor and one

Table 4.2: Transition probabilities from G major to D major and D minor chord in each model

Model	Transition Probability	
	G:maj→D:maj	G:maj→D:min
D major key (A)	0.0892	0.0018
Key-independent (B)	0.0332	0.0053
Ratio (A/B)	2.67	0.34

parallel major-minor). MIREX [2] scoring system assigns 1 point for correct keys, 0.5 for perfect fifth, 0.3 for relative keys and 0.2 for parallel keys. Using these scores, our system achieves 88.96% for key estimation. Although the test set is small to provide more complete results, these accuracies are similar to other state-of-the-art key-finding systems.[3]

4.3 Genre-Specific HMMs

4.3.1 System

When our previous systems [60] were tested with various kinds of input, the chord recognition performance was greatest when the input audio was of the same kind as the training data set, suggesting the need to build genre-specific models. This is because not only different instrumentations cause the feature vector to vary, but also the chord progression, and thus the transition probabilities, are very different from genre to genre. For example, blues music makes extensive use of I (tonic), V (dominant) and IV (subdominant) chords, and therefore the transitions between these chords are very frequent. Thus the state-transition probability matrix trained on blues music will be different from those trained on other types of music.

The overall system shown in Figure 4.6 is almost identical to key-dependent HMMs shown in Figure 4.1, except that key models are replaced with genre models. In addition, while there are 24 fixed number of key models, the number of genres may

[2]Music Information Retrieval Evaluation eXchange. `http://www.music-ir.org`

[3]The best system in MIREX 2005 Audio Key Finding achieved 89.55% accuracy using 1,252 MIDI-synthesized audio files as a test set.

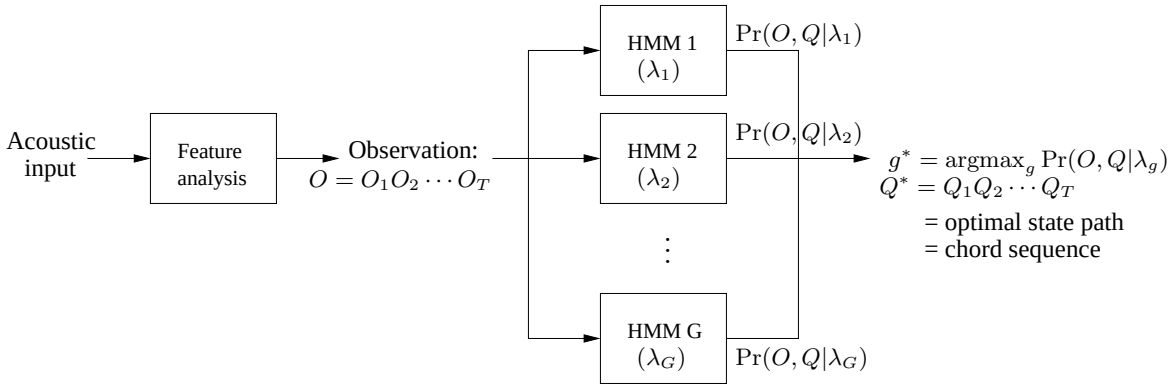


Figure 4.6: Chord recognition system using genre-specific HMMs ($G = 6$).

Table 4.3: Online sources for MIDI files

Genre	Online source
Keyboard	`http://www.classicalarchives.com`
Chamber	`http://www.classicalarchives.com`
Orchestral	`http://www.classicalarchives.com`
Rock	`http://www.mididb.com`
Jazz	`http://www.thejazzpage.de`
Blues	`http://www.davebluesybrown.com`

vary according to the training data.

4.3.2 Parameter Estimates

As has been described in Section 4.3.1, we build an HMM for each genre. While the genre information is not explicitly contained in the symbolic music files, as is the key information, most MIDI files are categorized by their genres, and we can use them to obtain different training data sets by genres. We define six musical genres including keyboard, chamber, orchestral, rock, jazz, and blues. Table 4.3 shows the online sources where we acquired the MIDI files.

The total number of MIDI files and synthesized audio files used for training is 4,212, which correspond to 348.73 hours of audio or 6,758,416 feature vector frames. Table 4.4 shows in more detail the data sets used to train each genre model.

Figure 4.7 shows the 36×36 transition probability matrices for rock, jazz, and

Table 4.4: Training data sets for each genre model

Genre	# of MIDI/Audio files	# of frames	Audio length (hours)
Keyboard	393	1,517,064	78.28
Chamber	702	1,224,209	63.17
Orchestral	319	1,528,796	78.89
Rock	1,046	1,070,752	55.25
Jazz	1,037	846,006	43.65
Blues	715	571,589	29.49
All	4,212	6,758,416	348.73

blues model after training. Although they are all strongly diagonal because the rate at which chord changes is usually longer than the frame rate, we still can observe the differences among them. For example, the blues model shows higher transition probabilities between the tonic (I) and the dominant (V) or subdominant (IV) chord than the other two models, which are the three chords almost exclusively used in blues music. This is indicated by darker off-diagonal lines 5 or 7 semitones apart from the main diagonal line. In addition, compared with the rock or blues model, we find that the jazz model reveals more frequent transitions to the diminished chords, as indicated by darker last third region, which are rarely found in rock or blues music in general.

We also see the difference in the observation distribution for each genre, as shown in Figure 4.8. Figure 4.8 displays the mean tonal centroid vectors and covariances of C major chord in the keyboard, chamber, and in the orchestral model, respectively, where the observation distribution of the chord was modeled by a single Gaussian.

We believe that using model parameters specific to each genre will increase the chord recognition accuracy when the correct genre model is selected. The success of this hypothesis will be described in Section 4.3.3.

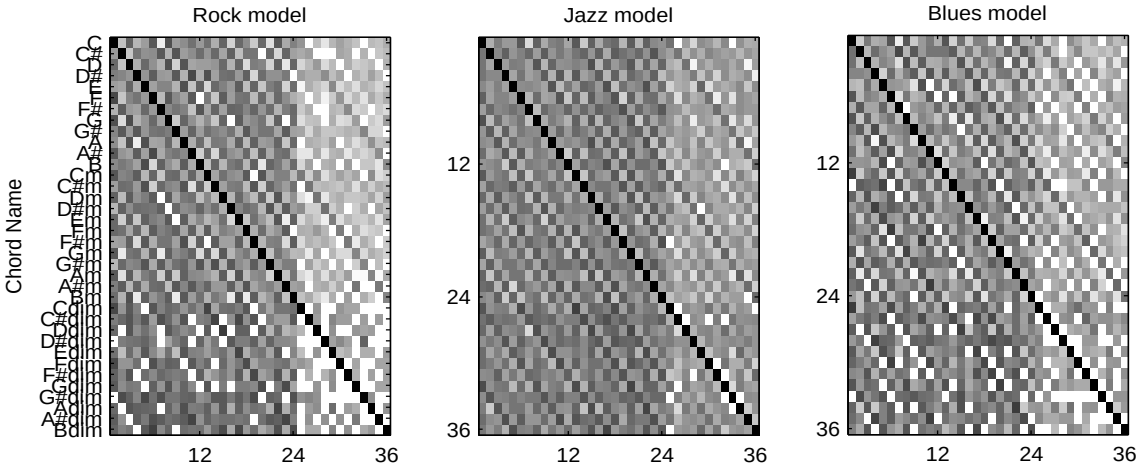


Figure 4.7: 36×36 transition probability matrices of rock (left), jazz (center), and blues (right) model. For viewing purpose, logarithm was taken of the original matrices. Axes are labeled in the order of major, minor, and diminished chords.

4.3.3 Experimental Results and Analysis

Evaluation

We tested our models' performance on the two whole albums of Beatles (CD1: *Please Please Me*, CD2: *Beatles For Sale*) as done by Bello and Pickens [8], each of which contains 14 tracks. Ground-truth annotations were provided by Harte and Sandler at the Digital Music Center at University of London in Queen Mary.[4]

In computing scores, we only counted exact matches as correct recognition. We tolerated the errors at the chord boundaries by having a time margin of one frame, which corresponds approximately to 0.19 second. This assumption is fair since the segment boundaries were generated by human by listening to audio, which cannot be razor sharp.

To examine the dependency of the test input on genres, we first compared the each genre model's performance on the same input material. In addition to 6 genre models described in Table 4.4, we built a universal model without genre dependency where all the data were used for training. This universal, genre-independent model

[4] `http://www.elec.qmul.ac.uk/digitalmusic/`

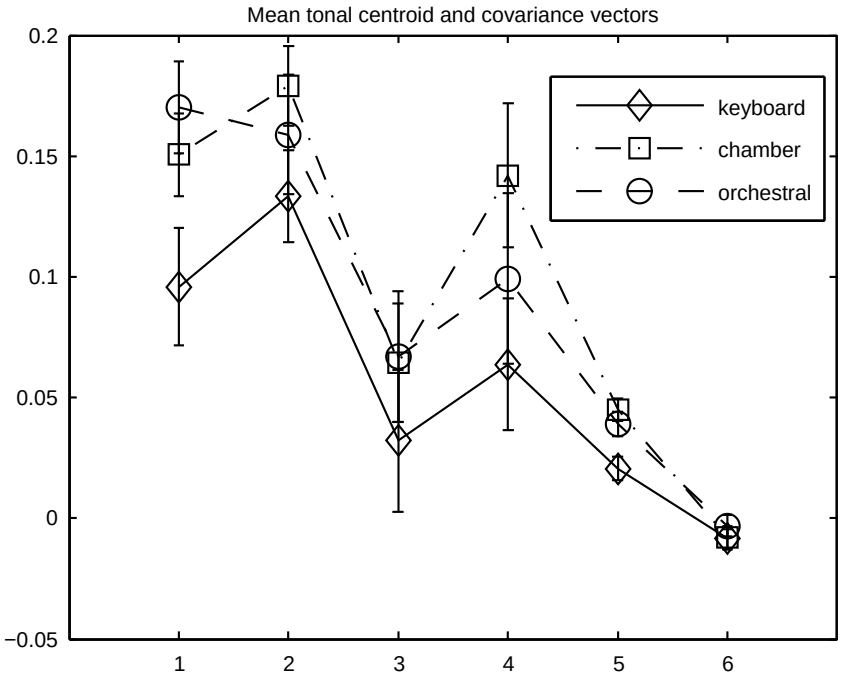


Figure 4.8: Mean tonal centroid vectors and variances of C major chord in keyboard, chamber, and orchestral model.

was to investigate the model's performance when no prior genre information of the test input is given.

Results and Discussion

Table 4.5 shows the frame-rate accuracy in percentage for each genre model when tested using the Beatles music. A single Gaussian was used to model the output probability distribution for each chord. The best results are shown in boldface.

From the results shown in Table 4.5, we notice a few things worthy of further discussion. First of all, the performance on the Beatles music of the classical models — keyboard, chamber, and orchestral model — is much worse than that of other models. Second, the performance of the rock model came in 2nd out of all 7 models, following the blues model. This is not surprising because even though the test material is generally classified as rock music, rock music has its root in blues music. Particularly, early rock music like The Beatles was greatly influenced by blues music. This supports

Table 4.5: Test results for each model with major/minor/diminished chords (36 states, % accuracy)

Model	Beatles CD1	Beatles CD2	Total
Keyboard	38.674	73.106	55.890
Chamber	30.557	73.382	51.970
Orchestral	18.193	57.109	37.651
Rock	45.937	77.294	61.616
Jazz	43.523	76.220	59.872
Blues	**48.483**	**79.598**	**64.041**
All	24.837	68.804	46.821

Table 4.6: Test results for each model with major/minor chords only (24 states, % accuracy)

Model	Beatles CD1	Beatles CD2	Total
Keyboard	43.945	73.414	58.680
Chamber	43.094	79.593	61.344
Orchestral	37.238	77.133	57.186
Rock	**60.041**	**84.289**	**72.165**
Jazz	44.324	76.107	60.216
Blues	52.244	80.042	66.143
All	51.443	80.013	65.728

our hypothesis that the corresponding or similar genre model will increase the chord recognition accuracy.

Knowing that the test material does not contain any diminished chords, we did another experiment with the class size reduced down to just 24 major/minor chords instead of full 36 chord types. The results are shown in Table 4.6.

With fewer chord types, we can observe that the recognition accuracy increased by as much as 20% for some model. Furthermore, the rock model outperformed all other models, again verifying our hypothesis on genre-dependency. This in turn suggests that if the type of the input audio is given, we can adjust the class size of the corresponding model to increase the accuracy. For example, we may use 36-state HMMs for classical or jazz music where diminished chords are frequently used, but use only 24 major/minor chord classes in case of rock or blues music, which rarely uses diminished chords.

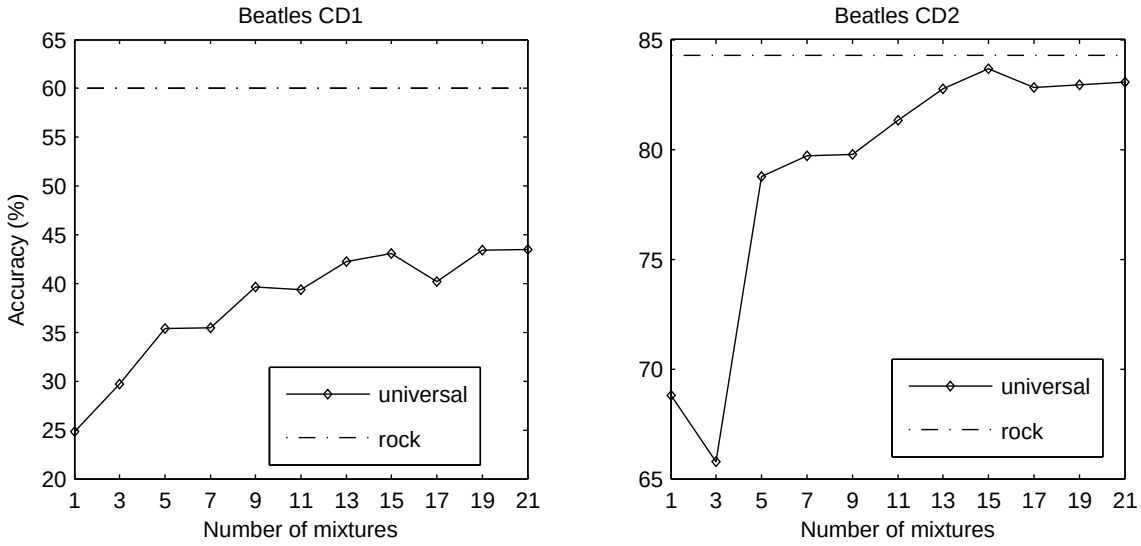


Figure 4.9: Chord recognition performance of a 36-state universal model with the number of mixtures as a variable (solid) overlaid with a 24-state rock model with one mixture (dash-dot).

Finally, we investigated the universal, genre-independent model in further detail to see the effect of the model complexity. This is because in practical situations, the genre information of the input is unknown, and thus there is no choice but to use a universal model. Although the results shown in Table 4.5 and Table 4.6 indicate a general, genre-independent model performs worse than a genre-specific model of the same kind as the input, we can build a richer model for potential increase in performance since we have much more data. Figure 4.9 illustrates the performance of a universal model as the number of Gaussian mixture increases.

As shown in Figure 4.9, the performance increases as the model gets more complex and richer. To compare the performance of a complex, genre-independent 36-state HMM with that of a simple, genre-specific 24-state HMM, we overlay the performance of a 24-state rock model with only a single mixture. Although increasing the number of mixtures also increases the recognition rate, it fails to reach the rate of a rock model with just one mixture. This comparison is not fair in that a rock model has only 24 states compared with 36 states in a universal model, resulting in less errors, particularly because not a single diminished chord is included in the test material.

As stated above, however, given no prior information regarding the kind of input audio, we can't take the risk of using a 24-state HMM with only major/minor chords because the input may be classical or jazz music in which diminished chords appear quite often.

The above statements therefore suggest that genre identification on the input audio must be done in advance in order to be able to use genre-specific HMMs for better performance. It turns out, however, that we don't need any other sophisticated genre classification algorithms or different feature vectors like MFCC, which is almost exclusively used for genre classification. Given the observation sequence from the input, when there are several competing models, we can select the correct model by choosing one with the maximum likelihood using a forward algorithm or Viterbi decoder. It is exactly the same algorithm as one used in isolated word recognition systems described by Rabiner [86]. Once the model is selected, we can apply the Viterbi decoder to find the most probable state path, which is identical to the most probable chord sequence. Using this method, our system successfully identified 24 tracks as rock music out of total 28 tracks, which is 85.71% accuracy. What is noticeable and interesting is that the other four mis-classified songs are all classified as blues music in which rock music is known to have its root. In fact, they all are very blues-like music, and some are even categorized as "bluesy". This suggests that perhaps rock and blues should be merged into one genre — *blues and rock*.

4.4 Summary

In this chapter, we have demonstrated that we can extend the basic chord recognition system using an HMM described in Chapter 3 to build key- or genre-specific HMMs and train them with synthesized audio. Easily generating a large amount of labeled training data from symbolic music files allows us to build such richer models.

Based on the close relationship between key and chord in Western tonal music, we built 24 key-dependent HMMs, one for each key, using key information derived from the symbolic music files. Given an acoustic input, our system performs feature analysis and a sequence of observation is input to the key-dependent HMMs. Using

a Viterbi decoder, we estimate the key by selecting the model with the maximum likelihood; at the same time, we recognize frame-level chord sequence because it is the same as the optimal state path in a selected key model. Experimental results on real acoustic recordings show that a key-dependent model not only gives the key information of an input, but also increases the chord recognition accuracy, yielding very promising results on both musical tasks.

Although genre information is not explicitly encoded in symbolic music files as key is, it is not difficult to find symbolic music files categorized by genres. We therefore build genre-specific HMMs, similar to key-dependent HMMs, resulting in improved performance with much simpler models than a more complex, genre-independent model.

Using synthesized audio to train the models successfully capture genre-specific musical characteristics seen in real acoustic recordings, and experimental results show that the performance is best when the model and the input are of the same kind, which supports our hypothesis on the need for building genre-specific models. Another great advantage of the present approach is that we can also predict the genre — even though the identification rate is not that of other state-of-the-art systems whose only purpose is genre identification — of the input audio by computing the likelihoods of different genre models as is done in isolated word recognizers. This way, we not only extract chord sequence but also identify musical genre at the same time, without using any other algorithms or feature vectors. This is important because we integrate two distinct musical tasks—*i.e.* chord recognition and genre classification—into one framework.

In the next chapter, we introduce an enhanced model for chord recognition — *discriminative HMM*.

Chapter 5

Advanced Model

5.1 Introduction

In this chapter, we present an advanced approach that combines a powerful discriminative algorithm for chord classification with the HMM smoothing for finding the optimal state path. This model is not finalized yet and we haven't had enough experiments to provide complete quantitative evaluations. However, we believe this model is based on more advanced statistical learning algorithms and therefore will outperform the basic models.

5.2 Discriminative HMM

In our HMM described in Chapter 3, we modeled each state (*i.e., chord*) using a multivariate Gaussian, which is a *generative* approach; that is, we try to build each chord model first, based on the labeled training data. Then we compute the posterior probabilities that the observation is generated by each model, given the input observation, using Bayes' rule [90]. Mathematically, we compute for each chord model

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)}, \tag{5.1}$$

where x is an input observation and y is a class or chord. In a generative model, we learn $p(x|y)$ and $p(y)$ from the data. $p(y)$ is called *class priors* and is given by

$$p(y) = \sum_{y} p(x|y). \tag{5.2}$$

In fact, we don't even need to know the marginal likelihood $p(x)$ to compute the posterior $p(y|x)$ since from Equation 5.1, we have

$$\begin{aligned} \underset{y}{\operatorname{argmax}}\, p(y|x) &= \underset{y}{\operatorname{argmax}}\, \frac{p(x|y)p(y)}{p(x)} \\ &= \underset{y}{\operatorname{argmax}}\, p(x|y)p(y). \end{aligned} \tag{5.3}$$

A generative model such as a Gaussian model is based on assumptions about how the data is generated. For example, if the distribution of the data is indeed Gaussian, the Gaussian model is asymptotically efficient, especially for a large amount of training data [72].

On the other hand, in a *discriminative* model no such assumptions are made and therefore it is more robust and less sensitive to incorrect modeling assumptions. Instead of computing the posterior distributions $p(y|x)$ from the model likelihood $p(x|y)$ and the prior $p(y)$ as in a generative model, the discriminative model tries to learn the boundary that discriminates $p(y|x)$ directly from the training data.

Support vector machines (SVMs) are among the most popular discriminative models and have recently been used with a great success in a number of applications, including music applications such as genre classification [5], chord recognition [69], mood classification [53] and instrument identification [44] and so on.

A hybrid model has been used in speech recognition to combine the discriminative power of SVMs with the ability of HMMs to efficiently represent the temporal dynamics of speech [47, 48]. However, the output of an SVM is a distance measure between the observation and the optimal decision boundary, which is not suitable for an HMM which requires posterior probabilities. Platt proposed an algorithm to

estimate these emission probabilities from the SVM output $f(x)$ using Gaussians of equal variance and then computing the probability of the class given the output by using Bayes' rule, yielding [83]

$$\begin{aligned} p(y=1|x) &= g(f(x), A, B) \\ &\equiv \frac{1}{1+\exp(Af(x)+B)}. \end{aligned} \tag{5.4}$$

Platt also proposed an algorithm to estimate the parameters A, B in Equation 5.4 using maximum likelihood estimation. Interested readers are encouraged to see [83].

Finally, since SVMs are *binary* classifiers, we need to find a method to classify more than two classes as in our chord recognition (we have 36 classes). There are two widely used approaches for multi-class SVMs: one-vs-rest approach and one-vs-one approach [38, 91]. Although the one-vs-one approach requires $N(N-1)/2$ number of SVMs for N-class classification (*e.g.,* 630 SVMs for our application), it takes much less time to train them all since the time to train one SVM depends super-linearly on the number of training samples.

Using the hybrid model described above, Krüger *et al.* achieved better results on phoneme recognition, even with less training samples than the HMM with Gaussian mixture models, which was trained on the full training set [47, 48].

We believe that this hybrid SVM/HMM approach, which has the best of both worlds, will help increase our chord recognition performance because recognizing the sequence of chords also does benefit from temporal modeling represented by an HMM through state-transition probabilities, and yet the powerful discriminative algorithms like SVMs are likely to help compute the posterior probabilities more accurately.

5.2.1 Experiments and Discussions

We performed an initial experiment on the discriminative HMMs for chord recognition. We used the same classical data for training and the Beatles' two albums for testing, which were also used in Chapter 3. The average % accuracy was 57.68% for

the CD1 (*Please Please Me*) and 76.72% for the CD2 (*Beatles For Sale*). These numbers are low compared to those by the basic models, which are 63.81% and 81.73% for each album, respectively.

However, we used only 1000 training samples for each class (chord) due to the extensive amount of time it takes to train 630 SVMs (we used one-versus-one approach for multi-class classification as explained in Section 5.2). This diluted training data set is just 7% of the full training set used in the original HMMs with Gaussian modeling. We hope that the performance of the discriminative HMMs increases as we increase the size of the training data.

One interesting observation worthy of further discussion is that the classification without HMM smoothing is better with the SVMs than with the Gaussian models; that is, the SVMs outperform the Gaussian models in classifying the given observations to correct classes (chords), using only the posterior probabilities. However, when we integrate the posterior distribution into an HMM framework with the state-transition probabilities and compute the optimal state path using a Viterbi decoder, the HMM with the Gaussian models yields the better results. We haven't fully investigated this yet, but a possible explanation is that the posterior probabilities estimated from the distance measures from the SVMs aren't accurate. We plan to try different estimation algorithms in the future.

For more complete and fair comparison, we built the Gaussian models using only 7% of the full training set as we did with the SVMs, and performed the experiments on the same test set. Table 5.1 shows the results.

Table 5.1: Comparison of SVMs and Gaussian models trained on the same amount of training data (% accuracy)

Model	Test data		
	CD1	CD2	Average
SVMs (diluted, 1/15)	57.68	76.72	**67.20**
Gaussian models (diluted, 1/15)	46.24	78.63	62.44
Gaussian models (full set)	63.81	81.73	72.77

As is shown in Table 5.1, SVMs outperform the Gaussian models by 5% under the same condition; *i.e.,* when the amount of training data to train both models is

the same. The sharp drop in accuracy (from 72.77% to 62.44%) with the Gaussian models trained on fewer data indicates the weakness of Gaussian model when the data is insufficient because in generative models, the less data we have, the more difficult it becomes to correctly estimate the model parameters.

5.3 Summary

In this chapter, we presented a hybrid model for chord recognition based on the original HMM. The hybrid model uses powerful discriminative algorithms to compute the posterior probabilities that the given observation vector belongs to each chord. To this end, we use SVMs which have proved to be very successful in many other classification problems, including several music applications. In order to estimate the emission probability distribution required in the HMM smoothing stage, we use a one-versus-one approach, which requires $36 \times (36 - 1)/2 = 630$ support vector machines to be trained. The experiments show promising results: when we train the SVMs and the Gaussian models with the same amount of diluted training data (approximately 1/15 of the full data), the SVMs outperform the Gaussian models by 5%.

In the following chapter, we demonstrate several practical applications where the key and the chord sequence recognized by the system are directly or indirectly used.

Chapter 6

Applications

6.1 Introduction

There are a number of practical applications where content-based music information retrieval becomes very useful. These applications include audio coversong finding, structural analysis, music segmentation/clustering, audio thumbnailing, query-by-humming (QBH), query-by-example (QBE), music recommendation, playlist generation, to name a few.

In this chapter, we demonstrate that we can use a sequence of chords, which is extracted using the models described in Chapter 3 and Chapter 4, in some of the applications stated above.

6.2 Audio Cover Song Identification

A cover song is defined as a song performed by an artist different from the original artist.[1] Identifying cover songs given an original as a seed/query or finding the original given a cover version from the raw audio is a challenging task, and it has recently drawn attention in the Music Information Retrieval community. A cover song is different from its original in terms of many musical attributes such as tempo, dynamics, melody, key, timbre, and/or instrumentation, and thus the raw waveform

[1] http://www.secondhandsongs.com/wiki/Guidelines/Cover

or its frequency-domain representation like spectrogram is very different from each other. Therefore, identifying cover songs requires a robust feature set that remains largely unchanged under various acoustical changes caused by various musical properties mentioned above.

Harmonic progression or chord sequence is a robust mid-level representation that is largely preserved under such musical variations. While other musical details such as melody, tempo, and/or timbre may vary from one to another, their harmonic progression over time undergoes minor changes compared with the others because it was built by conforming to fundamental rules found in Western tonal music. Using harmonic progression or chord sequence as a feature set has more advantages: first, once the chord sequence is extracted from the raw audio, it is just a uni-dimensional string, and thus is computationally very efficient to process. Second, in computing the distance between two chord sequences, we can also define the transition cost from a chord to another as well as the cost of being in aligned states since, for example, a transition from a G major chord to a C major chord is much more probable than a transition from a G major to a A♯ minor in Western tonal harmony. Defining such transition cost would not be possible or meaningful unless we use explicit chord names. Experiments show that adding a transition cost to an alignment cost improves performance of the proposed system.

In the next section, we review other related works on cover song finding.

6.2.1 Related Work

Pickens and Crawford proposed a system for polyphonic music retrieval using the harmonic description[82]. It was also based on the assumption that the underlying harmonies remain similar in variations on a piece of music. They chose 24 major/minor lexical chords, but used the distribution of chords rather than selecting a single chord to describe harmonic content. They then used this harmonic description to build a Markov model, and computed the Kullback-Lieber (KL) measure to create a ranked list between a query and the documents in a search collection. Even though they used the symbolic data to evaluate their system, their approach was unique in that

they used several important methods in combination such as harmonic description, statistical Markov modeling, and transposition invariant modeling.

Gomez and Herrera used tonal descriptors to identify cover versions from the audio [33]. They chose the Harmonic Pitch Class Profile (HPCP) as the feature set, which is similar to the chromagram. From the tonal descriptors, they obtained two types of similarity measures. First was a global similarity obtained by the correlation coefficient between two averaged HPCP vectors. The other was the instantaneous tonal similarity represented by diagonals in the similarity matrix from tonal descriptors. They also used the dynamic time warping (DTW) algorithm to compute the second measure. Using a transposed version of the HPCP and the DTW algorithm, they could obtain the accuracy of 55% at a recall level of 30%.

Ellis proposed a cover song identification system using a beat-synchronous chroma feature as a front end [25]. He first detects the beat of an input audio signal, and then averages the chroma frames within a beat to create a beat-synchronous chromagram. The main argument for using a beat-based representation is that variations in tempo in different cover versions are normalized, in addition to computational efficiencies. Given a beat-synchronous, normalized chromagram pair, he then performs cross-correlation between the two matrices to find the matches. Using a small set of test songs (15 cover song pairs), his best system successfully identified 10 out of 15 tracks.

Our system is similar to the aforementioned systems in that harmonic content is used as the feature set and the DTW algorithm is applied to compute the distance between a pair of songs. However, instead of using the raw PCP vectors, we extract from them more efficient and musically meaningful chord sequence, and use them as a front end to the DTW algorithm to compute the minimum alignment cost between a pair of chord sequences. This not only decreases the computation time by far but also allows the inclusion of a transition cost in computing the similarity matrix required by the DTW algorithm. Adding the transition cost turns out to increase the performance of the system.

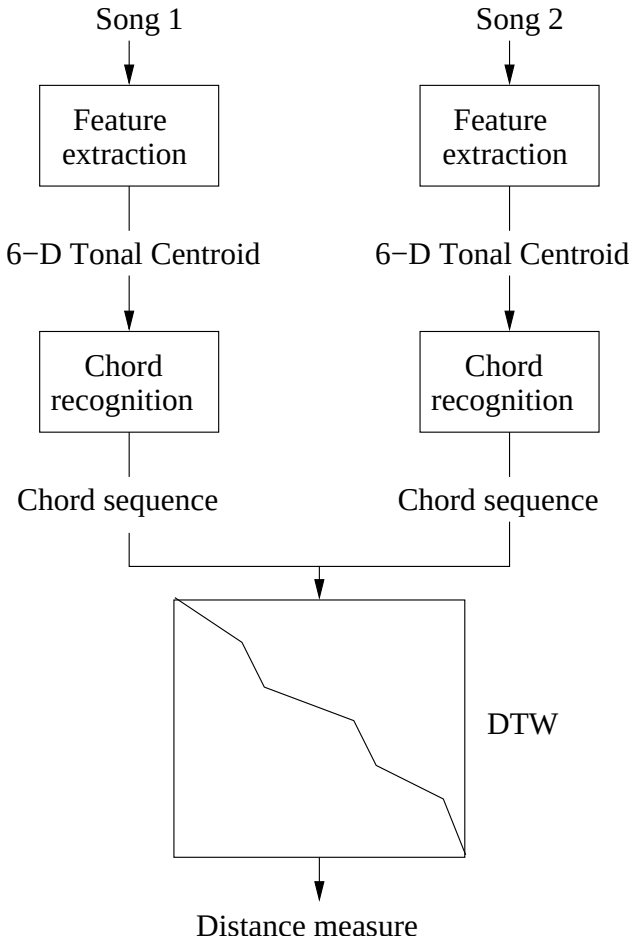


Figure 6.1: Overview of the coversong identification system.

6.2.2 Proposed System

Our system is composed of two main stages. First, we recognize musical chords at the frame rate using hidden Markov models (HMMs), as explained in Chapter 3 and Chapter 4. We then take the chord sequences as a front end to the distance computing algorithm, where we use the dynamic time warping (DTW) algorithm to find the minimum alignment cost between the two chord sequences. Figure 6.1 illustrates the system architecture.

In order to use the DTW algorithm, we first need to obtain two-dimensional cost matrix from the two inputs. We defined a cost of being in aligned states by computing

the chord-to-chord distance from the HMM parameters obtained above. In addition, we also defined a transition cost from the transition probability matrix in the HMM. Therefore, a combined cost at time step k for input a_k and b_k is given by

$$d(a_k, b_k) = d_S(a_k, b_k) + d_T(a_k, b_k | a_{k-1}, b_{k-1}), \tag{6.1}$$

where d_S is a cost of being in aligned states, and d_T is a transition cost from the previous states to the current states. The total cost of alignment between the two sequences a and b is then given by

$$D(a, b) = \sum_{k=1}^{K} d(a_k, b_k). \tag{6.2}$$

Transposition from one key to another is not rare in cover songs, and it may cause a serious problem in computing the distance because chord-to-chord distance becomes larger even though relative chord progression between the two sequences might be alike. To avoid this problem, key identification must precede the distance computation. Instead of designing a sophisticated key identification algorithm, we simply estimated the key of a song to be the most frequent chord in the chord sequence, and transposed every song to a C major chord before sending it to a distance computing algorithm.

6.2.3 Experimental Results

Test material

In order to evaluate the proposed system, our test material consisted of 19 cover songs and five originals. Five original songs were *Come Together* by John Lennon, *Knockin' on Heaven's door* by Bob Dylan, *Can't Explain* by The Who, *Eight Days a Week* by The Beatles, and *Live and Let Die* by Wings. Three to five cover songs were included for each original and we could obtain only cover songs for the last set. Cover songs vary from their original in tempo, duration, performance style (live or

Table 6.1: Test Material for Coversong Identification

Title	Original	Cover
Come Together	John Lennon	Aerosmith Tina Turner Tribute Band
Knockin' on Heaven's Door	Bob Dylan	Eric Clapton Guns 'N' Roses
Can't Explain	The Who	The Who (live) David Bowie Scorpions The Flammin Groovies
Eight Days a Week	The Beatles	The Premier String Quartet TheTribute (live)
Live and Let Die	Wings[a]	Guns'N' Roses Mig Ayesa (live) Starlite Orchestra

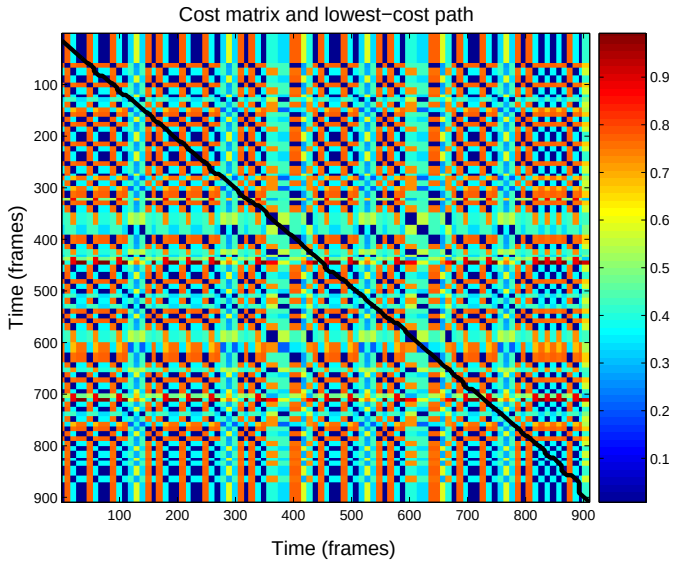
[a]Doesn't exist in the test collection because we couldn't obtain the original.

studio recording), and/or instrumentation. Table 6.1 shows the test material in more detail.

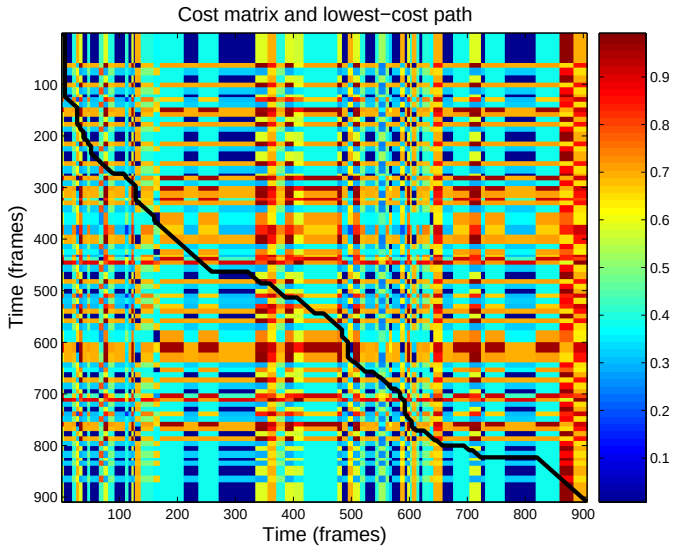
In addition to the original and cover songs, we added 19 non-cover songs to the search collection to verify the robustness of the algorithm in the presence of non-related, noisy items. Figure 6.2 displays the examples of the DTW for a cover-pair (on the left) and for a non-cover pair (on the right). As shown, the similarity matrix of a cover-pair is very symmetric and the lowest-cost path is almost diagonal, resulting in low alignment cost. On the other hand, the cost matrix of a non-cover pair appears very irregular and the minimum alignment path is also ziggy-zaggy, and therefore alignment cost is very high. We normalized the minimum alignment cost by dividing it by the sum length of the pair to compensate for the longer cover songs.

Evaluation

We evaluated the performance of our algorithm using a Cranfield-style recall-precision test. Given a query/seed, its cover songs are marked as relevant items, and all the



(a) Cover pair



(b) Non-cover pair

Figure 6.2: Similarity matrix and minimum alignment cost of (a) cover pair and (b) non-cover pair.

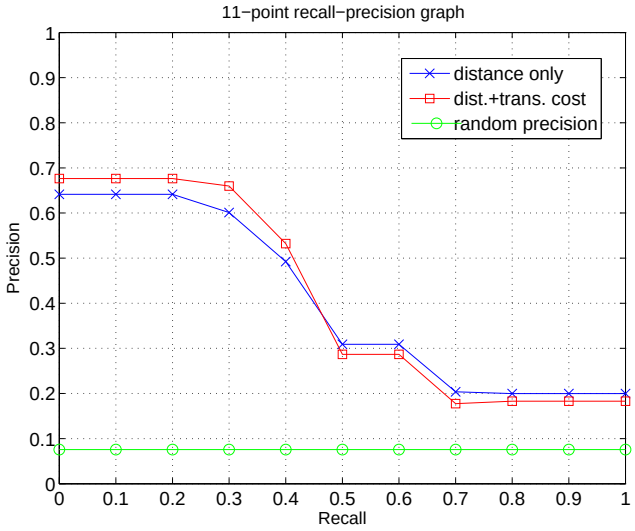


Figure 6.3: Averaged recall-precision graph.

others non-relevant. We then computed the distances between the query and all the other songs, and a ranked list was generated for this query. We included the original song in the query set as well since users may want to find its cover songs using the original as a seed just as they want to find the original using its covers. The query is then inserted into a search collection, and another relevant song is used as a query to generate another ranked list. We repeated this process for the remaining relevant items or cover songs, and the whole process is then repeated for all other query sets. We performed a recall-precision test on each ranked list for each query, and obtained an averaged 11-point interpolated recall-precision graph for 19 queries. This is shown in Figure 6.3.

Results

In Figure 6.3, two different results were displayed over the baseline (random precision). First of all, they both are way above the baseline, which is just the probability of randomly choosing a relevant item to a query. Secondly, the solid line with squares is the result obtained using a transition cost as well as a cost of being in aligned states (chord-to-chord distance) when computing the distance between the two chord

Table 6.2: Summary results of eight algorithms.

Measure	Total number of covers identified	Mean number of covers identified	Mean of maxima	MRR of first correctly identified cover
1	D.P.W. Ellis (761)	D.P.W. Ellis (2.31)	D.P.W. Ellis (4.53)	D.P.W. Ellis (0.49)
2	**K. Lee [1] (365)**	**K. Lee [1] (1.11)**	**K. Lee [1] (2.50)**	**K. Lee [1] (0.22)**
3	**K. Lee [2] (314)**	**K. Lee [2] (0.95)**	**K. Lee [2] (2.27)**	**K. Lee [2] (0.22)**
4	Sailer & Dressler (211)	Sailer & Dressler (0.64)	Sailer & Dressler (2.13)	Sailer & Dressler (0.21)
5	Lidy & Rauber (149)	Lidy & Rauber (0.45)	Lidy & Rauber (1.57)	Lidy & Rauber (0.12)
6	K. West [1] (117)	K. West [1] (0.35)	T. Pohle (1.50)	K. West [1] (0.10)
7	T. Pohle (116)	T. Pohle (0.35)	K. West [1] (1.30)	K. West [1] (0.10)
8	K. West [2] (102)	K. West [2] (0.31)	K. West [2] (1.23)	T. Pohle (0.09)

sequences whereas a dashed line with X's is the result of using just the cost of being in aligned states. As can be seen, the precision rate is 5 to 7% higher in the case of using both costs up to a recall rate of 0.4.

This is because even if two different chords have the same distance from a reference chord, if transition costs from the previous chords are different, then the chord sequence with more similar harmonic progression will have the shorter distance, and therefore will appear higher in a ranked list than the other.

For more complete and objective evaluation, we submitted our algorithm to Audio Cover Song Identification competition in MIREX 2006.[2] In the competition, test data consisted of 30 queries with each query having 11 different cover versions including themselves, resulting in $30 \times 11 = 330$ songs. In addition, 670 non-relevant songs were added to a collection to make the task more challenging. The collection includes a wide range of music from classical to hard rock.

Eight algorithms including these two by the author were evaluated. Four measures were used to evaluate the performance of the algorithms — (1) total number of cover songs identified; (2) mean number of cover songs identified; (3) mean of maxima; and (4) Mean reciprocal rank (MRR) of first correctly identified cover. Table 6.2 shows the results using these measures.

As shown in Table 6.2, the two algorithms described in this paper are ranked at 2nd and 3rd places, respectively, using all four measures. Raw results reveal that some

[2] `http://www.music-ir.org/mirex2006/index.php/Audio_Cover_Song`

songs are difficult to identify for all systems while other songs are system-specific. In addition, the top four algorithms were specifically designed only for cover song identification whereas the bottom four were originally used in the similarity finding task as well. This proves that the two tasks are quite different from each other.

However, although we used our system only to recognize the cover songs, we believe it can also be used to find musical similarity since cover songs are extreme examples of similar music. Therefore, those items having small distance to a query by the coversong identification system, even if they are not the cover versions, might be evaluated similar to the query by human subjects, especially harmonic content is one of key criteria when evaluating musical similarity.

6.3 Structural Analysis

As mentioned in Section 1.2.2, in Western tonal music, once we know harmonic content — musical key and chord progression — of a piece over time, it is possible to perform higher-level structural analysis from which we can define themes, phrases or forms. Although other musical properties like melody or rhythm are relevant, harmonic progression has the closest relationships with the musical structure. Finding structural boundaries automatically, or so-called *structural music segmentation*, can be very useful when, for example, shopping for items in large archives as in online music stores or navigating through a large collection of musical audio. It saves a great amount of time by allowing the users to skip to the next section without having to listen to the whole song from the beginning to the end.

Music segmentation is directly related to two other applications: *music clustering* and *music summarization*. Music clustering is referred to as grouping similar segments into a *cluster*. Then we can *summarize* a musical piece by finding the most repeating cluster, which in general is the most representative part and thus is likely to be remembered. Music summarization is also called *music thumbnailing*, where thumbnailing is a term used in the visual domain to describe a process to create a scaled-down image from the larger original, while preserving the overall content. For example, most popular music in general, although it may vary in detail, has a

structure as follows:

`Intro--Verse1--Chorus--Verse2--Chorus--Bridge--Chorus--Outro`

In such a case, the goal of music segmentation is to find segmental boundaries between the two contiguous segments. In the above example, there are eight segments in total. Clustering is used to group similar segments. If we don't distinguish the lyrics in verses, `Verse1` and `Verse2` are grouped as the same cluster, and so are the three `Chorus` segments. Therefore, we have five distinct clusters. Finally, we can summarize the above example by finding the most repetitive segment, which in this case is `Chorus`. Or we can add `Verse1` or `Verse2` as well to generate a bit longer thumbnail like `Verse1`–`Chorus`.

In the following section, we review several systems for structural analysis of tonal music in more detail.

6.3.1 Related Work

Analyzing the musical structure automatically from audio has recently attracted a number of researchers both from the academy and from the industries.

Foote proposed a system for automatic music segmentation using a measure of audio novelty based on the *self-similarity matrix* [27]. In his work, Foote first parameterized the raw audio using standard spectral analysis, and computed a 2-dimensional self-similarity matrix S, where an element $S(i, j)$ represents a similarity measure between the parameterized feature frame f_i and f_j. He used the cosine distance as a similarity measure and thus the matrix S is symmetric. He computed the distance between all pairs of frames. Figure 6.4 shows an example of a self-similarity matrix.

By visualizing a self-similarity matrix, he argues that not only local performance details are visible, as displayed as squares along the diagonal, but also the global structure such as repetitions can be found as off-diagonal lines. He then performed a kernel correlation on the similarity matrix S to compute a 1-dimensional novelty score, where peaks indicate significant changes in music and therefore denote potential segment boundaries. He later proposed other algorithms based on the similarity

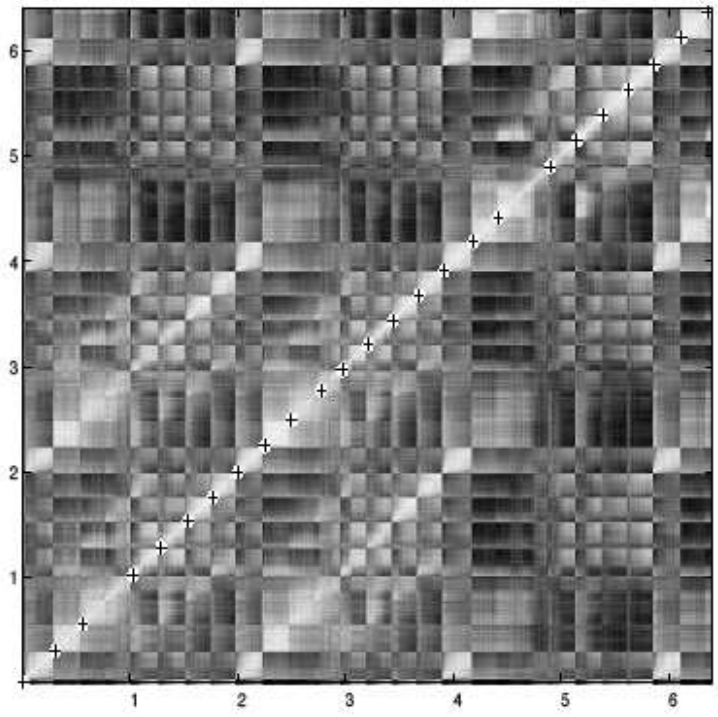


Figure 6.4: Self-similarity matrix of the first seconds from Bach's *Prelude No. 1 in C major* performed by Gould (from Foote [28]).

matrix to automatically find musical structures using MFCCs [28]. Foote and Cooper also used a self-similarity matrix to cluster the segments and then summarize music [29, 19, 20].

Although their work made significant contributions to visualizing the musical structure and to finding the segments, it is entirely dependent on the distance between individual low-level feature frames and thus fails to provide the relationships between the musical structure and high-level musical attributes such as melody or harmony, which is highly correlated with the structure.

Aucouturier and Sandler used a three-state HMM for music segmentation, where each state corresponds to {silence}, {voice + accordion + accompaniment} and {accordion + accompaniment} [1]. They trained the model on the sequence of feature vectors using the classic Baum-Welch algorithm to estimate the states' output distributions and state-transition probabilities. They then used Viterbi decoding to

find the most probable state path, which also unveils the musical structure.

Even though they use a statistical model to learn from the actual data the structure of music, it is based on the assumption that the "texture" or "polyphonic timbre" represented by low-level features of one segment are different from those of other segments, which may not be always true. For example, in music with very homogeneous instrumentation, such as piano solo, it is not likely that each section reveals distinct texture. Furthermore, they also assumed the fixed number of states or clusters, which is unknown.

Levy and Sandler addressed the problems of such unsupervised learning model and proposed a semi-supervised approach to segmentation [63]. They argue a simple division of a piece into two regions by a user — therefore called *semi*-supervising — helps initialize the states of the HMMs.

Goodwin and Laroche proposed a dynamic programming (DP) approach to audio segmentation and speech/music discrimination [35]. To this end, they first performed linear discriminant analysis (LDA) to condition feature space in a way that the classes — speech and music classes in their application — are maximally separable from each other, and apply dynamic programming to identify data clusters. The strength in their approach is that no prior information as to the number of clusters or states is required: DP finds it automatically as it computes the minimum cost path in feature space as shown in Figure 6.5.

All the aforementioned systems for structural analysis of music, whether they use a novelty measure between feature frames or a machine learning model to find the statistical properties within the clusters, depend on the (dis)similarity of low-level features, which barely have anything to do with more musically meaningful properties.

In the next section, we present a novel approach to finding structural boundaries in tonal music based on the sequence of chords and their time boundaries.

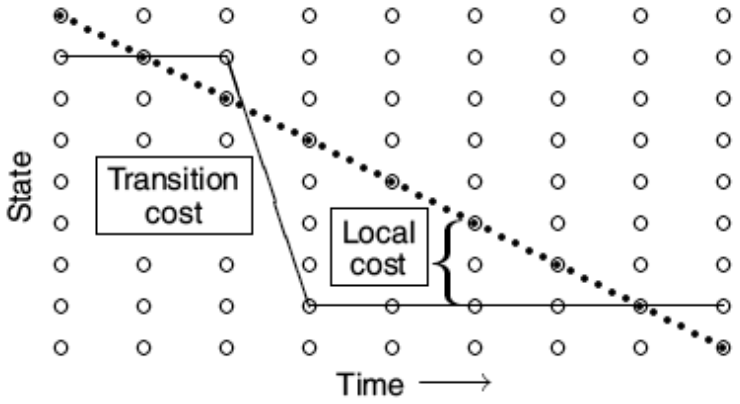


Figure 6.5: In dynamic programming, clustering in feature-space is done by finding the optimal state-path based on a transition cost and a local cost. The dotted diagonal is the nominal feature path. A candidate cluster path is shown in solid line (from Goodwin and Laroche [35]).

6.3.2 Proposed System

In tonal music, the tonality of a piece is very closely related with the *cadence*. Grove Music Online[3] defines cadence as follows:

> *Cadence: The conclusion to a phrase, movement or piece based on a recognizable melodic formula, harmonic progression or dissonance resolution; the formula on which such a conclusion is based. The cadence is the most effective way of establishing or affirming the tonality of an entire work or the smallest section thereof; it may be said to contain the essence of the melodic (including rhythmic) and harmonic movement, hence of the musical language, that characterizes the style to which it belongs.*

As is defined above, if we can identify the cadences in tonal music, we can also recognize the phrase or section boundaries since a cadence concludes a musical segment such as a phrase or movement. Furthermore, among many musical attributes, analyzing *harmonic progression* is the most definitive and effective way of identifying such cadences in tonal music. Therefore, we propose a system for finding structural

[3] `http://www.grovemusic.com`

boundaries in tonal music by recognizing the cadences from the key and chord progression. Figure 6.6 shows the overall procedure of the proposed system using the HMMs described in Chapter 3 and in Chapter 4. Each step is explained in more detail as follows:

1) Feature extraction: Tonal centroid feature is extracted from audio as described in Section 3.2.3.

2) Key & Chord recognition: Using key-dependent HMMs described in Section 4.2, key and the chord sequence with time boundaries are estimated from the feature vectors.

3) Harmonic-Beat detection: Most dominant length of the chord duration is computed by performing an autocorrelation analysis on the chord sequence. Because we are not interested in how but when the chord changes, we convert the chord sequence to an impulse train and the autocorrelation analysis is done on this impulse train. This process is explained in more detail in Section 6.3.3. Computation of harmonic beat is important in computing the tonal tension curve because it is impossible to define the length of cadential formulas without knowing the duration.

4) Tonal tension curve generation: Based on the harmonic-beat and key information, one or more cadential formulas are first created. We can use, for example, only a perfect authentic cadence (V-I), or a combination of a perfect authentic cadence and a half cadence such as IV-V. Once we have defined a cadential formula, we slide it through the estimated chord sequence and compute the distance between the cadential formula and the chord sequence at the frame level. For instance, assume that the estimated key was C major. Then if the harmonic beat was 10-frames long and if we used a V-I cadence, then we have a 20-frame long cadential formula whose first 10 frames are in G major chord and the last 10 frames are in C major chord. Therefore, if there are phrases that end with a V-I cadence, we'll have very low distance at those locations, indicating the potential phrase boundaries.

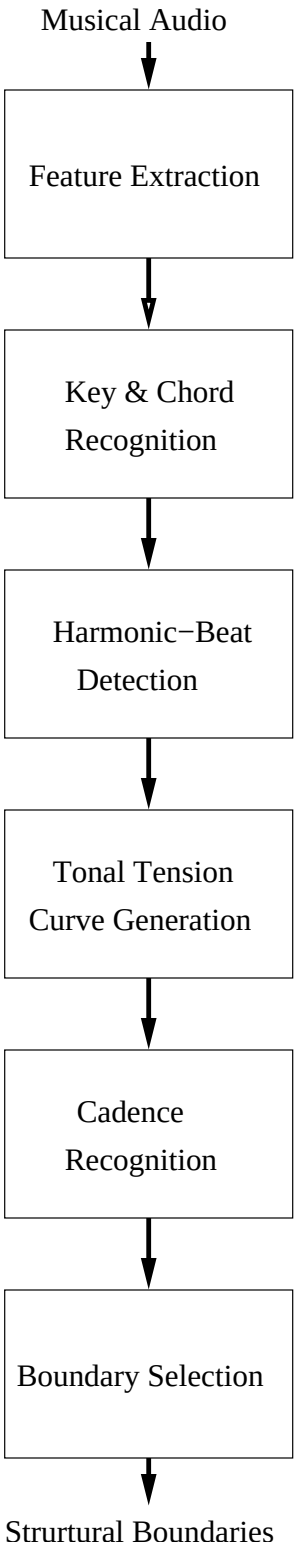


Figure 6.6: The overview of structural segmentation system

5) Cadence recognition: Once we compute the tonal tension curve, we can recognize potential cadences by finding the local minima in the curve. We can do this either by setting some threshold value heuristically or by finding the local valleys significantly lower than the neighboring points.

6) Boundary selection: Since step 5 can give false cadences as well, we need a pruning technique to select only true cadences. We prune such false cadences by finding the most dominant segment length as we did to detect the harmonic beat in step 3. This pruning is based on the assumption that most tonal music, particularly most pop music, has a fairly regular structure, and therefore most segments or phrases in a piece of music have a length that are related to the length of such a dominant, fundamental segment. Therefore, once we figure out the length of the most dominant segment, we can get rid of false boundaries using a simple correlation analysis.

6.3.3 Experimental Results

Segmentation

Figure 6.7 shows the frame-level chord recognition results obtained using key-dependent HMMs. The model correctly estimated the key, which is the key of C major.

The frame-level recognition accuracy was about 88.06%. We can observe in Figure 6.7 that the chord changes at a very regular rate most of the times. We can find the length of the most dominant chord duration, or *harmonic beat*, using a simple autocorrelation analysis. In doing so, we first generate an impulse train that has ones whenever chord changes and zeros otherwise. This is because we are not interested in how but *when* the chord changes. Then we compute the autocorrelation of this impulse train, which is shown in Figure 6.8.

We can see in Figure 6.8 that the peaks can be found at about every 11 frames. We can choose the first significant peak as the fundamental period of the harmonic beat. The next step is to generate a cadential formula. Among several cadential formulas, we use the perfect cadence or V-I movement since it is the most frequently used cadence in tonal music. In addition, we also use the half-cadence in IV-V motion

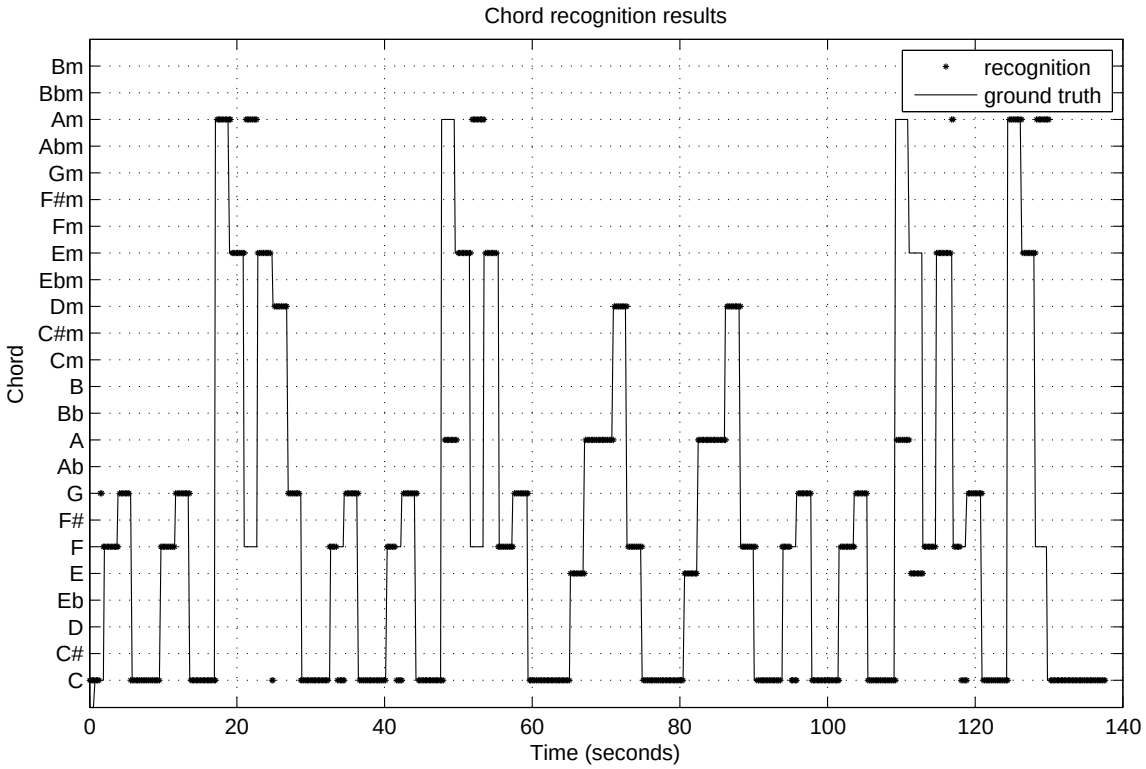


Figure 6.7: Frame-level chord recognition results (*The Beatles' "No Reply"*).

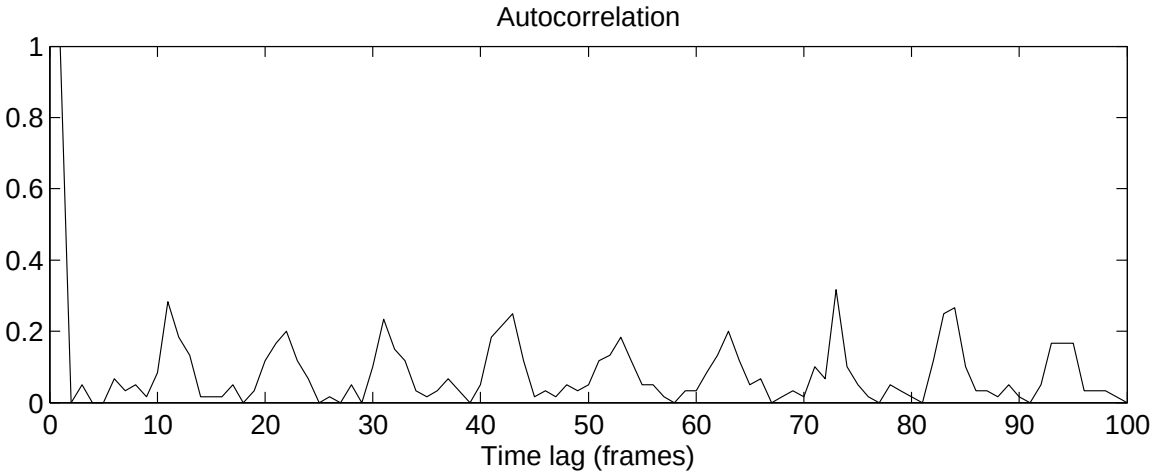


Figure 6.8: Normalized autocorrelation of an impulse train obtained from the chord sequence shown in Figure 6.7.

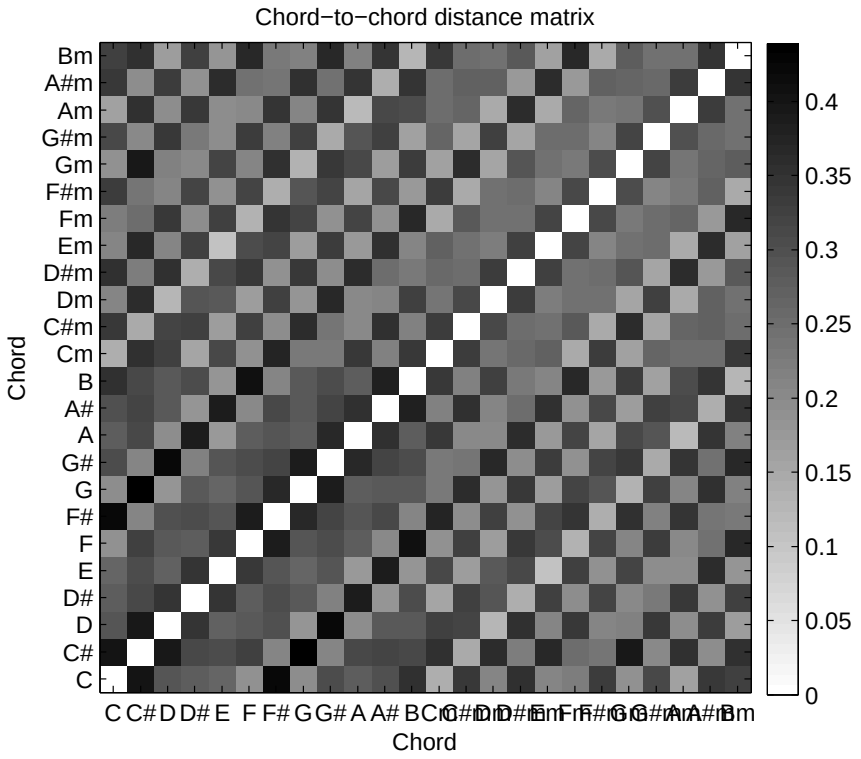


Figure 6.9: 24×24 distance matrix between 12 major/minor chords in tonal centroid space. Related chords are indicated by off-diagonals in lighter colors (*e.g.* C-F, C-G, C-Cm, C-Am).

as an additional cadential formula since it is also frequently used, especially in rock music.

We then compute the Euclidean distance between the pre-defined cadential formula and the chord sequence, for each cadential formula, and multiply them to obtain a combined tonal tension curve. In computing the distance, we use a lookup table of 24×24 matrix which contains every possible chord-to-chord distance for 24 chords–12 major and 12 minor chords. This table is pre-computed using the observation distribution parameters learned from the data. Figure 6.9 shows a 24×24 chord distance matrix.

Using such a matrix to compute the tonal tension curve has several advantages. First of all, it is meaningless to use the chord names in computing the distance

because the distance in the chord name space does not necessarily correspond to the distance between the chords in a perceptual or musical sense. We may use a circle of fifths to put related chords to close to each other, but computing the distance would still be completely heuristic. However, using the observation probability distribution of a feature vector such as tonal centroid, which is learned from the data, we can appropriately represent the chords in the feature space, and therefore can estimate the more meaningful distance between the chords.

A second advantage of this approach is that related chords such as tonic-dominant or relative major-minor are close to each other in the feature space, resulting in small distance, as shown in Figure 6.9 by off-diagonal lines in lighter colors. This is important because even when the chord recognition is incorrect, it is highly likely that the mis-recognized chord is related and thus the actual distance would still remain small. This adds more robustness to the segmentation algorithm.

Lastly, since all possible chord-to-chord distance is pre-computed, it is computationally very efficient using a table lookup method.

Using the distance matrix described above, we compute the tonal tension curve by sliding the cadential formula through the chord sequence, computing the distance at the frame rate. When we encounter a chord transition same as or similar to one defined in cadential formulas, we have very low tension or dissonance resolution, indicating a potential phrase boundary. Figure 6.10 shows the tonal tension curve overlaid with the chord recognition results.

From Figure 6.10, we observe several notches where tonal tension is minimal, indicating the potential cadences or boundaries. As mentioned above, however, it is possible that some of them are false cadences, and therefore we need to select only true ones by pruning. We prune false cadences by finding the most significant segment length, in a manner similar to how we find the harmonic beat. Once we find the most dominant segment length, we generate a periodic impulse train whose fundamental period is equal to the dominant segment length. We then slide this impulse train through the tonal tension curve and compute the inner product. The final structural boundaries are defined where the inner product is the smallest, namely the total tonal tension is minimal. Estimated boundaries shown in Figure 6.11 are close to

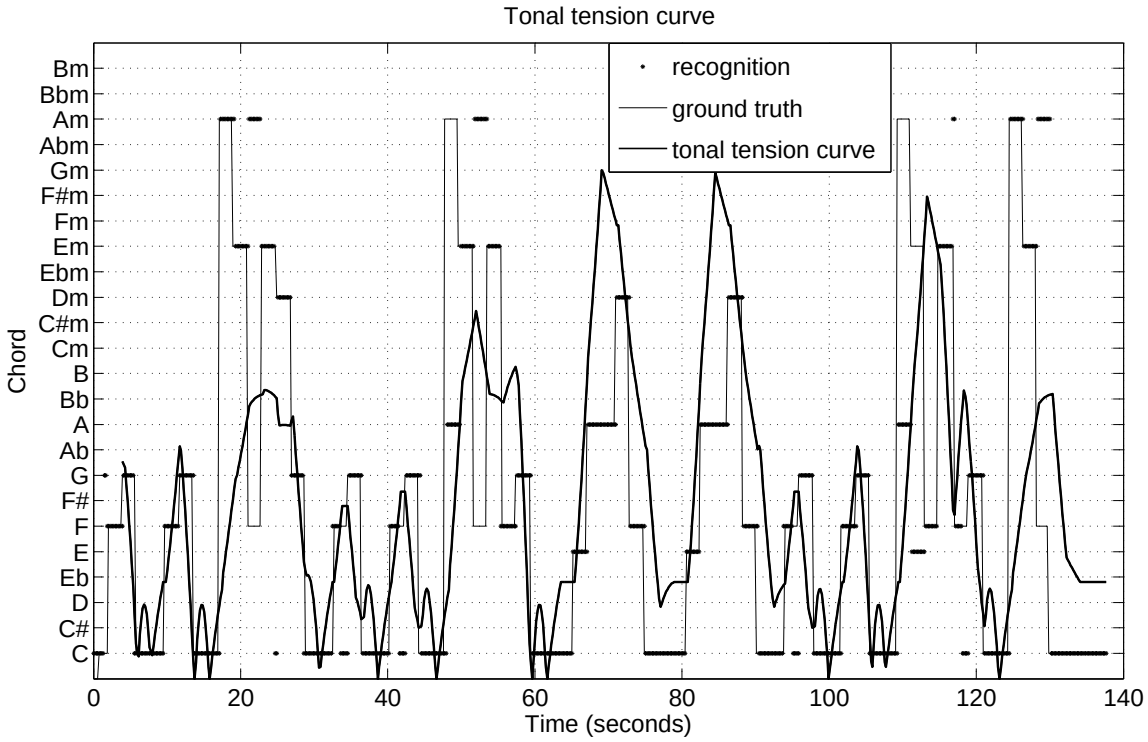


Figure 6.10: Tonal tension curve overlaid with the chord sequence (*The Beatles' "No Reply"*).

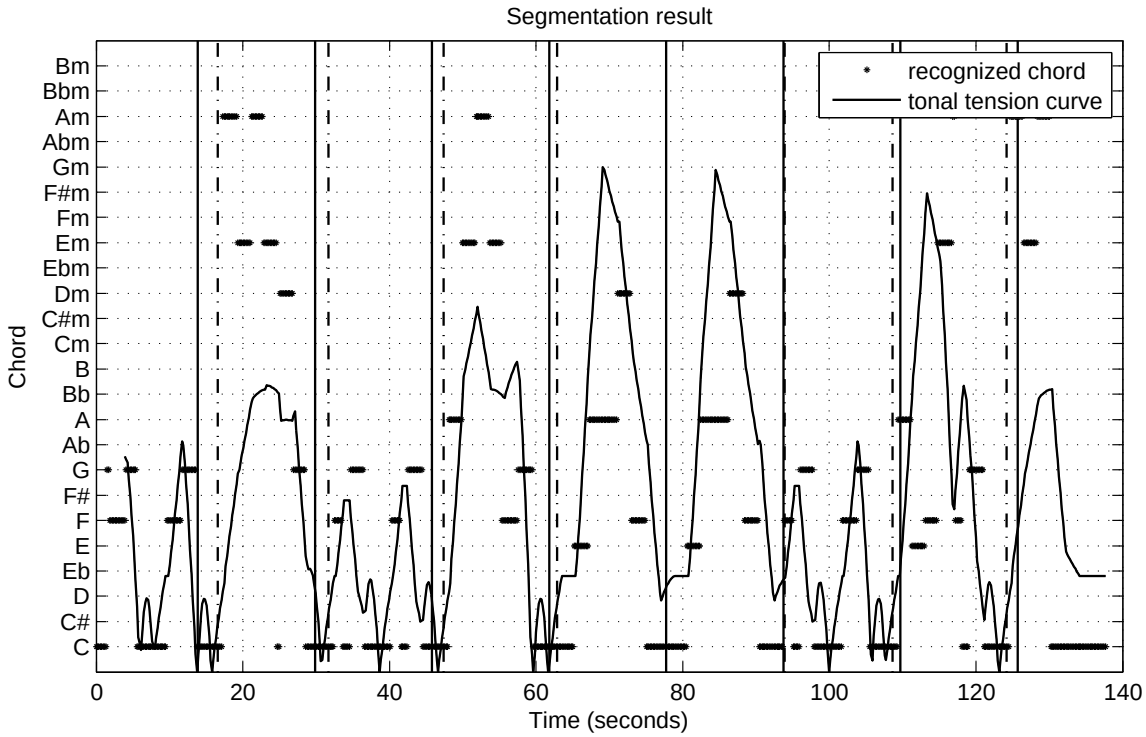


Figure 6.11: Final segmentation result. Estimated boundaries are shown in vertical solid lines and true boundaries in dashed-dot lines (*The Beatles' "No Reply"*).

true boundaries, except that there is another estimated boundary around 80 seconds in the middle of the bridge section. We can eliminate this in the clustering stage, as will be described shortly.

Clustering

Once we have found the structural boundaries, we can group the similar segments into a *cluster* by computing the distance between a pair of segments for all possible pairs. For example, we have nine segments in total in the above example. Since they are of the same length, we can directly compute the frame-level distance between a segment pair in the feature using the chord sequence. Figure 6.12 shows a 9×9 distance matrix of all segment pairs.

As is shown in Figure 6.12, the similar segments have smaller distance to each

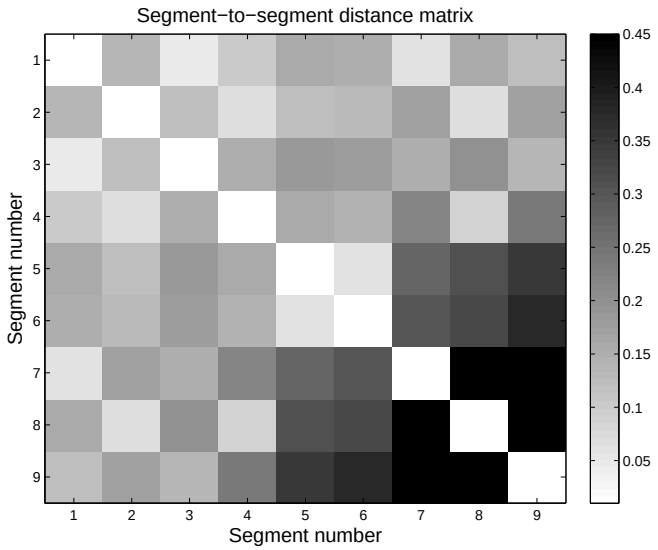


Figure 6.12: Distance matrix between segment pairs (*The Beatles' "No Reply"*).

other and appear as lighter color in the distance matrix. We then normalize the matrix and use a threshold value to determine if a segment pair is close enough to be grouped as a cluster. The final clustering result is as follows: *Cluster A:* {*1,3,7*}*; Cluster B:* {*2,4,8*}*; Cluster C:* {*5,6*}*; Cluster D:* {*9*}. If there are contiguous segments in the same cluster, such as segments 5 and 6 in the example, it implies a continuously repeating segment and thus we can group them into one segment. This will remove an extra boundary in Figure 6.11 and will result in segmentation closer to true segmentation,

Music Summarization

As has been described earlier, the most representative part in an entire piece of music is often the most frequently repeating part, such as a chorus section. Therefore, once we have found out the clustering information, we can easily *summarize* music simply by finding the cluster with the largest number of segments, because the segments in a cluster are similar to each other, or *repetitive*. So, in the above example, we can choose either a segment in Cluster A or one in Cluster B, or both since the number of repetitions is the same. Figure 6.13 illustrates the overall process of structural

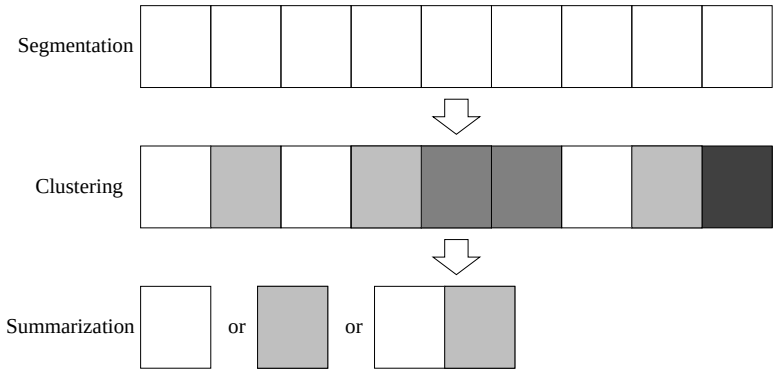


Figure 6.13: Structural analysis: segmentation, clustering and summarization.

analysis.

6.4 Miscellaneous Applications

The applications described in the previous sections — cover song identification and music segmentation/clustering/summarization — use the chord sequences as a front end to the systems to perform these tasks. However, a sequence of chords and corresponding boundaries by themselves, although not 100% correct, provide invaluable information to perform harmonic analysis of music. In addition, with the combination of musical key, which is estimated using key-dependent models as explained in Chapter 4, one can derive functional harmony in which each chord has a specific function with regard to its tonal center such as tonic, dominant or subdominant and so on. This will help many non-musicians or students without any formal music education understand the harmonic structure of music.

Chords can also be used in melody extraction to reduce the search space when estimating the pitch. This is because melody and chord are very closely related in tonal music, and therefore a melody tone is usually one of the chord tones, assuming enharmonic and octave equivalence. For example, if we know, at a certain time, that the current chord is a C major triad, then we can have more confidence in pitch classes such as C, E or G. Such melody-chord relationships can also be learned from the data to build a statistical model.

In applications like music recommendation or playlist generation, a mood is one of important criteria to find similarity or categorize in a collection of music. Mood is a complex state of mind and in music, several attributes such as melody, key, tempo or timbre contribute to defining the mood. We can infer simple relations like (fast tempo, major key $\leftrightarrow$ happy) or (slow tempo, minor key $\leftrightarrow$ sad), for instance. We believe harmonic progression also contributes significantly to building the mood in music. For example, even when a piece is in a major key, a frequent usage of minor chords can make it sound sad. The similar argument holds for a piece with a number of major chords but in a minor key. Therefore, knowing the harmonic content and its progression over time will help classify the mood in tonal music. Mood-specific HMMs, where each HMM is trained on a specific mood as in key-dependent or genre-specific HMMs, showed promising results in the Audio Music Mood Classification competition in MIREX 2007.[4]

Audio mixing is another interesting and practical application where a chord sequence is very useful. When mixing several or more clips from different musical sources, the process can't be arbitrary: it is very important to preserve consistency through a mixing process and placing the right chord at the right time is one of critical requirements to maintain such consistency. Furthermore, harmonic progression tells the degree of *closure* or cadence via specific sequence of chord functions such as dominant-tonic. Therefore, if we know the chord progression characteristic in the clips to be mixed, it is possible to edit and arrange them in a harmonically pleasing way.

6.5 Summary

In this chapter, we have demonstrated that a sequence of chords and chord time boundaries provide useful information in applications such as cover song identification and structural analysis.

In cover song finding, we use a pair of one-dimensional chord sequence as input

[4] `http://www.music-ir.org/mirex2007/index.php/Audio_Music_Mood_Classification`

to the dynamic time warping algorithm to score an alignment between the two songs abstracted through chord sequences. The rationale behind this idea was that harmonic content would remain largely intact under various acoustical changes found in different versions of cover songs. In computing the total cost, we not only used optimal state alignment but also a transition cost from chord to chord to reflect the theory of harmonic progression in Western tonal music. This increased the precision of the system evaluated using a recall-precision test. The evaluations performed by the author and by MIREX 2006 prove that the chord sequence is an efficient and robust feature in cover song identification

We have presented a novel approach to structural analysis by recognizing cadences in chord sequences. The hypothesis was, in tonal music, that a section such as a phrase, chorus or verse ends on some sort of cadential movement in general. Therefore, finding the cadences leads to finding the phrase boundaries. In doing so, we first detect the harmonic beat or the rate at which the chord is most likely to change, and define the cadential templates based on the key. We then slide these cadential templates through the recognized chord sequence and compute the distance, resulting in the tonal tension curve. When there are chord transitions the same as or similar to one defined in cadential templates, tonal tension will be low, which indicates potential phrase boundaries. False boundaries are removed using the dominant phrase length to give final segmentation results.

After segmentation, we compute the distance between each pair of segments and group those which are close to each other into a cluster. Finally, we summarize music by selecting the segment(s) that are most often repeated.

We have also described several potential applications, including analysis of functional harmony, melody extraction, mood classification and audio mixing, where harmonic content can be directly or indirectly used.

Chapter 7

Conclusions and Future Directions

7.1 Summary of Contributions

In this dissertation, we have presented a unified system for chord transcription and key extraction from audio. The main contribution of this work is its presentation of a framework for acquiring a large amount of training data, via a nearly labor-free process, that enables us to use a powerful supervised learning algorithm, leading to state-of-the-art performance in both musical tasks.

The key idea in this approach is using symbolic music data. Symbolic music files such as MIDI files contain noise-free information about pitch and duration of the notes and therefore it is much easier and more robust to perform harmony analysis to obtain the ground-truth data. In parallel, we synthesize the symbolic files to create audio from which we extract the feature vectors. The acoustic feature vectors and the chord labels are then in perfect alignment and we can use them to train the models in a supervised way.

The labeled training data allow us to directly estimate the model parameters; that is, we avoid the process of model initialization which is critical to accurately estimate the parameters in unsupervised learning. We also avoid incorporating any heuristics or musical knowledge that are required in many other key-finding or chord recognition systems for parameter estimation; we just give the data and the models will learn all the parameters.

We chose a first-order hidden Markov model for chord recognition and key estimation because of its abilities to describe temporal dynamics in observations, via transition probabilities, which are very important in music in general, and in chord progression in particular. The hidden state in the HMM represents a chord and is modeled by a single, multivariate Gaussian (12-dimensional for chroma feature or 6-dimensional for tonal centroid feature). We defined 36 states or chords in an HMM, including major, minor and diminished chords for each pitch class in a chromatic scale. We treated seventh chords as their corresponding root triads and disregarded augmented chords since they very rarely appear in Western tonal music. We allowed all possible transitions from chord to chord, resulting in an ergodic HMM.

As feature vectors, we compared the conventional 12-dimensional chroma vectors to the 6-dimensional tonal centroid vector. Chroma has been successfully used by others in chord recognition and in key finding. Tonal centroid is computed by a linear transformation from the chroma vector, and puts emphasis on important interval relations such as fifths, major and minor thirds. The experiments showed that the tonal centroid feature significantly outperformed the chroma feature.

After training the model with synthesized audio, we evaluated our system using various kinds of musical audio and demonstrated that the models trained just on synthesized audio perform very well on real recordings as well, in both musical tasks. We also compared our data-driven model with a state-of-the-art knowledge-based model to show that our model performs better than or comparable to the knowledge-based model.

A large training data set enabled us to build richer models, and we extended our HMM to be key-dependent as described in Section 4.2. The rational behind this idea was a close relationship between key and chord in tonal music. If we know what key a piece is in, recognizing chords is easier. We therefore built 24 key-dependent HMMs, one for each key, using key information derived from the symbolic music files. Given an acoustic input, our system performed feature analysis and a sequence of observation became an input to the key-dependent HMMs. Using a Viterbi decoder, we estimated the key by selecting the model with the maximum likelihood; at the same time, we recognized frame-level chord sequence because it is the same as the

optimal state path in a selected key model. Experiments on real acoustic recordings showed that a key-dependent model not only estimated the key of an input, but also increased the chord recognition accuracy, yielding very promising results on both musical tasks.

Based on the fact that each musical genre has chord progression characteristics of its own, we also extended our basic model to build genre-specific models in Section 4.3. Although genre information is not explicitly encoded in symbolic music files, it is not difficult to find symbolic music files categorized by genres. We therefore built genre-specific HMMs, similar to key-dependent HMMs, resulting in improved performance with much simpler models than a more complex, genre-independent model. Experimental results showed that the performance is best when the model and the unknown input are of the same kind, which supports our hypothesis on the need for building genre-specific models. Furthermore, a selected genre model using a maximum likelihood often gave a correct genre, suggesting the possibility of using the model as a genre classifier.

We also introduced a hybrid model for chord recognition in Chapter 5. In the hybrid model, we used powerful discriminative classifiers to compute the posterior probabilities given the observation feature vector. To this end, we used support vector machines (SVMs) which are widely used successfully in many other classification problems, including several music applications. Experiments showed the proposed discriminative approach using SVMs achieved the better performance when the same amount of data (1/15 of the full set) is used to train both SVMs and Gaussian models.

Another principal contribution of this dissertation is the demonstration that a harmonic description of music can be a compact, robust mid-level representation in several potential applications as described in Chapter 6. These applications are very important in music information retrieval because they allow efficient and effective methods in search/retrieval. One such application is finding cover versions of an original using the chord sequence as a front end to the system, as explained in Section 6.2. In order to identity cover songs, which can be acoustically very different from the original because of changes in instrumentation, tempo, key and/or dynamics, we need a robust representation that must remain largely invariant under such acoustical

transformations. Our hypothesis was that chord sequence is a robust mid-level representation that undergoes only minor changes under such musical variations. We then computed the optimal alignment of the original and cover song chord sequences using dynamic time warping algorithm. The results showed that the chord sequence can be successfully used to identify cover songs, proving our hypothesis. We also believe that it is usable in finding musical similarity as well.

Harmonic description of tonal music, as represented by key and chords, is a fundamental attribute for music analysis as described in a number of treatises. Music analysis often leads to finding a structure —*i.e.*, searching for smaller building blocks which all together form a whole piece. Therefore, we can use key and chords to perform structural analysis of music, which leads to three closely related practical applications, namely music segmentation, clustering and summarization.

In music segmentation, we tried to identify structural boundaries in musical audio. We presented a novel approach to this problem in Section 6.3 by recognizing cadences based on the fact that, in tonal music, a section usually ends with a cadence, which is a specific harmonic motion (*e.g.,* V-I movement in perfect cadence) characterized by its key. To this end, we first extracted key and frame-level chords and defined cadential templates based on the estimated key. We then located the positions where such cadential movements are found by computing the tonal tension curve using the pre-defined cadential templates and chord sequence. The valleys in the tonal tension curve indicated potential cadences, namely structural boundaries. We then used a pruning technique to remove false boundaries by finding the most dominant phrase length.

Some sections in tonal music, particularly in popular music, share the same harmonic content. For instance, the harmonic content of the first and the second verse or that of chorus sections seldom changes. Therefore, we can group similar segments into a cluster by finding *harmonically* similar segments. We computed segment-to-segment distances at the chord level and grouped segments that are close enough to each other to find a cluster. This clustering is useful in identifying repeating segments such as chorus or verse, which also leads to another immediate application of music summarization. Often the times, the most repeating section in music is also the most

remembered or representative section. Therefore, we could find a summary of music, sometimes called an audio thumbnail, by recognizing the most repeating section based on clustering information.

We also described other potential applications where harmonic descriptions are valuable such as melody extraction, music recommendation, playlist generation, mood classification or audio mixing.

We believe that the proposed system for chord recognition and key extraction achieves state-of-the-art performance in both musical tasks. Although the proposed hidden Markov models could successfully learn the first-order (*i.e.,* from the current to the next) chord transition probability distributions from the data, most tonal music employs higher-order chord progressions, which leads to a direction for further work.

7.2 Higher-Order HMM

7.2.1 Introduction

The HMMs described in Chapter 3 and in Chapter 4 are based on the first-order Markov process; that is, the current state q_t (chord) is only dependent on the predecessor state q_{t-1} (chord), namely

$$P(q_t = S_j | q_{t-1} = S_i, q_{t-2} = S_k, \cdots) = P(q_t = S_j | q_{t-1} = S_i). \tag{7.1}$$

Therefore, given the predecessor state q_{t-1} (chord), knowing the previous state(s) $q_{t-2}, q_{t-3}, \cdots$ (chords) has no effect in predicting the current state q_t (chord). However, this first-order property does not hold in music or in chord progression in particular. For example, a popular 12-bar blues[1] has a chord progression of {I-I-I-I-IV-IV-I-I-V-V(IV)-I-I} [43]. Such higher-order progression is also common in tonal music in general. For instance, {ii-V-I} or {iv-V-i} movement are most common cadential formulas [46].

[1] Although it is originated in blues music, the 12-bar blues progression is one of the most widely used chord progressions in popular music in general.

Therefore, if we know two or more preceding chords in tonal music, it is very likely that we can decide which chord to follow with more confidence. In fact, high-order HMMs perform better and thus have been successfully used in Natural Language Processing (NLP) [65, 22, 10, 34]. It is straightforward to solve the three basic problems of a high-order HMM[2] based on the first-order HMM. Interested readers are encouraged to read du Preez [23].

When applying high-order HMMs to chord recognition, however, we encounter a few problems. First, we can't recognize chords at the frame-rate any more because high-order chord transitions are not likely to happen at the frame rate; in other words, it is very unlikely that the chord changes at one frame and changes again at the very next frame. This in turn means that we can never model high-order chord transition characteristics like {ii-V-I}, for example, at the frame-level. Therefore, in order to use higher-order HMMs, we need to perform higher-level segmentation analysis than at the frame-level.

One possible solution to this problem is to use beat-synchronous analysis as done by Bello and Pickens in chord recognition [8]. Although they didn't do beat-synchronous analysis for modeling high-order chord progressions, we may be able to learn high-order chord transitions at the beat-level. In order to be more precise, however, we need to find out the *harmonic beat* rather than the rhythmic beat since the two can be different in general. We proposed an algorithm to compute the harmonic beat in Section 6.3.2 which can be used for harmonic beat-level analysis.

Another problem in high-order modeling is *data sparsity*. Because we increase the order of state transitions, the possible number of transitions increases exponentially, assuming an ergodic model[3] Therefore, if we had $36 \times 36 = 1,296$ possible chord-to-chord transitions with 36 different chords, in a 2nd-order HMM, we have $36 \times 36 \times 36 = 46,656$ possible transitions. This means we need even more data to appropriately fill this 3-dimensional matrix; in fact, even with a lot more data, most elements in the matrix will be zero. This leads to a serious overfitting problem since the model can never generalize to unseen input sequences. And yet this is in the case of only

[2]Refer to Section A.4 in Appendix A

[3]In an ergodic model, every state-to-state transition is possible.

2nd-order model.

This sparse-data problem becomes even more serious when we analyze audio at a higher level than at the frame-level such as beat-synchronous level, because the number of training samples becomes much less. To make this more concrete, we could train the 2nd-order model to learn the 2nd-order transition probabilities using the beat-level training data which contain more than 223,000 samples. The result was less than 20% of the matrix elements were nonzero; in other words, more than 80% of all possible chord-to-chord-to-chord transitions *never* happened!

The data sparsity is a central issue in language modeling as well. Researchers therefore proposed several *smoothing* algorithms to alleviate the sparsity problem [41, 42, 18, 45]. In smoothing algorithms, some probability is taken away from some occurrences because occurrences which appear only one time are usually greatly over-estimated [34]. By taking some probability from some words and redistributing it to other words, zero probabilities can be avoided.

We believe we can apply these smoothing techniques widely used in language modeling to our application although the language context and the musical context are different from each other.

7.2.2 Preliminary Results and Discussions

We experimented with the 2nd-order HMM which was trained on beat-synchronous data. For training data, we didn't perform beat-analysis on audio because Temperley's Melisma Music Analyzer also gives beat information [95]. For test audio. we used the beat-detection algorithm by Ellis [26]. The feature frames in a beat segment was averaged to give one feature vector per beat. Table 7.1 shows the chord recognition results with the song *No Reply* by the Beatles.

As shown in Table 7.1, the accuracy is highest with the 1st-order HMM trained on the frame-level data, followed by the 1st-order HMM trained on the beat-synchronous data. The proposed 2nd-order model trained on the beat-level data performed worse than the 1st-order models. One thing that is noticeable is a significant performance increase in the 2nd-order HMM when the data is smoothed (from 67% to 84%). This

Table 7.1: Test results on 2nd-order HMM (% correct)

Model	Analysis level	
	Frame	Beat-synchronous
1st-order	86.45	84.49
2nd-order (w/o smoothing)	N/A	66.99
2nd-order (w/ smoothing)	N/A	83.51

supports the idea that the smoothing techniques for language modeling are applicable to music application.

There are a few possible explanations why the 2nd-order model performs no better than the 1st-order models. First, the beat-analysis algorithm must be consistent between the training and the test data, which isn't the case in our experiments. While beat information in the training data is directly extracted from the symbolic data, not from the synthesized audio, we detect the beat from audio for the test data. We plan to perform beat-analysis on the synthesize training audio using the same beat-detection algorithm used for the test data.

Secondly, we may need more data to estimate the model parameters more accurately. As mentioned earlier, the number of parameters to be estimated grows exponentially as the order of Markov process in HMMs increases, although a smoothing technique helps avoid overfitting problems.

Finally, we are not sure of the applicability of the smoothing techniques used in language modeling to music application, especially in chord recognition. Although the fundamental idea in smoothing —*i.e.* redistributing the probabilities from rare occurrences to others to avoid zero probabilities — may also hold in chord progression modeling, the word space is far bigger than the chord space (thousands of words vs. 36 chords), and thus the same smoothing techniques may not be appropriate. This may require modifying the smoothing techniques to make them more suitable for chord progression modeling.

7.3 Musical Expectation

7.3.1 Introduction

As has been pointed out several times in this dissertation, harmonic description of tonal music, represented by key and chord names (and their onset/offset time), is a powerful attribute to describe musical semantics (*e.g.,* expectation, emotion/mood, storytelling, etc.). In particular, as we listen to music, we constantly build some sort of expectations — consciously or unconsciously — and our emotions, to a great degree, are governed by their realizations or violations.

Music listening involves continuous interactions between the stimulus, namely the listener's perception, and mental processes. In doing so, the listener builds "tendencies" to respond to the musical stimuli. According to Meyer,

> *. . . The tendency to respond becomes more conscious where inhibition of some sort is present, when the normal course of the reaction pattern is disturbed or its final completion is inhibited. Such conscious and self-conscious tendencies are often thought of and referred to as "expectations" [66].*

The musical expectations can be formed by melody, harmony, rhythm or by a combination of such attributes. However, in Western tonal music, expectations are particularly pronounced in the use of harmony as the most pervasive forms of music employ a small set of chords in highly constrained ways [9]. Bharucha and Stoeckig explored the cognitive processes underlying musical expectation by reaction time in a priming paradigm, and observed that a harmonic context primes the processing of chords related to the context, relative to chords unrelated to the context. They also proposed a spreading activation network model of the representation and processing of harmony, and proved the hypotheses using a set of experiments for chord priming.

Povel also investigated on musical expectations evoked by a sequence of chords in exploring the elementary harmonic force present in tonal music, based on the similar experiments done by Bharucha and Stoeckig [85].

7.3.2 Modeling Musical Expectation

These close relationships between harmony and musical expectations found in the literature suggest that we can induce the level and the type of expectations from chord progression or a sequence of chords. For example, if our chord recognition system tells a chord sequence like {C-d-G}, then we can say there is a very strong expectation that a C major chord will follow. If a C major chord follows, as expected, then our expectation is *realized*. If the next chord is instead a B♭ major for example, however, then we can say that our expectation is *violated*. This concept of implication-realization model in the context of melody expectation was studied by Narmour [70, 71].

Leistikow proposed a probabilistic framework to model musical expectations [62]. In his thesis, he presented a dynamic Bayesian inference engine to model an array of cognitive tasks performed by a listener when presented with musical sounds. He used a set of musical rules learned from musical corpora to predict upcoming notes and measured the surprise associated with the observed realizations.

As Leistikow suggested that his expectation models can be seamlessly integrated into probabilistic models that use audio signal features as input, we hope in the future to develop such a framework for modeling musical expectations from audio using our model to provide higher-level symbolic attributes — musical key and chords — extracted from audio. In addition, we can build a hierarchical model such as one described by Leistikow in which several musical attributes are mutually informative [62]. Although there are many more variables defined in his model, including meter, beat, duration, bar, harmony, and the observation is a (symbolic) musical note, we can build a similar hierarchical model with a reduced set of variables — that is, key, harmony and beat — and with the chroma or tonal centroid feature as observations. Our system doesn't have a beat-detection algorithm, but we believe it is needed because beat information is directly or indirectly related to many other musical properties.

Once we build such a hierarchical model, the evaluation of expectation realization/violation can be performed following these steps:

1. Estimate the prior distributions and the transition distributions from the labeled

training data as we do in our proposed system (assuming we perform beat-synchronous analysis).

2. Perform chord recognition and key extraction given an input audio. We may also consider local key-finding to examine more localized key-harmony relationships.

3. Compute a one-step observation prediction given as

$$P(X_{i+1}|x_{1:i}) = \sum_{KCB_i, KCB_{i+1}} P(KCB_i, KCB_{i+1}, X_{i+1}|x_{1:i}), \tag{7.2}$$

where K_i, C_i, B_i, X_i denote key, chord, beat and observation at time i.

4. Compute the violation or surprisal of that observation by evaluating

$$\text{Surprisal}(x_{i+1}|x_i) = -\log_2 P(X_{i+1} = x_{i+1}|x_{1:i}). \tag{7.3}$$

We can compute the quantities in Step 3 and 4 using the directed acyclic graph (DAG) for a hierarchical model proposed by Leistikow [62].

7.4 Final Remarks

It is our hope that the framework for extracting harmonic content from musical audio will make a significant contribution to the music information retrieval community in general and to those who try to apply machine learning techniques to solve music-related problems in particular. With our initial phase of research concluded, we wish to continue to work on more difficult, but even more interesting problems, such as those suggested in the previous sections, in the future.

Appendix A

Hidden Markov Model [1]

A.1 Discrete Markov Process

Figure A.1 illustrates a Markov chain with four states that may be described at any time as being in one of N distinct states, $S_1, S_2, \cdots, S_N$ ($N = 4$ in this example). If we denote the time instants associated with state changes as $t = 1, 2, \cdots$, and the actual state at time t as q_t, we can fully describe the behavior of such system by specifying the current state at time t and the previous states at time $t-1, t-2, \cdots$.

For the special case of a discrete, first-order, Markov chain, this probabilistic description simplifies to just the current and the preceding state, *i.e.*,

$$P(q_t = S_j | q_{t-1} = S_i, q_{t-2} = S_k, \cdots) = P(q_t = S_j | q_{t-1} = S_i). \tag{A.1}$$

Furthermore, only considering those process where the right-hand side of Equation A.1 is independent of time leads to a set of state transition probabilities a_{ij} of the form

$$a_{ij} = P(q_t = S_j | q_{t-1} = S_i), \qquad 1 \leq i, j \leq N \tag{A.2}$$

[1] This appendix is based on Rabiner [86] and Rabiner and Juang [87]

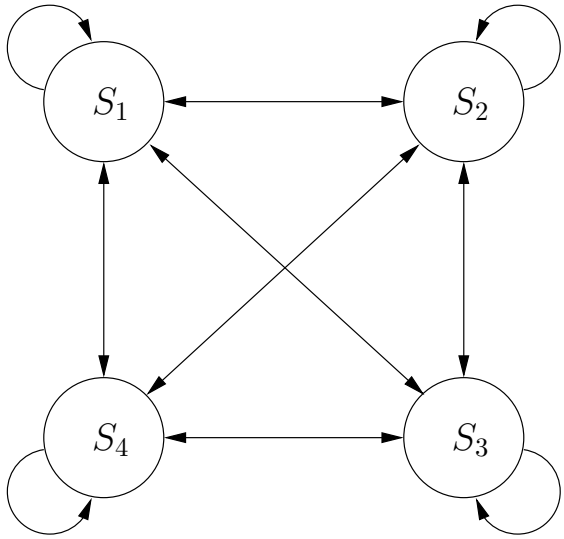


Figure A.1: A Markov chain with 4 states (labeled S_1 to S_4) with all possible state transitions.

with the following properties:

$$a_{ij} \geq 0 \tag{A.3a}$$

$$\sum_{j=1}^{N} a_{ij} = 1. \tag{A.3b}$$

The above Markov process may be called an *observable* Markov model since the output of the process is the set of states at each time, where each state corresponds to a physical (and thus observable) event.

For example, let's consider a simple three-state Markov model of the weather, where the weather is observed, once a day, as being one of the following:

State 1 (S_1): rain
State 2 (S_2): cloudy
State 3 (S_3): sunny.

We assume that the weather on day t undergoes a change of a state based on the

state transition probability matrix A given by

$$A = \{a_{ij}\} = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}. \tag{A.4}$$

We can then ask a question such as: *given* that the weather on day 1 ($t = 1$) is sunny ($S_3 = 3$), what is the probability that the weather for the next three days will be "sun-sun-cloudy-rain-rain"? Stated more formally, we define the observation sequence O as $O = \{S_3, S_3, S_2, S_1, S_1\}$ at $t = 1, 2, \cdots, 5$, respectively, and we wish to compute the probability of O, given the above model. This probability can be expressed and evaluated as

$$\begin{aligned} P(O|\text{Model}) &= P(S_3, S_3, S_2, S_1, S_1|\text{Model}) \\ &= P(S_3)P(S_3|S_3)P(S_2|S_3)P(S_1|S_2)P(S_1|S_1) \\ &= \pi_3 \cdot a_{33} \cdot a_{32} \cdot a_{21} \cdot a_{11} \\ &= 1 \cdot 0.8 \cdot 0.1 \cdot 0.2 \cdot 0.4 \\ &= 0.0064 \end{aligned} \tag{A.5}$$

where we use the notation

$$\pi_i = P(q_1 = S_i), \qquad 1 \leq i \leq N \tag{A.6}$$

to denote the initial state probabilities.

A.2 Extension to Hidden Markov Models

In the Markov model described above, each state (weather on a day) corresponds to an observable (physical) event (rain, cloudy or sunny). This model is too restrictive to be applicable to many problems since we may not be able to directly observe the

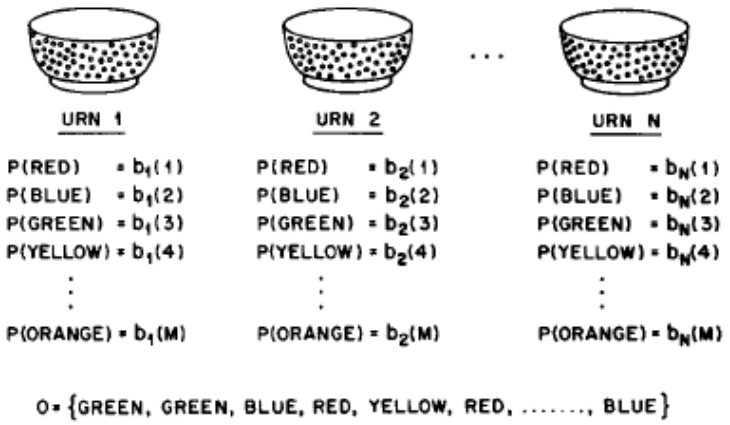


Figure A.2: An N-state urn and ball model of a discrete symbol HMM (adapted from Rabiner [86]).

states. Therefore, we extend the concept of Markov models to include the case where the model's states are *hidden* in that they are not directly observable, but can only be observable through another set of stochastic processes that generate the sequence of observations.

Let's consider the urn and ball system shown in Figure A.2. We assume that there are N glass urns in a room, each filled with a large number of balls with M number of distinct colors. The process of obtaining observations is as follows. According to some random process, one chooses an initial urn. From this urn, a ball is chosen at random, and its color is recorded as the observation. The ball is then replaced in the urn from which it was selected. A new urn is then selected according to the random process associated with the current urn, and the ball selection process is repeated. This entire process produces a finite observation sequence of colors, which is going to be modeled as the observable output of a hidden Markov model. In this example, the processes of choosing the urn and selecting the ball are *hidden*; they just generate a sequence of balls, according to some stochastic processes, that are observable.

A.3 Elements of an HMM

From the above example, we now formally define the elements of an HMM. An HMM is characterized by the following:

1) N, the number of states in the model. Although the states are hidden in HMMs, there is often physical significance in the states of the model. In the above urn and ball example, the states corresponded to the urns. The states in general are interconnected in such a way that any state can be reached from any other state; however, it is sometimes more practical and appropriate to put some restrictions on state transitions. We denote the individual states as $S = \{S_1, S_2, \cdots, S_N\}$, and the state at time t as q_t.

2) M, the number of distinct observation symbols per state. The observation symbols correspond to the physical output of the system to be modeled. In the ball and urn example, they were the colors of the balls. We denote the individual symbols as $V = \{v_1, v_2, \cdots, v_M\}$.

3) The state transition probability distribution $A = \{a_{ij}\}$ where

$$a_{ij} = P(q_{t+1} = S_j | q_t = S_i), \qquad 1 \leq i, j \leq N, \tag{A.7}$$

with the properties given in Equation A.3.

4) The observation probability distribution in state $j, B = \{b_j(k)\}$, where

$$b_j(k) = P(v_k \text{ at } t | q_t = S_j), \qquad 1 \leq j \leq N, \quad 1 \leq k \leq M. \tag{A.8}$$

5) The initial state distribution $\pi = \{\pi_i\}$ where

$$\pi_i = P(q_1 = S_i), \qquad 1 \leq i \leq N. \tag{A.9}$$

Given appropriate values of N, M, A, B, and π, the HMM can be used to generate an observation sequence

$$O = O_1 O_2 \cdots O_T, \tag{A.10}$$

where each observation O_t is one of the symbols from V, and T is the number of observations in the sequence, as follows:

1) Choose an initial state $q_1 = S_i$ according to the initial state distribution π.

2) Set $t = 1$.

3) Choose $O_t = v_k$ according to the symbol probability distribution in state S_i, *i.e.*, $b_i(k)$.

4) Transit to a new state $q_{t+1} = S_j$ according to the state transition probability distribution for state S_i, *i.e.*, a_{ij}.

5) Set $t = t + 1$; return to step 3) if $t < T$; otherwise terminate the procedure.

We can see from the above discussion that an HMM is completely characterized by the specification of two model parameters (N and M), observation symbols and the three probability measures A, B, and π. For convenience, we use the compact notation

$$\lambda = (A, B, \pi) \tag{A.11}$$

to indicate the complete parameter set of the model.

A.4 The Three Fundamental Problems of an HMM

Given the form of HMM described in the previous section, there are three basic problems that must be solved for the model to be useful in real-world applications. These problems are:

Problem 1: Given the observation sequence $O = O_1 O_2 \cdots O_T$ and a model $\lambda = (A, B, \pi)$, how do we efficiently compute $P(O|\lambda)$, the probability of the observation sequence, given the model?

Problem 2: Given the observation sequence $O = O_1 O_2 \cdots O_T$ and a model λ, how do we choose a corresponding state sequence $Q = q_1 q_2 \cdots q_T$ which is optimal in some meaningful sense (*i.e.*, best "explains" the observations)?

Problem 3: How do we adjust the model parameters $\lambda = (A, B, \pi)$ to maximize $P(O|\lambda)$?

Problem 1 is the evaluation problem; *i.e.*, given the observation sequence and a model, how do we compute the probability that the model produced the observation sequence. This is extremely useful when we try to choose among several competing models by allowing us to select the model with the highest probability.

Problem 2 is the one where we try to uncover the hidden part of the model, namely the "correct" state path. However, there is no "correct" state sequence except for the degenerate models, and therefore in practical applications, we usually use an optimality criterion to solve the problem as best as possible.

Problem 3 is the one in which we attempt to optimize the model parameters in order to best explain how a given observation sequence is generated. We refer to this procedure as "training" the HMM and call the observation sequence used to adjust the model parameters a training sequence. The training problem is very important for most applications because it allows us to build best models for real-world phenomena.

A.4.1 Solution to Problem 1

We wish to compute $P(O|\lambda)$, the probability of the observation sequence $O = O_1 O_2 \cdots O_T$, given the model λ. Let's consider a state sequence

$$Q = q_1 q_2 \cdots q_T \tag{A.12}$$

then the probability of the observation sequence O for the state sequence Q of Equation A.12 is

$$\begin{aligned} P(O|Q,\lambda) &= \prod_{t=1}^{T} P(O_t|q_t,\lambda) && \text{(A.13a)} \\ &= b_{q_1}(O_1)b_{q_2}(O_2)\cdots b_{q_T}(O_T), && \text{(A.13b)} \end{aligned}$$

where we have assumed statistical independence of observations.

The probability of such a state sequence Q can be written as

$$P(Q|\lambda) = \pi_{q_1} a_{q_1 q_2} a_{q_2 q_3} \cdots a_{q_{T-1} q_T}. \tag{A.14}$$

The joint probability of O and Q is simply the product of the above two terms, *i.e.*,

$$P(O,Q|\lambda) = P(O|Q,\lambda)P(Q|\lambda). \tag{A.15}$$

Therefore, the probability of O given the model λ is obtained by summing this joint probability over all possible state sequences q giving

$$\begin{aligned} P(O|\lambda) &= \sum_{\text{all } Q} P(O|Q,\lambda)P(Q|\lambda) && \text{(A.16a)} \\ &= \sum_{q_1,q_2,\cdots,q_T} \pi_{q_1} b_{q_1}(O_1) a_{q_1 q_2} b_{q_2}(O_2) \cdots a_{q_{T-1} q_T} b_{q_T}(O_T). && \text{(A.16b)} \end{aligned}$$

The direct calculation of $P(O|\lambda)$ following Equation A.16, however, involves on the order of $2T \cdot N^T$ calculations and is computationally infeasible, even for small values of N and T; *e.g.*, for $N = 5$ (states), $T = 100$ (observations), there are on the order of $2 \cdot 100 \cdot 5^{100} \approx 10^{72}$ computations! We can more efficiently solve this problem using a procedure called a *Forward-Backward algorithm.*

Let's consider the forward variable $\alpha_t(i)$ defined as

$$\alpha_t(i) = P(O_1 O_2 \cdots O_t, q_t = S_i|\lambda), \tag{A.17}$$

i.e., the probability of the partial observation sequence $O_1 O_2 \cdots O_t$ until time t *and* state S_i at time t, given the model λ. We then can solve for $\alpha_t(i)$ inductively, following these steps:

1) Initialization:

$$\alpha_1(i) = \pi_i b_i(O_1), \qquad 1 \leq i \leq N. \tag{A.18}$$

2) Induction:

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij}\right] b_j(O_{t+1}), \qquad 1 \leq t \leq T-1,\ 1 \leq j \leq N. \tag{A.19}$$

3) Termination:

$$P(O|\lambda) = \sum_{i=1}^{N} \alpha_T(i). \tag{A.20}$$

In a similar manner, we can define a backward variable $\beta_t(i)$ as

$$\beta_t(i) = P(O_{t+1} O_{t+2} \cdots O_T|q_t = S_i, \lambda), \tag{A.21}$$

i.e., the probability of the partial observation sequence from $t+1$ to the end, given state S_i at time t and the model λ. Using this Forward-Backward algorithm[2], we see that the calculation of $P(O|\lambda)$ requires on the order of N^2T calculations, compared with $2TN^T$ as required by the direct calculation. For $N = 5$ and $T = 100$, this is about 3000 computations versus 10^{72}, a savings of about 69 orders of magnitude.

[2]In order to solve Problem 1, we only need the forward *or* backward part of the forward-backward procedure, but both procedures are required to solve Problem 3.

A.4.2 Solution to Problem 2

While there exists an exact solution to *Problem 1*, there are several possible ways to solve *Problem 2*, *i.e.*, finding the "optimal" state sequence associated with the given observation sequence. Therefore, the solution may vary according to what optimality criterion is used. One of such optimality criteria that are mostly widely used is to find the *single* best state sequence or path to maximize $P(Q|O,\lambda)$ which is equivalent to maximizing $P(Q,O|\lambda)$. A formal technique for finding this single optimal state sequence exists, based on dynamic programming methods, and is called the *Viterbi* algorithm.

We wish to find the single best state sequence, $Q = q_1 q_2 \cdots q_T$, given the observation sequence $O = O_1 O_2 \cdots O_T$ and the model λ. Let's define the quantity

$$\delta_t(i) = \max_{q_1,q_2,\cdots q_T} P(q_1 q_2 \cdots q_t = i, O_1 O_2 \cdots O_t|\lambda); \tag{A.22}$$

i.e., $\delta_t(i)$ is the best score (highest probability) along a single path, at time t, which accounts for the first t observations and ends in state S_i. By induction we have

$$\delta_{t+1}(j) = \left[\max_i \delta_t(i) a_{ij}\right] \cdot b_j(O_{t+1}). \tag{A.23}$$

In order to retrieve the state sequence, we need to keep track of the argument which maximize Equation A.23, for each t and j. We do this via the array $\psi_t(j)$ which holds state information. The complete procedure can be explained as follows:

1) Initialization:

$$\delta_1(i) = \pi_i b_i(O_1), \qquad 1 \le i \le N \tag{A.24a}$$

$$\psi_1(i) = 0. \tag{A.24b}$$

2) Recursion:

$$\delta_t(j) = \max_{1\le i\le N} \left[\delta_{t-1}(i) a_{ij}\right] b_j(O_t), \qquad 2 \le t \le T,\ 1 \le j \le N \tag{A.25a}$$

$$\psi_t(j) = \operatorname*{argmax}_{1\le i\le N} \delta_{t-1}(i) a_{ij}, \qquad 2 \le t \le T,\ 1 \le j \le N \tag{A.25b}$$

3) Termination:

$$p^* = \max_{1\le i\le N} \delta_T(i) \tag{A.26a}$$

$$q_T^* = \operatorname*{argmax}_{1\le i\le N} \delta_T(i). \tag{A.26b}$$

4) Path (state sequence) backtracking:

$$q_t^* = \psi_{t+1}(q_{t+1}^*), \qquad t = T-1, T-2, \cdots, 1. \tag{A.27}$$

As can be shown in the above procedure, the Viterbi algorithm is similar in implementation, except for the backtracking step, to the forward calculation of Equation A.18–A.20.

A.4.3 Solution to Problem 3

The third, and by far the most difficult, problem of HMMs is to adjust the model parameters (A, B, π) to maximize the probability of the observation sequence given the model, and there is no analytical solution to this problem. In fact, given any finite training sequence, there is no optimal way of estimating the model parameters. However, we can choose $\lambda = (A, B, \pi)$ such that $P(O|\lambda)$ is *locally* maximized using an iterative approach such as the Baum-Welch method, also known as the EM (Expectation-Modification) algorithm.

Let's first define $\xi_t(i, j)$, the probability of being in state S_i at time t and in state S_j at time $t + 1$, given the model and the observation sequence, *i.e.*,

$$\xi_t(i, j) = P(q_t = S_i, q_{t+1} = S_j|O, \lambda). \tag{A.28}$$

Using the definitions of the forward and backward variables in Equation A.17 and

A.21, respectively, we can rewrite $\xi_t(i,j)$, using Bayes' rule, in the form

$$\begin{aligned} \xi &= \frac{\alpha_t(i)a_{ij}b_j(O_{t+1})\beta_{t+1}(j)}{P(O|\lambda)} \\ &= \frac{\alpha_t(i)a_{ij}b_j(O_{t+1})\beta_{t+1}(j)}{\sum_{i=1}^{N}\sum_{j=1}^{N}\alpha_t(i)a_{ij}b_j(O_{t+1})\beta_{t+1}(j)}. \end{aligned} \tag{A.29}$$

If we define another variable $\gamma_t(i)$, the probability of being in state S_i at time t, given the model and the observation sequence, as

$$\gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{P(O|\lambda)} = \frac{\alpha_t(i)\beta_t(i)}{\sum_{i=1}^{N}\alpha_t(i)\beta_t(i)} \tag{A.30}$$

we can relate $\gamma_t(i)$ to $\xi_t(i,j)$ by summing over j, giving

$$\gamma_t(i) = \sum_{j=1}^{N}\xi_t(i,j). \tag{A.31}$$

If we sum this quantity over time t, we obtain the expected number of times that state S_i is visited, or equivalently, the expected number of transitions made from state S_i, excluding $t = T$ for termination. Similarly, summation of $\xi_t(i,j)$ over t (from $t = 1$ to $t = T-1$) gives the expected number of transitions from state S_i to state S_j, *i.e.*,

$$\sum_{t=1}^{T-1}\gamma_t(i) = \text{expected number of transitions from } S_i \tag{A.32a}$$

$$\sum_{t=1}^{T-1}\xi_t(i) = \text{expected number of transitions from } S_i \text{ to } S_j \tag{A.32b}$$

Using the above formulas and definitions, we can give a method of re-estimating the parameters of an HMM. A set of reasonable re-estimation formulas for π, A and

B are

$$\begin{aligned}
\overline{\pi}_i &= \text{expected frequency (number of times) in state } S_i \text{ at time } (t=1) \\
&= \gamma_1(i) && \text{(A.33a)} \\
\overline{a}_{ij} &= \frac{\text{expected number of transitions from state } S_i \text{ to state } S_j}{\text{expected number of transitions from state } S_i} \\
&= \frac{\sum_{t=1}^{T-1} \xi_t(i,j)}{\sum_{t=1}^{T-1} \gamma_t(i)} && \text{(A.33b)} \\
\overline{b}_j(k) &= \frac{\text{expected number of times in state } S_j \text{ and observing symbol } v_k}{\text{expected number of times in state } S_j} \\
&= \frac{\sum_{t=1 \text{ s.t.} O_t = v_k}^{T} \gamma_t(j)}{\sum_{t=1}^{T} \gamma_t(j)}. && \text{(A.33c)}
\end{aligned}$$

The above formulas allow us to recursively re-estimate the model parameters $\overline{\lambda} = (\overline{A}, \overline{B}, \overline{\pi})$, as determined from the left-hand sides of Equation A.33 using the current model $\lambda = (A, B, \pi)$ on the right-hand sides. Baum and his colleagues [6, 3] have proven that the re-estimated model $\overline{\lambda}$ is always equally likely as or more likely than the current model λ in the sense that $P(O|\overline{\lambda}) \geq P(O|\lambda)$; *i.e.*, a new model $\overline{\lambda}$ is more likely to have produced the observation sequence.

Therefore, if we iteratively use $\overline{\lambda}$ in place of λ to estimate new model parameters, we can improve the likelihood of the observation sequence being generated by the model until convergence. The final result of this re-estimation procedure is called a maximum likelihood estimate of the HMM. It should be noted, however, that the forward-backward algorithm leads to local maxima only, and thus the initialization of the model is crucial in many real-world applications.

Bibliography

[1] Jean-Julien Aucouturier and Mark Sandler. Segmentation of musical signals using hidden markov models. In *Proceedings of the Audio Engineering Society*, 2001.

[2] Ricard Baeza-Yates and Berthier Ribeiro-Neto. *Modern Information Retrieval*. Addison-Wesley, New York, 1999.

[3] J. K. Baker. The dragon system — an overview. *IEEE Transactions on Acoustics and Speech Signal Processing*, 23(1):24–29, 1975.

[4] Roberto Basili, Alfredo Serafini, and Armando Stellato. Classification of musical genre: a machine learning approach. In *Proceedings of the International Conference on Music Information Retrieval*, 2004.

[5] Roberto Basili, Alfredo Serafini, and Armando Stellato. An investigation of feature models for music genre classification using the support vector classifier. In *Proceedings of the International Conference on Music Information Retrieval*, 2005.

[6] L. E. Baum and G. R. Sell. Growth functions for transformations on manifolds. *Pacific Journal of Math*, 27(2):211–227, 1968.

[7] Juan P. Bello. Audio-based cover song retrieval using approximate chord sequences: Testing shifts, gaps, swaps and beats. In *Proceedings of the International Conference on Music Information Retrieval*, Vienna, Austria, 2007.

[8] Juan P. Bello and Jeremy Pickens. A robust mid-level representation for harmonic content in music signals. In *Proceedings of the International Conference on Music Information Retrieval*, London, UK, 2005.

[9] Jamshed Jay Bharucha and Keiko Stoeckig. Reaction time and musical expectancy: Priming of chords. *Journal of Experimental Psychology: Human Perception and Performance*, 12(4):403–410, 1986.

[10] Thorsten Brants. TnT — a statistical part-of-speech tagger. In *Proceedings of the 6th Applied Natural Language Processing Conference*, Seattle, WA, 2000.

[11] Judith C. Brown. Calculation of a constant-Q spectral transform. *Journal of the Acoustical Society of America*, 89(1):425–434, 1990.

[12] Giordano Cabral, François Pachet, and Jean-Pierre Briot. Automatic x traditional descriptor extraction: The case of chord recognition. In *Proceedings of the International Conference on Music Information Retrieval*, London, UK, 2005.

[13] Giordano Cabral, François Pachet, and Jean-Pierre Briot. Recognizing chords with EDS: Part one. In *Computer Music Modeling and Retrieval (CMMR 2005), Lecture Notes in Computer Science Series*, pages 185–195. Springer-Verlag, 2006.

[14] Wei Chai and Barry Vercoe. Detection of key change in classical piano music. In *Proceedings of the International Conference on Music Information Retrieval*, 2005.

[15] Elaine Chew and Yun-Ching Chen. Real-time pitch spelling using the spiral array. *Computer Music Journal*, 29(2):61–76, 2005.

[16] Ching-Hua Chuan and Elaine Chew. Fuzzy analysis in pitch class determination for polyphonic audio key finding. In *Proceedings of the International Conference on Music Information Retrieval*, London, UK, 2005.

[17] Ching-Hua Chuan and Elaine Chew. Polyphonic audio key finding using the spiral array ceg algorithm. In *Proceedings of IEEE International Conference on Multimedia and Expo*, 2005.

[18] Kenneth W. Church and William A. Gale. A comparison of the enhanced good-turing and deleted estimation methods for estimating probabilities of english bigrams. *Computer Speech and Language*, 5:19–54, 1991.

[19] Matthew Cooper and Jonathan Foote. Automatic music summarization via similarity analysis. In *Proceedings of the International Conference on Music Information Retrieval*, Paris, France, 2002.

[20] Matthew Cooper and Jonathan Foote. Summarizing popular music via structural similarity analysis. In *Proceedings of IEEE Workshop on Applications of Signal Processing to Audio and Acoustics*, New Paltz, NY, 2003.

[21] Stephen Downie. Music information retrieval. *Annual Review of Information Science and Technology*, 37:295–340, 2003.

[22] J A. du Preez and D. M. Weber. Automatic language recognition using high-order HMMs. In *Proceedings of the 1998 International Conference on Spoken Language Processing*, Sydney, Australia, 1998.

[23] Johan A. du Preez. Efficient high-order hidden Markov modeling. *Ph.D. thesis*, 1997.

[24] Matthias Eichner, Matticas Wolff, and Rüdiger. Instrument classification using hmm. In *Proceedings of the International Conference on Music Information Retrieval*, 2006.

[25] Daniel P. W. Ellis. Identifying 'cover songs' with beat-synchronous chroma features. In *Extended abstract submitted to Music Information Retrieval eXchange task*, BC, Canada, 2006.

[26] Daniel P. W. Ellis. Beat tracking by dynamic programming. *Journal of New Music Research*, 36(1):51–60, 2007.

[27] Jonathan Foote. Visualizing music and audio using self-similarity. In *Proceedings of ACM International Conference on Multimedia*, Orlando, Florida, 1999.

[28] Jonathan Foote. Automatic audio segmentation using a measure of audio novelty. In *Proceedings of International Conference on Multimedia and Expo*, New York, NY, 2000.

[29] Jonathan Foote and Matthew Cooper. Visualizing musical structure and rhythm via self-similarity. In *Proceedings of International Conference on Computer Music*, Habana, Cuba, 2001.

[30] G. D. Forney. The viterbi algorithm. *Proceedings of the IEEE*, 61:268–278, 1973.

[31] Takuya Fujishima. Realtime chord recognition of musical sound: A system using Common Lisp Music. In *Proceedings of the International Computer Music Conference*, Beijing, 1999. International Computer Music Association.

[32] Emilia Gómez. *Tonal Description of Music Audio Signals*. 2006. Ph.D. thesis.

[33] Emilia Gómez and Perfecto Herrera. The song remains the same: identifying versions of the same piece using tonal descriptors. In *Proceedings of the International Conference on Music Information Retrieval*, Victoria, Canada, 2006.

[34] Joshua T. Goodman. A bit of progress in language modeling. *Microsoft Research Technical Report*, 2001.

[35] Michael M. Goodwin and Jean Laroche. A dynamic programming approach to audio segmentation and speech / music discrimination. In *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, Montreal, Canada, 2004.

[36] Christopher A. Harte and Mark B. Sandler. Automatic chord identification using a quantised chromagram. In *Proceedings of the Audio Engineering Society*, Spain, 2005. Audio Engineering Society.

[37] Christopher A. Harte, Mark B. Sandler, and Martin Gasser. Detecting harmonic change in musical audio. In *Proceedings of Audio and Music Computing for Multimedia Workshop*, Santa Barbara, CA, 2006.

[38] T. Hastie and R. Tibshirani. Classification by pairwise coupling. *Technical Report*, 1996. Stanford University and University of Toronto.

[39] David Huron. The Humdrum Toolkit: Software for Music Research. `http://musiccog.ohio-state.edu/Humdrum/`.

[40] Brian Hyer. Tonality. In *Grove Music Online*. ed. L. Macy, `http://www.grovemusic.com`.

[41] Frederick Jelinek and Robert L. Mercer. Interpolated estimation of markov source parameters from sparse data. In *Proceedings of the Workshop on Pattern Recognition in Practice*, Amsterdam, The Netherlands, May, 1980.

[42] Slava M. Katz. Estimation of probabilities from sparse data for the language model component of a speech recognizer. *IEEE Transactions on Acoustics, Speech and Signal Processing*, 35(3):400–401, 1987.

[43] Barry Kernfeld and Allan F. Moore. Blues progression. *Grove Music Online*. ed. L. Macy, `http://www.grovemusic.com`.

[44] Tetsuro Kitahara, Masataka Goto, Kazunori Komatani, Tetsuya Ogata, and Hiroshi G. Okuno. Instrument identification in polyphonic music: feature weighting to minimize influence of sound overlaps. *EURASIP Journal on Applied Signal Processing*, 2007(1), 2007.

[45] R. Kneser and H. Ney. Improved backing-off for m-gram language modeling. In *Proceedings of the International Conference on Acoustics, Speech, and Signal Processing*, pages 181–184, 1995.

[46] Stefan Kostka and Dorothy Payne. *Tonal harmony with an introduction to 20th-century music*. McGraw-Hill, 2004.

[47] Sven E. Krüger, Martin Schafföner, Marcel Katz, Edin Andelic, and Andreas Wendemuth. Speech recognition with support vector machines in a hybrid system. In *Proceedings of INTERSPEECH*, pages 993–996, 2005.

[48] Sven E. Krüger, Martin Schafföner, Marcel Katz, Edin Andelic, and Andreas Wendemuth. Mixture of support vector machines for hmm based speech recognition. In *Proceedings of 18th International Conference on Pattern Recognition*, 2006.

[49] Carol L. Krumhansl. *Cognitive Foundations of Musical Pitch*. Oxford University Press, 1990.

[50] Carol L. Krumhansl and Edward J. Kessler. Tracing the dynamic changes in perceived tonal organization in a spatial representation of musical keys. *Psychological Review*, 89(4):334–368, 1982.

[51] Carol L. Krumhansl and Mark A. Schmuckler. The petroushka chord. *Music Perception*, 4:153–184, 1986.

[52] Carol L. Krumhansl and Roger N. Shepard. Quantification of the hierarchy of tonal functions within a diatonic context. *Journal of Experimental Psychology: Human Perception and Performance*, 29:579–594, 1979.

[53] Cyril Laurier and Perfecto Herrera. Audio music mood classification using support vector machine. In *Proceedings of the International Conference on Music Information Retrieval*, Vienna, Austria, 2007.

[54] Kyogu Lee. Automatic chord recognition using a summary autocorrelation function. *Stanford EE391 Special Report*, 2005.

[55] Kyogu Lee. Automatic chord recognition using enhanced pitch class profile. In *Proceedings of the International Computer Music Conference*, New Orleans, USA, 2006.

[56] Kyogu Lee. Identifying cover songs from audio using harmonic representation. In *Extended abstract submitted to Music Information Retrieval eXchange task*, BC, Canada, 2006.

[57] Kyogu Lee. A system for automatic chord recognition from audio using genre-specific hidden Markov models. In *Adaptive Multimedia Retrieval (AMR 2007), Lecture Notes in Computer Science Series*. Springer-Verlag, 2007. to appear.

[58] Kyogu Lee and Malcolm Slaney. Automatic chord recognition from audio using a supervised HMM trained with audio-from-symbolic data. In *Proceedings of Audio and Music Computing for Multimedia Workshop*, Santa Barbar, CA, 2006.

[59] Kyogu Lee and Malcolm Slaney. Automatic chord recognition using an HMM with supervised learning. In *Proceedings of the International Conference on Music Information Retrieval*, Victoria, Canada, 2006.

[60] Kyogu Lee and Malcolm Slaney. Acoustic chord transcription and key extraction from audio using key-dependent HMMs trained on syntheisze audio. *IEEE Transactions on Speech, Audio, and Language Processing*, 2007. to appear.

[61] Kyogu Lee and Malcolm Slaney. A unified system for chord transcription and key extraction using Hidden Markov Models. In *Proceedings of the International Conference on Music Information Retrieval*, Vienna, Austria, 2007.

[62] Nadal J. Leistikow. Bayesian modeling of musical expectations via maximum entropy stochastic grammars. *Ph.D. thesis*, 2006.

[63] Mark Levy and Mark Sandler. New methods in structural segmentation of musical audio. In *Proceedings of European Signal Processing Conference*, Florence, Italy, 2006.

[64] Namunu C. Maddage, Mohan S. Kankanhalli, and Haizhou Li. A hierarchical approach for music chord modeling based on the analysis of tonal characteristics. In *Proceedings of IEEE International Conference on Multimedia and Expo*, 2006.

[65] Jean-François Mari, Jean-Paul Haton, and Abdelaziz Kriouile. Automatic word recognition based on second-order hidden Markov models. *IEEE Transacstions on Speech and Audio Processing*, 5(1):22–25, 1997.

[66] Leonard B. Meyer. *Emotion and Meaning in Music*. The University of Chicago Press, Chicago, 1956.

[67] Constantinos Michael. Apple iTunes sales statistics. *Online Blog*.

[68] Tom M. Mitchell. *Maching Learning*. WCB/McGraw-Hill, 1997.

[69] Joshua Morman and Lawrence Rabiner. A system for the automatic segmentation and classification of chord sequences. In *Proceedings of Audio and Music Computing for Multimedia Workshop*, Santa Barbar, CA, 2006.

[70] Eugene Narmour. *Beyond Schenkerism: The Need for Alternatives in Music Analysis*. The University of Chicago Press, Chicago, 1977.

[71] Eugene Narmour. *The Analysis and Cognition of Melodic Complexity: The Implication-Realization Model*. The University of Chicago Press, Chicago, 1992.

[72] Andrew Ng. *Machine Learning*. Stanford CS229 Lecture Notes, 2005.

[73] Katy Noland and Mark Sandler. Key estimation using a hidden markov model. In *Proceedings of the International Conference on Music Information Retrieval*, 2006.

[74] Bee Suan Ong, Emilia Gómez, and Sebastian Streich. Automatic extraction of musical structure using pitch class distribution features. In *Proceedings of 1st Workshop on Learning the Semantics of Audio Signals*, Athens, Greece, 2006.

[75] Gen Onishi, Masatoshi NIIZEKI, Ichiro KIMURA, and Hidemi YAMADA. A kansei model for musical chords based on the structure of the human auditory system. In *Proceedings of International Joint Conference on Neural Networks*, Washington, USA, 2001.

[76] Özgür İzmirli. Audio key finding using low-dimensional spaces. In *Proceedings of the International Conference on Music Information Retrieval*, Victoria, Canada, 2006.

[77] Özgür İzmirli. Negative matrix factorization for segmentation. In *Proceedings of the International Conference on Music Information Retrieval*, Vienna, Austria, 2007.

[78] Jean-François Paiement, Douglas Eck, and Samy Bengio. A probabilistic model for chord progressions. In *Proceedings of the International Conference on Music Information Retrieval*, London, UK, 2005.

[79] Steffen Pauws. Musical key extraction from audio. In *Proceedings of the International Conference on Music Information Retrieval*, Barcelona, Spain, 2004.

[80] Geoffroy Peeters. Chroma-based estimation of musical key from audio-signal analysis. In *Proceedings of the International Conference on Music Information Retrieval*, Victoria, Canada, 2006.

[81] Geoffroy Peeters. Musical key estimation of audio signal based on hidden markov modeling of chroma vectors. In *Proceedings of the 9th International Conference on Digital Audio Effects*, Montreal, Canada, 2006.

[82] Jeremy Pickens and Tim Crawford. Harmonic models for polyphonic music retrieval. In *Proceedings of the International Conference on Information and Knowledge Management*, Virginia, USA, 2002.

[83] John C. Platt. Probabilistic outputs for support vector machines and comparisons to regularized likeliehood methods. In *Advances in Large Margin Classifiers*. MIT Press, 1999. Alexander J. Smola, Peter Bartlett, Bernhard Schölkopf, Dale Shuurmans (eds.).

[84] Alan W. Pollack. Notes on ... series. In *soundscapes.info*, 2000. Available at: `http://www.icce.rug.nl/~soundscapes/DATABASES/AWP/awp-notes_on.shtml`.

[85] Dirk-Jan Povel. Exploring the elementary harmonic forces in the tonal system. *Psychological Research*, 58(4):274–283, 1996.

[86] Lawrence R. Rabiner. A tutorial on hidden Markov models and selected applications in speech recognition. *Proceedings of the IEEE*, 77(2):257–286, 1989.

[87] Lawrence R. Rabiner and Biing-Hwang Juang. *Fundamentals of speech recognition*. Prentice Hall, 1993.

[88] Leonard G. Ratner. *Harmony: Structure and Style*. McGraw-Hill, 1962.

[89] W. S. Rockstro, George Dyson/William Drabkin, and Harold S. Powers/Julian Rushton. Cadence. *Grove Music Online*. ed. L. Macy, `http://www.grovemusic.com`.

[90] Sheldon Ross. *A First Course in Probability*. Prentice Hall, 2001.

[91] B. Scholkopf and A. J. Smola. *Learning with Kernels*. MIT Press, 2002.

[92] Joan Serrà. A qualitative assessment of measures for the evaluation of a cover song identification system. In *Proceedings of the International Conference on Music Information Retrieval*, Vienna, Austria, 2007.

[93] Alexander Sheh and Daniel P.W. Ellis. Chord segmentation and recognition using EM-trained hidden Markov models. In *Proceedings of the International Conference on Music Information Retrieval*, Baltimore, MD, 2003.

[94] Daniel Sleator and Davy Temperley. The Melisma Music Analyzer. http://www.link.cs.cmu.edu/music-analysis/, 2001.

[95] David Temperley. *The cognition of basic musical structures*. The MIT Press, 2001.

[96] George Tzanetakis, Georg Essl, and Perry Cook. Automatic musical genre classification of audio signals. In *Proceedings of the International Conference on Music Information Retrieval*, 2001.

[97] A.L. Uitdenbogerd and J. Zobel. Matching techniques for large music databases. *Proceedings of the 7th ACM International Multimedia Conference*, pages 57–66, 1999.

[98] Steven van de Par, Martin McKinney, and André Redert. Musical key extraction from audio using pro le training. In *Proceedings of the International Conference on Music Information Retrieval*, Victoria, Canada, 2006.

[99] Emmanuel Vincent and Xavier Rodet. Instrument identification in solo and ensemble music using independent subspace analysis. In *Proceedings of the International Conference on Music Information Retrieval*, 2004.

[100] A. J. Viterbi. Error bounds for convolutional codes and an asymptotically optimal decoding algorithm. *IEEE Transactions on Information Theory*, 13:260–269, 1967.

[101] Takuya Yoshioka, Tetsuro Kitahara, Kazunori Komatani, Tetsuya Ogata, and Hiroshi G. Okuno. Automatic chord transcription with concurrent recognition of chord symbols and boundaries. In *Proceedings of the International Conference on Music Information Retrieval*, Barcelona, Spain, 2004.

[102] Youngwei Zhu, Mohan S. Kankanhalli, and Sheng Gao. Music key detection for musical audio. In *Proceedings of the 11th International Multimedia Modelling Conference*, 2005.